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APPLICATION OF ELZAKI TRANSFORM TO THE ANALYTICAL SOLUTION OF STIFF LINEAR SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS

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ABSTRACT

Stiff systems of ordinary differential equations (ODEs) are often difficult to solve because of their rapid changes and sensitivity to small variations in conditions. This study explores how the Elzaki transform can be used solve these types of equations analytically, offering an alternative to the traditional matrix method, which usually involves complex operations or exponentiation. By applying the Elzaki transform to the stiff system ODEs, we were able to find exact solutions and compare them with those from the matrix method. The outcome showed that both methods produce the same results, but the Elzaki transform approach is simpler and requires less computation. Elzaki transform proves to be a reliable and efficient method for solving stiff linear ODEs, especially in cases where traditional methods become too complicated or unstable. The Elzaki transform is a relatively new integral transform that is still not widely known, nor used.

Keywords: Elzaki transform, Stiff-ODE, Matrix, System of Differential Equation, Exact solution

INTRODUCTION

Stiff differential equations involve components that decay at vastly different rates, which often makes numerical solutions difficult. The Elzaki transform can be applied to solve systems of stiff ordinary differential equations by converting them into algebraic equations in the transform domain. This approach can be more efficient than traditional numerical methods, especially for stiff system where small-time steps are required to maintain stability (mathWorks). The Elzaki transform, being a new integral transform, offers advantages in solving differential equations and engineering control problems. Differential Equation plays important role in Engineering and Applied Sciences.

Triag and Salih (2011) worked on the Elzaki transform and ordinary differential equations. Nurettin (2012) applied the combine Laplace transform-Adomian decomposition method to solve differential equation system. The results obtained are in good agreement with exact solutions. Uvarova (2015) presented the properties of solutions to a system of ordinary differential equations of higher dimension by considering a class of systems of differential equations of dimension showing that if the number of equations is sufficiently large, the last component of the solution can be approximated by solutions to delay differential equations. Awari and Kumleng (2017) developed a self-starting family of three and five step continuous extended trapezoidal rule of second kind a block hybrid type method through interpolation and collocation for numerical solution of system of initial value problems for ordinary differential equations. Amurawaye *et. al* (2023a) demonstrated the use of the Aboodh transform method in solving linear stiff ordinary differential equations compared solution with Laplace transform solution.

Amurawaye *et. al* (2023b) considered the application of Aboodh transform to solution of nth order differential equations and compared results with Laplace Transform. Uncu and Cimen (2024) presented a novel approximation on the solution of systems of ordinary differential equations by constructing a fitted difference scheme using the finite method and improving the first order convergence for the method in the discrete maximum norm. Zabidi *et. al* (2014) presented the two - point block one-step method for solving stiff ODE. The proposed block method is A-stable and the order is three. The solutions are obtained simultaneously in block form and produce approximate solutions using a constant step size. Yatim *et. al* (2013) introduced an advanced method using block backward differentiation formula (BBDF) with efficient strategy in choosing the step size and order of the method. Pradip and Kirtiwant (2016) used the Elzaki transform to solve system of homogeneous and nonhomogeneous linear differential equations of first order and first degree with constant coefficients and satisfying some initial conditions. The aim of the study is to demonstrate the Elzaki transform as an efficient method in providing exact solution to stiff linear system ODE.

MATERIALS AND METHODS

Elzaki Tarig (2011) introduced the Elzaki transform defined by the formular:

$$E[f(t),v] = T(v) = v^2\int_0^\infty f(vt)e^{-t}dt \quad ,k_1 \le v \le k_2$$

and has the property:

$$E\left(\frac{d^n f(t)}{dt^n}\right)(v) = v^2\left[\frac{T(u)}{v^n} - \sum_{k=0}^{n-1}\frac{f^{(k)}(0)}{v^{n-k}}\right]$$

Elzaki Transform of Some Important Function

$E(1) = v^2$

$E(t) = v^3$

$E(t^n) = n!\, v^{n+2}$

$E\left(\frac{t^{b-1}}{\Gamma(b)}\right), b > 0 \ = v^{b+1}$

$$E(e^{bt}) = \frac{v^2}{1-bv}$$

$$E(te^{bt}) = \frac{v^3}{(1-bv)^2}$$

$$E\left(\frac{t^{n-1}e^{bt}}{(n-1)!}\right), \quad n = 1,2,3, \dots = \frac{v^{n+1}}{(1-bv)^n}$$

$$E(\sin bt) = \frac{bv^3}{1+b^2v^2}$$

$$E(\cos bt) = \frac{v^2}{1+b^2v^2}$$

$$E(\sinh bt) = \frac{bv^3}{1-b^2v^2}$$

$$E(\cosh bt) = \frac{bv^2}{1-b^2v^2}$$
$$E(e^{bt}\cos bt) = \frac{(1-bv)v^2}{(1-bv)^2+b^2v^2}$$
$$E(e^{bt}\sin bt) = \frac{bv^3}{(1-bv)^2+b^2v^2}$$
$$E(t\sin bt) = \frac{2vb^4}{1+b^2v^2}$$

System of Stiff First Order Ordinary Differential Equations

Consider the system

$$y_1' = a_1y_1 + a_2y_2 + c_1(x) \quad (1)$$
$$y_2' = b_1y_1 + b_2y_2 + c_2(x) \quad (2)$$

With the initial value $y_1(0) = k_1 and \ y_2(0) = k_2$

Where a_1, a_2, b_1, b_2 are constants. Rewriting the system of Stiff differential equation above as:

$Y' = \begin{pmatrix} y_1' \\ y_2' \end{pmatrix}$, $Y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$, $A = \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix}$

$C(x) = \begin{bmatrix} c_1(x) \\ c_2(x) \end{bmatrix}$ and $Y_0 = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$

The above system can be written in a matrix differential system as:

$$Y' = Ay + C(x), \quad y(0) = y_o \quad (3)$$

Taking the Elzaki transform of equation (3) with the initial condition, we have

$$E(Y') = E(Ay) + E[C(x)]$$
$$E(y_1') = E(a_1y_1) + E(a_2y_2) + E[c_1(x)]$$
$$E(y_2') = E(b_1y_1) + E(b_2y_2) + E[c_2(x)]$$
$$\frac{1}{v}E(y_1) - vy_1(0) = a_1E(y_1) + a_2E(y_2) + E[c_1(x)]$$
$$\frac{1}{v}E(y_2) - vy_2(0) = b_1E(y_1) + b_2E(y_2) + E[c_2(x)]$$

Multiply through by v and introducing the initial condition

$$E(y_1) - v^2k_1 = a_1vE(y_1) + a_2vE(y_2) + vE[c_1(x)] \quad (4a)$$
$$E(y_2) - v^2k_2 = b_1vE(y_1) + b_2vE(y_2) + vE[c_2(x)] \quad (4b)$$

Putting equation 4(a) and 4(b) in matrix form, we have

$$\begin{pmatrix} 1-a_1v & -a_2v \\ -b_1v & 1-b_2v \end{pmatrix}\begin{bmatrix} E(y_1) \\ E(y_2) \end{bmatrix} = \begin{bmatrix} v^2k_1 + vA[c_1(x)] \\ v^2k_2 + vE[c_2(x)] \end{bmatrix}$$

$$/A/ = \begin{vmatrix} 1-a_1v & -a_2v \\ -b_1v & 1-b_2v \end{vmatrix} = 1-(b_2+a_1)v + (a_1b_2 - -a_2b_1)v^2 \quad (5)$$

$$|Ay_1(x)| = \begin{vmatrix} v^2k_1 + vE[c_1(x)] & -a_2v \\ v^2k_2 + E[c_2(x)] & 1-b_2v \end{vmatrix}$$
$$= (a_2k_2 - b_2k_1)\ v^3 + vk_1 + (a_2E[c_2(x)] - b_2E[c_1(x)])v^2 + vE[c_1(x)] \quad (6)$$

$$|Ay_2(x)| = \begin{vmatrix} 1-a_1v & v^2k_1 + vE[c_1(x)] \\ -b_1v & v^2k_2 + vE[c_2(x)] \end{vmatrix}$$
$$= (b_1k_1 - a_1k_2)\ v^3 + v^2k_2 + (b_1E[c_1(x)] - a_1E[c_2(x)])v^2 + vE[c_2(x)] \quad (7)$$

$$E[y_1(x)] = \frac{|Ay_1(x)|}{|A|} = \frac{(a_2k_2 - b_2k_1)\,v^3 + v^2k_1 + (a_2E[c_2(x)] - b_2E[c_1(x)])v^2 + vE[c_1(x)]}{1-(b_2+a_1)v + (a_1b_2 - a_2b_1)v^2} \quad (8$$

$$E[y_2(x)] = \frac{|Ay_2(x)|}{|A|} = \frac{(b_1k_1 - a_1k_2)\,v^3 + v^2k_2 + (b_1E[c_1(x)] - a_1E[c_2(x)])v^2 + vE[c_2(x)]}{1-(b_2+a_1)v + (a_1b_2 - a_2b_1)v^2} \quad (9)$$

Substituting $c_1(x) = 0$ and $c_2(x) = 0$; we have:

$$E[y_1(x)] = \frac{(a_2k_2 - b_2k_1)\,v^3 + v^2k_1]}{1-(b_2+a_1)v + (a_1b_2 - a_2b_1)v^2} \quad (10)$$
$$E[y_2(x)] = \frac{(b_1k_1 - a_1k_2)\,v^3 + v^2k_2}{1-(b_2+a_1)v + (a_1b_2 - a_2b_1)v^2} \quad (11)$$

Equation (10) and (11) give the Elzaki transform for $y_1(x)$ and $y_2(x)$

In this section, we apply the Elzaki transform to provide exact solution to the system of Stiff differential equation.

RESULTS AND DISCUSSION

Example 1

Consider a stiff linear system of the form:

$$y_1' = -20y_1 - 19y_2, \quad y_1(0) = 2 \quad (12)$$
$$y_2' = -19y_1 - 20y_2, \quad y_2(0) = 0 \quad (13)$$

using Elzaki transform of equation (10) and (11)

$a_1 = -20$, $a_2 = -19$, $b_1 = -19$, $b_2 = -20$, $k_1 = 2$, $k_2 = 0$

Substituting the above values, we have

$$E[y_1(x)] = \frac{40v^3+2v^2}{1+40v+39v^2} = \frac{v^2}{39v+1} + \frac{v^2}{v+1}$$
$$E[y_2(x)] = \frac{-38v^3}{1+40v+39v^2} = \frac{v^2}{39v+1} - \frac{v^2}{v+1}$$

Taking the inverse Elzaki transform, we have

$y_1(x) = e^{-39x} + e^{-x}$

$y_2(x) = e^{-39x} - e^{-x}$

Using Matrix method for example 1

$$y_1' = -20y_1 - 19y_2, \quad y_1(0) = 2$$
$$y_2' = -19y_1 - 20y_2, \quad y_2(0) = 0$$

$0 \leq x \leq 20$

The system of equations above can be written in matrix form.

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} -20 & -19 \\ -19 & -20 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad \text{i.e} \quad y' = Ay$$

The characteristics equation $/A - \lambda I/ = 0$. Eigenvalues of A are $\lambda_1 = -1$ and $\lambda_2 = -39$.

The system has eigenvalues with significantly different magnitudes, making it stiff. The corresponding eigenvectors are the non-trival solution P of the solution.

$(A - \lambda_t I)P_t = 0$, $t = 1, 2, 3, \ldots$

when $t = 1$, $(A - \lambda_1 I)P_1 = 0$ and $\lambda_1 = -1$

$P_1 = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \rightarrow$ eigenvector

when $t = 2$, $(A - \lambda_2 I)P_2 = 0$ and $\lambda_2 = -39$

$P_2 = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow$ eigenvector

$y = c_1p_1e^{\lambda_1 x} + c_2p_2e^{\lambda_2 x}$

The general solution is therefore

$$y(x) = c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-x} + c_2\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{-39x}$$

Applying the initial value to find

$$y(0) = c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-x} + c_2\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{-39x} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

Then our $c_1 = 1$ and $c_2 = 1$

Consequently, the particular solution is

$$y(x) = 1\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-x} + 1\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-39x}$$

Then,

$y_1(x) = e^{-x} + e^{-39x}$ and $y_2(x) = -e^{x} + e^{-39x}$

Example 2

Consider a stiff linear system of the form:

$$y_1' = -100y_1 + 9.901y_2 \quad y_1(0) = 1 \quad (14)$$
$$y_2' = 0.1y_1 - y_2, \quad y_2(0) = 0 \quad (15)$$

From the system, $a_1 = -100$, $a_2 = 9.901$, $b_1 = 0.1$, $b_2 = -1$, $k_1 = 1$ and $k_2 = 10$

Substituting into the Elzaki transform of Equation (10) and (11), we have

$$E[y_1(x)] = \frac{100.01v^3+v^2}{99.0099v^2+101v+1} = \frac{v^2}{0.99v+1}$$
$$E[y_1(x)] = \frac{1000.01v^3+10v^2}{99.0099v^2+101v+1} = \frac{10v^2}{0.99v+1}$$

Taking the inverse Elzaki transform, we have

$y_1(x)= e^{-0.99x}$ and $y_2(x) = 10e^{-0.99x}$

Using Matrix method

$y_1' = -100y_1 + 9.901y_2$, $y_1(0) = 1$

$y_2' = 0.1y_1 - y_2$, $y_2(0) = 10$ $0\le x \le 10$

The system of equations above can be written in matrix form as:

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} -100 & 9.901 \\ 0.1 & -1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad \text{i.e} \quad y' = Ay$$

The characteristic equation /A - λI/ = 0 gives $\lambda_1 = -0.99$ and $\lambda_2 = -100.01$, which are the eigenvalues of A. The corresponding eigenvectors are the non-trival solution P of the system.

$(A - \lambda t)P_t = 0$, $t = 1, 2, 3, \dots$

when t = 1, $(A - \lambda_1 I)P_1 = 0$ and $\lambda_1 = -0.99$

$P_1 = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 10 \end{pmatrix} \rightarrow$ eigenvectors

when t = 2, $(A - \lambda_2 I)P_2 = 0$ and $\lambda_2 = -100.01$

$P_2 = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -9901 \\ 10 \end{pmatrix} \rightarrow$ eigenvector

$y(x) = c_1 p_1 e^{\lambda_1 x} + c_2 p_2 e^{\lambda_2 x}$

The general solution is therefore

$$y(x) = c_1 \begin{pmatrix} 1 \\ 10 \end{pmatrix} e^{-0.99x} + c_2 \begin{pmatrix} -9901 \\ 10 \end{pmatrix} e^{-100.01x}$$

Applying the initial value to find

$$y(0) = 1 \begin{pmatrix} 1 \\ 10 \end{pmatrix} e^{-0.99x} + c_2 \begin{pmatrix} -9901 \\ 10 \end{pmatrix} e^{-100.01x} = \begin{pmatrix} 1 \\ 10 \end{pmatrix}$$

Then our $c_1 = 1$ and $c_2 = 0$

Consequently, the particular solution is

$$y(x) = 1 \begin{pmatrix} 1 \\ 10 \end{pmatrix} e^{-0.99x} + 0 \begin{pmatrix} -9901 \\ 10 \end{pmatrix} e^{-100.01x}$$

Then,

$y_1(x) = e^{-0.99x}$ and $y_2(x) = 10e^{-0.99x}$

Example 3

Consider a stiff linear system of the form:

$y_1' = 198y_1 + 199y_2$, $y_1(0) = 1$ (16)

$y_2' = -398y_1 - 399y_2$, $y_2(0) = 0$ (17)

$0\le x \le 10$

Using Elzaki transform

$$E[y_1(x)] = \frac{(a_2k_2 - b_2k_1)v^3 + v^2k_1}{1 - (b_2 + a_1)v + (a_1b_2 - a_2b_1)v^2}$$

$$E[y_2(x)] = \frac{(b_1k_1 - a_1k_2)v^3 + v^2k_2}{1 - (b_2 + a_1)v + (a_1b_2 - a_2b_1)v^2}$$

$a_1 = 198, a_2 = 199, b_1 = -398$,

$b_2 = -399, k_1 = 1$ and $k_2 = -1$

Substituting the above values, we have

$$E[y_1(x)] = \frac{v^2 + 200v^3}{1+201v+200v^2} = \frac{v^2}{v+1}$$

$$E[y_2(x)] = \frac{-v^2 - 200v^3}{1+201v+200v^2} = \frac{-v^2}{v+1}$$

Taking the inverse Elzaki transform, we have

$y_1(x) = e^{-x}$ and $y_2(x) = -e^{-x}$

Using Matrix method

$y_1' = 198y_1 + 199y_2$, $y_1(0) = 1$

$y_2' = -398y_1 - 399y_2$, $y_2(0) = -1$

$0\le x \le 10$

The system of equations above can be written in matrix form as:

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 198 & 199 \\ -398 & -399 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

Characteristics equation is /A - λI/ = 0. We have our $\lambda_1 = -1$ and $\lambda_2 = -200$, which are the eigenvalues of A. The corresponding eigenvectors are the non-trival solution P of the system.

$(A - \lambda_t I)P_t = 0$, $t = 1, 2, 3, \dots$

when t = 1, $(A - \lambda_1 I)P_1 = 0$ and $\lambda_1 = -1$

$P_1 = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \rightarrow$ eigenvectors

when t = 2, $(A - \lambda_2 I)P_2 = 0$ and $\lambda_2 = -200$

$P_2 = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \rightarrow$ eigenvector

$y(x) = c_1 p_1 e^{\lambda_1 x} + c_2 p_2 e^{\lambda_2 x}$

The general solution is therefore

$$y(x) = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-x} + c_2 \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{-200x}$$

Applying the initial value to find

$$y(0) = c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-x} + c_2 \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{-200x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Then our $c_1 = -1$ and $c_2 = 0$

Consequently, the particular solution is

$$y(x) = -1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-x} + 0 \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{-200x}$$

Therefore,

$c_1(x) = e^{-x}$ and $c_2(x) = -e^{-x}$

Example 4

Let consider a stiff linear system of the form:

1. $y_1' = -80.6y_1 + 119.4y_2, y_1(0) = 0$ (18)

$y_2' = 79.6y_1 - 120.4y_2$, $y_2(0) = 1$ (19)

$0\le x \le 10$

Using Elzaki transform

$$E[y_1(x)] = \frac{(a_2k_2 - b_2k_1)v^3 + v^2k_1}{1 - (b_2 + a_1)v + (a_1b_2 - a_2b_1)v^2}$$

$$E[y_2(x)] = \frac{(b_1k_1 - a_1k_2)v^3 + v^2k_2}{1 - (b_2 + a_1)v + (a_1b_2 - a_2b_1)v^2}$$

$a_1 = -80.6$, $a_2 = 119.4$, $b_1 = 79.6$,

$b_2 = -120.4$, $k_1 = 0$ and $k_2 = 1$

Substituting the above values, we have

$$E[y_1(x)] = \frac{119.4v^3}{1+201v+200v^2} = \frac{3}{5}\left(\frac{v^2}{v+1}\right) - \frac{3}{5}\left(\frac{v^2}{200v+1}\right)$$

$$E[y_2(x)] = \frac{v^2 + 80.6v^3}{1+201v+200v^2} = \frac{2}{5}\left(\frac{v^2}{v+1}\right) + \frac{3}{5}\left(\frac{v^2}{200v+1}\right)$$

Taking the inverse Elzaki transform, we have

$y_1(x) = \frac{3}{5}e^{-x} - \frac{3}{5}e^{-200x}$ and $y_2(x) = \frac{2}{5}e^{-x} + \frac{3}{5}e^{-200x}$

Using Matrix method

$y_1' = -80.6y_1 + 119.4y_2$, $y_1(0) = 0$

$y_2' = 79.6y_1 - 120.4y_2$, $y_2(0) = 1$

$0\le x \le 10$

The system of equations above can be written in matrix form as:

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} -80.6 & 119.4 \\ 79.6 & -120.4 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

Characteristics equation is /A - λI/ = 0. We have our $\lambda_1 = -1$ and $\lambda_2 = -200$, which are the eigenvalues of A. The corresponding eigenvectors are the non-trivial solution P of the system.

$(A - \lambda_t I)P_t = 0$, $t = 1, 2, 3, \dots$

when t = 1, $(A - \lambda_1 I)P_1 = 0$ and $\lambda_1 = -1$

$P_1 = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \rightarrow$ eigenvector

when t = 2, $(A - \lambda_2 I)P_2 = 0$ and $\lambda_2 = -200$

$P_2 = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \rightarrow$ eigenvector

$y(x) = c_1 p_1 e^{\lambda_1 x} + c_2 p_2 e^{\lambda_2 x}$

The general solution is therefore

$$y(x) = c_1 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{-x} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-200x}$$

Applying the initial condition

$$y(0) = c_1 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{-x} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-200x} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Then our $c_1 = \frac{1}{5}$ and $c_2 = \frac{3}{5}$

Consequently, the particular solution is

$$y(x) = \frac{1}{5}\begin{pmatrix}3\\2\end{pmatrix} e^{-x} + \frac{3}{5}\begin{pmatrix}-1\\1\end{pmatrix} e^{-200x}$$

$$y_1(x) = \frac{3}{5} e^{-x} - \frac{3}{5} e^{-200x}$$

$$y_2(x) = \frac{2}{5} e^{-x} + \frac{3}{5} e^{-200x}$$

Discussion

The Elzaki transform gave the same accurate solutions as the matrix method but was much simpler to use. It avoids difficult calculations like matrix inversion, making it easier to handle more complex or large systems. It also offers clearer view of how the system behaves, without the numerical instability often seen in matrix-based approaches. Overall, it is a reliable and efficient alternative for solving stiff differential equations, especially when exact solutions are needed.

CONCLUSION

This study looked at how the Elzaki Transform can help solve stiff linear differential equations and compared it to the usual matrix method. After testing both approaches on example problems, we found that the Elzaki transform gave the same correct answers but was much easier to work with. It avoided complex matrix operations and made the solution process easier to follow, especially for more challenging systems.

Based on these findings, we conclude that the Elzaki transform is not only a valid alternative to the matrix method but a more user - friendly and efficient option for solving linear stiff system ODEs in analytical form.

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