FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645 - 2944
Vol. 9 No. 7, July, 2025, pp 126– 132
DOI: https://doi.org/10.33003/fjs-2025-0907-3699

UNIT PROBABILITY DISTRIBUTIONS: A COMPREHENSIVE REVIEW OF MODELS, PROPERTIES, AND APPLICATIONS

*[1]Sule Omeiza Bashiru, [2]Alhaji Modu Isa, [2]Ibrahim Ali, [3]Kingsley Chinedu Arum, [3]Henrietta Ebele Oranye, [3]Tobias Ejiofor Ugah and [4]Nkechi Grace Okoacha

[1]Department of Mathematics and Statistics, Confluence University of Science and Technology, Osara, Kogi State, Nigeria.
[2]Department of Mathematics and Computer Science, Borno State University, Maiduguri, Nigeria
[3]Department of Statistics, University of Nigeria, Nsukka, Nigeria.
[4]Basic Science Unit, School of Science and Technology, Pan-Atlantic University, Nigeria. Lagos, Nigeria.

*Corresponding authors' email: bash140@gmail.com

ABSTRACT
Unit probability distributions defined on the standard interval (0, 1) serve as foundational tools for modeling data constrained within this bounded domain. Such data frequently emerge in disciplines where proportions, rates, and probabilities are analyzed, including economics, finance, hydrology, environmental sciences, biomedical research, and reliability engineering. Traditional models such as the Beta and Kumaraswamy distributions have long provided flexible frameworks for these applications. However, the increasing complexity of real-world phenomena has spurred the development of more versatile and specialized unit distributions. This article presents a comprehensive review of the literature on unit probability distributions, encompassing both classical models and recent innovations. Emphasis is placed on transformation techniques used to generate new families, key analytical properties, and a comparative evaluation of estimation methods. A diverse array of real-world applications is examined, highlighting the practical relevance and empirical performance of modern unit distributions across multiple domains. By synthesizing these developments, the review offers a structured resource to support further methodological advancement and informed model selection for bounded data analysis.

Keywords: Unit probability distributions, Bounded data, Transformation techniques, Flexible models, Parameter estimation, Real-world applications

INTRODUCTION
Modeling bounded data, that is data confined to the unit interval (0, 1), is of vital importance across a wide spectrum of scientific and applied fields. Such data include proportions (such as market share or election turnout), rates (including infection or mortality rates), probabilities, and standardized indices (for instance, climate risk scores or financial stress indices). In these contexts, conventional probability distributions defined on the real line are unsuitable, necessitating the use of specialized models known as unit distributions.

Historically, the Beta distribution and its variants have been the cornerstone of unit data modeling due to their mathematical tractability and remarkable flexibility. The Kumaraswamy distribution, with its closed-form cumulative distribution function, further enriched the toolkit available for such tasks. Nevertheless, as empirical datasets have grown in size and complexity, often exhibiting features such as multimodality, extreme skewness, heavy tails, and diverse hazard rate behaviors, the limitations of these classical models have become more apparent.

To address these challenges, an array of new unit distributions has been proposed, often derived through transformations of unbounded parent distributions or via compounding strategies. These innovations aim to capture intricate patterns observed in real-world data more effectively. Consequently, the field has witnessed an upsurge in research devoted to theoretical advancements, estimation techniques, and empirical validations of these models.

This review systematically explores the evolution of univariate continuous unit probability distributions, with a primary focus on those introduced between 2017 and early 2025. Both classical distributions (such as the Beta and Kumaraswamy) and recent models are considered to provide historical context and demonstrate the trajectory of innovation in the field. Distributions included in this review were selected based on their theoretical novelty, modeling flexibility, and relevance to applied contexts. A targeted literature search was conducted using databases such as Scopus, Web of Science, and Google Scholar, with keywords including "unit distribution," "bounded support," "proportion modeling," and the names of specific proposed models. This review aims to provide a comprehensive and structured synthesis of developments in the area, highlighting key properties, estimation methods, and application domains.

MATERIALS AND METHODS
Key Unit Distributions
Beta Distribution
The Beta distribution, introduced by Johnson et al. (1955), is one of the most widely used unit distributions. The probability density function (PDF) is given by:

$$f(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}; x \in (0,1), \alpha, \beta > 0$$

This distribution is highly flexible, capable of modeling a variety of shapes including symmetric, skewed, and U-shaped distributions, depending on the values of α and β .

Kumarswamy Distribution
Proposed by Kumaraswamy (1980), this two-parameter distribution serves as an alternative to the Beta distribution. It has the advantage of a closed-form cumulative distribution function (CDF), which is convenient for analytical work. The PDF is given by:

$$f(x) = \alpha\beta x^{\alpha-1}(1-x^{\alpha})^{\beta-1}; x \in (0,1), \alpha, \beta > 0$$

The Kumaraswamy distribution has been applied to hydrological and economic datasets.

Musa Type II Distribution
Muhammad (2017) introduced a one-parameter unit distribution known as the Musa Type II distribution. The author showed that the hazard function of the model can exhibit different behaviors depending on the parameter value: a bathtub shape when the parameter is less than 1, and an increasing failure rate when the parameter is greater than or equal to 1. The probability density function (PDF) of the distribution can also display increasing or bathtub-shaped patterns. Several analytical properties were derived, including the quantile function, moments, and order statistics. The parameter of the distribution was estimated using the maximum likelihood estimation (MLE) method. To demonstrate its practical relevance, the model was applied to a dataset from a study on anxiety in a group of 166 normal women (i.e., outside a pathological clinical diagnosis) in Townsville, Queensland, Australia. A comparative analysis was performed with the generalized standard arcsine distribution, and the Musa Type II distribution showed a superior fit based on goodness-of-fit measures. The PDF of the model is given by:
$f(x) = \alpha In2 x^{\alpha-1} e^{x^{\alpha} In2}; x \in (0,1), \alpha > 0.$

Unit Gompertz Distribution
Mazucheli et al. (2019) introduced the Unit Gompertz distribution (UGD) by applying the transformation $x = e^{-y}$ to the classical Gompertz distribution, thereby restricting its support to the unit interval (0,1). The probability density function (PDF) of the model was shown to be capable of exhibiting increasing, unimodal, reversed J-shaped, and positively right-skewed forms. In terms of the hazard function, the distribution can accommodate constant, increasing, and upside-down bathtub shapes. The parameters of the model were estimated using the maximum-likelihood estimation (MLE) method. To demonstrate its practical utility, the model was applied to two real-world datasets, and its performance was compared with the Beta, Kumaraswamy, and McDonald distributions. Based on five widely-used goodness-of-fit statistics, the Unit Gompertz model provided a better fit than the Beta and Kumaraswamy distributions for both datasets. The PDF of the Unit Gompertz distribution is given as:
$f(x) = \alpha\beta x^{-(\beta+1)} \, exp\left[-\alpha\left(x^{-\beta} - 1\right)\right]; x \in (0,1), \alpha, \beta > 0.$

Transmuted Unit Rayleigh Distribution
Korkmaz et al. (2021) proposed the Transmuted Unit Rayleigh distribution by applying the quadratic transmutation scheme to the unit Rayleigh distribution. The resulting model is flexible, with a probability density function (PDF) that can exhibit right- or left-skewed, and unimodal shapes. The quantile function of the distribution was derived in closed form. The study also developed a quantile regression model based on the proposed distribution and obtained the maximum likelihood estimates for the unknown regression parameters. To demonstrate its practical utility, the model was applied to a real dataset linking educational attainment in OECD (Organization for Economic Co-operation and Development) countries with components of the Better Life Index, including life satisfaction, homicide rate, and voter turnout. The results showed that the proposed quantile regression model outperformed several well-known regression models, especially when the unit response variable exhibits skewness and contains outliers.
Its PDF is given as:
$f(x) = \alpha \frac{(-\log x)}{x} e^{-\alpha(-\log x)^2}\left(1 + \beta - 2\beta e^{-\alpha(-\log x)^2}\right); x \in (0,1), \beta \in (-1,1), \lambda > 0.$

Unit Teisser Distribution
Krishna et al. (2022) introduced the Unit Teissier distribution by transforming the Teissier distribution to the unit interval. The resulting model displays right-skewed and unimodal shapes in its probability density function (PDF), while the hazard rate function (HRF) can assume a range of behaviors, including increasing, decreasing, bathtub, and N-shaped patterns. Parameter estimation was carried out using maximum likelihood estimation (MLE), least squares estimation (LSE), weighted least squares estimation (WLSE), and Bayesian techniques. A simulation study was conducted to assess the performance of these methods. The practical usefulness of the distribution was demonstrated through its application to two real-world datasets: maximum flood levels (in million cubic feet per second) for the Susquehanna River at Harrisburg, Pennsylvania, and times between failures of secondary reactor pumps. In both cases, the unit Teissier distribution outperformed competing models based on several model evaluation metrics. The PDF of the distribution is given as:
$f(x) = \theta\left(x^{-\theta} - 1\right)x^{-(\theta+1)} e^{-x^{-\theta}+1}; x \in (0,1), \ \theta > 0.$

Unit XGamma Distribution
Hashmi et al. (2022) developed the Unit XGamma distribution by transforming the Xgamma distribution into a unit distribution using the transformation $X = Y/(1+Y)$. The study showed that the probability density function (PDF) can effectively model skewed data, as evidenced by the shape of its plot, while the hazard function exhibits an increasing failure rate pattern. Two key risk measures—value at risk (VaR) and tail value at risk—were derived to assess its applicability in risk analysis. Other statistical properties, including the moments, order statistics, and moment generating function, were also obtained. The model parameter was estimated using six techniques: the maximum likelihood estimation, ordinary least squares, weighted least squares, Cramér–von Mises method, maximum product of spacings, and Anderson–Darling method. All estimation techniques yielded reliable results. The practical utility of the proposed model was illustrated through its application to a real-life dataset involving monthly water capacity measurements from the Shasta Reservoir in California. When compared with competing models, the unit Xgamma distribution demonstrated superior fitting performance. The PDF is given as:
$f(x) = \left(\frac{\theta^2}{1+\theta}\right)\left[\frac{1}{(1-x)^2}\right]\left[1 + \frac{\theta}{2}\left(\frac{x}{1-x}\right)^2\right] e^{-\theta\left(\frac{x}{1-x}\right)}; x \in (0,1), \theta > 0.$

Marshall–Olkin Reduced Kies Distribution
Afify et al. (2022) introduced a two-parameter unit distribution called the Marshall–Olkin Reduced Kies distribution. This distribution was developed by applying the Marshall–Olkin generator to the reduced Kies distribution. The resulting model is capable of handling both positively and negatively skewed data, making it suitable for a variety of modeling situations. To estimate its parameters, the authors employed eight classical estimation methods, including maximum likelihood, least squares, and maximum product spacing methods. A comprehensive simulation study was conducted to evaluate the performance of these estimators. Based on partial and overall performance metrics, the maximum product spacing estimator was identified as the best-performing estimation method. To demonstrate the model's practical usefulness, it was applied to two COVID-19 datasets from Spain, focusing on recovery and death rates. The distribution showed strong flexibility and provided an

excellent fit to the datasets. Additionally, the expected values of the first and last order statistics were used to estimate the minimum and maximum recovery and death rates, respectively. The probability density function (PDF) of the distribution is given by:

$$f(x) = \frac{\theta\delta x^{\delta-1}(1-x)^{-\delta-1}\, exp\left[-\left(\frac{x}{1-x}\right)^{\delta}\right]}{\left\{1-(1-\theta)\, exp\left[-\left(\frac{x}{1-x}\right)^{\delta}\right]\right\}^{2}}; x \in (0,1),\ \theta,\delta > 0.$$

Log Hamza Distribution

Ahmad et al. (2022) proposed the log Hamza distribution by applying the transformation $y = e^{-x}$ to the classical Hamza distribution. The authors examined several statistical properties of the model, including moments, the moment generating function, order statistics, and reliability measures. Notably, the hazard rate function of the distribution exhibits a bathtub shape. The parameters of the model were estimated using the maximum likelihood estimation (MLE) method. The model was applied to a moderately skewed real-life dataset and demonstrated superior performance compared to several existing alternatives. The PDF of the distribution is given by:

$$f(x) = \frac{\beta^{6}}{\alpha\beta^{5}+120}\left(\alpha + \frac{\beta}{6}\left(In(x)\right)^{6}\right)x^{\beta-1}; x \in (0,1), \alpha,\beta > 0.$$

Unit-Exponentiated Lomax Distribution

Fayomi et al. (2023) introduced the Unit Exponentiated Lomax (UEL) distribution as the unit version of the classical exponentiated Lomax distribution, employing the transformation $Y = e^{-X}$. The resulting model exhibits a wide range of shapes for its probability density function (PDF), including left-skewed, U-shaped, unimodal, and J-shaped forms. Additionally, the hazard rate function (HRF) can display increasing, decreasing, J-shaped, or bathtub-shaped behaviors, making the model flexible for diverse applications. The authors derived several statistical properties of the distribution, including the probability-weighted moments, raw moments, incomplete moments, and quantile function. Parameter estimation was conducted using multiple well-established techniques, such as maximum likelihood estimation (MLE), Bayesian methods, and maximum product of spacing (MPS). A simulation study was carried out to evaluate the efficiency and consistency of the proposed estimators. The authors applied the UEL model to COVID-19 data, where it demonstrated improved fit compared to selected existing unit distributions based on various comparison criteria. The PDF of the unit exponentiated Lomax distribution is given by:

$$f(x) = \frac{\lambda\delta\vartheta}{x}\left(1-\lambda In(y)\right)^{-\delta-1}\left\{1-\left(1-\lambda In(y)\right)^{-\delta}\right\}^{\delta}; x \in (0,1), \lambda,\delta,\vartheta > 0.$$

Exponentiated Unit Exponential Half-Logistic Distribution

Genç and Özbilen (2023) proposed the Exponentiated Unit Exponential Half-Logistic (EUEHL) distribution by extending the unit exponential half-logistic (UEHL) distribution using the exponentiated transformation technique. The authors demonstrated that the probability density function (PDF) of the model can exhibit various shapes—unimodal, increasing, decreasing, and U-shaped—depending on the values of the parameters. A simulation study was conducted to evaluate the performance of the maximum likelihood estimation method for parameter estimation. In the real data analysis based on record values within the unit interval, the EUEHL distribution outperformed several well-known competing models according to standard comparison criteria. The PDF of the EUEHL distribution is given by:

$$f(x) = 2\theta\lambda\alpha\frac{x^{\theta-1}}{\left(1+x^{\theta}\right)^{2}}\left(\frac{1-x^{\theta}}{1+x^{\theta}}\right)^{\lambda-1}\left[1-\left(\frac{1-x^{\theta}}{1+x^{\theta}}\right)^{\lambda}\right]^{\alpha-1}; x \in (0,1), \theta,\alpha,\lambda > 0.$$

Alpha Power Topp-Leone Distribution

Ehiwario et al. (2023) proposed the Alpha Power Topp-Leone distribution by extending the classical Toppleone distribution using the alpha power transformation method. The authors derived several mathematical properties of the distribution, including the survival function, hazard rate function, quantile function, median, moments and their related measures, probability weighted moments, moment generating function, Rényi entropy, and the distribution of order statistics. The probability density function (PDF) of the model was shown to accommodate various shapes, including decreasing, left-skewed, right-skewed, and symmetric forms. The hazard function exhibits a range of behaviors such as increasing, bathtub-shaped, and upside-down bathtub patterns, demonstrating the model's versatility for modeling datasets with diverse failure rate trends and density shapes. Parameter estimation was conducted using the maximum likelihood estimation (MLE) method. The model was applied to two real-world datasets: (1) a dataset comprising trade share data, and (2) a dataset containing records of ordered component failures. In both cases, the alpha power Topp-Leone distribution exhibited superior fit compared to four other unit distributions based on standard goodness-of-fit criteria. The PDF of the model is given by:

$$f(x) = \frac{log\,\alpha}{\alpha-1}2\lambda(1-x)[1-(1-x)^{2}]^{\lambda-1}\alpha^{[1-(1-x)^{2}]^{\lambda}}; x \in (0,1),\ \lambda > 0, \alpha > 0, \alpha \neq 1.$$

Odd Beta Prime Kumaraswamy Distribution

Suleiman et al. (2024) introduced the Odd Beta Prime Kumaraswamy distribution by compounding the classical Kumaraswamy distribution with the odd beta prime generalized family of distributions. The authors demonstrated that the probability density function (PDF) of the model is highly flexible and can exhibit a variety of shapes, including (i) symmetric, (ii) right-skewed, (iii) left-skewed, (iv) bathtub-shaped, (v) N-shaped, (vi) reversed-J shaped, and (vii) J-shaped. Furthermore, the hazard rate function (HRF) of the distribution accommodates multiple shapes, such as (i) increasing, (ii) decreasing, (iii) bathtub-shaped, (iv) increasing–decreasing, (v) upside-down bathtub, and (vi) N-shaped. Several statistical properties of the distribution were derived, including the quantile function, moments, moment-generating function, stress–strength reliability function, and order statistics. The parameters were estimated using the method of maximum likelihood. To demonstrate the practical utility of the model, it was applied to four real-life datasets on COVID-19 mortality rates from Saudi Arabia, Canada, Senegal, and Italy. The odd beta prime Kumaraswamy distribution outperformed competing models based on various goodness-of-fit measures. The PDF of the model is given by:

$$f(x) = \frac{\gamma\lambda x^{\gamma-1}(1-x^{\gamma})^{\kappa\lambda-1}}{B(\varphi,\kappa)}\left\{1-(1-x^{\gamma})^{\lambda}\right\}^{\varphi-1}; x \in (0,1), \gamma,\lambda,\varphi,\kappa > 0.$$

Power New Power Function Distribution

Karakaya et al. (2024) introduced the Power New Power Function (PNPF) distribution by extending the new power function distribution through the transformation $X = T^{\frac{1}{\sigma}}$, resulting in a model defined on the unit interval. The distribution can exhibit both left-skewed and symmetric shapes in its probability density function, while the hazard rate function is capable of capturing bathtub, increasing, and decreasing patterns, thereby enhancing the model's applicability across various contexts. The study explored key statistical characteristics of the distribution, including the mean residual life, moments, and order statistics. For parameter estimation, the authors applied several techniques: maximum likelihood estimation, least squares estimation, weighted least squares estimation, Anderson–Darling estimation, and Cramér–von Mises estimation. A simulation study confirmed the consistency and reliability of the estimators under varying sample sizes. The practical performance of the distribution was evaluated using two real-life datasets related to firm risk management cost-effectiveness and recovery rates of viable CD34+ cells post-chemotherapy, where the model showed strong fitting capabilities. The probability density function of the PNPF distribution is given by:

$$f(x) = \frac{(\delta+1)\eta\sigma x^{\sigma-1}\left(\frac{1-x^{\sigma}}{\delta x^{\sigma}+1}\right)^{\eta}}{(1-x^{\sigma})(\delta x^{\sigma}+1)}; x \in (0,1), \delta, \sigma, \eta > 0.$$

Extended Unit Tiesser Distribution

Gemeay et al. (2024a) introduced the Extended Unit Tiesser distribution by adding an extra parameter to the unit Tiesser distribution, resulting in a new two-parameter unit distribution. The PDF of the distribution can be left-skewed or right-skewed, while the hazard function exhibits diverse shapes such as bathtub and N-shaped patterns. The study explored the fundamental properties of the model, including its quantile function, mode, moments, moment generating function, and conditional moments. Parameter estimation was conducted using the maximum likelihood method, supported by eleven alternative estimation approaches. A simulation study demonstrated the accuracy and consistency of these methods even in small samples. To illustrate its applicability, the model was fitted to two datasets: the first comprising 30 measurements of the tensile strength of polyester fibers, and the second representing the time in months until infection among kidney dialysis patients. Model comparisons based on various selection criteria and goodness-of-fit tests revealed that the extended unit Tiesser distribution offered a superior fit relative to existing models. The probability density function of the distribution is given by:

$$f(x) = \alpha x^{-2\alpha-1} e^{\beta-\beta x^{-\alpha}}(\beta - x^{\alpha}); x \in (0,1), \alpha > 0, \beta \geq 1.$$

Unit Compound Rayleigh Distribution

Gong et al. (2024) introduced the Unit Compound Rayleigh distribution (UCRD) by applying the transformation $T = X/(1-X)$ to the compound Rayleigh distribution. The probability density function of the model exhibits unimodality and general symmetry, while its hazard function consistently displays an increasing trend, reflecting the flexibility of the distribution in modeling data with such characteristics. Several statistical properties were derived, including the quantile function, K-order moments, mean, and variance. Parameter estimation was carried out using both maximum likelihood and Bayesian methods. Additionally, the authors calculated various entropy measures based on maximum likelihood estimates and assessed their performance using Monte Carlo simulations. These simulations reported the average entropy estimates, bias, mean squared errors, and mean relative estimates, providing a detailed evaluation of the entropy behavior under the UCRD framework. To demonstrate practical applicability, the model was fitted to two real datasets, and the results underscored its effectiveness in capturing and predicting patterns in real-world observations. The PDF of the distribution is given as:

$$f(x) = \frac{2\beta x}{\theta(1-x)^3}\left[1 + \frac{1}{\theta}\left(\frac{x}{1-x}\right)^2\right]^{-(\beta+1)}; x \in (0,1), \beta, \theta > 0.$$

Unit Zeghdoudi Distribution

The Unit Zeghdoudi distribution, proposed by Bashiru et al. (2025), was derived by transforming the Zeghdoudi distribution using the transformation $x = \frac{y}{y+1}$. The resulting model is defined on the unit interval and is capable of capturing both left- and right-skewed data distributions. Its hazard rate function exhibits an increasing failure rate, making it suitable for modeling lifetime data. The authors assessed sixteen classical parameter estimation techniques through a simulation study, all of which yielded consistent and reliable estimates. The distribution's practical applicability was demonstrated using datasets from biomedical research, hydrology, and radiation science. In each case, the unit Zeghdoudi distribution provided a superior fit compared to existing models based on standard goodness-of-fit metrics. The probability density function is given by:

$$f(x;\omega) = \frac{\omega^3 x}{(1-x)^4(\omega+2)} e^{-\frac{\omega x}{1-x}}; x \in 0, \omega > 0.$$

Transmuted Power Unit Inverse Lindley Distribution

Eldessouky et al. (2025) introduced the Transmuted Power Unit Inverse Lindley distribution (TPUILD) by applying the quadratic transmutation map to the power unit inverse Lindley distribution, resulting in a flexible three-parameter unit distribution. The probability density function (PDF) of the TPUILD accommodates various shapes, including left-skewed, right-skewed, and nearly symmetrical forms. The hazard rate function can exhibit both increasing and decreasing patterns, making the model adaptable for a wide range of applications, particularly in biomedical and economic research. Several analytical properties of the TPUILD were derived, including reliability measures, moments and incomplete moments, order statistics The parameters of the distribution were estimated using the maximum likelihood estimation (MLE) method. A simulation study was conducted to assess the efficiency and robustness of the estimators. To demonstrate the practical utility of the TPUILD, it was applied to two real-world datasets—one from the biomedical domain and the other from economics. The model's goodness-of-fit was evaluated using multiple statistical measures. Results showed that the TPUILD provided a significantly better fit than competing models, including:

Inverse Topp-Leone distribution
Power X-Lindley distribution
Truncated Power Lomax distribution
Truncated Weibull distribution
Exponential Pareto distribution
Kumaraswamy–Kumaraswamy distribution
Exponentiated Kumaraswamy distribution
Marshall–Olkin Kumaraswamy distribution

These findings suggest that the TPUILD is a versatile and effective choice for modeling bounded data in economic and biomedical contexts. The PDF of the distribution is given by:

$$f(x) = \frac{\vartheta_2\vartheta_1^2 e^{\vartheta_1}}{1+\vartheta_1} x^{-2\vartheta_2-1} e^{-\frac{\vartheta_1}{x^{\vartheta_2}}}\left[1 + \vartheta_3 - \frac{2\vartheta_3 e^{\vartheta_1}}{(1+\vartheta_1)}\left(1 + \frac{\vartheta_1}{x^{\vartheta_2}}\right) e^{-\frac{\vartheta_1}{x^{\vartheta_2}}}\right]; x \in (0,1), |\vartheta_3| \leq 1, \vartheta_1, \vartheta_2 > 0.$$

Two Parameter Log-Lindley Distribution

Altun et al. (2025) introduced the Two-Parameter Log-Lindley (TPLL) distribution using the transformation $x = exp(-y)$ where y follows a two-parameter Lindley distribution. The PDF exhibits increasing, decreasing, and increasing-decreasing shapes. The hazard function can exhibit both increasing and bathtub-shaped failure rates. The parameters were estimated using three classical techniques. To illustrate the model's practical relevance, it was applied to datasets from linguistics (geographical distribution of French speakers across 88 countries), materials engineering (failure times of Kevlar 49/epoxy strands under 90% stress), and biomedical science (antimicrobial resistance data from 24 individuals), where it consistently outperformed competing models based on various evaluation metrics. Its PDF is given by:

$$f(x) = \frac{\theta^2 x^{\theta-1}(1-\alpha\, log(x))}{\alpha+\theta}; x \in 0, \alpha, \theta > 0.$$

Unit Bilal Distribution

Sindhu et al. (2025) introduced the Unit Bilal distribution by transforming the classical unbounded Bilal distribution using the inverse exponential transformation $y = \exp(-x/\delta)$. The authors showed that the probability density function (PDF) of the model is unimodal and can exhibit both left and right skewness. The hazard rate function was found to follow increasing and bathtub-shaped failure rate patterns. Parameters of the unit Bilal distribution were estimated using maximum likelihood estimation (MLE), least squares estimation (LSE), and weighted least squares estimation (WLSE). A simulation study confirmed the consistency of these estimators. The model was applied to datasets with nearly symmetric and highly positively skewed characteristics and demonstrated superior performance compared to existing unit distributions based on standard goodness-of-fit criteria. The PDF of the model is given by:

$$f(x) = 6\delta/\beta\left(1 - x^{\delta/\beta}\right)x^{2\delta/\beta-1}; x \in 0, \delta, \beta > 0.$$

RESULTS AND DISCUSSION

Table 1: Summary of Additional Unit Distributions

Distribution	Key Features	Reference
Unit Weibull	Exhibits flexible hazard rates: increasing, decreasing, and bathtub-shaped.	Mazucheli et al. (2018)
One-parameter unit Lindley	Has closed-form moments and belongs to the exponential family. Ideal for parsimonious modeling with a single parameter; outperforms beta regression in modeling (0,1) data with covariates.	Mazucheli et al. (2020)
Bounded shifted Gompertz distribution	Derived from the shifted Gompertz distribution; represents the minimum of shifted Poisson variables. Offers closed-form expressions for moments and quantiles using gamma and Lambert W functions.	Jodrá (2020)
Bounded truncated Cauchy power exponential	Provides flexible PDF shapes (left/right-skewed, reversed-J, bathtub) and hazard rates (J-shaped, bathtub). Demonstrates superior fit to COVID-19 and quantile regression data.	Nasiru et al. (2022)
Unit extended exponential	Features highly flexible PDFs (decreasing, skewed, unimodal) and diverse hazard shapes (J-shaped, U-shaped, bathtub). Offers closed-form statistical properties. Suitable for modeling real-world unit data.	Rajab et al. (2024)
Log-cosine-power unit	Combines logarithmic, cosine, and power functions; tangent-based PDF. Models inverted-J, J, and decreasing-constant-increasing shapes. Performs well for proportion data.	Nasiru et al. (2024)
Transmuted unit exponentiated half-logistic	Based on the unit exponential half-logistic distribution. Supports diverse PDF shapes: right-skewed, decreasing, increasing, and U-shaped.	Genç and Özbilen (2024)
Power unit inverse Lindley	A two-parameter generalization of the unit inverse Lindley distribution. Exhibits unimodal, decreasing, increasing, and right-skewed PDFs with various hazard forms (increasing, U-shaped, N-shaped). Estimated using 15 methods. Outperforms several competing models on real-world datasets.	Gemeay et al. (2024b)
Power unit exponential	Power transformation of the unit exponential distribution. Hazard function shows increasing and bathtub shapes. Estimated using 12 methods (MLE, Anderson-Darling variants, Cramér-von Mises, etc.). Demonstrates superior fit in real-data applications.	Alsadat et al. (2024)
Unit-Chen	Derived from the Chen distribution. Supports flexible PDFs (U-shaped, unimodal, right-skewed) and hazard functions (bathtub, increasing, or increasing-decreasing-increasing). Allows both MLE and Bayesian estimation. Yields superior fit in real-data applications and quantile regression.	Sarhan (2025)

CONCLUSION

Unit probability distributions have emerged as indispensable tools for modeling data restricted to the interval (0, 1), offering a broad spectrum of applications in fields such as economics, biomedical science, hydrology, and reliability engineering. This review set out to examine both foundational and recent univariate continuous unit distributions, particularly those proposed between 2017 and early 2025, by cataloging their derivation methods, statistical properties, estimation techniques, and empirical applications. While the

Beta and Kumaraswamy distributions laid the groundwork for modeling bounded data, the increasing complexity of empirical datasets has led to the development of more flexible models. Recent advances, including transformed and compounded distributions, have enriched the modeling landscape by accommodating skewness, multimodality, and diverse hazard behaviors. Through this synthesis, the review provides researchers and practitioners with a structured overview of available tools, aiding informed model selection in practical applications. Nonetheless, the review is limited to univariate continuous models and recent literature, excluding multivariate, or earlier classical contributions beyond a few key references. Additionally, the review did not include quantitative meta-analyses or performance benchmarking. These constraints, however, highlight valuable opportunities for future research. From a practical standpoint, the findings of this review are relevant for statisticians, data scientists, and applied researchers seeking models suited for bounded data, such as proportions, risk indices, and rates, especially in contexts requiring high flexibility and interpretability.

REFERENCES

Afify, A. Z., Nassar, M., Kumar, D., & Cordeiro, G. M. (2022). A new unit distribution: Properties, inference, and applications. *Electronic Journal of Applied Statistical Analysis, 15*(2), 438–462.

Ahmad, A., Ahmad, A., Dar, I. H., & Tripathi, R. (2022). The Log-Hamza distribution with statistical properties and application: An alternative for distributions having domain (0,1). *Reliability: Theory & Applications, 17*(3[69]), 306–314.

Alsadat, N., Taniş, C., Sapkota, L. P., Rajitha, C. S., Bahloul, M. M., & Gemeay, A. M. (2024). Power unit exponential probability distribution: Statistical inference and applications. *Alexandria Engineering Journal, 107*, 332–346. https://doi.org/10.1016/j.aej.2024.07.038

Bashiru, S. O., Kayid, M., Sayed, R. M., Balogun, O. S., Abd El-Raouf, M. M., & Gemeay, A. M. (2025). Introducing the unit Zeghdoudi distribution as a novel statistical model for analyzing proportional data. *Journal of Radiation Research and Applied Sciences, 18*(1), 101204. https://doi.org/10.1016/j.jrras.2024.101204

Altun, E., Chesneau, C., & Alqifari, H. N. (2025). Two parameter log-Lindley distribution with LTPL web-tool. *AIMS Mathematics, 10*(4), 8306–8321. https://doi.org/10.3934/math.2025382

Ehiwario, J., Igabari, J., and Ezimadu, P. (2023). The alpha power topp-leone distribution: Properties, simulations and applications. *Journal of Applied Mathematics and Physics,* 11:316–331.

Eldessouky, E. A., Hassan, O. H. M., Aloraini, B., & Elbatal, I. (2025). Modeling medical and economic data using the transmuted power unit inverse Lindley distribution. *Alexandria Engineering Journal, 113*, 633–647. https://doi.org/10.1016/j.aej.2024.11.008

Fayomi, A., Hassan, A. S., & Almetwally, E. M. (2023). Inference and quantile regression for the unit-exponentiated Lomax distribution. *PLOS ONE, 18*(7), e0288635. https://doi.org/10.1371/journal.pone.0288635

Gemeay, A. M., Sapkota, L. P., Tashkandy, Y. A., Bakr, M. E., Balogun, O. S., & Hussam, E. (2024a). New bounded probability model: Properties, estimation, and applications. *Heliyon, 10*(23), e38965. https://doi.org/10.1016/j.heliyon.2024.e38965

Gemeay, A. M., Alsadat, N., Chesneau, C., & Elgarhy, M. (2024b). Power unit inverse Lindley distribution with different measures of uncertainty, estimation, and applications. *AIMS Mathematics, 9*(8), 20976–21024. https://doi.org/10.3934/math.20241021

Genç, M., & Özbilen, Ö. (2023). Exponentiated UEHL distribution: Properties and applications. *Recep Tayyip Erdogan University Journal of Science and Engineering, 4*(2), 232–241. https://doi.org/10.53501/rteufemud.1388416

Genç, M., & Özbilen, Ö. (2024). Transmuted Unit Exponentiated Half-Logistic distribution and its applications. *Ordu Üniversitesi Bilim ve Teknoloji Dergisi, 14*(2), 249–260. https://doi.org/10.54370/ordubtd.1512101

Gong, Q., Luo, L., & Ren, H. (2024). Unit compound Rayleigh model: Statistical characteristics, estimation and application. *AIMS Mathematics, 9*(8), 22813–22841. https://doi.org/10.3934/math.20241110

Hashmi, S., Ahsan-ul-Haq, M., Zafar, J., & Khalee, M. A. (2022). Unit Xgamma distribution: Its properties, estimation and application. *Proceedings of the Pakistan Academy of Sciences: A. Physical and Computational Sciences, 59*(1), 15–28. https://doi.org/10.53560/PPASA(59-1)636

Jodrá, P. (2020). A bounded distribution derived from the shifted Gompertz law. *Journal of King Saud University - Science, 32*(1), 523–536. https://doi.org/10.1016/j.jksus.2018.08.001

Johnson, N. L., Kotz, S., and Balakrishnan, N. (1955). Continuous Univariate Distributions, volume 2. Wiley, Hoboken, NJ, USA, 2nd edition.

Karakaya, K., Rajitha, C. S., Sağlam, Ş., & et al. (2024). A new unit distribution: Properties, estimation, and regression analysis. *Scientific Reports, 14*, 7214. https://doi.org/10.1038/s41598-024-57390-7

Korkmaz, M. Ç., Chesneau, C., & Korkmaz, Z. S. (2021). Transmuted unit Rayleigh quantile regression model: Alternative to beta and Kumaraswamy quantile regression models. *U.P.B. Scientific Bulletin, Series A*, *83*(3), 149–158.

Krishna, A., Maya, R., Chesneau, C., & Irshad, M. R. (2022). The Unit Teissier Distribution and Its Applications. *Mathematical and Computational Applications*, *27*(1), 12. https://doi.org/10.3390/mca27010012

Kumaraswamy, P. (1980). A generalized probability density function for double-bounded random processes. *Journal of Hydrology,* 46(1–2):79–88.

Mazucheli, J., Bapat, S. R., & Menezes, A. F. B. (2020). A new one-parameter unit-Lindley distribution. *Chilean Journal of Statistics, 11*(1), 53–67.

Mazucheli, J., Menezes, A. F. B., & Dey, S. (2019). Unit-Gompertz distribution with applications. *Statistica, 79*(1), 25–43.

Mazucheli, J., Menezes, A. F. B., & Ghitany, M. E. (2018). The unit-Weibull distribution and associated inference. *Journal of Applied Probability and Statistics, 13*(2), 1–22.

Muhammad, M. (2017). A new lifetime model with a bounded support. *Asian Research Journal of Mathematics, 7*(3), 1–11.
Nasiru, S., Abubakari, A. G., & Chesneau, C. (2022). New Lifetime Distribution for Modeling Data on the Unit Interval:

Properties, Applications and Quantile Regression. *Mathematical and Computational Applications*, *27*(6), 105. https://doi.org/10.3390/mca27060105

Nasiru, S., Chesneau, C., & Ocloo, S. K. (2024). The log-cosine-power unit distribution: A new unit distribution for proportion data analysis. *Decision Analytics Journal, 10*, 100397. https://doi.org/10.1016/j.dajour.2024.100397

Ragab, I. E., Alsadat, N., Balogun, O. S., & Elgarhy, M. (2024). Unit extended exponential distribution with applications. *Journal of Radiation Research and Applied Sciences, 17*(4), 101118. https://doi.org/10.1016/j.jrras.2024.101118

Sindhu, T. N., Shafiq, A., Riaz, M. B., & Abushal, T. A. (2024). A statistical framework for a new two-parameter unit Bilal distribution with application to model asymmetric data. *Heliyon*, *10*, e38340. https://doi.org/10.1016/j.heliyon.2024.e38340

Sarhan, A. M. (2025). Unit-Chen distribution and its quantile regression model with applications. *Scientific African, 27*, e02555. https://doi.org/10.1016/j.sciaf.2025.e02555

Suleiman, A. A., Daud, H., Ishaq, A. I., Othman, M., Alshanbari, H. M., & Alaziz, S. N. (2024). A novel extended Kumaraswamy distribution and its application to COVID-19 data. *Engineering Reports, 6*(12), e12967.