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# A COMPARATIVE STUDY OF STENCIL-BASED METHOD OF LINES FOR SOLVING THE BURGERS-HUXLEY EQUATION

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### Abstract

This study presents a numerical investigation of the Burgers-Huxley equation using the Method of Lines (MoL) with varying stencil points. The Burgers-Huxley equation models complex nonlinear phenomena involving reaction, diffusion, and convection processes and is prominent in biological and physical sciences. Spatial discretization was performed using central finite difference schemes with three, five, seven, nine, and eleven-point stencils to analyze their impact on solution accuracy and convergence. The resulting system of ordinary differential equations was solved using standard time integration methods. Comparative analysis across different stencil sizes and simulation times shows that higher-order stencils significantly reduce the approximation error, with the seven-point stencil offering an optimal balance between computational efficiency and accuracy. Notably, the method achieved an absolute error as low as  $7.63 \times 10^{-10}$  at x = 5.0 and T = 15 using the seven-point stencil. The findings highlight the MoL's robustness and offer guidance on stencil selection for accurate and efficient modeling of nonlinear partial differential equations.

Keywords: Method of Lines, Burgers-Huxley, Finite Difference Method, Numerical Simulation, Stencil Accuracy.

## INTRODUCTION

In numerical analysis, given a square grid in one, two, or three dimensions: A three-point stencil is made up of a point together with its two neighbors, a five-point stencil is made up of a point together with its four neighbors, a seven-point stencil is made up of a point with its six neighbors, a ninepoint stencil is made up of a point with its eight neighbors and an eleven-point so on.

A stencil is a geometric ordering of a nodal group that relates to the point of interest by using a numerical approximation routine. It refers to a formula that can be used to approximate derivatives at a given position using function values and its derivatives sampled at finite intervals around the point of interest. Stencils are the basis for many algorithms to numerically solve Partial Differential Equations (PDEs). Finite central difference stencils are often used to estimate the derivatives of a function represented on a one, two, or three-dimensional grid. The concept of stencil is used when the space under consideration includes a time-dimension. This concept can also be applied to non-regular grids, such as methods that use a grid of volumes.

This approach simplifies the computation of derivatives by structuring the combination of function values at specific locations, a critical aspect of numerical analysis and differential equation solutions. Stencils can be either one-sided or centered, depending on whether values from only one side of a grid point or from both sides are used in the derivative calculation.

The choice of stencil plays a crucial role in determining the numerical accuracy, as it affects how well the numerical scheme captures the solution's behavior. Oberman (2007) observed that wider stencils provide more accurate approximations, though at a higher computational cost. Griffiths and Schiesser (2012) used five-point stencils to solve several PDEs and validated their accuracy through error analysis. Bayona et al., (2019) noted that increasing stencil size smooths the cardinal function and enhances the diagonal dominance of the resulting differentiation matrix. Feng et al., (2022) focused on developing a narrow stencil framework, arguing that existing methods often use wider stencils, which are less efficient and limit high-order methods. Kolar-Pozuna (2024) high-



lighted that when solving PDEs on scattered nodes using the Radial-Basis Function Generated Finite Difference Method, stencil size is a critical parameter, as it influences approximation accuracy, particularly in terms of error oscillation with larger stencils.

The selection of a appropriate stencil influences the accuracy and stability of finite difference approximations, with more complex stencils often providing enhanced results. Common stencils include first-order and second-order approximations, which differ based on the number of grid points involved in the derivative estimation. Stencils can also be extended to multiple dimensions, requiring careful arrangement of values to preserve accuracy in higher-dimensional problems. The use of stencils play a crucial role in areas like Reaction-diffusion process, and other areas that rely on the numerical modeling of continuous phenomena.

Recent studies have focused extensively on the numerical solutions of nonlinear PDEs, particularly Reaction-Diffusion equations. Notable examples include the Burgers-Huxley equation, Burgers-Fisher equation, Fisher-Kolmogorov equation, FitzHugh-Nagumo equation, and Fisher-KPP equation. Researchers such as( Havindra, 2023; Mohammadi, 2013; B. Singh, 2016; and Chandraker 2016) have contributed to this area. Reaction-Diffusion equations serve as models that capture the complex interaction between reaction processes, convection dynamics, and diffusion-driven transport A. Singh *et al.*, . (2024).

Burgers-Huxley equation is a nonlinear partial differential equation that integrates characteristics of both the Burgers' equation and the Huxley equation. This equation models interactions involving reaction mechanisms, convection effects, and diffusion transport, such as nerve impulse propagation in excitable media, population dynamics and the spread of genetic traits or diseases, shock waves and turbulent fluid flow. It finds relevance in various scientific domains, including fluid mechanics, nonlinear wave analysis, and biological pattern formation. The equation's significance lies in its capacity to analyze the combined influence of advection, diffusion, and reaction on system behavior, offering insights into complex dynamical processes.

Numerous computational techniques have been applied to solve the Burgers-Huxley equation, including the Adomian Decomposition Method (Ismail *et al.*, 2004; *Hashim et al.*, 2006) and the Variational Iteration Method (Baitha *et al.*, 2008). Brastos (2011) implemented a fourthorder improved numerical scheme for the generalized Burgers-Huxley equation, while Mohammadi (2013) utilized a B-spline collocation algorithm. Additionally, Dehghan *et al.*, (2012) explored various methods involving interpolation scaling functions and mixed collocation finite difference approaches for the numerical solution of the nonlinear Burgers-Huxley equation. Singh,K *et al.*, (2016) introduced the modified cubic B-spline differential quadrature method (MCB-DQM), a refined numerical approach for solving the generalized Burgers-Huxley equation to model physical phenomena. Singh, A. *et al.*, (2024) proposed a Higher Order collocation method for solving Burgers-Huxley equation, demonstrating computational efficiency and accuracy through comparisons with existing methods.

One of the important techniques to solve a timedependent PDEs is the Method of Lines (MoL). The Method of Lines has formed a broad interest in science and engineering. It discretizes the spatial dimension by using techniques such as finite difference, finite element, and finite volume, spectral or meshless methods. It serves as a general procedure for the solution of partial differential equations (PDEs) Samir, William and Graham (2009). The use of MoL yields a system of first order differential equations with initial value Kazem and Deghan, (2017). This method could be described as a semi analytical procedure and a general way of viewing a partial differential equation as a system of ordinary differential equations (ODEs) Zafarullah (1970).

The partial derivatives with respect to the space variables are discretized to obtain a system of ODEs in the time variable,Gautham and Kaushal (2017). Sadiku and Obiozor(2000) described it as a special finite difference method and noted it to be more effective in terms of accuracy and computational time than the standard finite difference method Sadiku and Obiozor, (2000). For the PDEs to which MOL is applied, the method typically proves to be quite efficient.

Despite these advancements, a critical research gap remains: there is a lack of systematic analysis on the impact of varying stencil sizes within the Method of Lines framework when applied to nonlinear reaction-diffusion equations, particularly the Burgers-Huxley equation. Although numerous studies have optimized algorithms and basis functions, few have explored how different stencil configurations affect the accuracy, stability, convergence behavior, and computational efficiency of the resulting numerical solutions. Existing studies either fix the stencil size or do not compare different stencil choices within the same method and problem context.

This study addresses this gap by presenting a comparative investigation of stencil-based Method of Lines schemes for solving the Burgers-Huxley equation. Specifically, it explores the influence of varying stencil sizes (e.g., three-point, five-point, seven-point, etc.) on the performance of the numerical solution without altering the discretization points. The objective is to identify the optimal stencil size that balances computational efficiency and solution accuracy. This approach provides a more detailed understanding of how stencil configuration affects the solution behavior in timedependent nonlinear PDEs and contributes new insights to the design of efficient and accurate numerical schemes.

# MATERIALS AND METHODS

#### Algorithm of the Method of Lines

To evaluate the numerical solution of nonlinear equations using the Method of Lines, the following steps are considered;

- 1. Discretize the spatial derivatives in PDE
- 2. Formulate the approximate system of ODEs

3. Apply any integration algorithm for the initial value of ODE to compute an approximate numerical solution to the PDE.

# Method of Lines for Solving Partial Differential Equations

If y(x) and its derivatives are single-valued continuous function of x , then by Taylor's expansion we have;

$$y(x+h) = y(x) + hy'(x) + \frac{h^2}{2!}y''(x) + \frac{h^3}{3!}y'''(x) + \cdots$$
(1)
$$y(x-h) = y(x) - hy'(x) + \frac{h^2}{2!}y''(x) - \frac{h^3}{3!}y'''(x) + \cdots$$
(2)

From 
$$(1)$$

$$y'(x) = \frac{1}{h} \left[ y(x+h) - y(x) \right] - \frac{h}{2} y''(x) - \frac{h^2}{3!} y'''(x) + \cdot$$
(3)

$$y'(x) = \frac{1}{h} \left[ y(x+h) - y(x) \right] + O(h)$$
(4)

Equation (4) is the forward difference approximation of y'(x) with an error of order h.

Similarly, from (2), we have

$$y'(x) = \frac{1}{h} \left[ y(x) - y(x-h) \right] + O(h)$$
 (5)

Equation (5) is the backward difference approximation of y'(x).

Subtracting (2) from (1), we have

$$y(x+h) - y(x-h) = \left(y(x) + hy'(x) + \frac{h^2}{2!}y''(x) + \frac{h^3}{3!}y'''(x) + \cdots\right) - \left(y(x) - hy'(x) + \frac{h^2}{2!}y''(x) - \frac{h^3}{3!}y'''(x) + \cdots\right) y(x+h) - y(x-h) = 2hy'(x) y'(x) = \frac{1}{2h} [y(x+h) - y(x-h)] + O(h^2)$$
(6)

Equation (6 ) is the central difference approximation of y'(x).

Adding (1) and (2),

$$y(x+h) + y(x-h) = 2y(x) + 2h^2 y''(x) + \cdots$$
$$y''(x) = \frac{y(x+h) - y(x-h)}{2h^2} \quad (7)$$

Equation (7) is the central difference approximation of y''(x).

Thus, central approximations to higher derivatives can be derived.

Finite Central Difference Approximations for Numerical Differentiation for First and Second Derivatives

Two-point central difference:

$$U_x \approx \frac{U_{i+1} - U_{i-1}}{2h} + \mathcal{O}(h^2) \tag{8}$$

Five-point central difference:

$$U_x \approx \frac{-U_{i-2} + 8U_{i-1} - 8U_{i+1} + U_{i+2}}{12h} \qquad (9)$$

Three-point stencil:

$$U_{xx} \approx \frac{U_{i+1} - 2U_i + U_{i-1}}{h^2}$$
(10)

Five-point stencil:

$$U_{xx} \approx \frac{-U_{i-2} + 16U_{i-1} - 30U_i + 16U_{i+1} - U_{i+2}}{12h^2}$$
(11)

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Seven-point stencil:

$$U_{xx} \approx \frac{-U_{i-3} + 9U_{i-2} - 45U_{i-1} + 100U_i - 45U_{i+1} + 9U_{i+2} - U_{i+3}}{60h^2} \tag{12}$$

Nine-point stencil:

$$U_{xx} \approx \frac{3U_{i-4} - 32U_{i-3} + 168U_{i-2} - 672U_{i-1} + 122U_i - 672U_{i+1} + 168U_{i+2} - 32U_{i+3} + 3U_{i+4}}{840h^2}$$
(13)

#### Eleven-point stencil:

$$U_{xx} \approx \frac{-2U_{i-5} + 25U_{i-4} - 150U_{i-3} + 600U_{i-2} - 2100U_{i-1}}{2520h^2} + \frac{3600U_i - 2100U_{i+1} + 600U_{i+2} - 150U_{i+3} + 25U_{i+4} + 2U_{i+5}}{2520h^2}.$$
 (14)

#### Example

Consider the three-dimensional PDE satisfying the initial condition:

$$U_{t} = U_{xx} + U_{yy} + U_{zz}, \quad x \\ \in (x_{0}, L_{x}], \quad y \\ \in (y_{0}, L_{y}], \quad z \\ \in (z_{0}, L_{z}], \quad t \\ > 0$$
(15)

#### **RESULTS AND DISCUSSIONS**

## Results

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Given the Burgers-Huxley equation (Griffiths and Schiesser, 2012)

$$\frac{\partial u}{\partial t} + u^2 \frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} = \frac{2}{3} u^3 (1 - u^2), \quad t > 0, \quad 0 \le x \le 1$$
(19)

with the initial condition:

Evaluating the diffusion operators  $U_{xx}$ ,  $U_{yy}$ , and  $U_{zz}$  using five-point finite difference gives:

 $U(x, y, z, 0) = f(x, y, z), \quad x \in (x_0, L_x], \quad y \in (y_0, L_y],$ 

$$U'_{i,j,k} = \frac{1}{12h_x^2} \left( -U_{i-2,j,k} + 16U_{i-1,j,k} + 16U_{i+1,j,k} - U_{i+2,j,k} \right) \\ + \frac{1}{12h_y^2} \left( -U_{i,j-2,k} + 16U_{i,j-1,k} + 16U_{i,j+1,k} - U_{i,j+2,k} \right) \\ + \frac{1}{12h_z^2} \left( -U_{i,j,k-2} + 16U_{i,j,k-1} + 16U_{i,j,k+1} - U_{i,j,k+2} \right) \\ - 30 \left( \frac{1}{12h_x^2} + \frac{1}{12h_y^2} + \frac{1}{12h_z^2} \right) U_{i,j,k}$$
(16)

with the initial condition:

$$U_{i,j,k}(0) = f_{i,j,k} = f(x_i, y_j, z_k)$$
(17)

Where:

$$h_x = \frac{L_x - x_0}{M_x}, \quad h_y = \frac{L_y - y_0}{M_y}, \quad h_z = \frac{L_z - z_0}{M_z}$$

This system of ODEs has a solution of the form:

$$U(t) = e^{A_3 t} U(0)$$
 (18)

and the boundary conditions at x = 0.5 and x = 5.

The analytical solution is:

$$u(x,t) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{1}{9}(3x+t)\right)\right]^{1/2}$$
(21)

where:

$$\frac{\partial u}{\partial t}$$
 represents the temporal evolution

$$u^2 \frac{\partial u}{\partial x}$$
 is the nonlinear advection (convection) term

 $u^2$  is a nonlinear velocity for the transport of u

and

$$\frac{\partial^2 u}{\partial x^2}$$
 is the diffusion term

Substituting the finite difference approximations for three point stencil, five point stencil, seven point stencil, nine point stencil and eleven point stencil as shown in Section (2) into Equation (19)

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Spatial Domain	Analytical Solution	3PT Error	5PT Error	7PT Error	9PT Error	11PT Error
0.5	0.899529	6.45E-04	5.85E-06	6.75E-07	5.97E-07	5.86E-07
1.0	0.924889	4.41E-04	1.56E-06	9.40E-07	5.20E-07	4.04E-07
1.5	0.944440	2.42E-04	5.77E-06	8.21E-07	4.00E-07	3.29E-07
2.0	0.959240	8.18E-05	7.02E-06	5.51E-07	2.91E-07	1.93E-07
2.5	0.970284	3.16E-05	6.39E-06	2.95E-07	2.09E-07	1.89E-07
3.0	0.978437	1.02E-04	4.91E-06	1.16E-07	1.50E-07	7.39E-08
3.5	0.984408	1.37E-04	3.31E-06	1.65E-08	1.07E-07	1.19E-07
4.0	0.988754	1.49E-04	1.92E-06	2.92E-08	7.50E-08	1.15E-09
4.5	0.991904	1.46E-04	8.91E-07	4.20E-08	5.10E-08	9.97E-08
5.0	0.995819	1.33E-04	1.81E-07	4.06E-08	3.30E-08	7.83E-09

Table 1: Comparison of absolute error of Burgers-Huxley at different point stencils when T = 5

Table 2: Comparison of absolute error of Burgers-Huxley at different point stencils when T=10

Spatial Domain	Analytical solution	3PT Error	5PT Error	7PT Error	9PT Error	11PT Error
0.5	0.963123	1.76E-04	1.16E-05	2.07E-07	5.63E-08	3.88E-07
1	0.973075	2.15E-04	9.37E-06	4.99E-08	2.15E-08	3.12E-07
1.5	0.980415	2.29E-04	6.89E-06	3.97E-08	8.60E-09	2.40E-07
2	0.985793	2.23E-04	4.64E-06	7.49E-08	7.79E-09	1.88E-07
2.5	0.989716	2.05E-04	2.87E-06	7.81E-08	1.18E-08	1.42E-07
3	0.992566	1.81E-04	1.57E-06	6.75E-08	1.59E-08	1.09E-07
3.5	0.994632	1.55E-04	7.02E-07	5.27E-08	1.79E-08	8.06E-08
4	0.996127	1.29E-04	1.46E-07	3.90E-08	1.73E-08	5.74E-08
4.5	0.997207	1.06E-04	1.72E-07	2.75E-08	1.48E-08	4.13E-08
5	0.997987	8.60E-05	3.40E-07	1.90E-08	1.13E-08	1.25E-08

Table 3: Comparison of absolute error of Burgers-Huxley at different point stencils when T=15

Spatial Domain	Analytical solution	3PT Error	5PT Error	7PT Error	9PT Error	11PT Error
0.5	0.987459	2.94E-04	5.25E-06	2.30E-08	5.79E-08	1.81E-08
1	0.990966	2.53E-04	3.30E-06	2.15E-09	4.05E-08	9.93E-09
1.5	0.993502	2.13E-04	1.91E-06	2.82E-09	3.00E-08	1.38E-08
2	0.995331	1.76E-04	9.73E-07	3.67E-09	2.38E-08	4.52E-09
2.5	0.996648	1.43E-04	3.76E-07	4.27E-09	1.97E-08	1.16E-08
3	0.997595	1.15E-04	1.84E-08	5.01E-09	1.61E-08	5.75E-10
3.5	0.998275	9.12E-05	1.77E-07	5.07E-09	1.25E-08	7.67E-09
4	0.998763	7.18E-05	2.70E-07	4.28E-09	8.96E-09	7.19E-09
4.5	0.999113	5.61E-05	2.97E-07	2.59E-09	5.89E-09	4.44E-09
5	0.999364	4.36E-05	2.91E-07	7.63E-10	3.72E-09	1.94E-08

and solving numerically over a range of spatial domains and time intervals (T=5,10,15), the following tabulated results are obtained at different times



Figure 1: Convergence analysis showing absolute error at x = 5.0 for different stencil sizes and time levels T = 5, 10, 15.

#### Discussion

The results presented in the table analyze the accuracy of various point stencils (3-point, 5-point, 7-point, 9-point, and 11-point) for solving the Burgers-Huxley equation over a range of spatial domains and time intervals (T = 5, 10, 15). The results indicate that errors generally decrease with increased stencil points from 3 point to 7 point across spatial domains (0.5 to 5). At T=5,. However, error values tend to fluctuate at stencils higher than 7 point. Errors at each stencil compared across time T=5, 10, and 15 indicates that higher stencil points tend to stabilize the error reduction over time, with notable improvement in accuracy at longer time intervals for higher stencils. For instance, while the 3-point stencil at T=15 still shows significant errors, the 11-point stencil achieves near-zero error values.

In terms of Convergence, the trend in decreasing errors as the stencil size increases suggests convergence toward the analytical solution, especially for the 7PT and higher stencils. The error reduction as observed is not linear, this suggests that the method's efficiency at reducing error improves at a decreasing rate as the stencil size increases. This could be due to diminishing returns from higherpoint stencils as the solution approaches the analytical values, especially in smoother regions of the domain.

In terms of efficiency, increasing stencil points enhances accuracy but also likely raises computational costs. The 7-point stencil represents a balanced trade-off between accuracy and computational efficiency, as it achieves significant error reduction without the added complexity of the 7point and other higher stencils. Beyond this point, the method experiences an error plateau, indicating that larger stencils do not yield significant accuracy improvements. This insight is valuable for designing efficient numerical schemes for solving nonlinear PDEs like the Burgers-Huxley equation. The convergent rate analysis presented in Figure 1 demonstrates a consistent improvement in accuracy as stencil size increases, with slight variations in convergence behavior over time. This high convergence rate suggests that increasing stencil size significantly improves accuracy up to a point. The error initially drops rapidly (from 3-point to 7-point stencil), but then exhibits diminishing returns bevond that.

## CONCLUSION

This paper successfully applied the numerical Method of Lines (MOL) to solve the Burgers-Huxley equation across various point stencils at different spatial domains and time intervals. It highlights the critical role of stencil size in the accuracy and convergence of numerical solutions. The study contributes to a deeper understanding of how the MoL can be optimized for Burger's Huxley equation by selecting appropriate stencil sizes. It also validates MoL's capability in handling stability and convergence challenges, making it a versatile tool for solving complex partial differential equations in science and engineering. It establishes that modeling systems with higher stencils can ensure higher precision, especially for long-term simulations. Identifying the seven point stencil as a good balance between accuracy and efficiency can help in designing simulations that are both fast and reliable, important for real-time systems, embedded simulations, or resource-constrained environments.

One of the limitations of this work is that it employs a fixed grid and static stencil sizes, without adapting to local solution behavior, which may limit its effectiveness for problems with sharp gradients or discontinuities. While the method shows strong performance for smooth, nonlinear PDEs like Burgers-Huxley, its applicability to other equations—such as those with stiff terms or non-smooth solutions may require modifications.

Future work could explore adaptive stencil strategies that adjust stencil width based on local error or gradient, as well as mesh refinement to enhance resolution where needed. Hybrid approaches combining MoL with other numerical methods, and performance optimization through parallel computing or Graphic Processing Unit (GPU) acceleration, are also promising. Extending the method to multidimensional PDEs and conducting runtime benchmarking would further improve its generality and practical utility.

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