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ANALYSIS OF TRANSIENT AND FREQUENCY RESPONSES OF MODIFIED RLC ELECTRICAL CIRCUIT WITH DAMPING AND FILTERING CHARACTERISTICS

Okafor, U. L. and *Oyetoro, J. A.

Department of Mathematics, Nigerian Defence Academy, Kaduna Nigeria.

*Corresponding authors' email: juliusoyetoro@gmail.com

ABSTRACT

In this paper we address the problem occurring in electrical circuit with discontinuity in power source, using the Heaviside function to represent the switches (on/off) existing in the electrical circuit component. Thus, this paper presents an analytical and simulation-based study of modified RLC circuits, focusing on their transient and frequency responses, using Laplace transform techniques and Heaviside step input modeling, the research evaluates underdamped, undamped, overdamped, and critically damped behaviors and the frequency domain analysis using the Bode plots to reveals the filtering characteristics of RC and RLC configurations. The paper highlights how component values influence energy dissipation, signal attenuation, and resonance. The findings provide useful guidelines for circuit design in signal processing and communications.

Keywords: RLC circuit, Transient response, Frequency response, Damping, Filtering, Laplace transform, Heaviside function, MATLAB simulation

INTRODUCTION

Transient and frequency domain analyses are essential for understanding the dynamic behavior of electrical circuits. Circuits composed of resistors (R), inductors (L), and capacitors (C) find widespread application in electronics, from signal filtering to resonance control (Alexander & Sadiku, 2017). RC circuits exhibit first-order exponential charging and discharging behavior, governed by the time constant, which dictates how quickly the circuit responds to step inputs (Hayt et al., 2019). Andi, Ibeh, and Umar (2017) applied this analytical approach to investigate how instantaneous inputs affect the transient and frequency responses of RC and RLC circuits. Their use of both analytical modeling and numerical simulations highlighted the critical role of input type and component values in determining circuit behavior.

Andi, Ibeh, and Umar (2017) studied the behavior of RC and RLC circuits under instantaneous forcing functions, such as step and impulse inputs. Their results showed that the damping ratio and natural response of the circuit are highly dependent on the input type findings that support the use of Heaviside-based input modeling in this study. For instance, Ikechiamaka, Okpala, and Lawal (2017) conducted frequency response measurements of an audio amplifier, showcasing the relevance of such analytical methods in evaluating signal behavior in electrical systems. The analysis of electrical vibrations is vital to understanding circuit behavior, especially when discontinuities or abrupt changes in the power supply occur. Dorf and Svoboda (2018) examine the use alternating current analysis is particularly important for understanding filter design, signal modulation, power distribution networks and various types of time-varying signals that oscillate at specific frequencies

Hayt, Kemmerly, and Durbin (2019) further expanded the analytical tools for AC circuit analysis by introducing phasor representation, impedance, and resonance concepts. These tools are particularly helpful in understanding the frequency domain behavior of RLC circuits. Boylestad and Nashelsky (2020) emphasized the importance of foundational theorems in circuit analysis such as Ohm's Law, Kirchhoff's Laws, Thevenin's, and Norton's Theorems. These theorems are essential in determining current and voltage distributions across linear networks and serve as the basis for more advanced simulation models, including those applied in this research. This has led to significant studies on the application of the Heaviside step function in modeling switching operations within electrical circuits. Heaviside ((Heaviside, 1893; Operational Calculus, 2025) introduced a symbolic method known as operational calculus, which transforms differential equations into algebraic expressions. This laid the groundwork for modern Laplace-domain analysis, where the Heaviside function plays a critical role in defining step-input behaviors in RC and RLC circuits.

Governing Equation

The governing equation of the charging RC electrical circuit in the loop at time$t = 0$, switch S_1 closes and switch S_2 is open creating a new loop consisting of $R_1, C \text{ and } V_s$ as the voltage source.

$$V_s \cdot H(t) = R_1 i(t) + v_c(t) \tag{1}$$

The current through a capacitor is

$$i(t) = C\frac{dv_c(t)}{dt} \tag{2}$$

Substituting equation (2) into (1) gives

$$V_s \cdot H(t) = R_1 C\frac{dv_c(t)}{dt} + v_c(t) \tag{3}$$

Equation (3) is the governing equation for the RC loop

Charging of a Capacitor through a Resistor

A circuit consisting of a resistor R in series with a capacitor C, connected to a battery of voltage V, at time t = 0. The steady state current in this case is obviously zero, since no current will flow from a D.C supply through a capacitor hence $i_s = 0$

$$i = Be^{-(\frac{i}{CR})t} \tag{4}$$

When the switch is closed, there will be momentarily no voltage across the capacitor, so that the battery voltage V must all appear across the resistor R. Hence the initial current from the battery must be $i = \frac{V}{R}$ at t =0,

$$i = \frac{V}{R} = I,$$

$$i = \frac{V}{R}e^{-(\frac{i}{CR})t} = Ie^{-(\frac{i}{CR})t} \tag{5}$$

This equation represents the exponential decay curve and a charging capacitor through a resistor

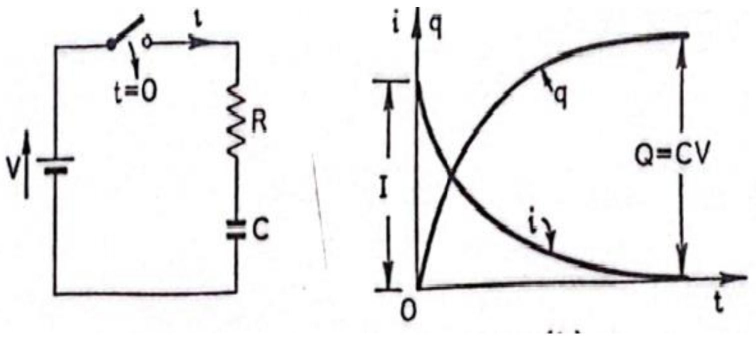

Figure 1: Charging a Capacitor through a Resistor

The voltage across the resistor at any instant is v_R

$$v_R = iR = IRe^{-(\frac{i}{CR})t} = Ve^{-(\frac{i}{CR})t} \quad (6)$$

The voltage across the capacitor is v_c

$$v_c = V - v_R = V\left(1 - e^{-(\frac{i}{CR})t}\right) \quad (7)$$

The charge q of the capacitor at any instant

$$q = v_cC = VC\left(1 - e^{-(\frac{i}{CR})t}\right) = Q\left(1 - e^{-(\frac{i}{CR})t}\right), \quad (8)$$

Where $Q = VC =$ final charge on the capacitor

Discharge of a capacitor through a resistor

Suppose that a capacitor which is originally charged to v_c volts is discharged through a resistor of R ohms, since there is no generator in the circuit the steady state current must be zero, so that the general equation for the circuit current is from equation (4).

$$i = Be^{-(\frac{i}{CR})t}$$

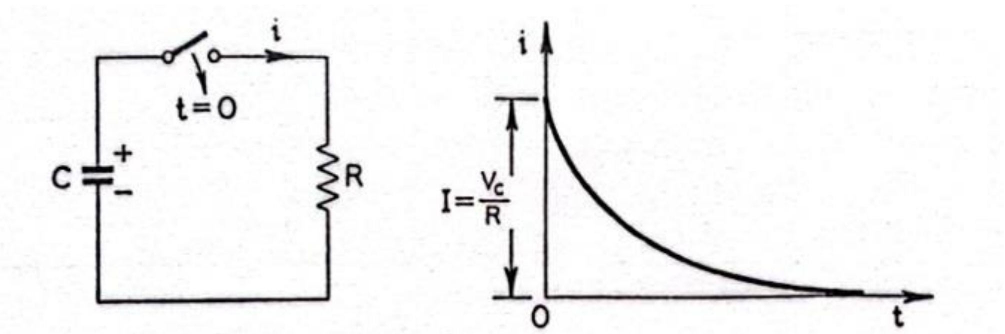

Figure 2: Discharging of a Capacitor through a Resistor

The initial condition is that the voltage across the capacitor must be the same after the switch is closed as it was before. The general equation for the current is from (5)

$$i = \frac{V}{R}e^{-(\frac{i}{CR})t} = Ie^{-(\frac{i}{CR})t}$$

This is the exponential decay curve shown above and the voltage at any instant is the same across both capacitor and the resistor

$$v = iR = V_ce^{-(\frac{i}{CR})t} \quad (9)$$

The charge on the capacitor at any instant is

$$q = Cv = CV_ce^{-(\frac{i}{CR})t} = Qe^{-(\frac{i}{CR})t} \quad (10)$$

Where $Q(= CV_c)$ is the initial charge on the capacitor.

The Governing Equation of the Discharging RLC Electrical Circuit in the Loop

At time$t = t_0$, switch S_2 closes, creating a new loop consisting of $R_2, L, and\ C$ the capacitor has already been charged to V_s

The charged capacitor $V_c(t_0)$ now acts like a voltage source V_s of magnitude V_0 that is suddenly applied to the RLC electrical circuit we model this using a Heaviside step function

The Kirchhoff's voltage law of the RLC series loop

$$v_{in}(t) = v_R(t) + v_L(t) + v_c(t)$$

$$V_0.H(t - t_0) = R_2i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(T)dT \quad (11)$$

In other to obtain the transient and frequency response, the Laplace transform method of solution is applied because it is a very powerful tool for solving linear differential equations with constant coefficient encountered in this study of electrical vibration problem. The use of Laplace transform methods in this project aligns with techniques outlined by Andi, Ibeh, and Umar (2017), who demonstrated that instantaneous inputs could be accurately represented and analyzed using this approach for transient and steady-state studies.

Mathematically, the definition of Laplace transform is given as;

$$L\{f(t)\} = F(s) = \int_0^\infty e^{-st}f(t), dt \quad t \geq 0 \quad (12)$$

Where t a real number and s is a complex number

While, the differentiation in time domain derivatives isturned into algebraic power of s

$$L\frac{df(t)}{d(t)} = sF(s) - f(0) \quad (13)$$

$$L\left\{\frac{d^2f(t)}{dt^2}\right\} = s^2F(s) - sf(0) - f'(0) \quad (14)$$

In circuit context it model behavior of inductors and capacitors

$$V_L = L\frac{di}{dt} \Rightarrow sLI(s) - Li(0) \quad (15)$$

$$I_c = C\frac{dv}{dt} \Rightarrow sCV(s) - Cv(0) \quad (16)$$

We will need to also defineIntegration becoming divisions$s$, helpful for systems with accumulating energy

$L\{\int_0^t f(T)dT\} = \frac{F(s)}{s}$ (17)

Appears when using

$v_C(t) = \frac{1}{C}\int i(t)dt$

The Initial Value Theoremis applied only of $f(t)$ is causal (zero before $t = 0$)

$\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)$ (18)

The Final Value Theorem also make all the poles of $sF(s)$to have negative real parts making the system stable

$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$ (19)

The Convolution Theorem in the time domain becomes multiplication in Laplace domain

$L\{f(t) * g(t)\} = F(s) \cdot G(s)$ (20)

The Heaviside step function, denoted $H(t-a)$ is used to model the sudden application or removal of a voltage or current source in electrical circuits.

$q''(t) + Rq'(t) + \frac{1}{C}q(t) = H(t-a)$ (21)

$E(t) = H(t-a)$The Heaviside forcing function define as; and it's used for switches in an electrical circuit $H(t-a) = \begin{cases}0\\1\end{cases} t < a, t \geq a$ (22)

The function represents the instantaneous switching of voltage source at specific times, convert time domain discontinuities into algebraic expression in the Laplace domain using transformation. This occurs when there is a discontinuity in the power source, and the unit-step function is used to represent on/off switches.

$L\{H(t-a)\} = \frac{e^{-as}}{s}$ (23)

Enable the analysis of circuit responses to step inputs such as $V_s u(t)$ or delayed step $V_0 H(t-t_0)$ and to model the charging and discharging phase of the RC and RLC circuit triggered by switching events.

Solution of the Governing Equation

The solution begins by getting the transient and frequency response of the RC charging stage and the Discharging into RLC loop $t \geq t_0$. RC starting from the charging stage with the Heaviside forcing function

Thus, in a loop, KVL gives

$V_s \cdot H(t) = R_1 i(t) + v_c(t)$ (24)

Current through a capacitor i

$$i(t) = C\frac{dv_c(t)}{dt}$$

$V_s \cdot H(t) = R_1 C\frac{dv_c(t)}{dt} + v_c(t)$ (25)

Solution using the Laplace transform

$$L\{u(t)\} = \frac{1}{s}$$

$$L\{v_c(t)\} = V_c(s)$$

$$L\left\{\frac{dv_c(t)}{dt}\right\} = SV_c(s) - v_c$$

Assume the capacitor is initially uncharged

$v_c(0) = 0$

$L\{V_S H(t)\} = L\left\{R_1 C\frac{dv_c(t)}{dt} + v_c(t)\right\}$ (26)

$$\frac{V_S}{s} = R_1 C(SV_c(s) - 0) + V_c(S)$$

$$\frac{V_S}{s} = V_c(S)(R_1 CS + 1)$$

$V_c(S) = \frac{V_S}{R_1 C} \cdot \frac{1}{S(S+\frac{1}{R_1 C})}$ (27)

Taking inverse Laplace Transform

$v_c(t) = V_s\left(1 - e^{-\frac{t}{R_1 C}}\right)H(t)$ (28)

The expression $v_c(t)$ describes the voltage across the capacitor starting from 0, gradually increases and approaches V_s as t→ ∞

Discharging into RLC Loop $t \geq t_0$

At time$t = t_0$, switch S_2 closes, creating a new loop consisting of $R_2, L, and\ C$

The capacitor has already been charged

$V_0 = V_c(t_0) = V_s(1 - e^{-\frac{t_0}{R_1 C}})$ (29)

The charged capacitor now acts like a voltage source of magnitude V_0 that is suddenly applied to the RLC circuit we model this using a Heaviside step function.

$v_{in}(t) = V_0 \cdot H(t-t_0)$ (30)

The KVL of the RLC series loop

$v_{in}(t) = v_R(t) + v_L(t) + v_c(t)$

$V_0 . H(t-t_0) = R_2 i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(T)dT$ (31)

To simplify, differentiate both sides with respect to time

$\frac{d}{dt}[V_0 . H(t-t_0)] = R_2\frac{di(t)}{dt} + L\frac{d^2 i(t)}{dt^2} + \frac{1}{C}i(t)$ (32)

Where:

$\frac{d}{dt}[V_0 . H(t-t_0)] = V_0 \cdot \delta(t-t_0)$

Hence, we have

$V_0 \cdot \delta(t-t_0) = L\frac{d^2 i(t)}{dt^2} + R_2\frac{di(t)}{dt} + \frac{1}{C}i(t)$ (33)

Instead of solving it in the time domain, we proceed with the Laplace Transform to handle both the time shift and solve efficiently.

Laplace of left-hand side of $V_0 \cdot \delta(t-t_0)$ give us equation (33)

$L\{V_0 \cdot \delta(t-t_0)\} = \frac{V_0 e^{-st_0}}{s}$ (34)

Laplace of right-hand side of $L\frac{d^2 i(t)}{dt^2} + R_2\frac{di(t)}{dt} + \frac{1}{C}i(t)$ give us equation (33)

$LS^2 I(S) + R_2 SI(S) + \frac{1}{C}I(S) = I(S)\left(LS^2 + R_2 S + \frac{1}{C}\right)$ (35)

Combination of both equations (34) and (35).

$$\frac{V_0 e^{-st_0}}{s} = I(S)\left(LS^2 + R_2 S + \frac{1}{C}\right)$$

$I(S) = \frac{V_0 e^{-st_0}}{s\left(LS^2 + R_2 S + \frac{1}{C}\right)}$ (36)

This is the Laplace domain current. Its time shifted due to the e^{-st_0}

Using the time-shifting theorem

$$L^{-1}\{e^{-st_0}F(s)\} = f(t-t_0)H(t-t_0)$$

$$F(S) = \frac{V_0}{S\left(LS^2 + R_2 S + \frac{1}{C}\right)} \Rightarrow f(t)$$

$i(t) = f(t-t_0)H(t-t_0)$ (37)

This means the current response is delayed by t_0 and the natural RLC response begins a $t = t_0$

$i(t) = L^{-1}\left\{\frac{V_0}{S\left(LS^2 + R_2 S + \frac{1}{C}\right)}\right\} \cdot H(t-t_0)$ (38)

Inverse Laplace and Damped Response**,** from (36) to get the time-domain current$i(t)$, we need to compute the inverse Laplace transform

$i(t) = L^{-1}\{I(s)\}$

Isolate the core expression from (36)

$I(s) = \frac{V_0}{S\left(LS^2 + R_2 S + \frac{1}{C}\right)}$ (39)

Where:

$\propto = \frac{R_2}{2L}$ (Damping coefficient)

$\omega_0 = \frac{1}{\sqrt{LC}}$ (Undamped natural frequency)

$\omega_d = \sqrt{\omega_0{}^2 - \propto^2}$ (Damped frequency, for underdamped case)

The nature of the roots depends on the relationship between $\alpha$and ω_0:

i. Overdamped Case ($\alpha > \omega_0$): Roots are real and distinct (two slow exponentials, no oscillation)

ii. Critically Damped Case ($\alpha = \omega_0$): Roots are real and equal (fastest non-oscillatory decay)
iii. Underdamped Case ($\alpha < \omega_0$): Roots are complex conjugates (oscillatory decay)

$$L^{-1}\left\{\frac{1}{s[(s+\propto)^2+\omega_d{}^2]}\right\} = \frac{1}{\omega_d}\left(1 - e^{-\propto t}\left[\cos(\omega_d t) + \frac{\propto}{\omega_d}\sin(\omega_d t)\right]\right) \quad (40)$$

$$LS^2 + R_2 s + \frac{1}{C} = L\left[s^2 + \frac{R_2}{L}s + \frac{1}{LC}\right] = L(s^2 + 2\propto s + \omega_0{}^2)$$

$$i(t) = \frac{V_0}{L\omega_d}(1 - e^{-\propto t}\left[\cos(\omega_d t) + \frac{\propto}{\omega_d}\sin(\omega_d t)\right] \quad (41)$$

Applying a time delay t_0 because ofe^{-st_0}, we use the time-shifting theorem

$$L^{-1}\{e^{-st_0}F(s)\} = f(t - t_0)H(t - t_0)$$

$$i(t) = \frac{V_0}{L\omega_d}\left(1 - e^{-\propto(t-t_0)}\left[\cos(\omega_d(t - t_0)) + \frac{\propto}{\omega_d}\sin(\omega_d(t - t_0))\right]\right) H(t - t_0) \quad (42)$$

Frequency Response for RC

The frequency response of the RC circuit was analyzed using Laplace transform methods, with the substitution $s = j\omega$ to enter the frequency domain. The resulting transfer functions showed that when the output is taken across the capacitor, the circuit behaves as a low-pass filter, attenuating high-frequency components. Conversely, when the output is across the resistor, the system operates as a high-pass filter, attenuating low-frequency signals. Magnitude and phase responses were derived for both configurations to fully characterize the system's filtering behavior.

Low-Pass Voltage

$Applying\ Laplace\ to\ KVL\ in\ the\ loop$

$$V_{in}(s) = I(s)\left(R + \frac{1}{C_S}\right) \Rightarrow I(s) = \frac{V_{in}(s)}{R+\frac{1}{C_S}} \quad (43)$$

$$V_{out} = V_c$$

$$V_C(s) = \frac{1}{C_S}\cdot I(s) = \frac{1}{C_S}\cdot\frac{V_{in}(s)}{R+\frac{1}{C_S}} = \frac{V_{in}(s)}{1+RC_S}$$

$$Transfer\ function{:}\ H_{LP}(s) = \frac{V_C(s)}{V_{in}(s)} = \frac{1}{1+RC_S} \quad (44)$$

$Substitute\ s = j\omega\ for\ frequency\ response$

$$H(j\omega) = \frac{1}{1+j\omega RC} \quad (45)$$

High-pass voltage

$$V_{out} = V_R$$

$$V_R(s) = R\cdot\frac{V_{in}(s)}{R+\frac{1}{C_S}} = \frac{RC_S}{1+RC_S}\cdot V_{in}(s) \quad (46)$$

$$Transfer\ function{:}\ H_{HP}(s) = \frac{V_R(s)}{V_{in}(s)} = \frac{RC_S}{1+RC_S} \quad (47)$$

$Substitute\ s = j\omega\ for\ frequency\ response$

$$H(j\omega) = \frac{j\omega RC}{1+j\omega RC} \quad (48)$$

Frequency Response for RLC

The frequency response of the series RLC circuit was analyzed by deriving the transfer function from Kirchhoff's voltage law and substituting $s = j\omega$ to enter the frequency domain. The output across the capacitor was expressed in terms of magnitude and phase. The resulting behavior showed a band-pass characteristic with maximum response occurring at the resonant frequency$\omega_0 = 1/\sqrt{LC}$. The circuit's bandwidth and sharpness were defined by the resistance and inductance values, measured via the bandwidth $\Delta\omega = R/L\omega$ and quality factor$Q = \omega_0 L/R$. This analysis reveals how the RLC circuit selectively amplifies signals at its natural resonant frequency while attenuating others, making it useful for filtering and signal selection

Using KVL on equation (2.35)

$$V_{in}(s) = I(s)\left(Ls + R + \frac{1}{C_S}\right) \Rightarrow V_{out}(s) = \frac{1}{C_S}\cdot I(s) \quad (49)$$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

$$H(s) = \frac{\frac{1}{C_S}}{Ls+R+\frac{1}{C_S}} = \frac{1}{LCs^2+RCs+1} \quad (50)$$

$$s = j\omega$$

$$H(j\omega) = \frac{1}{LC(j\omega)^2+RC(j\omega)+1} = \frac{1}{-LC\omega^2+jRC(\omega)+1} \quad (51)$$

In the frequency domain, the magnitude and phase of the response are

Magnitude

$$|H(j\omega)| = \left|\frac{1}{1-LC\omega^2+jRC\omega}\right| = \frac{1}{\sqrt{(1-LC\omega^2)^2+(RC\omega)^2}} \quad (52)$$

Phase

$$\angle H(j\omega) = -tan^{-1}\left(\frac{RC\omega}{1-LC\omega^2}\right) \quad (53)$$

RESULTS AND DISCUSSION

The study investigated the transient and frequency responses of both RC and RLC circuits subjected to a Heaviside step input, using Laplace transform analysis. For the RC circuit, the transient analysis revealed an exponential charging behavior of the capacitor voltage. The results confirmed that the circuit voltage rises asymptotically and approaches the supply voltage V_S with the rate of change governed by the time constant $\tau = RC$. A MATLAB simulation supported this behavior, producing a smooth curve that demonstrated how the voltage approaches V_S over time, with 63% of the supply voltage reached at t = τ. This behavior validated the theoretical model and illustrated the RC circuit's role as a low-pass filter that attenuates high-frequencysignals for the RLC circuit, the transient response depended heavily on the damping factor. In the underdamped case in Figure 3, the response showed decaying oscillations as expected, with current peaking and then settling to zero over time. This behavior was visually confirmed by the waveform plotted in MATLAB, showing sinusoidal oscillations enveloped by exponential decay. The critically damped case in Figure 4. Shows the return of the system to equilibrium fastest without overshoot, while the overdamped in Figure 5, the response also avoided oscillations but took more time to settle. An undamped case in Figure 6, simulated for theoretical comparison, displayed persistent sinusoidal oscillations with no energy loss due to the absence of resistance. In the frequency domain, Bode plot analysis was used to explore filtering behavior. The RC circuit's frequency response shown in Figure, 7 & 8 followed that of a first-order low-pass filter, with a clear -3 dB cutoff frequency at $f_C = 1/(2\pi RC)$, beyond which the gain steadily declined and phase lag increased. The RLC circuit exhibited a second-order band-pass filter behavior, where the gain peaked at the resonant frequency$f_0 = 1/(2\pi\sqrt{LC})$. From the Bode plot analysis in Figure, 9 & 10 as frequency moved away from resonance, attenuation increased. Phase shifts in both cases aligned with theoretical expectations, supporting the filtering models.These results highlight the impact of circuit parameters (R, L, and C) on energy dissipation, transient decay, and signal frequency selectivity. RC circuits are suitable for smoothing applications where a simple low-pass response is sufficient, while RLC circuits are better suited for selective filtering and resonance-based applications. The combined analytical and simulation-based approach confirmed the consistency of Laplace-transform methods with time-domain circuit behavior and underscored the effectiveness of the Heaviside step function for modeling switch-like transitions in circuit excitation.

Table 1: The Numerical Data Tables for the Critically Damped, Undamped, Overdamped and Underdamped RLC discharge

Time (s)	Critically Damped i(t) (A)	Undamped i(t) (A)	Overdamped i(t) (A)	Underdamped i(t) (A)
2	0	0	0	0
2.001	0	-0.054402	0.000001	0.115724
2 0.002	0	0.091295	0	0.115476
2.003	0	-0.098803	0	0.115473
2.004	0	0.074511	0	0.115473
2.005	0	-0.026237	0	0.115473
2.006	0	-0.030481	0	0.115473
2.007	0	0.077389	0	0.115473
2.008	0	-0.099389	0	0.115473
2.009	0	0.0894	0	0.115473
2.01	0	-0.050637	0	0.115473

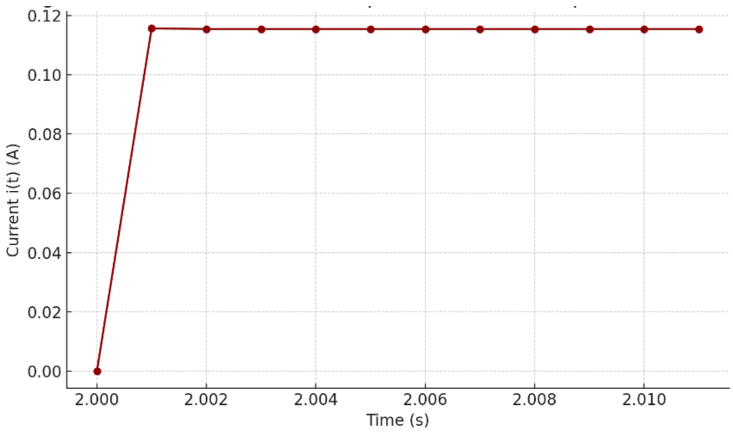

Figure 3: Underdamped Discharge Current in RLC Circuit

The current peaks sharply just after the switch is closed, then quickly stabilizes and the initial rise reflects the energy discharge stored in the capacitor-inductor system. The under-damping condition causes a small oscillation or overshoot, but the circuit is quickly settling from t=2.003 onward, the current remains nearly constant at 0.11547 A, indicating a pseudo steady-state due to minimal damping.

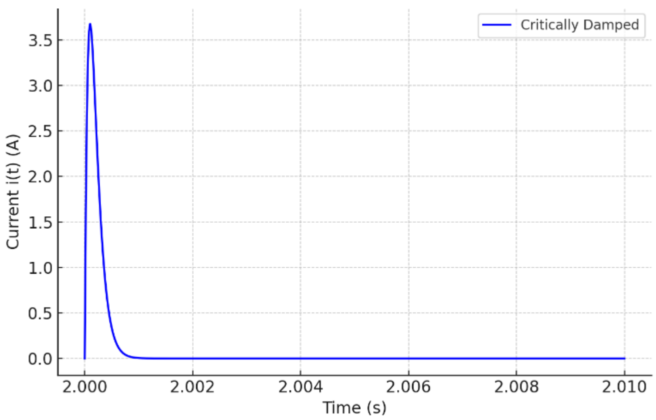

Figure 4: Critically Damped Discharge Current in RLC Circuit

In the critically damped condition, the circuit achieves the fastest non-oscillatory response. The current rapidly rises to a peak and decays smoothly to zero without crossing the time axis. This condition arises when the resistance R is precisely equal to $2\sqrt{L/C}$. The energy stored in the capacitor and inductor is efficiently dissipated through the resistor with no oscillatory delay.

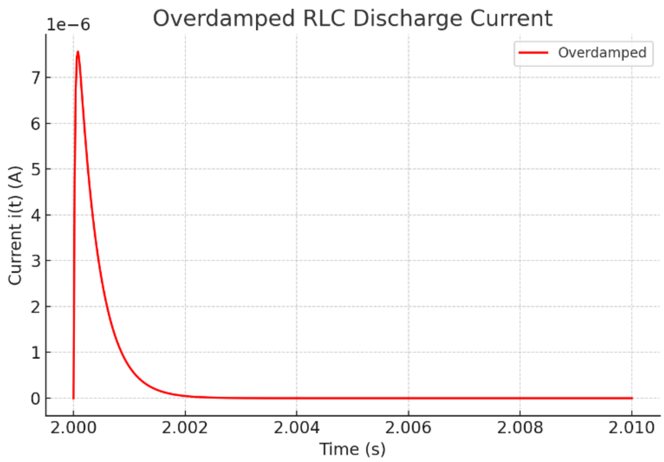

Figure 5**:** Overdamped Discharge Current in RLC Circuit

In the overdamped configuration, where $> \mathbf{2}\sqrt{\boldsymbol{L}/\boldsymbol{C}}$, the discharge is slow and non-oscillatory. The current increases gently and returns to zero over an extended time period without overshooting. The system's return to equilibrium is highly stable, albeit at the cost of speed.

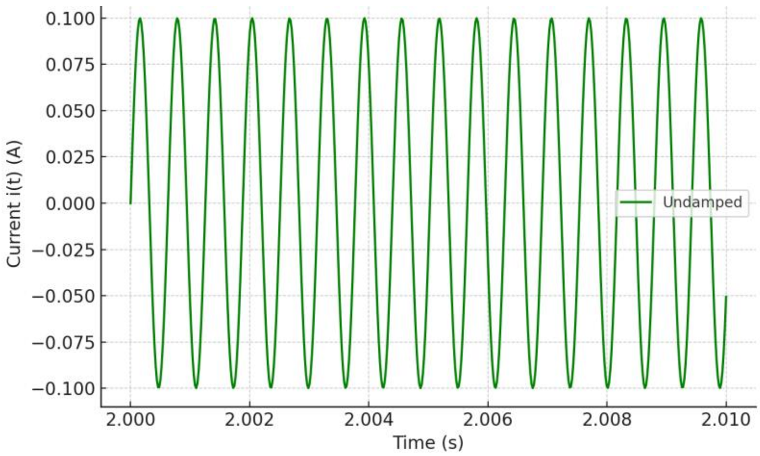

Figure 6**:** Undamped Discharge Current in RLC Circuit

The undamped case represents an idealized scenario where $\boldsymbol{R} = \mathbf{0}$, meaning no energy is dissipated as heat. As a result, the current oscillates sinusoidally with constant amplitude and no decay. While physically unrealistic due to inevitable resistive losses in real components, it serves as a benchmark for resonance-based applications.

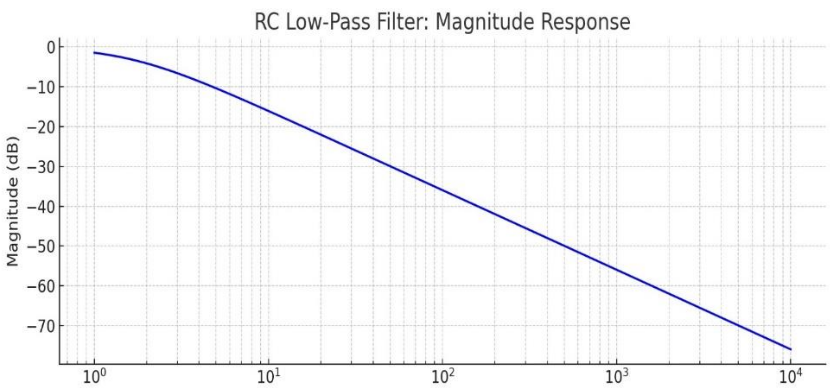

Figure 7: Bode Magnitude Response plot for RC Low-Pass Filter

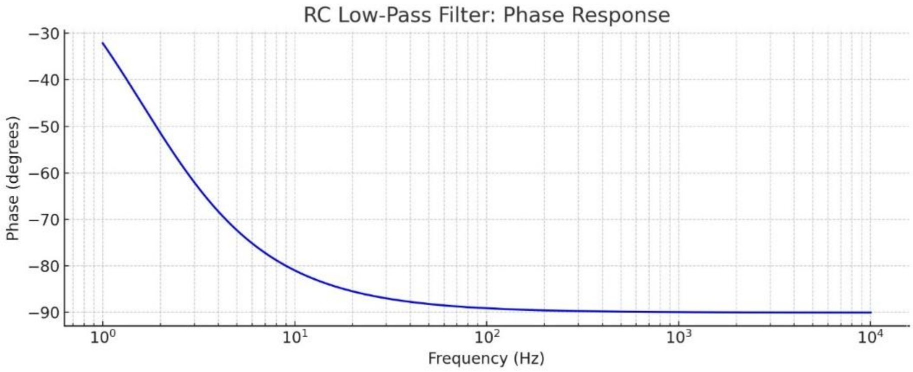

Figure 8: Bode Phase Response plot for RC Low-Pass Filter

The RC circuit, configured as a low-pass filter, demonstrates a classic first-order response. It passes low-frequency signals with minimal attenuation and gradually attenuates higher-frequency components beyond its cutoff frequency. The magnitude response shows a smooth –20 dB/decade roll-off after the cutoff point, and the phase shifts from 0° toward –90° as frequency increases. This makes the RC filter effective for noise suppression, analog signal conditioning, and anti-aliasing in data acquisition systems.

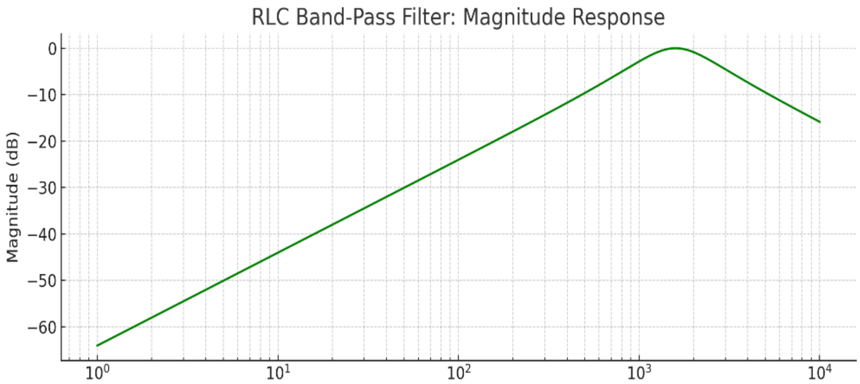

Figure 9: Bode Magnitude Response Plot for RLC Band-Pass Filter

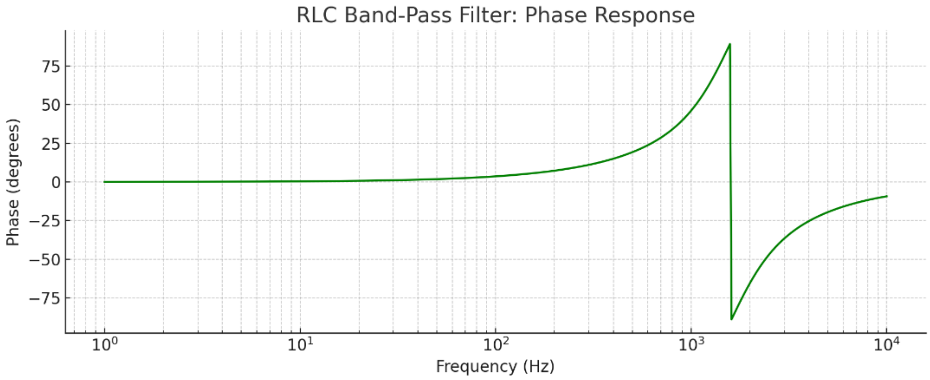

Figure 10: Bode Phase Response Plot for RLC Band-Pass Filter

In contrast, the RLC circuit, acting as a band-pass filter, exhibits a second-order resonant behavior. It allows signals within a narrow frequency band centered at the resonant frequency to pass while attenuating both lower and higher frequencies. The magnitude response peaks sharply at resonance, and the phase shifts rapidly from 0° to 180°, reflecting the circuit's resonant and reactive nature. This property makes the RLC filter well-suited for frequency selection, radio communications, and signal tuning applications.

Table 2: Pole-Zero Map of RC and RLC Circuit Behaviors

Circuit	Poles	Zeros	Behavior
RC Low-Pass	$S = -10$	None	Stable, non-oscillatory
RLC Band-Pass	$S = -5000 \pm j8660$	$S = 0$	Resonant, underdamped

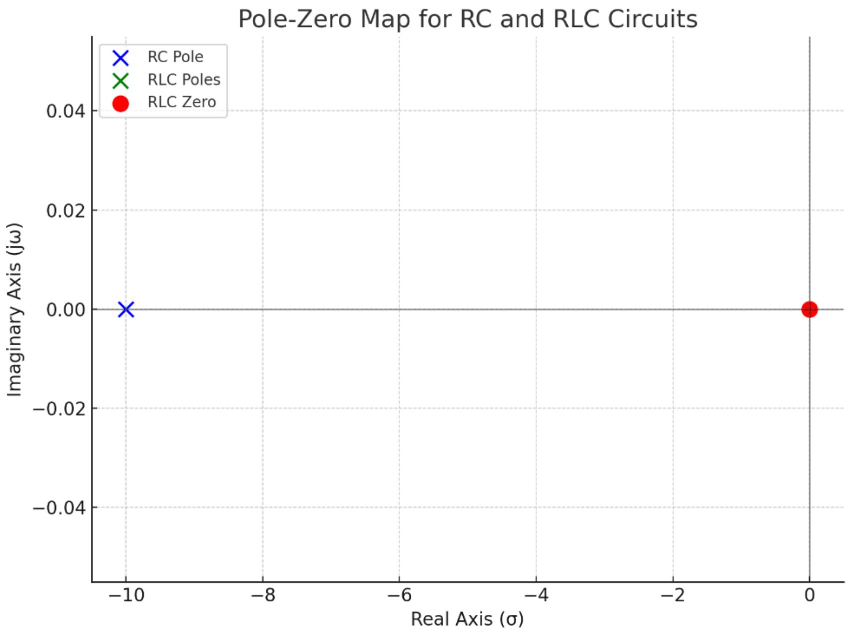

Figure 11: Pole-Zero Map for RC and RLC Circuit

One real negative pole means a stable, exponentially decaying system, no zeros and two complex conjugate poles mean underdamped system, oscillatory behavior.

CONCLUSION

This study explored the transient and frequency response behavior of modified RC and RLC circuits subjected to a Heaviside step function using analytical methods and MATLAB simulations. Through the application of Laplace transforms, the time-domain behavior of each circuit configuration was successfully modeled, capturing the effects of component values on damping, energy dissipation, and system dynamics. The RC circuit exhibited first-order exponential charging behavior, verifying its role as a fundamental low-pass filter. Conversely, the RLC circuit revealed second-order responses across various damping regimesunderdamped, critically damped, and overdampeddemonstrating its suitability for applications requiring selective frequency filtering and resonance control. The frequency-domain analysis further reinforced these findings. The RC circuit attenuated high-frequency signals beyond its -3 dB cutoff, while the RLC configuration displayed a clear band-pass response centered around its natural resonant frequency. These observations validate the analytical models and emphasize the importance of selecting appropriate circuit parameters to achieve desired performance outcomes in electrical design. The results highlight the effectiveness of combining symbolic modeling through Laplace techniques with simulation-based tools like MATLAB to fully understand the behavior of linear circuits under step-input conditions. The Heaviside step function served as a robust representation of switching operations, accurately capturing real-world circuit behavior at activation points.

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