FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645 - 2944
Vol. 9 No. 7, July, 2025, pp 288 – 293
DOI: https://doi.org/10.33003/fjs-2025-0907-3706

CONTINUOUS TIME MARKOV MODEL OF KANJI DAM WATER OUTFLOW AS A PANACEA TO FLOODING IN NIGERIA

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ABSTRACT

This paper examines the application of a continuous time Markov model with non-stationary transition probabilities to study the water outflow level of Kainji Dam. The results show that state 2 (Medium water outflow) has the optimal water outflow of about 36%. This consolidates the reality on the phenomenon that the water outflow stays in state 2 (Medium outflow) most of the time and in other states some other time. Also, High water outflow is obtained for about 32%, which shows that flooding is not occur every year in Kainji hydroelectric Dam. These variations of the water outflow directly affect the hydroelectric power generation, periodic floods experienced, and availability of other dam resources. Continuous-time Markov model could be used as a predictive technique for studying the reservoir outflow of the Kainji Hydro Dam. These projections might be useful for the management of the dam resources, readiness and control of periodic floods being experienced in this era in Nigeria.

Keywords: Continuous -Time Markov, Outflow, Kainji Hydro-dam, Dam Resources, Flood

INTRODUCTION

Dam can be natural or man-made and can be constructed to solve a lot of socio-economic problems which include among others irrigation and electricity generation.Several related researches have been made on the application of the Markov model to dam dynamics and other areas.

Markov chain models have been applied to many areas of dam-related problems, and stochastic models have been widely used in reservoir management and health care. Akyuz et al. (2012) examined the application of first and second-order Markov chain models to dry and wet periods of annual stream flow series to produce a stochastic structure of hydrological droughts and discovered that, generally speaking, the second-order Markov (MC2) model produces results that are more in line with simulation results than the first-order (MC1) model.

A continuous-time Markov chain (CTMC) is coupled with a Bayesian networks (BN) model to enable the dynamic assessment of dam failure risk. Ahmed et al. (2021) utilized continuous-time Markov chain (CTMC) to describe the transition of the system variables over their various states, while BN is used to simulate the propagation of uncertainty throughout the system and reflect the correlation among the system variables. The created coupled BN-CTMC model was used to forecast the likelihood of the Daisy Lake Dam in British Columbia, Canada, failing under the uncertainty of reservoir water level, inflow, and wind speed conditions in order to show its applicability. By (1) accurately representing the propagation of hydrological (such as inflow and reservoir water level) and meteorological (such as wind speed) variable uncertainties through dam system dynamical processes, (2) accurately quantifying dam failure risk under various operational conditions and failure scenarios, (3) accurately defining the critical periods of dam system operational safety, and (4) offering a thorough understanding of the relationships between the failure of the dam system and associated variables over time, the developed BN-CTMC modeling approach can help in the development of dependable dam operation schemes and risk mitigation strategies.

The structure of reservoir dams has always drawn attention from all across the world and is essential for protecting property and public lives. Reservoir dam structure (RDS) safety risks still need to be evaluated methodically, though. This study creates an evaluation index system and offers a comprehensive framework for assessing the safety risk of RDS. A risk assessment model is created based on the cloud and Dempster-Shafer (D-S) evidence theories. The model's validity is proved through an empirical investigation of the XY reservoir project. This study supports the development of industry standards and helps managers make decisions by providing them with useful answers and theoretical insights (Dingying et al., 2024).

The three-state Markov model for reservoir elevation has been examined in both discrete and continuous time. In the model, the impact of the changing season conditions was also examined. The model was used to the analysis of the dam data. The outcome demonstrates how Nigeria's dry and wet seasons affect the reservoir height. According to Mohammed (2013), it was also noted that the reservoir will eventually be at a high elevation of 49% and 56%, respectively, in discrete and continuous time.

Arash and Mohammadi (2009) Used Markov chain approach to generate arrival flow discharges. The yield model was used to determine the ideal of reservoir volume for the dam. The yield model uses the water needed downstream of the dam and the discharges generated by the Markov chain approach. Due to the scarcity of hydrometric data, these data must be generated using an appropriate technique. This approach ought to commit stochastic parameter properties to memory. The Markov chain model may learn the properties of stochastic parameters and uses correlation between data. The dam's reservoir's arrival flow exhibits stochastic properties. In The yield model optimizes the dam's reservoir volume by taking into account the key year. During the critical year, the reservoir's inflow discharge is the lowest value. The yield model's results are reliable under various circumstances. A simulation was used to determine the number of deficits, or the number of months in which the reservoir's released flow was less than the amount of water needed downstream of the dam. Dynamic programming was used in this study's simulation software. A number of possibilities were taken into account for the simulation program and yield model. In some cases, 60% to 70% of the water needed for farms was provided, while the essential consumable water was prepared flawlessly. The Amirkabir dam on Iran's Karaj River was taken into consideration for the assessment of the study.

Paraskevi (2006) investigated a system of three interconnected dams a supply, storage, and capture dam using a discrete-time Markov chain. He came to the conclusion that the system converges to a steady state after modeling it as a discrete time Markov chain with three states: state one represents the discrete amount of water in the supply dam, state two represents a storage dam, and state three represents the capture dam.

The scenario of two connected dams a capture and supply dam with a random input into the capture dam and a regular demand of one unit from the supply dam was examined (Piantadosi, 2004). She solved for the invariant state probability vector of the water held in the dams by modeling a system as a Markov chain and deriving a transition probability matrix with a broad block structure. She went on to decrease the problem's size to one of the supply dam's orders of magnitude using matrix reduction techniques.

A Markov steady-state model of the plant was built to ascertain the probability output of the turbines, and over 200,000 pieces of data regarding the performance of the machines were used to estimate values of the failure and repair rates for each machine. The Kainji hydropower station, which consists of seven turbo-alternators, was the base load supply for the Nigerian power grid for many years. Additionally, the clusters of probability that define the system state due to the different output capacities of the units show that the hydropower plant has not performed to its maximum capacity; further evaluation reveals that 60% of the KT machine units are operating, which is consistent with the observed robustness of the output; additionally, the model not only conforms to observations but reasonably provides a means of studying the effects of various actions that may be taken to improve the performance of hydropower plants (Thomas, 2023). This result demonstrated that Kaplan turbine (KT) 12 is more likely to fail than any other KT unit in the hydropower plant in comparison to the other KT units.

This work aims to develop a continuous time Markov model that might be utilized to quantitatively forecast the dam outflow in light of these uncertainties.

MATERIALS AND METHODS

Study Area and Data Source

The data used in this paper is collected from the Kainji Dam Niger State, Nigeria. Kainji Dam was commissioned in the year 1968 and the dam was built primarily for the production of Hydroelectric power (H E P). The Kainji Lake itself has a surface water level of approximately $1{,}243KM^2$. With a maximum depth of 12.1metres. Since only eight of the dam's twelve turbines have been constructed, the 960 megawatts (MW) (1,290,000 horsepower) of generating capacity that was intended has been reduced to 760 MW (1,020,000 horsepower).

We shall consider the three state model of dam water outflow in continuous time, which will enable us to obtain information about the water outflow of the Dam at any given point in time.

Formulation of the Model

It has been observed that the normal range of water outflow of Kainji dam that can tolerate turbine operation is between $622m^3$and $1{,}400m^3$levels. Any water excess above $1{,}400m^3$ will lead to the opening of spillways for overflows that result in flooding. Periodic flooding affects downstream farming and ecological activities. This water outflow level rises and falls unpredictably every year. It therefore, interests us to model this process using Markov principle.

Let us consider the water outflow of Kainji Dam been modeled by a three state Continuous Time Markov Model thus.

STATE 1: Lower Dam Water Outflow (Water Outflow< $800m^3$)

STATE 2:Medium Dam Water Outflow (Water Outflow between $800m^3$ and$\leq 1400m^3$)

STATE 3: High Dam Water Outflow (Water Outflow> $1400m^3$) Figure 1 below depicts a potential transition between the states.

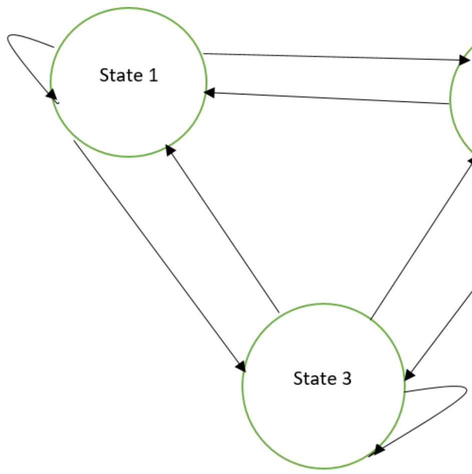

Figure 1: The State Transition Diagram for the Dam Water Outflow

We have thus considered the dam water outflow at a given time to be a random variable X and a collection of this random variable indexed by time parameter 'n' constitutes a stochastic processX_n, That is

$$P(X_{n+1} = j / X_n = i) = P_{ij} \quad (1)$$

Suppose that the dam water outflow, which is currently in state$i$, has a probability of being in state j during the next transition. The following transition probability matrix P describes the change between states.

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \quad (2)$$

Equation (2) is the transition matrix for the dam water outflow. The above model shall be used to study the data recorded from Kainji Dam.

A continuous stochastic process known as the Continuous Time Markov Chain is the transition between states based on an exponential random variable and subsequently to a new state determined by the probabilities of a stochastic matrix. Following Howard (1960), we let a_{ij}represent the transition rate of the water outflow from state$i$ to state$j$,$i \neq j$in a short time interval $(t, t + \Delta t)$ the water outflow currently in state $i$will make a transition to state $j$with probability$a_{ij}\Delta t \quad i \neq j$.If$X_i$ is the state of the process at time$t$, then we have

$$P(X_{i+1} = j/X_i = i) = a_{ij}\Delta t \qquad (3)$$

The probability of two or more state transitions is of order$(\Delta t)^2$ or more and it is negligible if (Δt) is sufficiently small.

Suppose that the transition rate does not change with time ($a_{ij}^{s}$are constants) and

$$a_{ij} = -\sum_{i \neq j} a_{ij} \quad i, j = 1,2,3 \qquad (4)$$

A transition rate matrix A with components $a_{ij}$is how we characterize the process.

Suppose$P_i(t)$is the probability that the water outflow is in the state$i$ at time $t$after the start of the process and let$P_j(t + \Delta t)$be the probability that the water outflow will be in state j a short timeΔtlater. Then

$$P_j(t + \Delta t) = P_j(t)\left[1 - \sum_{i \neq j} a_{ij}\Delta t\right] + \sum_{i \neq j} P_i(t)a_{ij}\Delta t \quad j = 1,2,3 \qquad (5)$$

Since the dam water outflow may have reached j from any other state$i$, equation (5) is produced by multiplying the probabilities and adding the total i that are not equal to$j$. Rearranging the components and inserting (4) into equation (5) provides

$$P_j(t + \Delta t) - P_j(t) = \sum_{i=1}^{3} P_i(t)a_{ij}\Delta t \qquad (6)$$

Thus, we have

$$\frac{dp_j(t)}{dt} = \sum_{i=1}^{3} p_i(t)a_{ij} \quad ij = 1,2,3 \qquad (7)$$

In matrix form, we have

$$\frac{dp(t)}{dt} = p(t)A \qquad (8)$$

Equation (8) is an exact (not approximate) differential equation for $P_{ij}(t)$in

$$\frac{dp_{ij}(t)}{dt} = \sum p_{ik}a_{kj} \qquad (9)$$

Chapman-kolmogorov differential equation) which is a linear; first order differential equation with constant coefficientsa_{ij}^{s}.

The initial condition needs to be provided in order to get the solution of (9), where $P(t)$ is a row vector containing the state probabilities at time$t$. Using (8)'s Laplace transform, we have.

$$p(s) = p(0)(SI - B)^{-1} \qquad (10)$$

Thus,$P(t)$ is obtained as the inverse transform of $P(s)$(Korve 2000).

RESULTS AND DISCUSSION

A summary statistic for 10 years for the monthly dam water outflow of Kainji dam is contained in Table 1.

Table 1: A Summary of Statistics for 10 Years for the Monthly Dam Water Outflow in Cubic Meters of Kainji Dam from 2015 to 2024

Class Interval	State	Frequency
Dam Water Outflow $< 800m^3$151	1	51
Dam Water Outflow between $800m^3$and $\leq 1400m^3$261	2	61
Dam Water Outflow $> 1400m^3$38	3	8
Total		120

Table 1 presents the summary of the monthly water outflow of the Kainji Dam for the period of ten (10) years that is from 2015-2024.

Using Equations (1) and (2), respectively, we provide the following transition count matrix as:

$$M = \begin{bmatrix} 22 & 7 & 2 \\ 9 & 54 & 8 \\ 2 & 8 & 14 \end{bmatrix} \qquad (11)$$

By applying equation (4) to normalize this matrix, we obtain

$$B = \begin{bmatrix} -9 & 7 & 2 \\ 9 & -17 & 8 \\ 2 & 8 & -10 \end{bmatrix} \qquad (12)$$

As a result, the matrix B can be expressed as the expected value of the exponential distribution $\frac{1}{\lambda}$ thus

$$B = \begin{bmatrix} -0.6429 & 0.1429 & 0.5000 \\ 0.1111 & -0.2361 & 0.1250 \\ 0.5000 & 0.1250 & -0.6250 \end{bmatrix} \qquad (13)$$

FFrom equation (10), we have

$$P(s) = \begin{pmatrix} S + 0.6429 & -0.1429 & -0.5000 \\ -0.1111 & S + 0.2361 & -0.1250 \\ -0.5000 & -0.1250 & S + 0.6250 \end{pmatrix}^{-1} \qquad (14)$$

Solving equation (14) by Maple software, and for$t = 0, 1, \ldots, 24$, we obtained the following Tables

Table 2: The Values of the Transition Probabilities for $P_{12}(t)$ $P_{13}(t)$ $P_{23}(t)$ $P_{21}(t)$ *and* $P_{31}(t)$

	$P_{12}(t)$	$P_{13}(t)$	$P_{23}(t)$	$P_{21}(t)$	$P_{31}(t)$
0	0.0000000000	0.0000000000	0.000000000	0.0000000	0.000000
1	0.116268890	0.2837760763	0.1026640337	0.0947138705	0.2825391329
2	0.1935983054	0.3552522655	0.1718875167	0.1618512523	0.3520127210
3	0.2819956620	0.3645512578	0.2191560194	0.2087745458	0.3595591012
4	0.2460648452	0.3580810858	0.2516282694	0.2413609073	0.3517595746
5	0.3067132598	0.349463839	0.2739994910	0.2639243783	0.342189328
6	0.3237530557	0.3421773262	0.2894323560	0.2795265241	0.3342265746
7	0.3355115711	0.3367152807	0.3000854104	0.2903082394	0.322958569
8	0.3436294269	0.3328046465	0.3074411786	0.2977566480	0.3240602551
9	0.3492350483	0.3300592529	0.3125209106	0.3029015791	0.3210900093
10	0.3531062861	0.3281488086	0.3160290835	0.3064551738	0.3190241367
11	0.3557798848	0.3268247366	0.3184519751	0.3089095623	0.3175926741
12	0.3576263967	0.3259087715	0.3201253488	0.3106047300	0.3166025250

13	0.3589016969	0.3252756745	0.3212810741	0.3117755208	0.3159181877
.	.		.		.
.	.		.		.
.	.		.		.
23	0.3616787046	0.3238963684	0.3237977096	0.3143249811	0.3144272969
24	0.3617004574	0.3238855625	0.3238174228	0.3143449515	0.3144156169

Table 3: The Values of the Virtual Transition Probabilities for $P_{32}(t)$ $P_{11}(t)$ $P_{22}(t)$ and $P_{33}(t)$

	$P_{32}(t)$	$P_{11}(t)$	$P_{22}(t)$	$P_{33}(t)$
0	0.0000000	1.00000000	1.00000000	1.0000000000
1	0.1076118075	0.5999670346	0.8026220957	0.6098490596
2	0.1848456947	0.4511494291	0.6662612308	0.4631415843
3	0.2391246458	0.3893838970	0.5720694347	0.4013162530
4	0.2769143142	0.3599232522	0.5070108233	0.3713261112
5	0.3031110951	0.3438229064	0.4620761306	0.3547029721
6	0.3212353624	0.3340696182	0.4310411200	0.3445380630
7	0.3337631051	0.327773648	0.4096063502	0.3379410378
8	0.3424187440	0.3235629267	0.3948021734	0.3335210009
9	0.3483978848	0.3207056988	0.384577104	0.3305121060
10	0.3525277712	0.3187449053	0.3775157427	0.3284480922
11	0.3553802250	0.3173953786	0.3726384626	0.3270271009
12	0.3573503347	0.316468317	0.3692699212	0.3260471403
13	0.358711213	0.3158226286	0.366434051	0.3253707910
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
23	0.361673995	30.3144249270	0.3618773093	0.3238987078
24	0.361697204	90.3144139801	0.3618376256	0.3238871782s

The graph of Table 2 and 3 are presented below

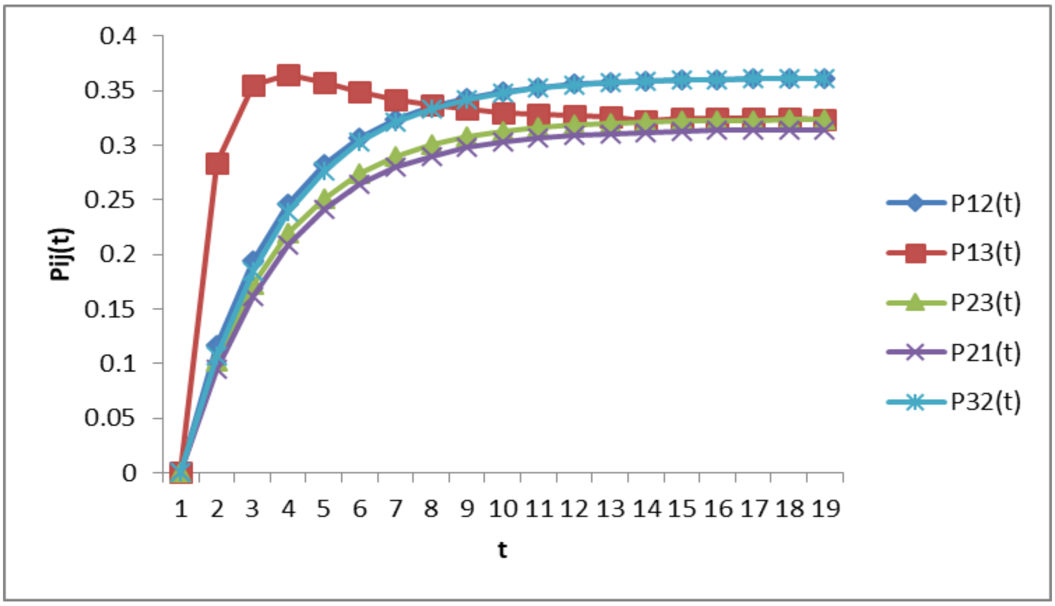

Figure 2: The graph of transition probabilities $P_{12}(t)$, $P_{13}(t)$, $P_{23}(t)$, $P_{21}(t), P_{31}(t)$ and $P_{32}(t)$ for $t =$ 0,1,..24

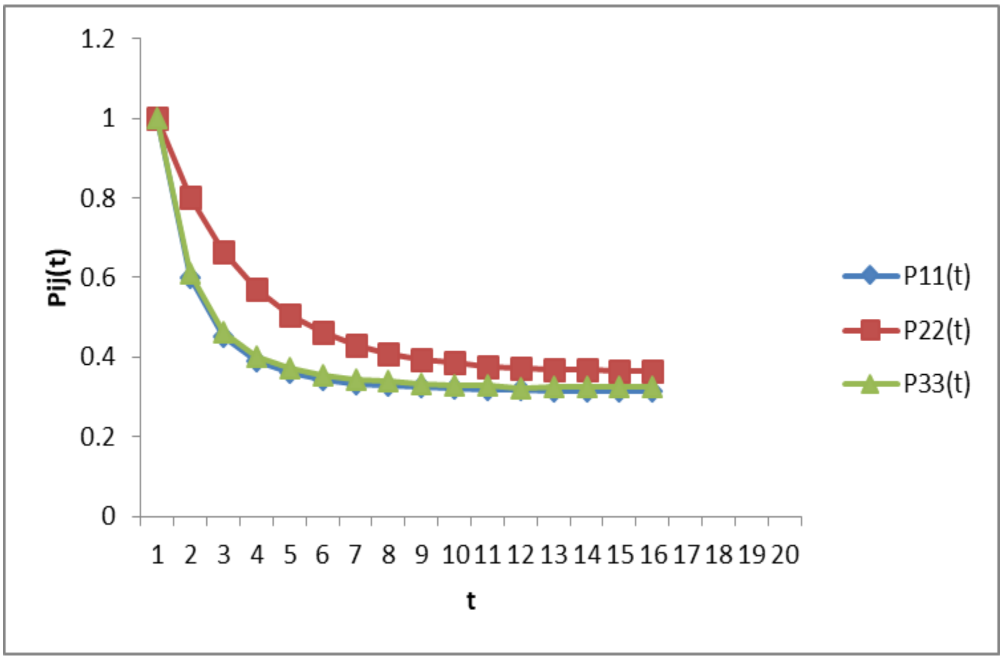

Figure 3: The virtual transition probabilities $P_{11}(t),\ P_{22}(t)\ and\ P_{33}(t)\quad for\ t = 0,1,\dots 24$

Dams and hydrological resources management are critical to power generation in hydropower stations such as Kainji hydroelectric power station. This study dwells on Markov model using continuous time to study the three state water outflow levels of a dam. From the empirical analysis of our collected data on water outflow of Kainji Dam; the results of the model were presented in Tables 2 and 3 and illustrated with Figures 2 and 3 above. The model enables us to determine the values of$P_{12}(t)$,$P_{13}(t)$,$P_{23}(t)$, $P_{21}(t)$, $P_{31}(t)$, $P_{32}(t)$, $P_{11}(t)$, $P_{22}(t)$ and $P_{33}(t)$at any time t. From Table 2 and Figure 2, the result shows that the transition probabilities from state 1 to state 2 and state 3 to state 2 have the highest water outflow of about 0.36 in 24 months. This shows that $P_{12}(t)$and$P_{32}(t)$ has the optimal probabilities of about 0.36 respectively irrespective of the seasons. The outcome shows that, considering the low water outflow this month, a medium water outflow level is anticipated, with an ideal probability of 0.36 for$t = 0,1,\dots,24$. It gives the second higher transition probability by $P_{13}(t)$ and $P_{23}(t)$ with a maximum value of about 0.32 in 24 months. This shows that flooding does not occur every year in Kainji hydroelectric power station. Similarly, $P_{21}(t)$ and $P_{31}(t)$ has a water outflow of about 0.31 for$t = 0,1,\dots,24$. Also from Table 3 and Figure 3 the virtual transition probabilities$P_{11}(t)$,$P_{22}(t)$, and $P_{33}(t)$ are about 0.31, 0.36, and 0.32 respectively for $t = 0,1,\dots,24$.This shows that state 2 (Medium water outflow) has the optimal water outflow. This implies that the water outflow shall be in state 2 (Medium outflow) most of the time and in other states some other time.

In summary, the non-stationary (continuous time) model shows that the optimal water outflow shall be available by about 36% irrespective of the seasons (dry/wet). The model also shows that Kainji hydroelectric Dam do not experience flooding every year. Moreover, the water outflow level with regard to the states may be easily predicted using the continuous time Markov model at any given time. The prediction is the information that could be useful in the management and control of the Dam resources, which includes power supply and generation, and overflows could boost dam stream aquatic and agricultural activities.

CONCLUSION

The Kainji dam's monthly water outflow has been analyzed and predicted using a stochastic model that was developed using the continuous time Markov model principle. The results indicate that there shall be a higher water outflow of about 36% in state 2 (Medium water outflow) water irrespective of the seasons (dry/wet). Conversely, lower water outflow is obtained in state one (1) of about 31% (Lower water outflow). These erratic variations in water outflow have a significant effect on the amount of hydroelectric power produced by the dam as well as the availability of other dam resources like agriculture cultivation and freshwater fishing along the dam basins. Due to water discharge from Kainji Dam, the model could be used as a predictive tool to forecast potential floods that may occur on a regular basis. This prediction can also be used for the management of the dam resources.

REFERENCES

Ahmed Badr, Ahmed Y., Sonia H, and Wael El-D. (2021) Coupled Continuous-Time Markov Chain–Bayesian Network Model for Dam Failure Risk Prediction. Journal of Infrastructure System 27(4) https://doi.org/10.1061/(ASCE)IS. 1943-555X.0000649

Akyuz, D.E, Bayazit.M., & Onoz, B. (2012): Markov Chain Models for Hydrological Drought Characteristics, American Meteorological Society (AMMS) journal, 13(1)25-29

Arash Adib and Ali Reza Mohammad Majd (2009) Optimization of Reservoir Volume by Yield Model and Simulation of it by Dynamic Programming and Markov Chain Model. American-Eurasian J. Agric. & Environ. Sci., 5 (6): 796-803, 2009. ISSN 1818-6769 IDOSI

Bhat, U. N (1984). Elements of Applied Stochastic Processes, New York: John Wiley

Dingying Yang, Jiamei Wu, Zhenxu Guo, Xiaoye Zeng & Qianqian Zhang (2024) Safety risk Assessment of reservoir dam structure: an empirical study in China. Fuzhou

University, Fuzhou 350116, China.2School Content courtesy of Springer Nature, terms of use app

Jain, R.K. (1988). The Use of a Time. Varying Markov Model to Study the Effect of Weather on Asthma. Biomedical Journal, 30, 93-97. Korve, K.N. (2000). A Three State Continuous Time Markov Model for the Asthma Process. (Abacus) the Journal of the Mathematical Association of Nigeria, 207, (2), 33-46.

Mohammed A. (2013): Application of Markov Model to Reservoir Elevation of Shiroro Dam, Unpublished M Tech thesis, Department of Mathematics and Statistics, FUT Minna.

Paraskevi, T. (2006). Calculation of the Steady state Probabilities for a Water Management Problem with Three Connected Dams. University of South Australia.

Piantadosi, J. (2004). Optimal Policies for Storage of Urban Storm Water, Unpublished PhD Thesis, University of South Australia.

Thomas, O. A., Benjamin, J. O., & Mathew, O. O. (2023). A Markov model of a generator Performance at the Kainji Hydro-power Station in Nigeria International Journal of Electrical and Computer Engineering (IJECE) Vol. 13, No. 4, August 2023, pp. 3585~3592 ISSN: 2088-8708, https://doi.org/10.11591/ijece.v13i4.pp3585-3592