FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645 - 2944
Vol. 9 No. 6, June, 2025, pp 182 – 185
DOI: https://doi.org/10.33003/fjs-2025-0906-3546

SOME GRAPH PROPERTIES OF Γ1- NONDERANGED PERMUTATION GRAPHS

***[1]Sefinat Bola Jaiyeola, [2]Abdulazeez Suleiman, [2]Ahmad Rufai Tasiu, [2]Rahinat Ahmad and [2]Kazeem Olalekan Aremu**

[1]Department of Physical Sciences, Faculty of Natural and Applied Sciences, Al-Hikmah University, Ilorin, Kwara State, Nigeria.
[2]Department of Mathematics, Usmanu Danfodiyo University Sokoto. P. O. Box, 2346, Sokoto State, Nigeria

*Corresponding authors' email: sbjaiyeola@alhikmah.edu.ng

ABSTRACT

Γ_1-nonderanged permutation ($G_p^{\Gamma_1}$) consists of permutations obtained from a modulo p function (p is prime and $p \geq 5$), and the permutation graphs of these permutations exhibit interesting graph properties which were investigated in this work. The permutation graph of every $\omega_i \in G_p^{\Gamma_1}$ is simple and ω_{p-1} adjacency and path matrices coincide. Furthermore, it was shown that the graph union of the permutation graph without the vertex v_1 and its inverse without the vertex v_1 is a complete graph.

Keywords: Permutation graph, Graph operations, Graph matrix, Graph distance

INTRODUCTION

Permutation graphs are graphs whose vertices represent the element of a permutation and whose edge represent pairs of element that are reversed by the permutation. Permutation graph may also be defined geometrically, as the intersection graph of the line segment whose endpoint are on two parallel lines. Permutation graphs were first introduced by Chartrand and Harary in 1967. Subsequently, (Pnueli et al., 1971) gave a different definition of permutation graph which he defined as a simple graph on n vertices, say$[n] = \{1, 2, 3, \ldots, n\}$, which is isomorphic to the graph G_π on vertices $[n]$, associated with a given permutation $\pi = \pi(1)\ \pi(2)\ldots\pi(n)$ by joining a pair of vertices i and j if $(i-j)(\pi^{-1}(i)$-$\pi^{-1}(j))$. There are various notable research paper on permutation graph. In the work of Chartrand et al. (1971) they established that a graph is outer plannar if and only if it contains no subgraph homeomorphic to K_4 or $K_{2,3}$. Furthermore (Koh & Ree, 2005) extended the notion of permutation graphs by determining whether a given labelled graph is a permutation graph or not and when a graph is a permutation graph. With no dispute, permutation and graph are combinatorial structures. The connectedness of graphs were also studied and it was discovered that almost all permutations are asymptotically connected if uniformly chosen at random (Koh & Ree, 2007). As such, graph of permutation have been obtained. However, the study of graph on specific families of permutation are yet to be studied. Recently Aminu (2016) suggested a permutation group called Γ_1 - nonderanged permutation group denoted as $G_p^{\Gamma_1}$. A permutation in this group is expressed as a sequence of 1 and numbers of integer modulo p where p is prime and $p \geq 5$.

A new method of constructing permutation group from existing one through a composition operation was established by Suleiman et al. (2020) then Garba et al. (2021) employed a combinatorial process to establish the intransitivity of a non-deranged permutation group. Aremu et al. (2023) observed that not all element of $\Gamma 1$-non deranged permutation group generate the elements of the group therefore concluded that the generating set is a proper subset of $G_p^{\Gamma_1}$ and are not unique and also the undirected Cayley graphs of $\Gamma 1$-non deranged permutation group is not simple. Other researchers such as Yusuf & Ejima, (2023), Yusuf & Umar, (2025) have also worked in this area. Precisely, the focus of this work will be on the Γ1- nonderanged permutation as the notion of graph has not been studied on this set of permutations. In this work, we investigated the structure of permutation graphs of$\omega_i \in G_p^{\Gamma_1}$. Some graph operations, graph distance and graph matrix on Γ1- nonderanged permutation.

A permutation ω on the set $\{\tau_1, \tau_2, \ldots, \tau_n\}$ is a sequence of distinct letters $\omega(1), \omega(2), \ldots, \omega(n)$ such that $\omega_i \in \{\tau_1, \tau_2, \ldots, \tau_n\}and\omega$has letter n. We say thatω is a Γ1-nonderanged permutation if $\omega = \omega_1, \omega_2, \ldots, \omega_n$ is of the form $\omega_i = (1+i)_{mp}$, where $n = p$ and p is a prime and $p \geq 5$. A graph G is a pair *(V,E)*, where *V*is a finite set of vertices and *E* is the set of edges not necessary non-empty. A loop in *G* is an edge in *G* that connect a vertex to itself. Two vertices u and w of a G are adjacent if there is an edge uw joining them. For any vertex u in a G, the degree of the vertex is the number of edge incident with vertex u. A vertex v∈ G, is said to be an isolated vertex if the degree of v is zero. The order of a graph G is the number of vertex in the graph. The size of the graph is the number of edges in the graph. A path is a U-V walk in which vertices are repeated but edges are not repeated. Cycle in a graph is a path such that no vertices and edges is repeated and it start and end with the same vertex. A graph$G_1=(V_1,E_1)$ is a subgraph$G_2=(V_2,E_2)$ where $V_2 \subseteq V_1$ and $E_2 \subseteq E_1$. A graph is said to be connected, if for any two vertices u; v, there exist a path between vertex u and v. Otherwise the graph is disconnected. A complete graph is a graph in which each pair of distinct vertices are adjacent. A graph G is said to be a null graph if the edge set is empty and is denoted as N_n. A graph G is a regular graph if all the vertices in G has equal number of degree.

Γ1-Nonderanged Permutation Graph

Definition 1: (Γ_1-Nonderanged Permutation)

One line notation of Γ1-nonderanged permutation is giving permutation by

$\omega_i = (1\ (1+i)_{mp}\ \ (1+2i)_{mp}\ \ \ldots(1+(p-1)i)_{mp}$

Where $p \geq 5$, which is extended by Aremu et al. (2017b) to two line notation given by

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & \ldots & p \\ 1 & (1+i)_{mp} & (1+2i)_{mp} & \ldots & (1+(p-1)i)_{mp} \end{pmatrix}$$

Such that $\omega_i \in G_p^{\Gamma_1}$, for $\mathrm{i} = 1, 2, 3, \ldots, \mathrm{p}-1$

Example 1: Let $\omega_i \in G_5^{\Gamma_1}$ and let $i = 2$

the one line notation of $\omega_2 = 1\ 3\ 5\ 2\ 4$ and two line notation of ω_2 is

$\omega_2 = \begin{pmatrix} 12345 \\ 13524 \end{pmatrix}$

Definition 2: (Permutation graph)
If $\pi = \pi(1)\,\pi(2)\,\pi(3)\ldots\pi(n)$is any permutation of the number 1 to n, then one may define a permutation graph from πin which there are edges v_i, v_j for any two indices i and j for which i < j andπ(i) >π(j). That is two indices i and j determine an edge in the permutation graph exactly when they determine an inversion in the permutationπ.

Example 2: Let $\omega_2 \in G_5^{\Gamma_1}$
$\omega_2 = \begin{pmatrix} 12345 \\ 13524 \end{pmatrix}$
The vertex set: V(G_{ω_2})= {1, 2, 3, 4, 5} and The edge set: E(G_{ω_2}) = {(2,3),(2,5),(4, 5)}.

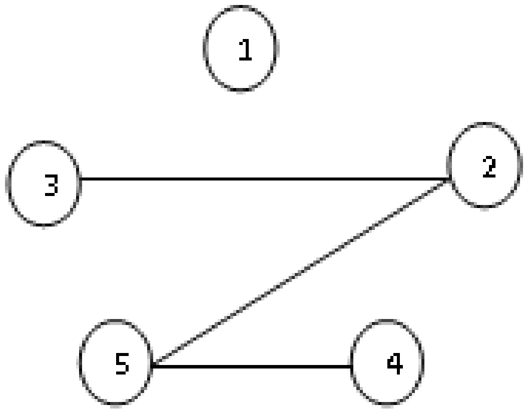

Figure 1: G_ω (the permutation graph of ω_2)

From Figure 1 above, we have that all Γ1-nonderanged permutation graph have an isolated vertex because of the fix 1 in Γ1-nonderanged permutation.

Definition 3: A connected graph *H* is a component of a graph *G*, if it is not a proper subgraph of any connected subgraph of *G*.

Proposition 1: Let G_{ω_i} and $G_{\omega_{p-1}}$ be Γ1-nonderanged permutation graph of $\omega_i$and ω_{p-1} respectively, then

1. $G_{\omega_i} - \{v_1\}$, where $i \neq 1$ is a connected graph.
2. $G_{\omega_{p-1}} - \{v_1\}$is complete, connected and regular graph.
3. $G_{\omega_i}$is a null graph, if i = 1.

Proposition 2: LetG_{ω_i}be Γ1-nonderanged permutation graph of ω_i, then every Γ1-nonderanged permutation graph of ω_i, where $i \neq 1$ has two connected component, that is *c(G)* = 2.

Proof. Since all the permutation graph of Γ1-nonderanged permutation has an isolated vertex and there exist a path between each and every vertex remaining in the graph, then this shows us that every Γ1 non-deranged permutation graph has 2 connected component.

Proposition 3: Let $G_{\omega_i}$and $G_{\omega_{p-i}}$ be Γ1-nonderanged permutation graph of $\omega_i$and ω_{p-1} respectively, then the permutation graph of$G_{\omega_i}$and $G_{\omega_{p-i}}$has no common edges.
Proof. $\omega_i$and $\omega_{p-i}$of Γ1-nonderanged permutation are inverse of each other and it is immediate to see it in their permutation graph structure that they have no common edges.

Graph Operations on Γ1-Nonderanged Permutation Graph
In this section we give some results on graph operations

Definition 4: Let V(G) and E(G) be the vertex set and edge set of a simple graph respectively, then the graph isomorphism from a simple graph G to a simple graph H is a bijection f:V(G) →V(H) such that u, v ∈ E(G) if and only if f(u) ∘ f(v) ∈ E(H).

Definition 5: Let G be a simple graph, it complement G is the graph of the same vertex set such that two vertices are adjacent in G if and only if they are not adjacent in G.
Definition 6: A graph G is called a tree if it is a connected graph without a tree.

Definition 7: A subgraph H of a graph G is a spanning subgraph if V (H) = V (G)
and V (H) ⊆ V (G).

Definition 8: A matching is a set of edges without common vertices.

***Definition 9*:** A matching number is the number of edge of the maximal matching.

Proposition 4: Let $G_{\omega_i}$and $\boldsymbol{G_{\omega_{p-i}}}$ be Γ1-nonderanged permutation graph of $\omega_i$and ω_{p-i} respectively, then

$$\boldsymbol{G_{\omega_i} \bigcup G_{\omega_{p-i}} = G_{\omega_{p-1}}}$$

Proof. By Proposition 3, since G_{ω_i} and $G_{\omega_{p-i}}$has no common edges, therefore the graph union gives the permutation graph $G_{\omega_{p-1}}$.

Proposition 5: Let G_{ω_i} be Γ1-nonderanged permutation graph of ω_i and $G_{\omega_{p-1}} - \{v_1\}$ denote permutation induced subgraph of ω_{p-1}, then

$$\boldsymbol{G_{\omega_{p-1}} - \{v_1\} \cong K_{p-1}}$$

Proof. A complete graph Kn has $\frac{n(n-1)}{2}$ edges and since G_{ω_i}has p vertices, then by Proposition 1 is a complete graph with p-1 vertices. The number
of edges of $G_{\omega_{p-1}} - \{v_1\}$ is $\frac{(p-1)(p-2)}{2}$, then the permutation graph $G_{\omega_{p-1}} - \{v_1\}$is isomorphic to$K_{p-1}$.

Proposition 6: Let G_{ω_i} and $G_{\omega_{p-i}}$ be Γ1-nonderanged permutation graph of ω_i and ω_{p-i} respectively, then
$(\boldsymbol{G_{\omega_i} \cup G_{\omega_{p-i}}}) - \{\boldsymbol{v_1}\} \cong \boldsymbol{K_{p-1}}$

Proof. Suppose $G_{\omega_i} - \{v_1\}$ and $G_{\omega_{p-1}} - \{v_1\}$ denote the permutation induced subgraph of G_{ω_i} and $G_{\omega_{p-i}}$ respectively, such that
$(\boldsymbol{G_{\omega_i}} - \{\boldsymbol{v_1}\})(\boldsymbol{G_{\omega_{p-i}}} - \{\boldsymbol{v_1}\}) = (\boldsymbol{G_{\omega_i} \cup G_{\omega_{p-i}}}) - \{\boldsymbol{v_1}\}$
$= \boldsymbol{G_{\omega_{p-1}}} - \{\boldsymbol{v_1}\}\ (Proposition\ 4.)$
$= \boldsymbol{K_{p-1}}\ (Proposition\ 5.)$

Proposition 7: Let G_π be Γ1-nonderanged permutation graph of π, such thatthee is a fix point $\pi(1)$ in π, then the graph complement $\overline{G}$has no isolated vertex.
Proof. Suppose that G_π and G_σ are permutation graph of π andσrespectively, then G_π= $G_\sigma$where $\sigma(i) = \pi(n - i + 1)$, for $i = 1, 2, 3 \ldots n$such that

$$\pi = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ \pi(1) & \pi(2) & \pi(3) & \cdots & \pi(n) \end{pmatrix}\ and\ \sigma = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ \sigma(1) & \sigma(2) & \sigma(3) & \cdots & \sigma(n) \end{pmatrix}$$

$$\overline{\pi} = \sigma = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ \pi(n) & \pi(n-1) & \pi(n-2) & \cdots & \pi(1) \end{pmatrix}$$

Where $\sigma = \pi(n), \sigma(2) = \pi(n-1), \sigma(2) = \pi(n-2), \ldots, \sigma(n) = \pi(1)$, that is, the permutation complement graph $\overline{G}$ will have no isolated vertex.

Proposition 8: Let $G_{\omega_{p-1}}$ be Γ1-nonderanged permutation graph of ω_{p-1}, then the graph complement of permutation graph $\overline{G_{\omega_{p-1}}}$ is a tree.
Proof. Since there exist an isolated vertex in permutation graph of $G_{\omega_{p-1}}$, then by the permutation inversion of it complement graph $\overline{G_{\omega_{p-1}}}$, has every vertex adjacent to only the isolated vertex in $G_{\omega_{p-1}}$, then $\overline{G_{\omega_{p-1}}}$ is a tree.

Proposition 9: Let G_π be a permutation graph of π, if $\pi(1)$ and π(n) is fix, then the permutation graph will have two isolated vertices, then it complement graph has no isolated vertices.

Proof. It follows from the proof of Proposition 7. the complement graph $\overline{G_{\omega_i}}$will have no isolated vertex.

Proposition 10: Let G_{ω_i} and $G_{\omega_{p-1}}$ be Γ1-nonderanged permutation graph of$\omega_i and\ \omega_{p-1}$ respectively, then for every permutation graph of $G_{\omega_i}$is a spanning subgraph of the permutation graph $G_{\omega_{p-1}}$.
Proof. The graph order $|V(G_{\omega_i})|$and$|V(G_{\omega_{p-1}})|$ are equal and the graph size $|E(G_{\omega_i})|$is less than the graph size of $|E(G_{\omega_{p-1}})|$.

Proposition 11: Let G_{ω_i} be Γ1-nonderanged permutation graph of ω_i where $1 < i \leq p - 1$ and $\alpha'(G_{\omega_i})$denote the matching number of the permutation graphG_{ω_i}, then

$$\boldsymbol{\alpha'(G_{\omega_i}) = \frac{p-1}{2}}$$

Proof. Suppose G_{ω_i} be Γ1-nonderanged permutation graph of ω_i and $G_{\omega_i} - \{v_1\}$denote the permutation induced subgraph of G_{ω_i}, since $G_{\omega_i} - \{v_1\}$is connected graph, then the order will be $G_{\omega_i} - \{v_1\} = p - 1$ and half of $p - 1$ gives the matching number of the permutation graph G_{ω_i} for every *i*.

Graph Distance of Γ1-Nonderanged Permutation Graph
Definition 10: Let G be a graph, the distance between two vertex u and v length
of the shortest possible path .If G is a connected graph then the following holds.

i. $d(u, v) \geq 0, \forall u, v \in V.$
ii. $d(u, v) = 0\ if\ and\ only\ if\ u = v,\ \forall u, v \in V.$
iii. $d(u, v) = d(v, u),\ \forall u, v \in V.$
iv. $d(u, v) + d(v, w) \geq d(u, w), \forall u, v \in V.$

Definition 11: The eccentricity of a vertex in a graph G is the distance from v to a vertex farthest from v.
$ecc(v) = \max\{d(u, v)\}, where\ u, v \in V$ (Harary & Buckley, 1994, p. 32)

Definition 12: The radius of a graph *G* denoted by *rad(G)* is minimum eccentricity.
$rad(G) = \min\{ecc(v)\}, where\ v \in V$

Definition 13: A diameter of a graph *G* denoted by *diam(G)* is the maximum of a vertex eccentricity in *G*.
$diam(G) = \max\{ecc(v)\}, where\ v \in V$

Definition 14: The circumference of a graph is the length of any longest cycle in a graph

Definition 15: The girth of a graph is the length of any shortest cycle in a graph

Proposition 12: Let G_{ω_i} be Γ1-nonderanged permutation graph of ω_i and let$rad(G_{\omega_i})$denote the graph radius ofG_{ω_i}, then
$\boldsymbol{rad(G_{\omega_i}) = 0}$
Proof. All Γ1-nonderanged permutation graph has an isolated vertex, then the graph radius of the permutation graph G_{ω_i} is 0.

Proposition 13: Let G_{ω_i} be Γ1-nonderanged permutation graph of ω_i, where $1 < i < p - 1$ and let $diam(G_{\omega_i})$ denote the graph diameter of G_{ω_i} , then
$\boldsymbol{diam(G_{\omega_i}) = 3}$

Proposition 14: Let $G_{\omega_{p-1}}$ be Γ1-nonderanged permutation graph and let $circum(G_{\omega_{p-1}})$ denote the graph circumference of the permutation graph $G_{\omega_{p-1}}$, then
$\boldsymbol{circum(G_{\omega_{p-1}}) = p - 1}$
Proof. The graph circumference of a complete graph K_n is n, then the graph circumference of $G_{\omega_{p-1}}$is $p - 1$.

Corollary 1: The graph circumference of $G_{\omega_{p-1}}$ is equal to the cardinality of Γ1-nonderanged permutation group of *p*, that is $\boldsymbol{circum(G_{\omega_{p-1}}) = |G_p^{\Gamma_1}|}$.

Proposition 15: Let $G_{\omega_{p-1}}$ be Γ1 non-deranged permutation graph and let $g(G_{\omega_{p-1}})$
denote the graph girth of the permutation graph $G_{\omega_{p-1}}$, then
$\boldsymbol{g(G_{\omega_{p-1}}) = 3}$
Proof. There exist a cycle C_n, where n ≥ 3 in $G_{\omega_{p-1}} - \{v_1\}$, then the graph girth of

permutation graph $G_{\omega_{p-1}}$.

Graph Matrix of Γ1-Nonderanged Permutation Graph

Definition 16: The adjacency matrix $A = [a_{ij}]$ of a graph G is a square matrix given by

$$a_{ij} = \begin{cases} 1, & if\ v_i\ is\ adjacent\ to\ v_j \\ 0, & otherwise \end{cases}$$

Definition 17: The graph sum of a graph *G* and *H*is the graph with adjacency matrix given by the sum of adjacency matrix of *G* and *H*. (Jonathan & Jay, 2005)

Definition 18: The path matrix $P = [p_{ij}]$ of a graph G is a square matrix given by

$$a_{ij} = \begin{cases} 1, & if\ there\ is\ a\ path\ v_i\ to\ v_j \\ 0, & otherwise \end{cases}$$

Proposition 16: Let G_{ω_i} and $G_{\omega_{p-i}}$ be Γ1-nonderanged permutation graph of $\omega_i$and ω_{p-i} respectively, let $A(G_{\omega_i})$ and $A(G_{\omega_{p-i}})$ denote the adjacency matrix of $G_{\omega_i}$and $G_{\omega_{p-i}}$ respectively, then

$$A(G_{\omega_i}) + A(G_{\omega_{p-i}}) = A(G_{\omega_{p-1}})$$

Proof.

$$A(G_{\omega_i}) + A(G_{\omega_{p-i}}) = A(G_{\omega_i} \cup G_{\omega_{p-i}})$$
$$= A(G_{\omega_{p-1}})\ \ (Proposition\ 4.)$$

Proposition 17: Let $\boldsymbol{G_{\omega_i}}$ be Γ1-nonderanged permutation graph of ω_i,$P(G_{\omega_i})$ and $\chi\{P(G_{\omega_i})\}$ denote the path matrix and the character of path matrix of permutation graph $\boldsymbol{G_{\omega_i}}$, then

$$\boldsymbol{\chi\{P(G_{\omega_i})\} = 0}$$

Proof. Since all the Γ1-nonderanged permutation graphs are simple graph, there exist no loop, then the character of Γ1-nonderanged permutation graph path matrix is zero.

Proposition 18: Let $G_{\omega_{p-1}}$ be Γ1-nonderanged involution permutation graph of ω_{p-1}, let $A(G_{\omega_{p-1}})$ and $P(G_{\omega_{p-1}})$ denote the adjacency and path matrix of the permutation graph $G_{\omega_{p-1}}$, then

$$A(G_{\omega_{p-1}}) = P(G_{\omega_{p-1}})$$

Proof. Since every complete graph is a connected graph, then the $A(G_{\omega_{p-1}})$ is equal with the $P(G_{\omega_{p-1}})$.

REFERENCES

Aremu, K. O., Issa, F. A., Muhammad. A.. Tasiu, A. R. & Muhammed I. M. (2023). On Generators and Undirected Cayley graphs of *Γ*1-Non-Deranged Permutation Groups. *International Journal of Science for Global Sustainability*, 9(2), 108-112. https://doi.org/10.57233/ijsgs.v9i2.466

Aminu, I.A., Ojonugwa, E. And Aremu, K.O. (2016). On the Representations of Γ1-Nonderanged Permutation Group, *Advances in pure mathematics,* 6(9), 608-614. https://doi.org/10.4236/amp.2016.69049

Aremu, K.O.,Ejima O. & Abdullahi M.S.(2017b). On the Fuzzy Γ1-Non deranged Permutation Group G_p $^{\Gamma 1}$. *Asian Journal of Mathematics and Computer Research*,18(4), 152-157. https://ikprress.org/index.php/AJOMCOR/article/view/1060

Chartrand, G. & Harary, F. (1967). Planar Permutation Graph. *Ann. Ins. Henri Poincare*, 3(4), 433-438. https://www.numdam.org/item?id=AIHPB_1967__3_4_433_0

Chartrand, G., Geller, D. & Hedetniemi, S. (1971). Graph with Forbidden Subgraphs, *Journal of Combinatorial Theory, Series B*, 10, 12-41. https://cdn.isr.umich.edu/pubFiles/historicPublications/Graphswithforbiddensubgraphs_2632.PDF

Garba, A.I., Haruna, M., Maryam, S. & Suleiman (2021). Counting the Orbits of **Γ1**− Non-deranged permutation group. *Academic Journal of Statistics and Mathematics*, 7(11), 1-7. https://www.cirdjournal.com/index.php/ajsm

Harary, F. & Buckley, F. (1994). Distance in Graph. Redwood city, CA: Addison Wesley. https://www.emgywomenscollege.ac.in

Jonathan, L. G. & Jay, Y. (2005). Graph Theory and its Applications. Chapman and Hall,

FL:CRC Press. https://doi.org/10.1201/9781420057140

Koh, Y. & Ree, S. (2005). Determination of permutation Graph. *Honam* Mathematical Journal, 27(2), 183-194. https://www.researchgate.net/publication/266939196

Koh, Y. & Ree, S. (2007). Connected Permutation Graph. *Discrete Mathematics*, *307*(21), 2628-2635. https://doi.org/10.1016/j.disc.2006.11.014

Pnueli, A., Lempel, A. & Even, S. (1971). Transitive Orientation of Graphs and Identification of Permutation Graph. *Canadian Journal of Mathematics*. 23, (1), 160-175. https://doi.org/10.4153/cjm-1971-016-5

Suleiman I., Garba A.I, & Mustafa A. (2020). On Some Non-Deranged Permutation: A New Method of Construction. *International Journal of Granthaalayah*, 8(3), 309-314. https://doi.org/10.29121/granthaalayah.v8.i3.2020.162

Yusuf, A. & Ejima, O. (2023). Some Properties of Extended Γ1-Nonderanged Permutation Group. *Federal University Dutsinma Journal of Sciences (FUDMA)*, 7(3), 332-336. https://doi.org/10.33003/fjs-2023-0703-2031

Yusuf, A. & Umar, A., (2025). On Power Graph Representation of **Γ1**-nonderanged Permutation Group, *UMYU Scientifica*, 4(1), 53-61. https://doi.org/10.56919/usci.2541.006