FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645 - 2944
Vol. 10 No. 9, May, 2026, pp 199 – 204
DOI: https://doi.org/10.33003/fjs-2026-1009-3780

Coprime Graph of Multigroup Over the Dihedral Group D_{2n}, $n \geq 3$ Where n is Prime

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Received: 05 May 2026
Accepted: 25 May 2026
Published: 23 June 2026

ABSTRACT

Multigroups, derived from multisets, generalize groups by allowing element repetition. The coprime graph of a multigroup G, denoted $\Gamma_{cop}(G)$, has G as its vertex set, with two distinct vertices x and y adjacent if and only if their orders are coprime, i.e., $(|x|, |y|) = 1$. This paper studies the coprime graph of multigroups defined over the dihedral group D_{2n} for prime $n \geq 3$. We analyze fundamental properties such as vertex degree, connectivity, completeness, and determine the graph's clique and chromatic numbers. We prove the graph is always connected but never complete and provide explicit formulas for its structural parameters.

Keywords: Multiset, Multigroup, Coprime Graph, Complete Graph, Multipartite

INTRODUCTION

The graph theory has been used to study several properties of an algebraic structure through associating the structure as a graph. Specific graphs have been defined on algebraic structures by mathematicians and these graphs are used as representing these structures geometrically and study its properties (Juliana, Masriani, Wardhana, Switrayni, and Irwansyah, 2020).

A multiset is a mathematical structure similar to the classical set in which repetition of elements is allowed. In the classical set theory, elements are not allowed to repeat and hence occur only once in a set. However, most situations in real life involve repetition which creates limitations for the classical set in representing these situations for example polynomial roots that are repeated, statistical sample observation that are repeated, repeated atoms in a molecule and so on. This challenges brought about the multisets (Sowaity, Sharada, and Naji, 2019). The multigroup are structures formed from multiset which generalizes group structures formed from the classical sets (Shyamal and Debjani, 2018).

The coprime graphs of finite groups have been studied severally by different authors. Juliana et al. (2020) studied the coprime graphs of $Z_n$and its subgroups forall n. They showed that for any prime number n, the graph is bipartite and multipartite if $n$is not a prime power. They also consider the coprime ofits subgroups. In (Syarifudin, 2021), the coprime graph of dihedral group wherethey discussed some properties of the graph which include its radius, its diameter and degree of vertex as well as its completeness. Another study by Gayatri et al.(2023) considered the coprime graph of the generalized quaternion group where they studied clique number and chromatic number of the coprime graph. The graph of multigroups has been studied. The study by Sowaity et al. (2019) discussed the identity graph of multigroups in which some basic graph properties were discussed which include vertex degree, connectedness, clique, clique number, chromatic number, covering number etc. were studied. They also obtained the condition for completeness of the graph. Magami and Ashafa (2023) studied the commuting graph of multigroups and it was proved that the identity element multiplicity is the maximum in a multigroup and the maximum degree of vertex was found as well as the its completeness.

Despite the considerable attention given to coprime graphs of finite groups (Juliana et al., 2020; Syarifudin, 2021) and graph representations of multigroups (Sowaity et al., 2019; Magami and Ashafa, 2023), the coprime graph associated with multigroups remains largely unexplored. Existing studies have focused mainly on identity graphs and commuting graphs of multigroups, while coprime relations among multigroup elements have received little investigation. Furthermore, dihedral groups constitute one of the most important classes of finite non-abelian groups and have been widely studied in algebraic graph theory due to their rich structural properties (Syarifudin, 2021). Since multigroups generalize classical groups by allowing multiplicities of elements (Shyamal and Debjani, 2018), it is natural to investigate how these multiplicities influence the structure and properties of coprime graphs when the underlying algebraic system is a dihedral group. Therefore, the study of coprime graphs of multigroups over dihedral groups bridges two active research directions, namely coprime graphs of finite groups and graph-theoretic investigations of multigroups, and provides further insight into the interplay between multiplicity, group structure, and graph-theoretic properties.

Motivated by this research gap, this study investigates some basic properties of the coprime graph of multigroups over the dihedral group D_{2n}, where n is a prime number. The obtained results extend existing works on coprime graphs of groups and graph representations of multigroups.

MATERIALS AND METHODS

Preliminaries

In this section with some basic multigroup and graph theoretic terminologies are discussed. The graphs considered in this work are simple graphs; they are undirected graphs that do not have multiple edges or loops.

A graph Γ is formed by finite non-empty pair of sets V and E. V is called the set of vertices and E the set of edges connecting the vertices. The number of vertices in Γ is known as its order while the total number of edges in Γ is known as its size. $d(v)$ denotes the degree of a vertex $v \in V$in the graph Γ which is the number of edges connected to it. $\delta(\Gamma)$ and $\Delta(\Gamma)$ denote the minimum and maximum vertex degree in a graph Γ respectively (Garba, Magami, and Ejima, 2021). $\Gamma[S]$ is called the induced subgraph of Γ with $S \subset V$and is the graph with vertices set $S$and edges with both ends in S. A subset of Vin which no two vertices are adjacent iscalled an independent set. A complete graph Γ is a graph in which every pair of its distinct vertices are adjacent. A complete graph with n vertices is represented byK_n. A connected graph Γ is said a graph in which any two distinct vertices of Γ are joined by a path. Γ is said to be disconnected if Γ is not connected (Ajay, Lavanya, Peter, and Tamizh, 2021).

Definition 2.1
(Magami and Ashafa, 2023). Let X be a finite set, a multiset Mover X is a set that contains repeated elements of X.

Example 2.1
Let $X = \{1, 2, 3, 4, 5\}$then the following are examples of multisetsover X,
1. $M_1 = [1, 1, 1, 2, 2, 3, 3, 3, 4, 4, 5, 5, 5, 5]$
2. $M_2 = [1, 1, 2, 2, 2, 3, 3, 4, 4, 5, 5]$
3. $M_3 = [1, 1, 1, 1, 1, 2, 3, 3, 4, 5]$
The number of occurrences of an element in a multiset is its multiplicity and we write $C_{M_1}(1) = 3$ to mean that element 1 in the multiset (mset for short) M_1 occurs 3 times (Gambo and Tella, 2022). $MS(X)$ denotes the set of all multisets over X. (Ibrahim, Awolola and Alkali, 2016). There are many ways of representing mset depending on the author's choice for instance, the mset $M_3 = [1, 1, 1, 1, 1, 2, 3, 3, 4, 5]$ can be represented as$M_3 = [1_5, 2_1, 3_2, 4_1, 5_1]$ with the subscript of an element representing its multiplicity (Gambo and Tella, 2022).

Definition 2.2
Let $M \in MS(X)$, a subset $S \subseteq X$is called the support or root of M if for every $x \in M$ with $C_M(x) > 0, \exists x \in S$ (Gambo and Tella, 2022).

Example 2.2
Let $M = [a_4, b_2, c_8, d_1]$, then clearly $S = \{a, b, c, d\}$is a support of M.

Definition 2.3
A multiset M over X is said to be r - regular if all elements in M have same multiplicity r; M is irregular if otherwise (Gambo and Tella, 2022), (Awolola and Michael, 2023).

Example 2.3
Let $M = [a_4, b_4, c_4, d_4]$, then $M$is a 4 - regular multiset.

Definition 2.4
Let $(X,*)$ be a group and $G \in MS(X)$, then $G$is a multigroup (mgroup for short) over X if $\forall x, y \in X$ we have
1. $C_G(x * y) \geq C_G(x) \cap C_G(y)$ where $C_G(x) \cap C_G(y) = min\{C_G(x), C_G(y)\}$,
2. $C_G(x^{-1}) = C_G(x)$
The set of all mgroup over the group X is denoted by $MG(X)$ (Ejegwa and Ibrahim, 2020), (Awolola, 2019a), (Nazmul, Majumdar and Samanta, 2013).

Example 2.4
Let $(X,*) = (Z_4, \oplus_4)$ then $G = [0_3, 1_2, 2_3, 3_2]$ is a mgroup over Z_4.

Definition 2.5
Let $(X,*)$ be group, then the Coprime Graph of the multigroup $G \in MG(X)$, $\Gamma_{cop}(G)$, is the graph whose vertices are the elements of $G$and two distinct vertices $x, y \in G$ are connected if and only if $(|x|, |y|) = 1$ (Juliana et al., 2020), (Awolola, 2019b).

Example 2.5
The Coprime Graph of multigroup $G = [0, 0, 0, 1, 1, 2, 2, 2, 3, 3]$ over Z_4 is given in figure 1.

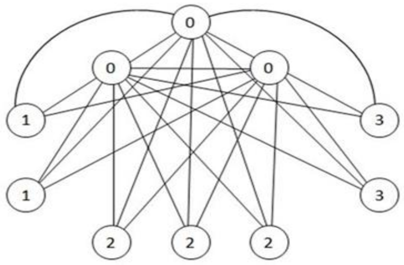

Figure 1: Commuting Graph of Multigroup G

Definition 2.6
(Syarifudin, 2021). A dihedral group $D_{2n}$is a group with order$2n, n \geq 3$ generated by 2 elements $a$and $b$with the property

$$D_{2n} = < a, b | a^n = e, b^2 = e, bab^{-1} = a^{-1} >$$

From the above definition, it is not difficult to see that

$$D_{2n} = \{e, a, a^2, a^3, \cdot\cdot\cdot, a^{n-1}, b, ab, a^2b, a^3b, \cdot\cdot\cdot, a^{n-1}b\}$$

Definition 2.7
Let D_{2n} be a dihedral group, a multigroup Gover $D_{2n}$is a mset over D_{2n} satisfying the following properties,
1. $C_G(x * y) \geq C_G(x) \cap C_G(y), \forall x, y \in D_{2n}$ where $C_G(x) \cap C_G(y) = min\{C_G(x), C_G(y)\}$
2. $C_G(x^{-1}) = C_G(x), \forall x \in D_{2n}$
By the above definition the multigroup $G \in MG(D_{2n})$ can be represented as

$$G = [M_e, M_a, M_{a^2}, M_{a^3}, \cdot\cdot\cdot, M_{a^{n-1}}, M_b, M_{ab}, M_{a^2b}, M_{a^3b}, \cdot\cdot\cdot, M_{a^{n-1}b}]$$

with M_{x_i} represents the mset containing repeated elements $x_i$and $|M_{x_i}| = C_G(x_i)$.

Example 2.6
Consider the dihedral group $D_6 = \{e, a, a^2, b, ab, a^2b\}$ and let $G_1 = [e, e, e, e, a, a, a^2, a^2, b, ab, ab, a^2b, a^2b]$, then it can be verified that $G$is a multigroup over $D_6$and can be represented more compactly as $G_1 = [M_e, M_{a^2}, M_b, M_{ab}, M_{a^2b}]$
with $|M_e| = 4; |M_a| = 2;\ |M_{a^2}| = 2;\ |M_b| = 1;\ |M_{ab}| = 2;\ |M_{a^2b}| = 2$.

Example 2.7
Another example of multigroup over $D_6$is

$$G_2 = [e,e,e,e,a,a,a,a^2,a^2,a^2,b,b,b,b,ab,ab,a^2b,a^2b,a^2b]$$
here $|M_e| = 4$; $|M_e| = 3$; $|M_{a^2}| = 3$; $|M_b| = 4$; $|M_{ab}| = 2$; $|M_{a^2b}| = 3$.

Example 2.8
The coprime graph $\Gamma_{cop}(G_1)$ of the multigroup
$$G_1 = [e,e,e,e,a,a,a^2,a^2,b,ab,ab,a^2b,a^2b]$$
defined in example (2.6) above is

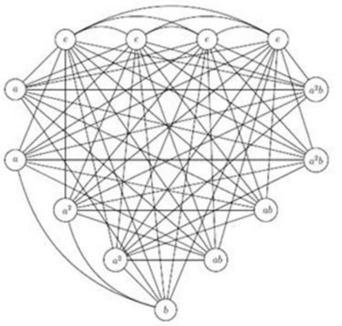

Figure 2: Coprime Graph of G_1

Definition 2.8
Let $A \in MG(X)$ and $x \in X$. If there exists a positive integer n such that $C_A(x^n) = C_A(e), e =$ identity element, then the least such positive integer is called the order of an element x with respect to A. If no such n exists, x is said to be of infinite order with respect to A. The order of an element x with respect to A is denoted by $|A(x)|$. (Awolola and Ibrahim, 2016), (Awolola and Ejegwa, 2017).

RESULTS AND DISCUSSION

Some Properties of $\Gamma_{cop}(G), G \in MG(D_{2n})$

Let G be a multigroup over $D_{2n}, n \geq 3$, then
$G = [M_e, M_a, M_{a^2}, M_{a^3}, \cdot\cdot\cdot, M_{a^{n-1}}, M_b, M_{ab}, M_{a^2b}, M_{a^3b}, \cdot\cdot\cdot, M_{a^{n-1}b}]$
where $M_{x_i} = [x_i, x_i, x_i, \cdot\cdot\cdot, x_i]$ is the mset of repeated $x_i \in D_{2n}$.
The elements of dihedral group $D_{2n}, n \geq 3$ with n prime can be partitioned into 3 disjoint subsets whose union gives the whole group. These subsets are:
$P_1 = \{e\}$
$P_2 = \{a, a^2, a^3, \cdot\cdot\cdot, a^{n-1}\}$
$P_3 = \{b, ab, a^2b, a^3b, \cdot\cdot\cdot, a^{n-1}b\}$
such that, element in P_1 has order 1 i.e. $|e| = 1$, all elements in P_2 each have order n i.e. $|a| = |a^2| = |a^3| = \cdot\cdot\cdot = |a^{n-1}| = n$ since $|a| = n$, all elements in P_3 each have order 2 i.e. $|b| = |ab| = |a^2b| = |a^3b| = \cdot\cdot\cdot = |a^{n-1}b| = 2$ since $|b| = 2$.
If we partition multigroup G in the same manner, we will get 3 msets
$$G = [M_e, M_{\Delta_1}, M_{\Delta_2}]$$
where
$M_{\Delta_1} = [M_a, M_{a^2}, M_{a^3}, \cdot\cdot\cdot, M_{a^{n-1}}]$ and
$M_{\Delta_2} = [M_b, M_{ab}, M_{a^2b}, M_{a^3b}, \cdot\cdot\cdot, M_{a^{n-1}b}]$.
Clearly, all elements in M_e have order 1, all elements in M_{Δ_1} have order n and all elements in M_{Δ_2} have order 2.

Theorem 3.1
Let $G \in MG(D_{2n})$ where $n \geq 3$ and n is prime, then
1. $\Gamma_{cop}(G)$ is a connected graph
2. $\Gamma_{cop}(G)$ is not complete

Proof
Suppose $G \in MG(D_{2n})$ where $n \geq 3$ and n is prime then G can be partitioned into 3 disjoint partitions of msets $M_e, M_{\Delta_1}, M_{\Delta_2}$ as defined earlier. Let $e \in M_e$ then $|e| = 1$.

Claim
e is connected to all vertices of $\Gamma_{cop}(G)$.

Proof of Claim
Let $v \in G$ then $v \in M_e$ or $v \in M_{\Delta_1}$ or $\in M_{\Delta_2}$. If
1. $v \in M_e$ then $e \sim v$ because $(|e|, |v|) = (1,1) = 1$.
2. $v \in M_{\Delta_1}$ then $e \sim v$ because $(|e|, |v|) = (1,n) = 1$.
3. $v \in M_{\Delta_2}$ then $e \sim v$ because $(|e|, |v|) = (1,2) = 1$.
which proves our claim and therefore $\Gamma_{cop}(G)$ is connected.
Secondly, to prove that $\Gamma_{cop}(G)$ is not complete, it suffice to show any two vertices
that are not adjacent. Let v_1 and v_2 be two vertices of $\Gamma_{cop}(G)$ such that $v_1, v_2 \in M_{\Delta_1}$ then v_1 and v_2 are not connected because $(|v_1|, |v_2|) = (n,n) = n \neq 1$. Hence the result.

Theorem 3.2.
Let $G \in MG(D_{2n})$ where $n \geq 3$ and n is prime, then
1. The degree $d(v)$ of a vertex v in $\Gamma_{cop}(G)$ is given by
$$d(v) = \begin{cases} |G| - 1 & , v \in M_e \\ |M_e| + |M_{\Delta_2}|, & v \in M_{\Delta_1} \\ |M_e| + |M_{\Delta_1}|, & v \in M_{\Delta_2} \end{cases}$$
2. The minimum degree of vertex in $\Gamma_{cop}(G)$ is
$$\delta(\Gamma_{cop}(G)) = |M_e| + min\{|M_{\Delta_1}|, |M_{\Delta_2}|\}$$
3. The maximum degree of vertex in $\Gamma_{cop}(G)$ is $\Delta(\Gamma_{cop}(G)) = |G| - 1$.

Proof
Suppose $G \in MG(D_{2n})$ where $n \geq 3$ and n is prime then G can be partitioned into 3 disjoint partitions of msets M_e, M_{Δ_1} and M_{Δ_2} as defined above.
i. Let $v \in G$ be a vertex of $\Gamma_{cop}(G)$ then we have 3 cases:

Case 1
($v \in M_e$): Since $v \in M_e$ then $|v| = 1$ and v is adjacent to
- i. all elements of $M_e \backslash \{v\}$ because $(|v|, |x|) = (1,1) = 1$ for $x \in M_e \backslash \{v\}$.
- ii. all elements of M_{Δ_1} because $(|v|, |x|) = (1,n) = 1$ for $x \in M_{\Delta_1}$.
- iii. all elements of M_{Δ_2} because $(|v|, |x|) = (1,2) = 1$ for $x \in M_{\Delta_2}$.

therefore,
$$\begin{aligned} d(v) &= |M_e \backslash \{v\}| + |M_{\Delta_1}| + |M_{\Delta_2}| \\ &= |Me| - 1 + |M_{\Delta_1}| + |M_{\Delta_2}| \end{aligned}$$

$$= |Me| + |M_{\Delta_1}| + |M_{\Delta_2}| - 1$$
$$= |G| - 1$$

Case 2

($v \in M_{\Delta_1}$): Since $v \in M_{\Delta_1}$ then $|v| = n$ and v is adjacent to

i. all elements of M_e because $(|v|,|x|) = (n,1) = 1$ for $x \in M_e$.

ii. all elements of M_{Δ_2} because $(|v|,|x|) = (n,2) = 1$ for $x \in M_{\Delta_2}$.

however, v is not adjacent to elements M_{Δ_1} because $(|v|,|x|) = (n,n) = n \neq 1$ for $x \in M_{\Delta_1 v}$, therefore

$$d(v) = |Me| + |M_{\Delta_2}|$$

Case 3

($v \in M_{\Delta_2}$): Since $v \in M_{\Delta_2}$ then $|v| = 2$ and v is adjacent to

i. all elements of M_e because $(|v|,|x|) = (2,1) = 1$ for $x \in M_e$.

ii. all elements of M_{Δ_1} because $(|v|,|x|) = (2,n) = 1$ for $x \in M_{\Delta_1}$.

however, v is not adjacent to elements M_{Δ_2} because $(|v|,|x|) = (2,2) = 2 \neq 1$ for $x \in M_{\Delta_2}$, therefore

$$d(v) = |Me| + |M_{\Delta_1}|$$

This proves part (1) of the theorem.

ii. $\delta(\Gamma cop(G)) = \min_{v \in G}\{d(v)\}$. $d(v)$ assumes 3 different values based on the partition v belongs to. These values are:

$|G| - 1$ or $|Me| + |M_{\Delta_1}|$ or $|Me| + |M_{\Delta_2}|$ for $x \in M_e$ or $x \in M_{\Delta_1}$ or $x \in M_{\Delta_2}$ respectively. We will obtain the minimum among these values. Since

$$|G| = |Me| + |M_{\Delta_1}| + |M_{\Delta_2}|$$

then $|G| - 1 > |Me| + |M_{\Delta_1}|$

and $|G| - 1 > |Me| + |M_{\Delta_2}|$

therefore $|G| - 1$ cannot be the minimum value. Hence

$$\delta(\Gamma_{cop}(G)) = |M_e| + min\{|M_{\Delta_1}|, |M_{\Delta_2}|\}$$

iii. $\Delta(\Gamma cop(G)) = \max_{v \in G}\{d(v)\}$. $d(v)$ assumes 3 different values based on the partition v belongs to. These values are:

$|G| - 1$ or $|Me| + |M_{\Delta_1}|$ or $|Me| + |M_{\Delta_2}|$ for $x \in M_e$ or $x \in M_{\Delta_1}$ or $x \in M_{\Delta_2}$ respectively. We will obtain the maximum among these values. Since

$$|G| = |Me| + |M_{\Delta_1}| + |M_{\Delta_2}|$$

then $|G| - 1 > |Me| + |M_{\Delta_1}|$

and $|G| - 1 > |Me| + |M_{\Delta_2}|$

therefore $|G| - 1$ is the maximum value. Hence

$$\Delta(\Gamma_{cop}(G)) = |G| - 1$$

Theorem 3.3

Let $G \in MG(D_{2n})$ where $n \geq 3$ and n is prime, then the size of $\Gamma_{cop}(G)$ is given by

$$\left|E\left(\Gamma_{cop}(G)\right)\right| = \frac{M_e(M_e - 1)}{2} + \sum_{M_x \in M_{\Delta_1}} \sum_{M_y \in M_{\Delta_2}} |M_x||M_y| + |M_e|(|M_{\Delta_1}| + |M_{\Delta_2}|)$$

Proof

Suppose $G \in MG(D_{2n})$ where $n \geq 3$ and n is prime then G can be partitioned into 3 disjoint partitions of msets M_e, M_{Δ_1} and M_{Δ_2} as defined earlier. By definition of $\Gamma_{cop}(G)$, the connected components of the graph can be classified into 3 as follows:

1. Since elements in M_e have order of 1 then any two vertices of M_e are connected

Therefore, the induced subgraph $\Gamma[M_e]$ is complete.

2. Since the elements in M_{Δ_1} and M_{Δ_2} have orders n and 2 respectively then no two elements of M_{Δ_1} are connected likewise no two elements of M_{Δ_2} are connected. However, since $(n,2) = 1$ then all vertices in M_{Δ_1} re adjacent to all the vertices in M_{Δ_2} and therefore the induced subgraph $\Gamma[M_x \cup M_y]$ with $M_x \in M_{\Delta_1}, M_y \in M_{\Delta_2}$ is acomplete bipartite graph.

3. Since elements in M_e, M_{Δ_1} and M_{Δ_2} have orders of $1, n$ and 2 respectivelythen all vertices of M_e are adjacent to all the vertices of M_{Δ_1} as well as all vertices of M_{Δ_2}.

Based on the above classification of connected components of $\Gamma_{cop}(G)$ above, edges of $\Gamma_{cop}(G)$ can be divided into 3 sets

1. The first set are the edges of the complete induced subgraph $\Gamma[M_e]$,

$$|E(\Gamma[M_e])| = \frac{|M_e|(|M_e| - 1)}{2}$$

2. The second set are the edges of the induced subgraphs $\Gamma[M_x \cup M_y]$ with $M_x \in M_{\Delta_1}, M_y \in M_{\Delta_2}$ which is a complete bipartite graph,

$$\left|E\left(\Gamma\left[M_x \cup M_y\right]\right)\right| = \sum_{M_x \in M_{\Delta_1}} \sum_{M_y \in M_{\Delta_2}} |M_x||M_y|$$

3. Finally the edges connecting vertices of M_e with all other vertices of $\Gamma_{cop}(G)$,

$$|M_e|(|M_{\Delta_1}| + |M_{\Delta_2}|)$$

Taking the sum of these edges gives the result.

Theorem 3.4

Let $G \in MG(D_{2n})$ where $n \geq 3$ and n is prime, then an induced subgraph $\Gamma[H]$ of $\Gamma_{cop}(G)$ where $H \subset G$ is complete iff

1. $H = M_e$
2. $H = M_e \cup \{v\}, v \in G \setminus M_e$.
3. $H = M_e \cup \{v_1\} \cup \{v_2\}, v_1 \in M_{\Delta_1}, v_2 \in M_{\Delta_2}$.

Proof

Suppose $G \in MG(D_{2n})$ where $n \geq 3$ and n is prime, then the vertices of $\Gamma_{cop}(G)$ can be partitioned into M_e, M_{Δ_1} and M_{Δ_2} as defined earlier.

($\Rightarrow$) Assume that $\Gamma[H]$ is a complete induced subgraph of $\Gamma_{cop}(G)$ where $H \subset G$ then $x, y \in H$ implies $(|x|,|y|) = 1$ which means $x \sim y$.

Observe that all elements in M_e have order 1 and hence the induced subgraph $\Gamma[M_e]$ is complete because for all $v_1, v_2 \in M_e$, $(|v_1|,|v_2|) = (1,1) = 1$ and $v_1 \sim v_2$. Therefore, we can have $H = M_e$.

Secondly, lets expand M_e by adding one vertex to see if the resulting subgraphwillbe complete. Consider the subset $M_e \cup \{v\}, v \in G \setminus M_e$.

Claim

$\Gamma[M_e \cup \{v\}]$ for $v \in G \setminus M_e$ is complete.

Proof of Claim

Let $x, y \in M_e \cup \{v\}$ with $v \in G \setminus M_e$ then we the following cases:

Case 1 ($x, y \in M_e$)

Since $x, y \in M_e$ then $|x| = |y| = 1$ and $(|x|,|y|) = 1$. Therefore $x \sim y$.

Case 2 ($x \in M_e, y = v$)

Since $x \in M_e$ and $y = v \in G \backslash M_e$ then $|x| = 1$ and $|y| \in \{n, 2\}$ depending on whether y belongs to M_{Δ_1} or M_{Δ_2}. We see that $(|x|, |y|) = 1$. Therefore $x \sim y$.

Case 3 **$(y \in M_e, x = v)$**
Since $y \in M_e$ and $x = v \in G \backslash M_e$ then $|y| = 1$ and $|x| \in \{n, 2\}$ depending on whether x belongs to M_{Δ_1} or M_{Δ_2}. We see that $(|x|, |y|) = 1$. Therefore $x \sim y$.
This proves our claim and hence we can have $H = M_e \cup \{v\}, v \in G \backslash M_e$.
Thirdly, lets again expand M_e by adding two vertices to see if the resulting subgraphwill be complete. Consider the subset $M_e \cup \{v_1\} \cup \{v_2\}$ $with$ $v_1, v_2 \in G \backslash M_e$
Since any two elements $x, y \in M_{\Delta_1}$ are not adjacent because $(|x|, |y|) = n \neq 1$ and similarly any two $r, s \in M_{\Delta_2}$ are not adjacent because $(|r|, |s|) = 2 \neq 1$ then $\Gamma[M_e \cup \{v_1\} \cup \{v_2\}]$ cannot be complete if either $v_1, v_2 \in M_{\Delta_1}$ or $v_1, v_2 \in M_{\Delta_2}$. So H cannot be the $M_e \cup \{v_1\} \cup \{v_2\}$ with $v_1, v_2 \in M_{\Delta_1}$ or $v_1, v_2 \in M_{\Delta_2}$. Therefore, weare left with two options: $v_1 \in M_{\Delta_1}, v_2 \in M_{\Delta_2}$ and $v_2 \in M_{\Delta_1}, v_1 \in M_{\Delta_2}$.
Assume that $x, y \in M_e \cup \{v_1\} \cup \{v_2\}$ then we have two cases:

Case 1 **$(v_1 \in M_{\Delta_1}, v_2 \in M_{\Delta_2})$**
Since $x, y \in M_e \cup \{v_1\} \cup \{v_2\}$ with $v_1 \in M_{\Delta_1}, v_2 \in M_{\Delta_2}$ then we have the following sub cases:

Sub case 1.1 **$(x, y \in M_e)$**
This means $|x| = |y| = 1$ and $(|x|, |y|) = 1$. Therefore $x \sim y$.

Sub case 1.2 **$(x \in M_e, y \in \{v_1, v_2\})$**
This means $|x| = 1, |y| \in \{n, 2\}$ and $(|x|, |y|) = 1$. Therefore $x \sim y$.

Sub case 1.3 **$(y \in M_e, x \in \{v_1, v_2\})$**
This means $|y| = 1, |x| \in \{n, 2\}$ and $(|x|, |y|) = 1$. Therefore $x \sim y$.

Sub case 1.4 **$(x = v_1, y = v_2$ or $x = v_2, y = v_1)$**
This means $|x| = n, |y| = 2$ or $|x| = 2, |y| = n$ and $(|x|, |y|) = 1$. Therefore $x \sim y$.

Case 2 **$(v_1 \in M_{\Delta_2}, v_2 \in M_{\Delta_1})$**
Since $x, y \in M_e \cup \{v_1\} \cup \{v_2\}$ with $v_1 \in M_{\Delta_2}, v_2 \in M_{\Delta_1}$ then we have the following sub cases:

Sub Case 2.1 **$(x, y \in M_e)$:**
This means $|x| = |y| = 1$ and $(|x|, |y|) = 1$. Therefore $x \sim y$.

Sub Case 2.2 **$(x \in M_e, y \in \{v_1, v_2\})$**
This means $|x| = 1, |y| \in \{n, 2\}$ and $(|x|, |y|) = 1$. Therefore $x \sim y$.

Sub Case 2.3 **$(y \in M_e, x \in \{v_1, v_2\})$:**
This means $|y| = 1, |x| \in \{n, 2\}$ and $(|x|, |y|) = 1$. Therefore $x \sim y$.

Sub case 2.4 **$(x = v_1, y = v_2$ or $x = v_2, y = v_1)$**
This means $|x| = n, |y| = 2$ or $|x| = 2, |y| = n$ and $(|x|, |y|) = 1$. Therefore $x \sim y$.
This proves the third part which is $H = M_e \cup \{v1\} \cup \{v2\}, v_1 \in M_{\Delta_1}, v_2 \in M_{\Delta_2}$.
$(\Leftarrow)$

1. Assume $H = M_e$ and let $x, y \in H$ then $(|x|, |y|) = 1$ and $x \sim y$. Therefore
$\Gamma[H]$ is complete.

2. Assume $H = M_e \cup \{v\}, v \in G \backslash M_e$ and let $x, y \in H$ then we have the following
cases:

Case 1 **$(x, y \in M_e)$**
Since $x, y \in M_e$ then $(|x|, |y|) = 1$ and $x \sim y$. Therefore $\Gamma[H]$ is complete.
Case 2 **$(x \in M_e, y = \{v\})$**
Since $x \in M_e$ and $y = v \in G \setminus M_e$ then $|x| = 1, |y| \in \{n, 2\}$ which implies that $(|x|, |y|) = 1$ and $x \sim y$. Therefore $\Gamma[H]$ is complete.
3. Assume $H = Me \cup \{v_1\} \cup \{v_2\}, v_1 \in M_{\Delta_1}, v_2 \in M_{\Delta_2}$ and let $x, y \in H$ then wehave the following cases:

Case 1 **$(x, y \in M_e)$**
Since $x, y \in M_e$ then $(|x|, |y|) = 1$ and $x \sim y$. Therefore $\Gamma[H]$ is complete.

Case 2 **$(x \in M_e, y = \{v_1, v_2\})$**
Since $x \in M_e$ and $y \in \{v_1, v_2\}$ then $|x| = 1, |y| \in \{n, 2\}$ which implies that $(|x|, |y|) = 1$ and $x \sim y$. Therefore $\Gamma[H]$ is complete.

Case 3 (x = v1, y = v2 or x = v2, y = v1)
Since $x = v_1, y = v_2$ or $x = v_2, y = v_1$ then $|x| = n$, $|y| = 2$ or $|x| = 2$, $|y| = n$ respectively. This implies that $(|x|, |y|) = 1$ and $x \sim y$. Therefore $\Gamma[H]$ is complete.
This completes the proof.

Theorem 3.5
Let $G \in MG(D_{2n})$ where $n \geq 3$ and n is prime, then the cliquenumber of $\Gamma_{cop}(G)$ is $\omega(\Gamma_{cop}(G)) = |M_e| + 2$

Proof
Suppose $G \in MG(D_{2n})$ where $n \geq 3$ and n is prime then G can be partitioned into 3 disjoint partitions of msets M_e, M_{Δ_1} and M_{Δ_2} as defined earlier.
The clique number of $\Gamma_{cop}(G)$ is the number of vertices of the maximum induced complete subgraph. By Theorem (3.4), the complete induced subgraph $\Gamma[H]$ canonly be one out of the following 3 options, $\Gamma[M_e]$ or $\Gamma[M_e \cup \{v\}]$, $v \in G \setminus M_e$ or $\Gamma[M_e \cup \{v_1\} \cup \{v_2\}], v_1 \in M_{\Delta_1}, v_2 \in M_{\Delta_2}$. The maximum among these option is $\Gamma[M_e \cup \{v_1\} \cup \{v_2\}], v_1 \in M_{\Delta_1}, v_2 \in M_{\Delta_2}$ and the number of vertices is given by $|M_e| + 2$ as required.

Lemma 3.6
(Sowaity, Sharada, and Naji, 2020). The chromatic number for the complete graph K_n is $\chi(K_n) = n$. Also for the complete bipartite graph $K_{a,b}$ is $\chi(K_{a,b}) = 2$.

Theorem 3.7
Let $G \in MG(D_{2n})$ where $n \geq 3$ and n is prime, then the chromatic number of $\Gamma_{cop}(G)$ is $\chi(\Gamma_{cop}(G)) = |M_e| + 2$

Proof
Suppose $G \in MG(D_{2n})$ where $n \geq 3$ and n is prime then G can be partitioned into 3 disjoint partitions of msets M_e, M_{Δ_1} and M_{Δ_2} as defined earlier. The chromatic number of $\Gamma_{cop}(G)$ is the smallest number of colours which can be used to colour it such that no two adjacent vertices have same

colour. By Theorem (3.5), the maximum induced complete subgraph is $\Gamma[M_e \cup \{v_1\} \cup \{v_2\}], v_1 \in M_{\Delta_1}, v_2 \in M_{\Delta_2}$has vertices $|M_e| + 2$. Therefore we can use the minimum of $|M_e| + 2$ colours to colour the maximum induced complete subgraph by lemma (3.6).Assuming we use $|M_e|$colours to colour the vertices in M_e, use the $(|M_e| + 1)^{\text{th}}$colour to colour $v_1 \in M_{\Delta_1}$and $(|M_e| + 2)^{\text{th}}$ colour to colour $v_2 \in M_{\Delta_2}$then we can use the $(|M_e| + 1)^{\text{th}}$colour to colour all vertices in $M_{\Delta_1}$and the $(|M_e| + 2)^{\text{th}}$ colour to colour all vertices in M_{Δ_2}. This is possible because no two vertices in $M_{\Delta_1}$are adjacent; similarly, no two vertices in $M_{\Delta_2}$are adjacent. This completes the proof.

CONCLUSION

In this investigation, we investigated the graph theoretic properties of coprime graph$\Gamma_{cop}(G)$ of multigroup over the dihedral group $D_{2n}$with $n$prime number greater than 2. The vertex degree of a vertex was obtained by classifying the vertex set into3 partitions. The maximum degree and minimum degree of vertex in the graph were obtained. The total number of edges of the graph was also obtained. It was shown that the graph is always a connected graph but never a complete graph. Conditions for completeness of an induced subgraph were also found and the clique number and chromatic number of the graph were computed and were found to be the same. Further research can be conducted on other graph properties which include: radius, diameter, independence number and so on.

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