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MODELLING THE VOLATILITY OF COTTON AND RUBBER STOCK RETURNS IN NIGERIA USING GARCH TYPE MODELS

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ABSTRACT

This research models the volatility of stock returns for cotton and rubber in Nigeria over the period 1960 to 2022 using monthly secondary data obtained from Central Bank of Nigeria (CBN). The study utilized GARCH (1,1) family models to analyze the conditional variance equation for the two stock prices, focusing on GARCH, EGARCH, IGARCH, and PARCH models. The GARCH model shows a positive coefficient for mean, constant, ARCH, and GARCH terms indicate positive relationships, with high z-statistics and low p-values suggesting statistical significance. The EGARCH model introduces a leverage effect (γ), revealing a negative impact of past negative shocks on future volatility for cotton. Significant coefficients emphasize the model's ability to capture asymmetry. The IGARCH model, with positive coefficients for mean, ARCH, and GARCH terms, exhibits significance, reflecting persistent volatility. The PARCH model includes asymmetry (γ) and long memory (δ) parameters, but the constant term is not statistically significant for Cotton. The IGARCH model, with the lowest Information Criteria values, is identified as providing a relatively better fit for both stocks. The findings provide valuable insights into the conditional variance dynamics of the stock prices, incorporating factors like asymmetry and long memory effects to enhance the understanding of market volatility and the implications of the forecasting results for investors, policymakers, and stakeholders in the agricultural sector.

Keywords: ARCH, Diagnostic Test, EGARCH, GARCH, PARCH, Skewness, Stationarity

INTRODUCTION

The stock market is an essential part of the economic system, and the global financial landscape has changed significantly over time (Ajayi & Aladesulu 2018). The economy of Nigeria, like that of many other developing countries, is heavily dependent on agriculture. Particularly, cash crops make a significant contribution to the GDP and foreign exchange revenues of the nation. Nigeria, a prominent participant in the world agricultural scene, depends heavily on the production and sale of cash crops to fuel its economic expansion. Cotton and rubber are two of these cash crops that are strategically significant, making significant contributions to the nation's agricultural industry and foreign exchange profits. Through the volatility of their stock returns, the stock market, a crucial part of the financial system, represents the economic performance of these crops. Researchers, decision-makers, and market players have all taken notice of the volatility of stock returns linked to these cash crops. Investors, portfolio managers, financial institutions, and policymakers must all comprehend and appropriately model volatility. Measures of volatility aid in risk assessment, investment strategy development, derivatives pricing determination, and risk management technique implementation. Good volatility modelling boosts the overall stability and effectiveness of the financial system, improves decision-making, and offers insightful information about market dynamics (Awujola, *et al.* 2015; Dewi, *et al.* 2023). Financial time series data is complicated and dynamic, making it difficult for traditional models like historical volatility or simple moving averages to adequately represent it (Christoffersen, 2012). These models frequently make the assumption that volatility is constant or neglect to take into consideration the non-linearities, asymmetries, and clustering of volatility that exist in financial returns (Caporale & Pittis 2019). They might therefore offer erroneous projections and predictions, which could result in less-than-ideal investment choices. Advanced econometric models have been created to better capture the features of volatility in order to get beyond these restrictions. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is one example of such a model. Since their introduction by Robert Engle in the early 1980s, GARCH models have gained popularity in the field of financial econometrics (Engle, 1982). The foundation of GARCH models is the idea that volatility is persistent and reliant on historical data. Both the short-term and long-term dynamics of volatility are captured by these models, which accommodate time-varying volatility (Ding *et al.* 2014). GARCH models have been effectively implemented in a number of global financial markets, offering enhanced forecasting and volatility prediction capabilities. GARCH model-based volatility modeling is a relatively understudied topic in relation to the Nigerian stock exchange. Although research on Nigeria's stock market volatility have been conducted, they have frequently used traditional volatility metrics or concentrated on descriptive analysis. Therefore, in order to capture the distinctive features of volatility in the Nigerian environment, particularly in the agricultural sector, extensive research that explicitly uses GARCH models is required. The dynamic and intricate character of stock return volatility for specific cash crops (rubber and cotton) in Nigeria from 1960 to 2022 is the main subject of this research. As a primary measure of risk and uncertainty, volatility is essential to financial market stability, portfolio management, and investment decision-making (Alexander, 2008). Investors, legislators, and other stakeholders in the agriculture industry can benefit greatly from an understanding of and ability to model the volatility of cash crop stock returns.

The Random Walk Theory, the Efficient Market Hypothesis (EMH), and the idea of volatility as a gauge of risk are all covered in (Nyakurukwa & Seetharam, 2023). It looks at the function of volatility in financial markets, how it affects investors and other market players, and how crucial precise volatility modeling is for risk assessment and judgment. A key idea in financial markets, volatility is important for risk management, financial instrument pricing, and investment decision-making. The basis for comprehending volatility and its consequences is provided by theoretical frameworks. Important theories and ideas pertaining to volatility modeling are examined in this section (Glosten, *et al.* 1993). According to the Efficient Market Hypothesis (EMH), asset prices accurately reflect all available information because financial markets are informationally efficient. There is no place for systematic patterns or predictability in an efficient market since asset values react to new information quickly and precisely. The EMH suggests that fresh information and unforeseen occurrences are the main drivers of volatility, which has significant ramifications for volatility modeling (Garcin, 2023).

Volatility can be viewed as a gauge of risk or uncertainty under the EMH, it can change as a result of price fluctuations brought on by fresh information entering the market. In order to shed light on the risk connected to financial assets, volatility modeling seeks to identify and measure this uncertainty (Garcin, 2023).The Random Walk Theory, which claims that future price fluctuations in financial markets are unpredictable and follow a random pattern, is strongly associated with the Efficient Market Hypothesis (EMH) (Nie *et al*., 2020).This theory holds that asset price fluctuations are autonomous and unaffected by historical price movements, making it pointless to try to predict future volatility or pricing. Since it questions the notion of serial correlation in asset returns, the Random Walk Theory has significant ramifications for volatility modeling (Mohammed, *et al.* 2022). Traditional time series models might not be suitable for capturing volatility dynamics if returns are actually random. Nevertheless, empirical data indicates that financial time series show some traits that depart from strict randomness, suggesting the existence of correlations and patterns that statistical models such as GARCH can capture. In financial markets, volatility is frequently employed as a gauge of risk. A higher degree of risk is indicated by higher volatility, which is linked to increased uncertainty and the possibility of bigger price swings (Liu *et al*., 2023). Because it influences asset allocation choices, risk management tactics, and portfolio performance, investors and market participants are concerned with controlling and comprehending this risk (Benavente, 2022; Liu, 2022; Kumar, 2022).

Financial crises provide difficulties for volatility modeling and have a significant effect on volatility dynamics. Market participants encounter greater volatility and unpredictability during times of financial turmoil. The significant oscillations and non-linearities linked to financial crises may be difficult for traditional models to reflect (Nguyen, *et al.*, 2022). Understanding the dynamics of market stress and evaluating systemic risks require an understanding of volatility modeling in the context of financial crises. To reflect the shifting character of volatility during crisis periods, researchers have investigated a variety of modeling approaches, including asymmetric volatility models and regime-switching models (Bilgili, *et al*, 2019). The empirical examination and implementation of GARCH models in the context of the Nigerian daily stock market are firmly based on an understanding of the theoretical underpinnings of volatility modeling. Researchers can learn more about the fundamental ideas and driving forces behind volatility modeling in financial markets by looking at the Random Walk Theory, the Efficient Market Hypothesis, the use of volatility as a risk measure, and the effects of financial crises on volatility dynamics (Garcin, 2023).

Engle (1982) established the model of heteroskedasticity (ARCH), the ARCH model acknowledges that the variation of a financial asset's return is dependent on historical data and is not constant. The current conditional variance is represented as a function of previous squared residuals or shocks in an ARCH model. Important aspects of volatility, including persistence, leverage effects, and volatility clustering, can be captured using the GARCH model. The tendency for times of high volatility to be followed by other periods of high volatility, and vice versa, is known as volatility clustering. According to leverage effects, volatility is more affected by negative shocks than by positive ones. According to Bollerslev (1986), persistence of volatility means that either high or low volatility levels typically continue over time. The field has benefited greatly from empirical research on volatility modeling in developed stock markets, including the Tokyo Stock Exchange (TSE), London Stock Exchange (LSE), and New York Stock Exchange (NYSE).Numerous facets of volatility dynamics have been examined in these research, such as long memory, asymmetry, volatility clustering, and the influence of macroeconomic factors Empirical research has looked at how macroeconomic factors like inflation, interest and exchange rates affect the volatility of the stock market. These studies have helped with risk management and investment decision-making by shedding light on the connections between macroeconomic variables and asset price volatility (Engle, *et al.* 1987). Since emerging markets have distinct dynamics and characteristics, volatility modeling in these markets has attracted a lot of attention. Research on the volatility of emerging markets has looked at investor mood, market microstructure impacts, volatility spillovers, and the existence of volatility asymmetry (Engle, 1982). The significance of taking into account country-specific characteristics and market peculiarities in volatility modeling has been illustrated by empirical research on emerging markets, such as those conducted in Brazil, India, China, and South Africa. This research has improved our knowledge of emerging market volatility patterns, which has aided in risk assessment and portfolio management techniques because currency volatility has a big impact on international trade, investment choices, and risk management, market participants in foreign exchange (forex) markets need to be able to simulate market volatility. The patterns of exchange rate volatility, the influence of economic news releases, and the effectiveness of volatility forecasting models have all been the subject of empirical research in forex markets (Chan *et al.,* 2023; Ajayi, *et al.* 2019).

Ibrahim & Isiaka (2020) argued that exchange rate volatility has been extensively modeled using GARCH models in a variety of formats. The existence of currency market volatility spillovers, the influence of macroeconomic variables on exchange rate volatility, and the efficiency of volatility forecasting models in predicting forex volatility have all been investigated in this research (Hansen & Lunde 2016). Commodity markets, such as those for gold, crude oil, and agricultural commodities, exhibit distinct patterns of volatility that are impacted by macroeconomic conditions, supply-demand dynamics, and geopolitical considerations (Andersen, *et al.* 2003). The objectives of empirical research in commodity markets have been to evaluate risk, model and predict volatility in commodity prices, and look into the connections between commodities and other financial assets (Wang, 2021). Commodity market volatility dynamics have been captured using GARCH models. These studies have investigated the impact of macroeconomic factors and geopolitical events on commodity price volatility, the existence of long memory in commodity pricing, and the transmission of volatility between commodities and financial markets. Volatility modeling in these markets has gained attention since the rise of cryptocurrencies like Bitcoin (Yahaya, *et. al.* 2022). The distinctive features of digital assets, the influence of market variables and regulatory actions, and the effectiveness of volatility forecasting models have all been studied in relation to bitcoin volatility. GARCH models and other cutting-edge techniques have been used in empirical research on cryptocurrency markets to capture the volatility patterns of Bitcoin and other cryptocurrencies. The nature of cryptocurrency volatility, risk assessment, and the difficulties of modeling volatility in this dynamic asset class have all been clarified by these studies (Chan *et al*., 2023). In conclusion, empirical research on financial market volatility modeling has greatly advanced our knowledge of asset price dynamics, risk assessment, and forecasting precision. To capture the distinctive features of volatility in diverse financial markets, these studies have used a variety of modeling methodologies, such as the ARCH and GARCH models. These studies' conclusions have real-world ramifications for risk management, investment plans, and policy choices in the international financial system (Adams & Bello, 2022). Researching volatility modeling in the context of the Nigerian daily stock market requires an understanding of the Nigerian Stock market's organization, trading procedures, and regulatory environment. Researchers can efficiently study the volatility patterns and create suitable modeling techniques for capturing and forecasting volatility in the Nigerian stock market by understanding the distinctive features and dynamics of the NSE (Yadudu, 2021).The review includes research that have forecasted and captured market volatility in Nigeria using GARCH models and other comparable models. It also points out any shortcomings or holes in the body of current research, including the requirement for more thorough modeling techniques or the investigation of certain market abnormalities. In order to comprehend the dynamics of volatility, evaluate risk, and offer guidance for investment decision-making, a number of research have looked at volatility modeling in the context of the Nigerian Stock Exchange (NSE) (Azevedo *et al.,* 2023). In order to capture the market's volatility patterns, early research on volatility modeling in the NSE concentrated on using conventional time series models like GARCH and ARCH. These investigations looked at persistence, leverage effects, and volatility clustering in the stock returns of the NSE. For example, (Anwar & Beg 2012) used GARCH models to study volatility modeling. They discovered evidence of persistence and clustering of volatility, suggesting that stock returns have long memory. Similarly, to capture the dynamics of volatility and leverage effects on the NSE, Akpokodje & Osamwonyi, 2012) used GARCH models. Researchers have begun examining the features of intraday volatility in the NSE since high-frequency data became available in recent years. These studies concentrate on identifying intraday patterns, investigating how trading volume and liquidity affect volatility, and evaluating market efficiency. For instance, (Ogbonna & Ilo, 2016; Kehinde, *et al.* 2021) used GARCH models to examine intraday volatility trends in the NSE. They discovered that stock returns at various intraday time intervals showed signs of asymmetry and volatility clustering. Their research highlighted how crucial it is to take high-frequency data into account in order to have a more precise knowledge of the dynamics of volatility in the NSE. The transfer of volatility shocks from one market or asset to another is known as volatility spillover. The existence of volatility spillover effects between the NSE and other foreign stock markets has been the subject of numerous studies. Using GARCH models, (Adebiyi *et al*. 2014) investigated the volatility spillover between the NSE and the international stock markets. The NSE and marketplaces like the Johannesburg Stock Exchange (JSE), London Stock Exchange (LSE), and New York Stock Exchange (NYSE) were discovered to have bidirectional volatility spillovers. These results demonstrate how the NSE and international markets are interdependent and interrelated.

MATERIALS AND METHODS

Source of Data

Data for this study was gathered from secondary sources, such as the Central Bank of Nigeria (CBN, 2023). Monthly stock prices for rubber and cotton from 1960 to 2022 are included in the dataset. Conditional variance models are fitted to continuously compounded daily stock returns, y_t:

$$y_t = 100(lnk_t - lnk_{t-1}) \quad (1)$$

Where k_t = current period of stock market exchange, k_{t-1}= previous period stock market exchange, $y_t =$ current period stock returns (stock market exchange -RT), and $\Omega_{t-1} =$ All stock returns up to the immediate past.

Model Specification

The Family of Autoregressive Conditional Heteroskedasticity (ARCH) Models. Every ARCH or GARCH family model requires two distinct specifications: the mean and variance equations. According to Engel, conditional heteroskedasticity in a return series can be modeled using ARCH model expressing the mean equation in the form:

$$y_t = E_{t-1}(y_t) + \varepsilon_t \quad (2)$$

Such that $\varepsilon_t = \varphi_t \sigma_t$

Equation 1 is the mean equation which also applies to other GARCH family model. $E_{t-1}[.]$is expectation conditional on information available at time $t-1, \varepsilon_t$ is error generated from the mean equation at time t and φ_t is a sequence of independent, identically distributed (iid) random variables with zero mean and unit variance. $E\left\{{\varepsilon_t}/{\Omega_{t-1}}\right\} = 0; and\ \sigma_t^2 = \{{\varepsilon_t^2}/{\Omega_{t-1}}\}$ is a nontrivial positive valued parametric function of Ω_{t-1}. The variance equation for an ARCH model of order q is given as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \mu_t \quad (3)$$

Where $\alpha_0 > 0, \alpha_i \geq 0, i = 1, \ldots, q, and\ \alpha_q > 0$

In practical application of ARCH (q) model, the decay rate is usually more rapid than what actually applies to financial time series data. To account for this, the order of the ARCH must be at maximum, a process that is strenuous and more cumbersome (Adams, *et al.*2023; Adams, *et al.* 2024).

The unconditional kurtosis of ARCH (1)

Suppose the innovations are normal, then

$$E(a_t^4 | F_{t-1}) = 3[E(a_t^2 | F_{t-1})]^2 \quad (4)$$

$= 3(\alpha_0+\alpha_1 a_{t-1}^2)^2$, it follows that

$$Ea_t^4 = 3\alpha_0^2(1+\alpha_1) / [(1-\alpha_1)(1-3\alpha_1^2)] \quad (5)$$

and

$$Ea_t^4/(Ea_t^2)^2 = 3(1-\alpha_1^2)/(1-3\alpha_1^2)>3. \quad (6)$$

This shows that the tail distribution of a_t is heavier than that of a normal distribution.

Generalized ARCH (GARCH) Model

The conditional variance for GARCH (p, q) model is expressed generally as:

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (7)$$

where p is the order of the GARCH terms, and q is the order of the ARCH terms, ε^2. Where $\beta_0 > 0, \alpha_i \geq 0, i = 1, \ldots, q -$

$1,\ j = 1, \ldots, p-1\ and\ \beta_p, \alpha_q > 0.$ σ_t^2 is the conditional variance and ε_t^2, disturbance term. The reduced form of equation 3 is the GARCH (1, 1) represented as:

$$\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \quad (8)$$

The three parameters (β_0 , $\beta_1$and β_2) are nonnegative and β_1 + β_2<1 to achieve stationartiy.

Exponential GARCH model

A different that also captures the leverage is the exponential GARCH MODEL OR EGARCH

$$ln\sigma^2{}_{t+1} = \omega + \alpha(\varphi R_t + \gamma[|R_t| - E|R_t|] + \beta ln\sigma^2{}_t \quad (9)$$

Which displays the usual leverage effect if $\alpha\varphi < 0$. The EGARCH model has the advantage that the logarithmic specification ensures that variance is always positive, but it has the disadvantage that the future expected variance beyond one period cannot be calculated analytically.

Weekend effect

It is always known that days that followed a weekend or a holiday have higher variance than average day. We can try the following model:

$$\sigma_{t+1}^2 = \omega + \beta\sigma_t^2 + \alpha\sigma_t^2 Z_t^2 + \gamma IT_{t+1}, \quad (10)$$

where IT_{t+1} takes value 1 if day t+1 is a Monday, for example.

More General EGARCH

The exponential GARCH, or EGARCH model is

$$\log(\sigma_t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i\, g(\varepsilon_{t-1}) + \sum_{i=1}^{p} \beta_i\, log(\sigma_{t-1}) \quad (11)$$

$$where\ g(\epsilon_t) = \theta\epsilon_t + \gamma\{|\epsilon_t| - E(|\epsilon_t|)\} \quad (12)$$

Integrated GARCH model

A GARCH (p, q) process is called an I-GARCH process if

$$\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i = 1 \quad (13)$$

The IGARCH processes are either non-stationary or have an infinite variance.

Fractional GARCH Model

The generalized specification for the conditional variance using FIGARCH (p, d, q) is given as:

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^{q} \alpha_i\, \varepsilon_{t-1}^2 + \sum_{i=1}^{q} \gamma_i\, I_{t-1}\varepsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_j\, \sigma_{t-j}^2 \quad (14)$$

Where I_{t-1} =1, if ε_{t-1}^2< 0 and 0 otherwise

In this model, good news implies that ε_{t-1}^2> 0 and bad news implies that ε_{t-1}^2< 0 and these two shocks of equal size have differential effects on the conditional variance. Good news has an impact of α_i and bad news has an impact of$\alpha_i + \gamma_i$. Bad news increases volatility when $\gamma_i > 0$, which implies the existence of leverage effect in the i-th order and when $\gamma_i \neq 0$ the news impact is asymmetric. However, the first order representation is of FIGARCH (p, d, q) is

$$\sigma_t^2 = \beta_0 + \alpha_1\varepsilon_{t-1}^2 + \gamma_1 I_{t-1}\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \quad (15)$$

Then, good news has an impact of α_1 and bad news has an impact of $\alpha_1 + \gamma_1$.

Unit Root Test

According to (Gujarati & Porter, 2009), works based on time series assume that the series are stationary; however, not all economic variables are stationary in their states; some variables are non-stationary, which means their mean, variance, and covariance are not constant over time. A non-stationary variable is one that has no trend; the trend can be stochastic or deterministic, and if it is completely predictive, it is no longer considered a variable but rather a determinist. When a trend is non-predictive, the variable is referred to as stochastic. The unit root test was used to determine whether the variables in the model were stationary or non-stationary. The unit root test was used to avoid spurious regression, which occurs when one nonstationary variable is regressed against another nonstationary variable. The Augmented Dickey Fuller test was used in the study to look for unit roots. To perform the unit root test using the ADF method, the researcher specified the equation as follows:

$$Y_t = \beta y_{t-1} + \mu \quad (16)$$

If β is less than 1, Y_t is stationary, while if β is greater than or equal to 1, y is not stationary. The null and alternative hypotheses for testing the presence of unit root in the variable Y_t were:

Ho: β = 0 Vs H1: β < 0

According to the null hypothesis, a unit root exists. The time series, in other words, is not stationary. The unit root does not exist, according to the alternative hypothesis. This indicates the stationary nature of the time series. If, following statistical testing, the time series is determined to be stationary but not otherwise, the Augmented Dickey Fuller (ADF) test rejects the null hypothesis. Dickey and Fuller showed that the predicted t value of the Y_{t-1} coefficient obeys the τ(tau) statistic on the null hypothesis that β = 0. The null hypothesis is rejected if the computed absolute value of the tau statistics is greater than the critical values; if not, it is not rejected. A stationary time series is Yt-1 if the null hypothesis is disproved. If there is non-stationarity, the time series would be made stationary by difference.

Co-integration test

After handling the varying lag length recommendations, Johansen test of co-integration through Unrestricted Co-integration Rank Test (Maximum Eigenvalue) (Lakshmi & Tuwajri, 2014) and trace statistics for the lags chosen was conducted to discover if there is long run equilibrium between the exchange rate, import rate and export rate.

$$Y_t = A_t Y_{t-1} + \ldots + A_p Y_{t-1} + B_y + e_t \quad (17)$$

Where; Yt represents the dimensional vector of non-stationary I(I) variable, y=y- dimensional vector of deterministic variable and et stochastic error residual. Therefore, the hypothesis for Johansen co-integration test is:

H_0: there is no co-integration between the variables

H_1: there is co-integration between the variables

The null hypothesis ought to be disproved if the statistical value of the variable exceeds the critical value. This indicates that over time, there is co-integration between series that move in tandem. In order to estimate and test for the presence of several co-integration relationships in a single step, this study employed the (Johansen, 1991) approach. The long-term link between the variables is shown by the co-integration test.

Pre-Diagnostic Tests

We can run a GARCH model only if we can fulfill the following conditions:

The purpose of clustering volatility in the residual is to ascertain whether high volatility periods are followed by high volatility periods and low volatility periods are followed by low volatility for an extended length of time. The residual (error term) may be conditionally heteroscedastic and representable by the ARCH and GARCH models if this criterion is met.

ii. ARCH effect: The second (2nd) criterion for judging the suitability of the GARCH model is the ARCH effect. We define the null and alternative hypotheses in order to conduct this test. While the H1 states that there is an ARCH effect, the H0 states that there is none. The GARCH model is appropriate if the probability value of the chi-square is less than (<) 5%, in which case we reject H_0 and accept H_1.

Error Distributions

To further prove that modelling of the return series is inefficient with a Gaussian process for high frequency financial time series, equations 14, 15, 16 and 17 above are estimated with a normal distribution by maximizing the likelihood function

$$L(\theta_t) = {}^{-1}/_{2}\sum_{t-1}^{T}(ln2\pi + ln2\sigma_t^2 + \frac{\varepsilon_t^2}{\sigma_t^2}\ d\ 0\ otherwise (18)$$

σ_t^2 is specified in each of the GARCH models.

The assumption that GARCH models follow GED2 tends to account for the kurtosis in returns, which are not adequately captured with normality assumption. As in (18) above, the volatility models are estimated with GED by maximizing the likelihood function below:

$$L(\theta_t) = -\frac{1}{2}\log(\frac{\Gamma 1/_{v^3}}{\Gamma(3/v)({}^{v}/_{2}{}^{2})}) - \frac{1}{2}\log\sigma_t^2 - (\frac{({}^{v}/_{3})(y_t - \grave{X}_t\theta^2)^{v/_2}}{\sigma_t^2\Gamma(1/v)} \quad (19)$$

v is the shape parameter which accounts for the skewness of the returns and v > 0. The higher the value of v, the greater the weight of tail. GED reverts to normal distribution if v = 0. In the case of t distribution, the volatility models considered are estimated to maximize the likelihood function of a student's t distribution:

$$L(\theta_t) = -\frac{1}{2}\log(\frac{\pi(r)^{\Gamma r}/_{2^2})}{(\Gamma(\frac{r+1}{2}^2)} - \frac{1}{2}\log\sigma_t^2 - (1 + \frac{(y_t - \grave{X}_t\theta^2}{\sigma_t^2(r-2)} \quad (20)$$

Here, r is the degree of freedom and controls the tail behavior. r > 2.

RESULTS AND DISCUSSION

Data Preparation

This section focuses on the historical trends and patterns of stock returns for cotton and rubber in Nigeria from 1960 to 2022. Additionally, the section includes the results of the econometric modeling using GARCH models to capture and forecast the volatility of stock returns for these selected cash crops.

Descriptive Statistics

To provide a preliminary overview of the stock return data for cotton and rubber, descriptive statistics were calculated. Table1 presents the mean, median, standard deviation, and skewness for the stock returns of both crops. Cotton and Rubber exhibit differences in their mean returns, volatility, skewness, and kurtosis. Rubber shows higher volatility and less negative skewness compared to Cotton. The high positive kurtosis for both stocks suggest a distribution with heavy tails and potential for extreme returns.

Table 1: Descriptive Statistics of the stock Returns for Cotton and Rubber in Nigeria

Stocks	Cotton	Rubber
Mean	0.001779	0.001672
Standard Error	0.00223	0.002724
Median	0	-0.00032
Mode	0	0
Standard Deviation	0.061141	0.074705
Sample Variance	0.003738	0.005581
Kurtosis	96.34317	43.47891
Skewness	-5.64802	-2.98765
Range	1.222141	1.267513
Minimum	-1	-1
Maximum	0.222141	0.267513
Sum	1.337615	1.257266
Count	752	752

Historical Trends and Patterns of Stock Returns

To understand the historical trends and patterns of stock returns for cotton and rubber in Nigeria from 1960 to 2022, time-series analysis was conducted. Figures 1 and 2 display the graphical representation of the time plot and stock returns volatility for cotton and rubber, respectively, highlighting any discernible patterns, trends, or seasonality. Figure 1 reveals notable trends and fluctuations in the stock prices of cotton and rubber over the studied period. An initial examination of the graphical representations provides insights into potential correlations with historical events, economic shifts, and global market trends. Figure 2 shows the returns series presented from 1960 to 2022. The volatility of all stock prices clustered, taking both positive and negative values of varying magnitude. These movements in returns during the course of the study are a sign of stock series volatility. However, until a thorough statistical analysis is performed, a solid conclusion may not be derived from simply examining the plots.

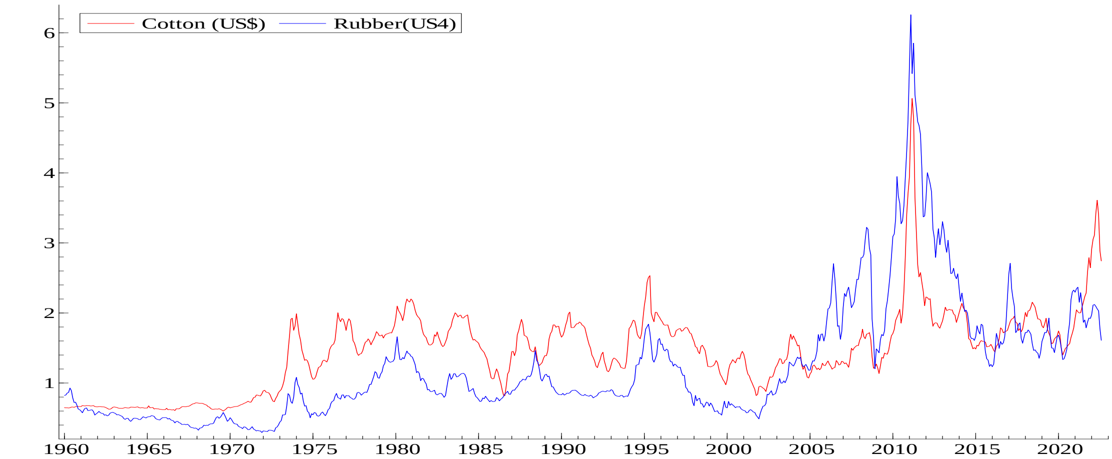


Figure 1: Time Plot of Monthly Stock Prices for Cotton and Rubber (1960-2022)

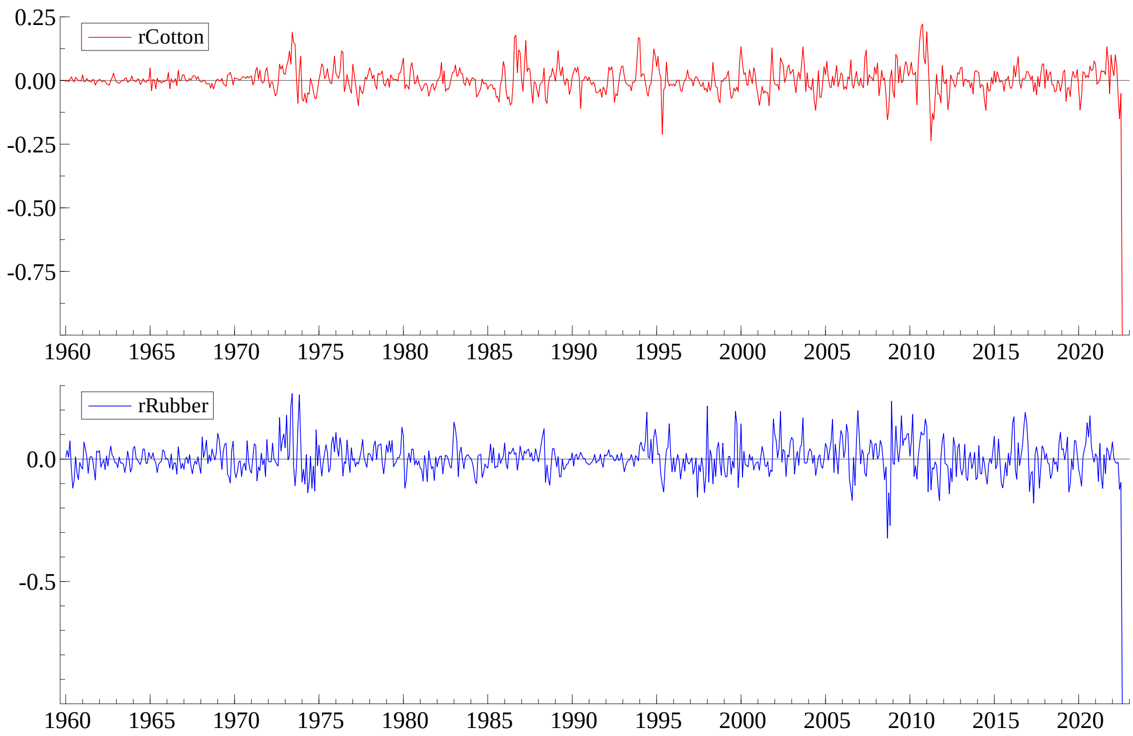


Figure 2: Time Plot of Stock Returns for Cotton and Rubber (1960-2022)

Test for Stationarity

Table 2a presents the results of the Augmented Dickey-Fuller (ADF) test statistic for the original data of Cotton and Rubber stock prices. There is evidence against the presence of a unit root in the original data for Cotton Stock Prices, but the evidence is weaker for Rubber Stock Prices. The results suggest that the Cotton stock prices are more likely to be stationary, while the Rubber stock prices may require further investigation or transformation to achieve stationarity. Table 2b presents the results of the Augmented Dickey-Fuller (ADF) test statistic for the first differences of Cotton and Rubber stock prices. The extremely negative ADF test statistics along with low p-values and the rejection of the null hypothesis at different significance levels indicate that the first differences of both Cotton and Rubber stock prices are stationary.

Table 2a: Augmented Dickey-Fuller Test Statistic for the Original Data

		Cotton Stock Price	Rubber Stock Price
		t-Statistic	t-Statistic
Augmented Dickey-Fuller test statistic		-3.563190	-2.858285
Test critical values:	1% level	-3.438854	-3.438854
	5% level	-2.865183	-2.865183
	10% level	-2.568766	-2.568766
	Prob.*	0.0067	0.0509

Table 2b: Augmented Dickey-Fuller Test Statistic at First Difference

		Cotton Stock Price	Rubber Stock Price
		t-Statistic	t-Statistic
Augmented Dickey-Fuller test statistic		-15.20709	-14.80851
Test critical values:	1% level	-3.438854	-3.438854
	5% level	-2.865183	-2.865183
	10% level	-2.568766	-2.568766
	Prob.*	0.0000	0.0000

ARCH Effect Test

The test statistics for all stock returns are extremely significant, according to Table 3. We agree that there is the presence of the ARCH effect in the residuals of the time series since p-values < 0.05 allow us to reject the null hypothesis of "no arch effect" at the 5% level. As a result, we can now proceed with the estimate of the GARCH family Model.

Table 3: Heteroskedasticity Test: ARCH

		Cotton	
F-statistic	143.4661	Prob. F(1,749)	0.0000
Obs*R-squared	120.7251	Prob. Chi-Square(1)	0.0000
		Rubber	
F-statistic	56.59312	Prob. F(1,749)	0.0000
Obs*R-squared	52.75794	Prob. Chi-Square(1)	0.0000

Parameter Estimation of the GARCH Models

The estimated parameters of the GARCH models for cotton and rubber, including coefficients for the lagged squared returns (ARCH terms) and lagged conditional variances (GARCH terms). The significance of these coefficients is assessed to determine the presence of volatility clustering and persistence. Table 4 presents the results of various GARCH (1.1) family models applied to the conditional variance equation for cotton stock prices. Each model includes coefficients for the mean equation (μ), the constant term (C), the ARCH term (β1), and the GARCH term (α1). The models considered are GARCH, EGARCH, IGARCH, and PARCH.

Exponential GARCH Model: In addition to the GARCH terms, there is a leverage effect (γ). Negative γ indicates a negative impact of past negative shocks on future volatility. The model has significant coefficients based on low p-values.

Integrated GARCH Model: The Positive coefficients for μ, β1, and α1 suggest positive relationships. The model has relatively high z-statistics and low p-values, indicating significance.

Power GARCH Model: Besides the GARCH terms, there are additional parameters, asymmetry (γ) and long memory (δ). The model includes a constant term (C), but it is not statistically significant based on the p-value. On overall considerations, the choice of the best model depends on the goals and characteristics of the data. The model with the lowest Information Criteria values is the IGARCH. The significance of coefficients and their economic interpretation should be carefully considered. This result provides insights into the conditional variance dynamics of cotton stock prices, considering different aspects such as asymmetry and long memory effects.

Table 5 presents the results of various GARCH (1.1) family models applied to the conditional variance equation for rubber stock prices.

Exponential GARCH Model: At the 5% level of significance (p-value < 0.05), all of the coefficients in the exponential GARCH model—μ, Constant, ARCH term, GARCH term, and γ—are statistically significant. Asymmetric responses to shocks are also shown by the relatively substantial leverage term (γ). The asymmetric volatility dynamics in rubber stock prices are well captured by the EGARCH model.

Integrated GARCH Model: At a 5% level of significance (p-value < 0.05), all coefficients (μ, ARCH term, and GARCH term) are statistically significant. The substantial coefficients for the ARCH and GARCH elements in the IGARCH model predict that rubber stock prices will continue to fluctuate.

Strength GARCH Model: At a 5% level of significance (p-value < 0.05), every coefficient (μ, Constant, ARCH term, GARCH term, γ, δ) is statistically significant.

Power GARCH Model: The PARCH model adds two further parameters (γ and δ) to account for the important impacts of long memory and asymmetry. It is evident that each model's parameters are important in explaining the conditional variation of rubber stock prices because they are generally significant. The leverage effect and extended memory are two additional dynamics introduced by the EGARCH and PARCH models that are statistically significant and add to the explanatory power of the models. In comparison to the other GARCH family models taken into consideration in this study, the IGARCH model has the lowest AIC, suggesting that it offers a comparatively better fit to the data on rubber stock prices.

Table 4: Results of the Conditional Variance Equation in the GARCH (1.1) Family Models on Cotton Stock Price

Models	Coefficient	Std. Error	z-Statistic	Prob.	Information Criterion	
					Akaike	Schwarz
Exponential GARCH						
μ	0.393084	0.568828	0.691043	0.4895		
Constant (C)	-0.688360	0.079414	-8.668024	0.0000		
ARCH term (β_1)	0.591526	0.042856	13.80260	0.0000	-3.474642	-3.443906
GARCH term (α_1)	-0.053231	0.024826	-2.144140	0.0320		
Γ	0.955721	0.008370	114.1832	0.0000		

Models	Coefficient	Std. Error	z-Statistic	Prob.	Information Criterion	
					Akaike	Schwarz
Integrated GARCH						
μ	0.985608	0.471943	2.088404	0.0368		
ARCH term (β_1)	0.161997	0.005037	32.16210	0.0000	-3.334282	-3.321988
GARCH term (α_1)	0.838003	0.005037	166.3728	0.0000		
Power GARCH						
μ	0.397949	0.549427	0.724298	0.4689		
Constant (C)	0.000663	0.000654	1.013765	0.3107		
ARCH term (β_1)	0.355912	0.032952	10.80108	0.0000	-3.475673	-3.438789
GARCH term (α_1)	0.062315	0.046924	1.328012	0.1842		
Γ	0.740522	0.021880	33.84434	0.0000		
δ	1.189822	0.232677	5.113632	0.0000		

Table 5: Results of the Conditional Variance Equation in the GARCH (1,1) Family Models on Rubber Stock Price

Models	Coefficient	Std. Error	z-Statistic	Prob.	Information Criterion	
					Akaike	Schwarz
Exponential GARCH						
μ	0.058656	0.503106	0.116589	0.9072		
Constant (C)	-1.257823	0.154459	-8.143435	0.0000		
ARCH term (β_1)	0.835132	0.045933	18.18162	0.0000	-2.694566	-2.663829
GARCH term (α_1)	-0.130294	0.030284	-4.302402	0.0000		
Γ	0.880285	0.023615	37.27731	0.0000		
Integrated GARCH						
μ	1.205351	0.357791	3.368874	0.0008		
ARCH term (β_1)	0.164829	0.006172	26.70576	0.0000	-2.497313	-2.485019
GARCH term (α_1)	0.835171	0.006172	135.3148	0.0000		
Power GARCH						
μ	0.179559	0.557533	0.322059	0.7474		
Constant (C)	0.000684	0.000721	0.948755	0.3427		
ARCH term (β_1)	0.541877	0.050110	10.81370	0.0000	-2.666919	-2.630035
GARCH term (α_1)	0.147654	0.041319	3.573508	0.0004		
Γ	0.559422	0.040637	13.76642	0.0000		
δ	1.798354	0.365344	4.922364	0.0000		

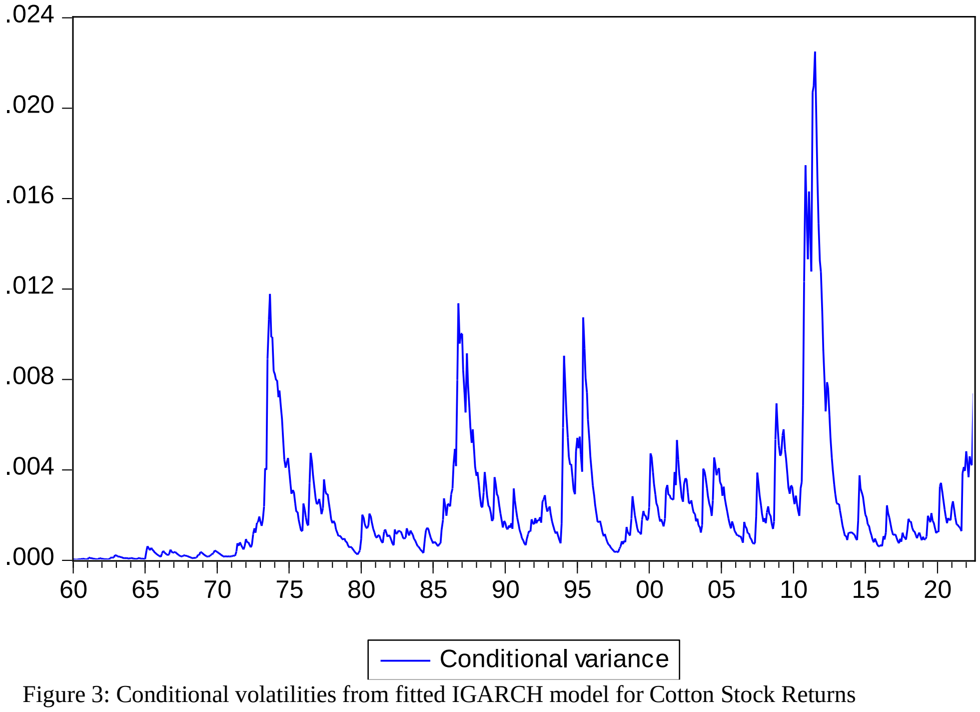


Figure 3: Conditional volatilities from fitted IGARCH model for Cotton Stock Returns

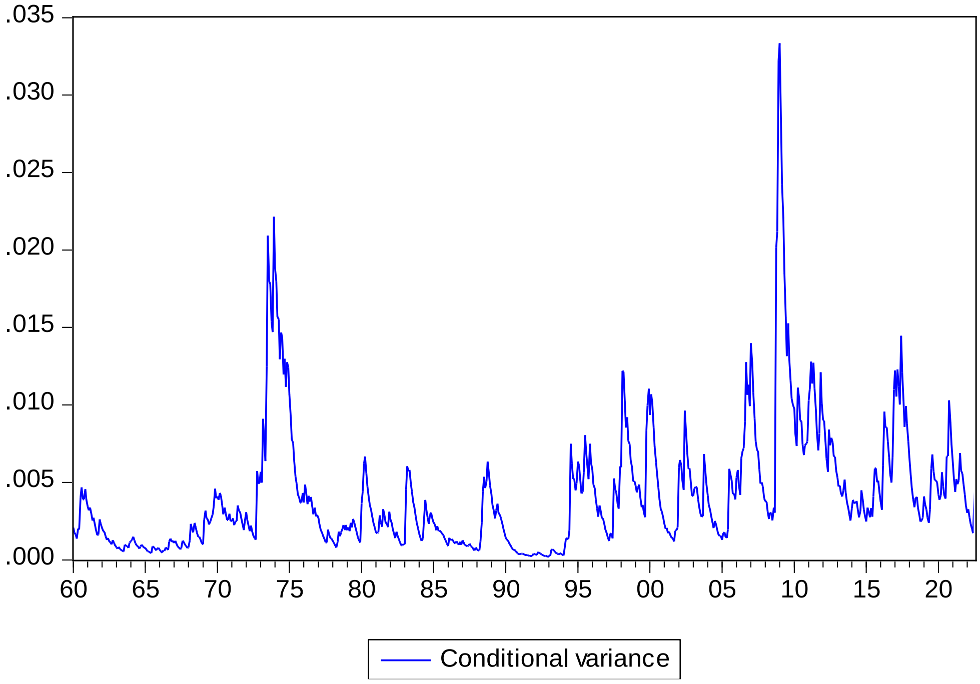


Figure 4: Conditional volatilities from fitted IGARCH model for Rubber Stock Returns

According to the plots of the conditional volatilities of the fitted GARCH models, shown in Figures 3 and 4, the volatility models chosen represent the main trends as well as periods of high and low equity returns.

Model Diagnostics

The adequacy of the GARCH models is evaluated through diagnostic tests, including the examination of residuals for autocorrelation and heteroskedasticity. These tests ensure the reliability of the models in capturing the volatility patterns of stock returns for cotton and rubber.

Table 6: Diagnostic Test for the Two Best Fitted GARCH Family Models

Heteroskedasticity Test: ARCH			
	IGARCH(1,1) Cotton Returns		
F-statistic	0.945070	Prob. F(1,749)	0.3313
Obs*R-squared	0.946399	Prob. Chi-Square(1)	0.3306
	IGARCH (1,1) Rubber Returns		
F-statistic	0.653396	Prob. F(1,749)	0.4192
Obs*R-squared	0.654570	Prob. Chi-Square(1)	0.4185

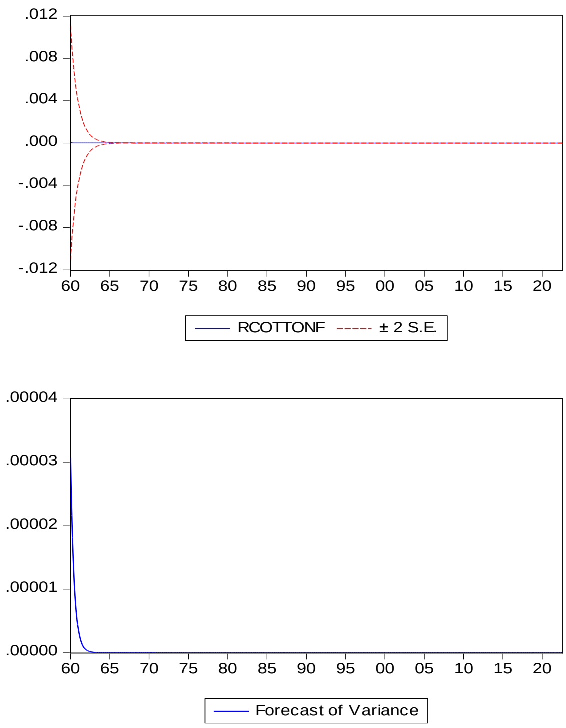
Based on the Chi-squared statistic, the null hypothesis that there is no residual ARCH impact in the models is not ruled out at the 5% level of significance. The estimated model's residuals' adherence to homoscedasticity is a sign of its strong fit. There is no serial correlation in the standardized residuals of the estimated models at the 5% significant level, according to the probability value of the Q-statistics in Table 7 for all Lags.

Table 7: Results of the Serial Correlation Tests for the Two Best Fit Volatility Models

Lag	**IGARCH (1,1) Cotton**				**IGARCH (1,1) Rubber**			
	AC	**PAC**	**Q-Stat**	**Prob***	**AC**	**PAC**	**Q-Stat**	**Prob***
1	0.016	0.016	0.1995	0.655	0.010	0.010	0.0737	0.786
2	0.014	0.013	0.3379	0.845	0.036	0.035	1.0268	0.598
3	-0.012	-0.013	0.4507	0.930	-0.009	-0.010	1.0869	0.780
4	-0.014	-0.014	0.6029	0.963	-0.008	-0.009	1.1401	0.888
5	-0.004	-0.004	0.6176	0.987	0.001	0.002	1.1408	0.950
6	-0.015	-0.014	0.7794	0.993	-0.011	-0.011	1.2395	0.975
7	-0.006	-0.005	0.8026	0.997	-0.003	-0.003	1.2479	0.990
8	0.002	0.002	0.8059	0.999	-0.006	-0.005	1.2751	0.996
9	-0.002	-0.002	0.8077	1.000	-0.007	-0.007	1.3164	0.998
10	-0.004	-0.005	0.8216	1.000	-0.010	-0.009	1.3858	0.999
11	0.054	0.054	3.0816	0.990	-0.005	-0.005	1.4061	1.000
12	-0.014	-0.016	3.2311	0.994	-0.005	-0.005	1.4266	1.000
13	0.013	0.012	3.3614	0.996	-0.011	-0.011	1.5160	1.000
14	-0.003	-0.001	3.3663	0.998	0.007	0.007	1.5501	1.000

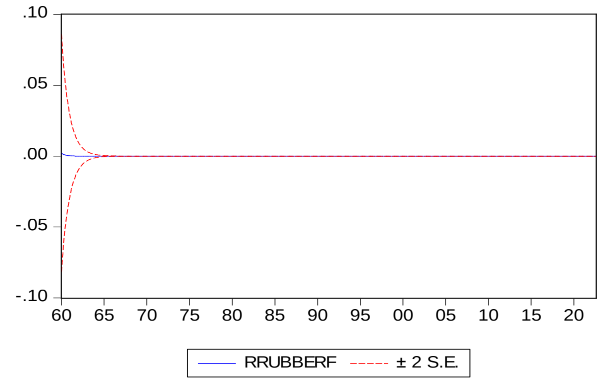
Lag	IGARCH (1,1) Cotton				IGARCH (1,1) Rubber			
	AC	PAC	Q-Stat	Prob*	AC	PAC	Q-Stat	Prob*
15	-0.010	-0.010	3.4497	0.999	0.000	0.001	1.5502	1.000
16	-0.007	-0.006	3.4825	1.000	0.002	0.001	1.5533	1.000
17	-0.001	0.001	3.4834	1.000	-0.002	-0.003	1.5570	1.000
18	-0.011	-0.011	3.5783	1.000	-0.007	-0.007	1.5938	1.000
19	0.001	0.001	3.5797	1.000	-0.009	-0.010	1.6633	1.000
20	0.012	0.013	3.6976	1.000	-0.006	-0.006	1.6942	1.000
21	-0.011	-0.011	3.7846	1.000	-0.001	-0.000	1.6947	1.000
22	-0.005	-0.008	3.8030	1.000	-0.009	-0.009	1.7545	1.000
23	0.001	0.004	3.8038	1.000	0.011	0.011	1.8528	1.000
24	-0.011	-0.013	3.8949	1.000	-0.005	-0.004	1.8690	1.000
25	-0.013	-0.013	4.0244	1.000	0.015	0.014	2.0547	1.000
26	0.005	0.007	4.0418	1.000	-0.005	-0.005	2.0740	1.000
27	-0.008	-0.008	4.0858	1.000	0.005	0.004	2.0905	1.000
28	-0.013	-0.014	4.2155	1.000	-0.007	-0.007	2.1279	1.000
29	-0.001	0.001	4.2159	1.000	0.007	0.007	2.1647	1.000
30	0.062	0.062	7.2709	1.000	-0.004	-0.005	2.1802	1.000
31	0.006	0.001	7.2984	1.000	-0.006	-0.007	2.2105	1.000
32	-0.004	-0.004	7.3096	1.000	-0.008	-0.008	2.2572	1.000
33	-0.016	-0.015	7.5195	1.000	-0.006	-0.005	2.2854	1.000
34	-0.009	-0.008	7.5879	1.000	-0.006	-0.006	2.3152	1.000
35	-0.008	-0.005	7.6338	1.000	-0.006	-0.005	2.3434	1.000
36	-0.012	-0.010	7.7503	1.000	-0.002	-0.001	2.3452	1.000

Forecasting
The GARCH models are employed for forecasting future stock returns volatility for cotton and rubber. This involves generating volatility forecasts based on the estimated parameters of the models. The forecasting process provides insights into potential risks and uncertainties associated with the selected cash crops in the coming periods.



Forecast: RCOTTONF	
Actual: RCOTTON	
Forecast sample: 1960M01 2022M08	
Included observations: 752	
Root Mean Squared Error	0.061126
Mean Absolute Error	0.035418
Mean Abs. Percent Error	98.93822
Theil Inequality Coefficient	0.999967
Bias Proportion	0.000847
Variance Proportion	0.999088
Covariance Proportion	0.000066

Figure 5: Forecast for Cotton Stock Returns



Forecast: RRUBBERF	
Actual: RRUBBER	
Forecast sample: 1960M01 2022M08	
Included observations: 752	
Root Mean Squared Error	0.074676
Mean Absolute Error	0.048419
Mean Abs. Percent Error	99.65822
Theil Inequality Coefficient	0.998142
Bias Proportion	0.000491
Variance Proportion	0.995714
Covariance Proportion	0.003795

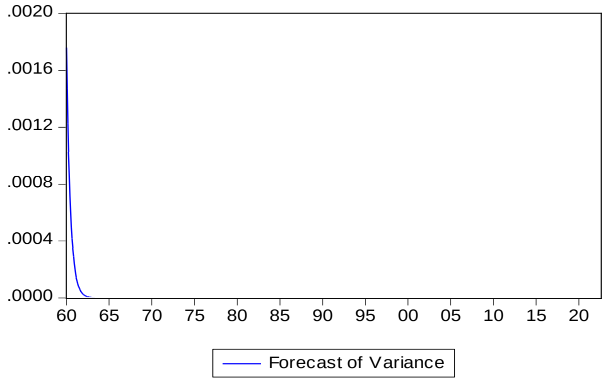


Figure 6: Forecast for Rubber Stock Returns

Discussion of Findings

The chapter ends with an overview of the empirical results, emphasizing significant patterns in the historical stock returns of rubber and cotton as well as how well the GARCH models capture volatility. The results of analyzing the conditional variance equation for the two stock prices using GARCH (1.1) family models, with a particular emphasis on GARCH, EGARCH, IGARCH, and Power GARCH models. High z-statistics and low p-values imply statistical significance, while positive coefficients for the mean, constant, ARCH, and GARCH components show positive connections for the GARCH model. The leverage effect (γ), which is introduced by the EGARCH model, indicates that previous negative shocks have a negative effect on cotton's future volatility. The model's capacity to capture asymmetry is highlighted by significant coefficients. With positive coefficients for the mean, ARCH, and GARCH factors, the IGARCH model shows importance and reflects ongoing volatility. Asymmetry (γ) and long memory (δ) factors are included in the PARCH model; however, the constant term is not statistically significant for Cotton. It is determined that the IGARCH model, which has the lowest Information Criteria values, offers a comparatively superior fit for both stocks. The importance of coefficients and how to interpret them economically, however, need careful thought. All things considered, the findings offer insightful information about the conditional variance dynamics of stock prices, taking into account elements such as asymmetry and long memory effects to improve comprehension of market volatility and the forecasting results' implications for stakeholders in the agricultural industry, investors, and policymakers.

CONCLUSION

This study provides a thorough analysis of the volatility of cotton and rubber stock returns whose dynamics are effectively captured by the GARCH models, providing stakeholders with a useful tool for navigating market uncertainties. The study concludes by highlighting the necessity of flexible tactics and a sophisticated approach to risk management. The report offers information that can help stakeholders in agriculture, investors, and politicians make wise choices. The results advance both academic understanding and real-world applications in the agriculture industry. The study suggests that scholars concentrate on, the use of GARCH (1,1) family models and the IGARCH model's superiority in terms of Information Criteria.

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