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ON THE EXISTENCE AND STABILITY OF COLLINEAR EQUILIBRIUM POINTS IN THE PHOTOGRAVITATIONAL CIRCULAR RESTRICTED THREE-BODY

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ABSTRACT

We have studied the positions and stability of collinear equilibrium points in the circular restricted three-body problem for Luyten 726-8 and Achird systems. We observed that the location of the collinear equilibrium points L_i (i=1,2,3) changes positions due to the oblateness and radiating factors for the binary systems under review. The changes in the positions of the collinear equilibrium points does not change the status of the collinear equilibrium points as they remain unstable and unchanged. As the oblateness increases, the region of stability of the collinear equilibrium points decreases. It is found that, the positions of the collinear equilibrium points are greatly affected by the oblateness and radiation factors of both primaries for the aforementioned binary systems. Our study reviewed that, at least one characteristic root has a positive real part and a complex root which in the sense of Lyapunov, the stability of the collinear equilibrium points is unstable for the stated binary systems.

Keywords: Oblateness, Positions, Stability, Radiation factors, Restricted Three-Body Problem

INTRODUCTION

There is still a lot of theoretical, practical, historical, and pedagogical value in the restricted three-body problem (R3BP). Numerous scientific disciplines, including as celestial mechanics, chaos theory, galaxy dynamics, molecular physics, and many others have benefited from the study of this issue. This problem is still a stimulating and active research field that is receiving considerable attention of scientists and astronomers due to its applications in dynamics of the stellar and solar systems, artificial satellites and lunar theory.

Researchers have remained fascinated and intrigued by the restricted three-body problem (R3BP), which is based on the assumptions that the participating bodies are spherical and that their orbits are circular. Under the impact of their mutual gravitational pull, two spherically massive masses (the primaries) move in circular orbits, influencing but not being influenced by the third massless body. In such a system, five co-planar equilibrium points exist; three collinear with the line joining the primary bodies and two form equilateral triangles with respect to the primary bodies. The collinear equilibrium points have been shown to be generally unstable, while the triangular points are conditionally stable by Bhatnagar and Hallan (1979), Kunitsyn (2001), Abdulraheem and Singh (2008), Singh and Begha (2011), Singh and Leke (2014), Singh and Amuda (2016), Singh and Tyokyaa (2017), Hussain *et al.* (2018).

The shape of the bodies, the effects of perturbing forces other than their mutual gravitational attractions, and other factors were taken into consideration in order to generalize the classical problem of the three bodies. In the solar system, planets like Saturn and Jupiter are suitably oblate. It has been noted that the oblateness of the planetary bodies plays a vital role in the investigation of the restricted three-body problem. Hence, the radiation pressure factors and the oblateness of these bodies generate great concern in the study of both the collinear and triangular stability in the restricted problem of three bodies.

The photogravitation effects with radiation pressure factors in the restricted three-body problem were formulated by Radzievskir (1950). In his study, he considered one of the interacting masses as intense emitter of light in the Sun-Planet and a dust particle bodies.

Simmons *et al.,* (1985) considered all values of radiation pressures of both primaries and all values of mass ratios in the study of the existence and linear stability of equilibrium points. Singh and Ishwar (1999) studied the stability of triangular equilibrium points in the generalized photogravitational restricted three-body problem. They concluded that the position and stability of triangular equilibrium points are affected by the radiation pressure factors and oblateness of the primaries.

Hassan *et al.,* (2013) studied the positions and velocity sensitivities at the triangular libration points. They considered the bigger primary as an oblate spheroid. Their result show that the value of the critical mass parameter reduces as a result of the oblateness of the bigger primary and the region of stability decreases with oblateness increase, hence the order of commensurability increases. Recently, Singh and Tyokyaa (2021) considered both primaries as sources of radiation and oblate spheroid in the study of the positions and velocity sensitivities in the study of the positions and velocity sensitivities in the restricted problem of three bodies. Sharma (1982) in his study of linear stability of triangular equilibrium points of the photogravitational restricted three body problem when the more massive primary (Sun) is a source of radiation and an oblate spheroid establishes that the collinear equilibrium points retrograde elliptical periodic orbits around the triangular points in the linear sense, while that of the triangular points have long or short-periodic retrograde elliptical orbits for the mass parameter.

Numerous researches are carried out on the position and stability of collinear equilibrium points in both circular and elliptic restricted three-body problem. Some affirmed that the collinear libration point remain unstable in the axisymmetric restricted three-body problem with both primaries as sources of radiation (Abouelmagd and El-Shaboury, 2012). Kunitsyn (2001) and Kunitsyn *et al.,* (1985) studied the characteristics of collinear equilibrium points. Their results confirmed that the collinear points are stable under certain conditions. In the case of a fourth-order resonance taking into account the radiation of both primaries, the collinear equilibrium points can be stable in the sense of Lyapunov (Tkhai 2012).

Singh and Leke (2012) observed stable points of collinear equilibrium points with the Einstein's gravitational constant k (kappa). However, the out-of-plane equilibrium points, it remains unstable even with the introduction of the constant (k). Singh and Tyokyaa (2017) investigated the positions and stability of collinear equilibrium points in the elliptic restricted three-body problem with oblateness of the primaries up to second even zonal harmonic. They stated that, the collinear equilibrium points remain unstable for the binary systems: HD188753 and Gliese 667.

Our aim in this study is to establish the positions and velocity sensitivities of collinear equilibrium points in the circular restricted three-body problem with radiating and oblate primaries.

MATERIALS AND METHODS

Equations of Motion

Using dimensionless variables and a barycentric Synodic coordinate system (x, y), the equations of motion for the restricted three-body problem under the effects of oblateness and radiation pressure factors of the primaries as in Singh and Ishwar (1999), can be written as

$$\ddot{x} - 2n\dot{y} = \Omega_x, \qquad \ddot{y} + 2n\dot{x} = \Omega_y \qquad \ddot{z} = \Omega_z \quad (1)$$

with the force function

$$\Omega = \frac{n^2}{2}(x^2 + y^2) + \frac{(1-\mu)q_1}{r_1} + \frac{\mu q_2}{r_2} + \frac{(1-\mu)A_1 q_1}{2{r_1}^3} + \frac{\mu A_2 q_2}{2{r_2}^3}. \quad (2)$$

where

$${r_1}^2 = (x - x_1)^2 + y^2 + z^2, {r_2}^2 = (x - x_2)^2 + y^2 + z^2. \quad (3)$$

The mean motion, n, is given as

$$n^2 = 1 + \frac{3}{2}A_1 + \frac{3}{2}A_2. \quad (4)$$

where m_1 and m_2 are the masses of the bigger and smaller primaries respectively positioned at the points $(x_i, 0), i = 1,2$; where $x_1 = \mu,\ x_2 = -(1-\mu)$ and $0 < \mu = \frac{m_2}{m_1 + m_2} < \frac{1}{2}$ is the mass ratio. r_1 and r_2 are respectively the distances of m_1 *and* m_2 from the infinitesimal body; q_1 and q_2 are their radiation factors; and A_1 and A_2 are their oblateness coefficients of the bigger and smaller primaries respectively.

Locations of Collinear Equilibrium Points

The equilibrium points are those points at which the velocity of the infinitesimal particle is zero. These points are the solutions of the equations:

$\Omega_x = \Omega_y = \Omega_z = 0.$

These equations yield;

$$xn^2 - \frac{(1-\mu)(x-\mu)q_1}{{r_1}^3} - \frac{\mu(x+1-\mu)q_2}{{r_2}^3} - \frac{3(1-\mu)(x-\mu)A_1 q_1}{2{r_1}^5} - \frac{3\mu(x+1-\mu)A_2 q_2}{2{r_2}^5} = 0, \quad (5)$$

$$y[n^2 - \frac{(1-\mu)q_1}{{r_1}^3} - \frac{\mu q_2}{{r_2}^3} - \frac{3(1-\mu)A_1 q_1}{2{r_1}^5} - \frac{3\mu A_2 q_2}{2{r_2}^5}] = 0 \quad (6)$$

$$z\left(\frac{-(1-\mu)q_1}{{r_1}^3} - \frac{\mu q_2}{{r_2}^3} - \frac{3(1-\mu)A_1 q_1}{2{r_1}^5} - \frac{3\mu A_2 q_2}{2{r_2}^5}\right) = 0. \quad (7)$$

To locate collinear equilibrium points on the x-axis, we put $y = z = 0$ in equations (3) and substituting the values of ${r_1}^2$ and ${r_2}^2$ into equation (5) we have;

$$xn^2 - \frac{(1-\mu)(x-x_1)q_1}{(r_1)^3} - \frac{\mu(x-x_2)q_2}{(r_2)^3} - \frac{3(1-\mu)(x-x_1)A_1 q_1}{2(r_1)^5} - \frac{3\mu(x-x_2)A_2 q_2}{2(r_2)^5} = 0. \quad (8)$$

Thus, collinear equilibrium points lie on the line joining the primaries. To obtain their positions on the $x - axis$, we divide the orbital plane into three parts; $x > x_1,\ x_2 < x < x_1$ and $x_2 > x$ with respect to their primaries, given that $y = z = 0$. The three points are considered in Cases I, II and III respectively.

Case 1: Let the collinear point L_1 be on the right side of the bigger primary at a distance ρ from it on the $x - axis$ (i.e $x > x_1$).

where $x_1 = \mu,\ x_2 = \mu - 1$.

Then

$x = x_1 + \rho = \mu + \rho$

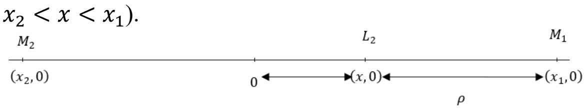
$(x - x_1) = (\mu + \rho) - \mu = \mu + \rho - \mu = \rho$ which implies $r_1 = |\rho|$

$(x - x_2) = (\mu + \rho) - (\mu - 1) = \mu + \rho - \mu + 1 = 1 + \rho$ which implies $r_2 = |1 + \rho|$

Substituting the values of $x, r_1, r_2, (x - x_1)$ *and* $(x - x_2)$ into equation (8), we get

$$2n^2(\mu+\rho)\rho^4(1+\rho)^4 - 2(1-\mu)\rho^2(1+\rho)^4 q_1 - 2\mu\rho^4(1+\rho)^2 q_2 - 3(1-\mu)(1+\rho)^4 A_1 q_1 - -3\mu\rho^4 A_2 q_2 = 0 \quad (9)$$

Case 2: Let the collinear point L_2 be on the lefthand side of the bigger primary at a distance ρ from it on the $x - axis$ (i.e $x_2 < x < x_1$).



where $x_1 = \mu,\ x_2 = \mu - 1$.

Then

$x = x_1 - \rho = \mu - \rho$

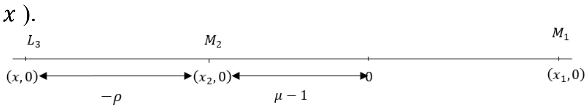
$(x - x_1) = (\mu - \rho) - \mu = \mu - \rho - \mu = -\rho$ which implies $r_1 = |\rho|$

$(x - x_2) = (\mu - \rho) - (\mu - 1) = \mu - \rho - \mu + 1 = 1 - \rho$ which implies $r_2 = |1 - \rho|$

Substituting the values of $x, r_1, r_2, (x - x_1)$ *and* $(x - x_2)$ into equation (8), we obtain

$$2n^2(\mu-\rho)\rho^4(1-\rho)^4 + 2(1-\mu)\rho^2(1-\rho)^4 q_1 - 2\mu\rho^4(1-\rho)^2 q_2 + 3(1-\mu)(1-\rho)^4 A_1 q_1 - -3\mu\rho^4 A_2 q_2 = 0 \quad (10)$$

Case 3: Let the collinear point L_3 be on the left side of the bigger primary at a distance $\rho$from it on the $x - axis$ (i.e $x_2 > x$).



where $x_1 = \mu,\ x_2 = \mu - 1$.

Then

$x = -\rho + x_2 = -\rho + \mu - 1 = \mu - 1 - \rho,$

$(x - x_1) = (\mu - 1 - \rho) - \mu = \mu - 1 - \rho - \mu = -1 - \rho = -(1 + \rho)$ which implies $r_1 = |1 + \rho|$.

$(x - x_2) = (\mu - 1 - \rho) - (\mu - 1) = \mu - 1 - \rho - \mu + 1 = -\rho$ which implies $r_2 = |\rho|$.

Substituting the values of $x, r_1, r_2, (x - x_1)$ *and* $(x - x_2)$ into equation (8), we have

$$2n^2(\mu-1-\rho)\rho^4(1+\rho)^4 + 2(1-\mu)\rho^4(1+\rho)^2 q_1 + 2\mu\rho^2(1+\rho)^4 q_2 + 3(1-\mu)\rho^4 A_1 q_1 + 3\mu(1+\rho)^4 A_2 q_2 = 0 \quad (11)$$

Stability of Collinear Equilibrium Points

Szebehely (1967) stated that, the motion which remains in the small neighbourhood of the collinear equilibrium point after it has been disturbed is termed "stable".

To examine the stability of the collinear equilibrium points, we consider the points lying in $x > x_1$, $x_2 < x < x_1$ and $x_2 > x$ respectively.
Considering the stability of a collinear equilibrium point for which $x > x_1$ we have that

$$r_1 = |x-\mu|,\ r_2 = |x+1-\mu|. \qquad (12)$$

Given the second partial derivatives as;

$$\Omega_{xx} = n^2 - \frac{(1-\mu)q_1}{r_1{}^3} - \frac{\mu q_2}{r_2{}^3} - \frac{3(1-\mu)A_1q_1}{2r_1{}^5} - \frac{3\mu A_2 q_2}{2r_2{}^5} + \frac{3(1-\mu)(x-\mu)^2 q_1}{r_1{}^5} + \frac{3\mu(x+1-\mu)^2 q_2}{r_2{}^5} + \frac{15(1-\mu)(x-\mu)^2 A_1 q_1}{2r_1{}^7} + \frac{15\mu(x+1-\mu)^2 A_2 q_2}{2r_2{}^7}, \qquad (13)$$

$$\Omega_{yy} = \left(n^2 - \frac{(1-\mu)q_1}{r_1{}^3} - \frac{\mu q_2}{r_2{}^3} - \frac{3(1-\mu)A_1q_1}{2r_1{}^5} - \frac{3\mu A_2 q_2}{2r_2{}^5}\right) + y^2\left(\frac{3(1-\mu)q_1}{r_1{}^5} + \frac{3\mu q_2}{r_2{}^5} + \frac{15(1-\mu)A_1q_1}{2r_1{}^7} + \frac{15\mu A_2 q_2}{2r_2{}^7}\right), \qquad (14)$$

$$\Omega_{xy} = y\left[\frac{3(1-\mu)(x-\mu)q_1}{r_1{}^5} + \frac{3\mu(x+1-\mu)q_2}{r_2{}^5} + \frac{15(1-\mu)(x-\mu)A_1q_1}{2r_1{}^7} + \frac{15\mu(x+1-\mu)A_2q_2}{2r_2{}^7}\right]. \qquad (15)$$

Substituting equation (12) into equation (13), we have

$$\Omega_{xx}^0 = n^2 + \frac{2(1-\mu)q_1}{|x-\mu|^3} + \frac{2\mu q_2}{|x+1-\mu|^3} + \frac{6(1-\mu)A_1q_1}{|x-\mu|^5} + \frac{6\mu A_2q_2}{|x+1-\mu|^3} > 0. \qquad (16)$$

From equation (8) with $r_1 = (x - x_1)$ and $r_2 = (x - x_2)$

$$\frac{(1-\mu)q_1}{r_1{}^2} = xn^2 - \frac{\mu q_2}{r_2{}^2} - \frac{3(1-\mu)A_1q_1}{2r_1{}^4} - \frac{3\mu A_2q_2}{2r_2{}^4}. \qquad 17)$$

Substituting equation (17) into equation (14) with $y = z = 0$ we obtain

$$\Omega_{yy} = n^2\left(1 - \frac{x}{r_1}\right) + \frac{\mu q_2}{r_1.r_2^2} + \frac{3\mu A_2q_2}{2r_1.r_2^4} - \frac{\mu q_2}{r_2{}^3} - \frac{3\mu A_2q_2}{2r_2{}^5}. \qquad (18)$$

Given that $n^2 = 1 + \frac{3}{2}(A_1 + A_2)$, in equation (4) and $x = r_1 + \mu$ from equation (12), we substitute the values of n^2 and x in equation (18) and by the virtue of $\mu < \frac{1}{2},\ A_i, q_i \ll 1, r_1 > 1, r_2 < 1$ where $i = 1,2$. we have

$$\Omega_{yy}^0 = \frac{\mu}{r_1}\left(1 + \frac{3A_1}{2} + \frac{3A_2}{2}\right) + \frac{\mu q_2}{r_1.r_2^2}\left(1 + \frac{3A_2}{r_2^2}\right) - \frac{\mu q_2}{r_2^3}\left(1 + \frac{3A_2}{2r_2^2}\right) < 0. \qquad (19)$$

Now, from equation (15) since $y = 0$ we have

$$\Omega_{xy}^0 = 0. \qquad (20)$$

Likewise, for the collinear equilibrium points lying in the interval $(x_2 < x < x_1)$ and $(x_2 > x)$ respectively with respect to their primaries, given that $y = z = 0$, we have $\Omega_{xx}^0 > 0$, $\Omega_{yy}^0 < 0$ and $\Omega_{xy}^0 = 0$.
We consider the characteristic equation of the system given below by Singh and Tyokyaa (2021);

$$\lambda^4 - \left(\Omega_{xx}^0 + \Omega_{yy}^0 - 4n^2\right)\lambda^2 + \Omega_{xx}^0\Omega_{yy}^0 - \left(\Omega_{xy}^0\right)^2 = 0 \qquad (21)$$

Since, $\Omega_{xx}^0\Omega_{yy}^0 - \left(\Omega_{xy}^0\right)^2 < 0$ in equation (21), its discriminant is positive and the roots can be expressed as $\lambda_{1,2} = \pm a$ and $\lambda_{3,4} = \pm ib$ where a and b are real. This confirms that, the motion in the neighbourhood of the collinear equilibrium points is unstable since it is not bounded.

Numerical Applications

The collinear equilibrium points denoted by L_1, L_2, L_3 are evidenced by cases I, II and II respectively. Using Equations (9), (10) and (11), for various oblateness (A_1, A_2), mass ratio (μ), radiation factors (q_1, q_2), mean motion (n), we compute numerically using MATHEMATICA software, the positions of the collinear equilibrium points as given in tables…. to show the effects of the aforementioned parameters for the systems: Luyten 726-8 and Achird.

Table 1: Binary Systems Data for Luyten 726-8 and Achird

Binary system	Masses ($M_\odot$)		Luminosity ($L_\odot$)		Spectral Type (v)
	M_1	M_2	L_1	L_2	
Luyten 726-8	0.1	0.1	6.0×10^{-5}	6.0×10^{-5}	$M_{5.5}/M_6$
Achird	0.95	0.62	1.29	0.06	G_0/K_7

Table 2: Dimensionless Data For the Binary Systems Luyten 726-8 and Achird

Binary system	Mass ratio (μ)	Radiation factors	
		q_1	q_2
Luyten 726-8	0.5	0.999998	0.99999
Achird	0.3949	0.9971	0.9997

Table 3: Effects OF Oblateness AND Radiation Factors on the Positions OF Collinear Equilibrium Points For the Binary Systems: Luyten 726-8 and Achird

Binary system	Mass ratio (μ)	Radiation factors		Oblateness		Collinear equilibrium points positions		
		q_1	q_2	A_1	A_2	L_1	L_2	L_3
Luyten 726-8	0.5	0.999998	0.99999	0.00	0.00	1.198405	−0.000001	−0.198404
				0.015	0.001	1.202632	−0.009076	−0.192659
				0.030	0.002	1.206388	−0.016846	−0.187138
				0.045	0.003	1.209753	−0.023637	−0.181826
				0.060	0.004	1.212792	−0.029663	−0.176714
				0.075	0.005	1.215552	−0.035072	−0.17179
				0.090	0.006	1.218072	−0.039972	−0.167045
				0.105	0.007	1.220386	−0.044444	−0.162470
Achird	0.3949	0.9971	0.9997	0.00	0.00	1.264425	−0.148571	−0.232106
				0.015	0.001	1.267379	−0.156206	−0.226908
				0.030	0.002	1.270040	−0.162847	−0.221922
				0.045	0.003	1.272451	−0.168714	−0.217137

0.060	0.004	1.274649	−0.173959	−0.212542
0.075	0.005	1.276662	−0.178690	−0.208126
0.090	0.006	1.278513	−0.182992	−0.203880
0.105	0.007	1.280222	−0.186928	−0.199794

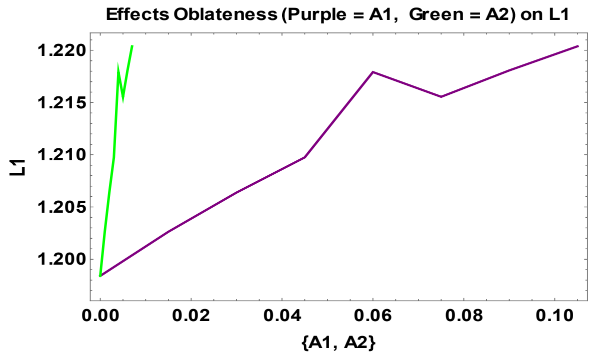


Figure 1: Effects of Oblateness on L_1 for **Luyten 726-8** System with $\mu = 0.5, q_1 = 0.999998, q_1 = 0.99999$

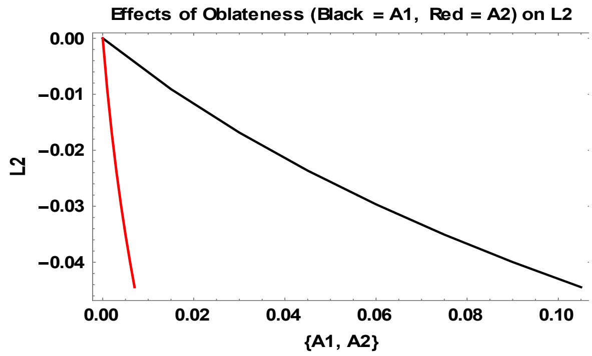


Figure 2: Effects of Oblateness on L_2 for **Luyten 726-8** System with $\mu = 0.5, q_1 = 0.999998, q_1 = 0.99999$

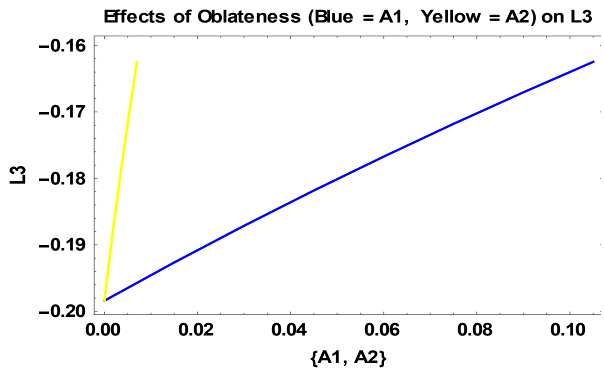


Figure 3: Effects of Oblateness on L_3 for **Luyten 726-8** System with $\mu = 0.5, q_1 = 0.999998, q_1 = 0.99999$

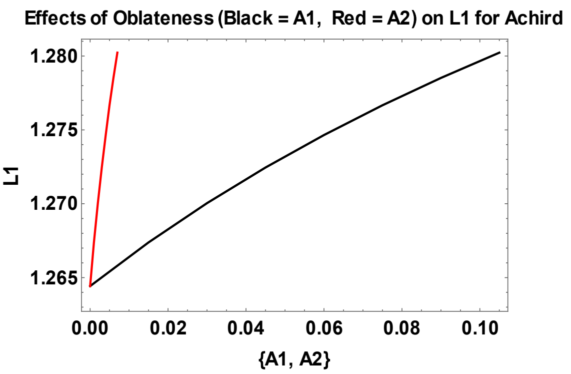


Figure 4: Effects of Oblateness on L_1 for **Achird** System with $\mu = 0.3949, q_1 = 0.9971, q_1 = 0.9997$

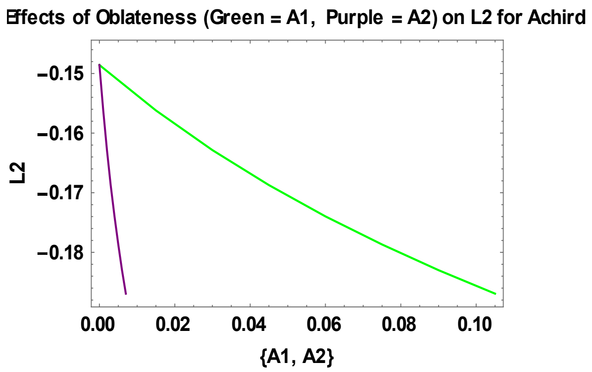


Figure 5: Effects of Oblateness on L_2 for **Achird** System with $\mu = 0.3949, q_1 = 0.9971, q_1 = 0.9997$

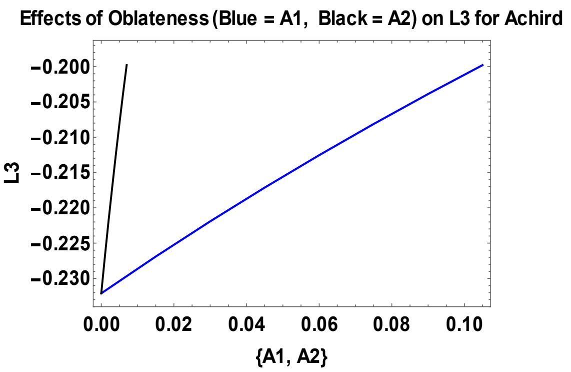


Figure 6: Effects of Oblateness on L_3 for **Achird** System with $\mu = 0.3949, q_1 = 0.9971, q_1 = 0.9997$

Table 4: The Characteristic Roots $(\lambda_{1,2};\ \lambda_{3,4})$ of Collinear Points for the Binary System Luyten 726-8.

Oblateness		Location	Characteristic Roots	
A_1	A_2	L_1	$\lambda_{1,2}$	$\lambda_{3,4}$
0.00	0.00	1.198405	$-1.09313 \pm 0.833138i$	$1.09313 \pm 0.833138i$
0.015	0.001	1.202632	$-1.12439 \pm 0.830387i$	$1.12439 \pm 0.830387i$
0.030	0.002	1.206388	$-1.1538 \pm 0.828066i$	$1.1538 \pm 0.828066i$
0.045	0.003	1.209753	$-1.18168 \pm 0.826099i$	$1.18168 \pm 0.826099i$
0.060	0.004	1.212792	$-1.20829 \pm 0.824429i$	$1.20829 \pm 0.824429i$
0.075	0.005	1.215552	$-1.23381 \pm 0.823008i$	$1.23381 \pm 0.823008i$
0.090	0.006	1.218072	$-1.25841 \pm 0.821799i$	$1.25841 \pm 0.821799i$
0.105	0.007	1.220386	$-1.28217 \pm 0.820777i$	$1.28217 \pm 0.820777i$
A_1	A_2	L_2	$\lambda_{1,2}$	$\lambda_{3,4}$

$\mathbf{0.00}$	0.00	-0.000001	$-1.83952 \pm 0.295922i$	$1.83952 \pm 0.295922i$
$\mathbf{0.015}$	0.001	-0.009076	± 2.1041	± 1.72625
$\mathbf{0.030}$	0.002	-0.016846	± 2.35723	± 1.60059
$\mathbf{0.045}$	0.003	-0.023637	± 2.52329	± 1.54583
$\mathbf{0.060}$	0.004	-0.029663	± 2.65502	± 1.51376
$\mathbf{0.075}$	0.005	-0.035072	± 2.76647	± 1.4932
$\mathbf{0.090}$	0.006	-0.039972	± 2.86417	± 1.47956
$\mathbf{0.105}$	0.007	-0.044444	± 2.95185	± 1.4705
$\boldsymbol{A_1}$	$\boldsymbol{A_2}$	$\boldsymbol{L_3}$	$\boldsymbol{\lambda_{1,2}}$	$\boldsymbol{\lambda_{3,4}}$
$\mathbf{0.00}$	0.00	-0.198404	$-1.09313 \pm 0.833137i$	$1.09313 \pm 0.833137i$
$\mathbf{0.015}$	0.001	-0.192659	$-1.1535 \pm 0.820698i$	$1.1535 \pm 0.820698i$
$\mathbf{0.030}$	0.002	-0.187138	$-1.21389 \pm 0.805638i$	$1.21389 \pm 0.805638i$
$\mathbf{0.045}$	0.003	-0.181826	$-1.27434 \pm 0.787739i$	$1.27434 \pm 0.787739i$
$\mathbf{0.060}$	0.004	-0.176714	$-1.33483 \pm 0.766743i$	$1.33483 \pm 0.766743i$
$\mathbf{0.075}$	0.005	-0.17179	$-1.39536 \pm 0.742327i$	$1.39536 \pm 0.742327i$
$\mathbf{0.090}$	0.006	-0.167045	$-1.45594 \pm 0.714087i$	$1.45594 \pm 0.714087i$
$\mathbf{0.105}$	0.007	-0.16247	$-1.51656 \pm 0.681496i$	$1.51656 \pm 0.681496i$

Table 5: The characteristic roots $(\lambda_{1,2};\ \lambda_{3,4})$ of collinear points for the binary system Achird

Oblateness		Location	Characteristic roots	
$\boldsymbol{A_1}$	$\boldsymbol{A_2}$	$\boldsymbol{L_1}$	$\boldsymbol{\lambda_{1,2}}$	$\boldsymbol{\lambda_{3,4}}$
$\mathbf{0.00}$	0.00	1.264425	$-0.93813 \pm 0.80759i$	$0.93813 \pm 0.80759i$
$\mathbf{0.015}$	0.001	1.267379	$-0.965976 \pm 0.806348i$	$0.965976 \pm 0.806348i$
$\mathbf{0.030}$	0.002	1.27004	$-0.992429 \pm 0.80535i$	$0.992429 \pm 0.80535i$
$\mathbf{0.045}$	0.003	1.272451	$-1.0177 \pm 0.804556i$	$1.0177 \pm 0.804556i$
$\mathbf{0.060}$	0.004	1.274649	$-1.04195 \pm 0.803939i$	$1.04195 \pm 0.803939i$
$\mathbf{0.075}$	0.005	1.276662	$-1.06532 \pm 0.803471i$	$1.06532 \pm 0.803471i$
$\mathbf{0.090}$	0.006	1.278513	$-1.08789 \pm 0.803133i$	$1.08789 \pm 0.803133i$
$\mathbf{0.105}$	0.007	1.280222	$-1.10977 \pm 0.802908i$	$1.10977 \pm 0.802908i$
$\boldsymbol{A_1}$	$\boldsymbol{A_2}$	$\boldsymbol{L_2}$	$\boldsymbol{\lambda_{1,2}}$	$\boldsymbol{\lambda_{3,4}}$
$\mathbf{0.00}$	0.00	-0.148571	$-1.68359 \pm 0.0794827i$	$1.68359 \pm 0.0794827i$
$\mathbf{0.015}$	0.001	-0.156206	± 2.07113	± 1.43306
$\mathbf{0.030}$	0.002	-0.162847	± 2.25762	± 1.36576
$\mathbf{0.045}$	0.003	-0.168714	± 2.40109	± 1.3287
$\mathbf{0.060}$	0.004	-0.173959	± 2.52119	± 1.30542
$\mathbf{0.075}$	0.005	-0.17869	± 2.62604	± 1.28999
$\mathbf{0.090}$	0.006	-0.182992	± 2.71991	± 1.2796
$\mathbf{0.105}$	0.007	-0.186928	± 2.80549	± 1.27267
$\boldsymbol{A_1}$	$\boldsymbol{A_2}$	$\boldsymbol{L_3}$	$\boldsymbol{\lambda_{1,2}}$	$\boldsymbol{\lambda_{3,4}}$
$\mathbf{0.00}$	0.00	-0.232106	$-1.34149 \pm 0.610147i$	$1.34149 \pm 0.610147i$
$\mathbf{0.015}$	0.001	-0.226908	$-1.43032 \pm 0.536497i$	$1.43032 \pm 0.536497i$
$\mathbf{0.030}$	0.002	-0.221922	$-1.5185 \pm 0.440182i$	$1.5185 \pm 0.440182i$
$\mathbf{0.045}$	0.003	-0.217137	$-1.60617 \pm 0.299887i$	$1.60617 \pm 0.299887i$
$\mathbf{0.060}$	0.004	-0.212542	± 1.84763	± 1.53922
$\mathbf{0.075}$	0.005	-0.208126	± 2.16437	± 1.39628
$\mathbf{0.090}$	0.006	-0.20388	± 2.39728	± 1.33656
$\mathbf{0.105}$	0.007	-0.199794	± 2.60537	± 1.30115

Discussion

We have studied the stability of collinear equilibrium points in the circular restricted three-body problem with radiating and oblate primaries. Analytical solutions are drawn from equations (9), (10), (11), (16), (19), (20) and (21). Using the software MATHEMATICA, we computed numerical values from equations (9), (10), (11) and (21) which are presented in Tables 3-5 and Figures 1-6.

Equation (9, 10, 11) indicate that, the collinear equilibrium points are affected by oblateness and the radiation pressure factors of the primaries. This is confirmed in the locations of the collinear equilibrium points of the systems: Luyten 726-8 and Achird as witnessed in table 3.

As represented in Table 3, the positions of collinear equilibrium points are affected by the oblateness of the primaries, radiation pressure factors for the binary systems: Luyten 726-8 and Achird. Our study agrees with the result of Kumar and Ishwar (2011), Singh and Umar (2013), Singh and Tyokyaa (2017) in the absence of eccentricity of the orbits and semi-major axis. As witnessed in Table 3 and Figures 1-6, the effects of the perturbed parameters on the positions of the collinear equilibrium points do not change its non-uniform movement. As the oblateness increases, the collinear equilibrium points L_1 and L_2 move away from the origin while the collinear equilibrium point L_3 move closer to the origin for both systems.

CONCLUSION

The stability study of collinear equilibrium points in the circular restricted three-body problem with radiating and oblate primaries have been carried out in this research for the binary systems: Luyten 726-8 and Achird. Analytical and numerical solutions are clearly outlined and computed respectively.

We observed that the location of the collinear equilibrium points $L_i (i = 1, 2, 3)$ changes positions due to the oblateness and radiating factors under review. The changes in the positions of the collinear equilibrium points does not change the status of the collinear equilibrium points as they remain unstable and unchanged.

As the oblateness increases, the region of stability of the collinear points decreases. Tables 4 and 5 depict the effects of the parameters for the aforementioned binary systems. The effects indicate that, the stability of collinear points is unstable for the stated binary systems. This is evidence as at least one characteristic root has a positive real part and a complex root. The stability behaviour of this study affirms with those of Singh and Umar (2012, 2013), and Abdulraheem and Singh (2006), Singh and Tyokyaa (2017).

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