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MODELING THE EFFECT OF BLOOD MASS FLOW AND METABOLIC RATE DURING PHYSICAL EXERCISE WITH A CONSTANT THERMAL CONDUCTIVITY

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ABSTRACT

Engaging in physical exercise is crucial for maintaining a healthy life, and those who participate in regular workouts typically experience improved health and a better quality of life. As a person engages in physical activities, their metabolism shifts in response to the intensity of the exercise, displaying different patterns depending on the specific activities involved. At the onset of exercise, the metabolic rate rises sharply, and after some time, it gradually levels off to remain relatively stable. By altering the existing mathematical model regulating physical activity, this study aims to model the effects of blood mass flow and metabolic rate during physical exercise with a constant thermal conductivity. The Olayiwola Generalized Polynomial approximation approach is used to solve the governing problem and provide an analytical solution for the suggested model. The results indicate that an increase in activity level causes the rate of metabolism to increase or decrease over time, as well as increase over the specified distance. As this trend continue until the exercise approaches to the stopage time, the tissue temperature profile θ(x,t) at time t and along the distance x is enhanced.

Keywords: Physical exercise, Thermal conductivity, Blood mass flow, Metabolic rate, OGPAM

INTRODUCTION

It is highly recommended to engage in physical activity in order to keep a healthier lifestyle. Being physically active helps us to effectively manage certain unique health issues in cases of Non-Communicable Diseases (NCD), including cardiovascular issues, type B diabetes, obesity, depression, anxiety, burning off exercise calories, and many more. When the body is at rest, any activity that occurs can be classified as a life-sustaining function, or simply put, a rest metabolic rest (RMR), according to the 2020 World Health Physical Activity Guideline for Sedentary Behavior. According to Encyclopedia of Science Clarified (2013), an increase in ones level of physical activity helps prevent chronic diseases so you live healthier and enjoy greater quality of life.

When a person participates in different forms of physical activity, the body demands more fuel, which raises the metabolic rate and the rate of heat generation. In order to maintain a healthy internal temperature, the body must employ additional methods to eliminate the extra heat created. Even in the same environment, various persons have varied thermal behavior (Shrestha et al., 2020). At least 150 minutes a week of moderate-intensity activity such as brisk walking or at least 75 minutes a week of vigorous-intensity activity such as hiking, jogging, or running. Also at least 2 days a week of activities that strengthens muscles should be observe. Hargreaves and Spriet (2023) wrote on the skeletal muscle energy metabolism during exercise and discovered that for most events at Olympics, carbohydrate is the primary fuel for anaerobic and aerobic metabolism. Garcia et al. (2023) wrote on teaching physical fitness and exercise using computer-assisted instruction: a school-based public health intervention and results shows that empirical evidence has demonstrated the benefits of physical activity in preventing chronic diseases and premature death. Unfortunately, there is a global trend of insufficient physical activity, which was aggravated by the recent global pandemic. Although physical education is often used to promote physical activity, the transition to online education made it difficult to teach fitness and exercise from a distance due to several limiting factors. Another finding is a study that looked at individual variances and how the body regulates its temperature during physical activity. It was discovered that intense exercise causes the body to produce a lot of heat energy, which raises body temperature considerably. As a result, vigorous activity can produce more than 1000 watts of heat, raising the body's core temperature by a few degrees (Chen, 2023).

Shrestha (2020) asserts that the temperature differential between the skin and the surrounding environment determines the total amount of heat loss from the human body. Normal blood flow occurs when the body is at rest, however during physical activity, this flow becomes extremely abnormal. Kljajević et al. (2022) investigated in a similar narrative that schools are a perfect place to undertake interventions that encourage adequate physical activity. Our goal is to find out how computer-assisted instruction (CAI) can be used to teach and carry out these activities instead of using traditional pedagogies. Using a mathematical modeling technique, Saidu et al. (2024) examined the impact of temperature-dependent thermal conductivity during physical activity. The results indicate that there is a minor increase in thermal conductivity during periodic exercise, which subsequently declines at time t and at the specified distance x. Consequently, the tissue temperature profile $\theta(x,t)$ at time t and along the specified distance x is reduced. Sun et al. (2024) conducted a study titled Physical education and student well-being: Promoting health and fitness in schools. The study's findings demonstrated that health education had a major influence on Chinese schoolchildren's psychological health. The study also showed that involvement in sports and sustainable health activity play a crucial moderating effect in the association between psychological wellness and health education. Similar to this, Saidu et al. (2025) investigate how physical activity can mimic the effects of an exponential rise in blood mass flow and metabolic rate. There results indicate that the tissue temperature profile $\theta(x,t)$ at time t and along distance x is improved by an increase in blood mass flow m_0 and metabolic rate s_0 during physical exercise. The long term impact of thermoregulation on human sleep was modeled by

Banuelosa et al. (2021) who founds out that higher activity level can cause increased weariness, which inturn improves sleep. Additionally, Guthold et al. (2020) noted that there are notable disparities in physical activity levels between countries and regions, as well as between higher and lower income classes and that girls and women are generally less active than boys and males.

Kwaghkor et al. (2022) conducted a research on the mathematical model for Diabetes management, results obtained shows that when physical exercise is combined with recommended dietary in the management of diabetes, the excess glucose, insulin and epinephrine concentrations returns to their normal level faster with time compared to when only physical exercise is used as a control measure.

Exercise conducted in a hot, humid atmosphere might put str ess on the body's regulatory system, according to Rowland (2008). For competitive athletes and active individuals, the effective dispersal of the heat load generated by contracting muscle is of considerable importance. Any failure in the control mechanisms to effectively remove body heat during such stressed exercise would result in substantial decrease in physical performance of the athletes while posing risk for eventual circulatory collapse, brain dysfunction, and generalized organ failure. According to Siregar (2016), the amount of electrolytes lost through perspiration must be taken into account while administering electrolytes. These requirements, however, are contingent upon the length and level of training, as well as the temperature and surrounding circumstances. The aim of this study is to model the effect of blood mass flow and metabolic rate during physical exercise with a constant thermal conductivity and also to provide analytical solution to the proposed model utilizing Olayiwola's generalize polynomial approximation method (OGPAM).

MATERIALS AND METHODS

Model Modification

Following the work of Saidu et al. (2024) describing the effect of temperature dependent thermal conductivity the mathematical modeling approach thus:

$$\left.\begin{aligned}\frac{\partial\theta}{\partial t} &= \varepsilon\frac{\partial}{\partial x}\left((1+\sigma\theta)\frac{\partial\theta}{\partial x}\right) + m_0\cos\left(\pi t()(\gamma-\theta)\left(s_0+\frac{E-s_0}{1+e^{-\beta(t-t_{si}}\ )}\right)\right)\\ \theta(x,0) &= 1,\ \frac{\partial\theta}{\partial x}|_{x=0} - \gamma_1\theta = \gamma_2 LE, \theta|_{x=1} = 1\end{aligned}\right\} \tag{1}$$

Where,

$\theta =$ dimensionless temperature, $\sigma =$ dimensionless thermal conductivity, $E =$ rate of sweat evaporation, $L =$ latent heat of evaporation $t =$ time taking during exercise, $x =$ distance covered during exercise, $m_0 =$ initial value of blood mass flow rate before the exercise commences, $s_0 =$ initial value of metabolism before the exercise commences, $\gamma =$ radius of the skin the dermis, $\gamma_1 =$ radius of the skin at the epidermis, $\gamma_2 =$ radius of the skin at the subcutaneous tissue and $\varepsilon =$ diffusion term .

The periodic increase in blood mass flow rate during physical exercise is given as:

$$m(t) = m_0\cos(\pi t) \tag{2}$$

And, the logistical increase in metabolic rate during physical exercise is given as:

$$s(\,t) = s_0 + \frac{E-s_0}{1+e^{-\beta(t-t_{si})}} \tag{3}$$

When thermal conductivity is constant, equation (1) together with the intitial and boundary conditions reduces to the form:

$$\frac{\partial\theta}{\partial t} = \varepsilon\frac{\partial^2\theta}{\partial x^2} + m_0\cos\left(\pi t()(\gamma-\theta)\left(s_0+\frac{E-s_0}{1+e^{-\beta(t-t_{si})}}\right)\right)$$
$$\theta(x,0) = 1,\ \frac{\partial\theta}{\partial x}|_{x=0} - \gamma_1\theta = \gamma_2 LE, \theta|_{x=1} = 1. \tag{4}$$

The above equation (4) is the proposed model describing: simulation of the effect of blood mass flow and metabolic rate during physical exercise with a constant thermal conductivity

Methodology

Olayiwola's Generalized Polynomial Approximation Method (OGPAM, 2022)

OGPAM has been selected as the solution method for the given model equation. GPAM, a polynomial approximation methods, be to parabolic equations involving constant variable when slab, cylindrical, spherical geometries are taken into consideration. One of the most straightforward and, in some situations, precise techniques for resolving parabolic equations is the Generalized polynomial approximation method developed by Olayiwola. There are five steps in the procedure.

The steps are applied to the dimensionless governing equation as shown below:

The parabolic equations are defined by

$$\frac{\partial\varphi}{\partial t} = \frac{k}{r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial\varphi}{\partial r}\right) + F(r,t,\varphi), t>0, r\in\Omega(\Omega\subset R^1, R^2 or R^3) \tag{5}$$

with the initial condition

$$\varphi(r,0) = f(r) \tag{6}$$

and the boundary conditions

$$\alpha_1\frac{\partial\varphi}{\partial r}\Big|_{r=a} + \beta_1\varphi|_{r=a} = g_1(t), \alpha_2\frac{\partial\varphi}{\partial r}\Big|_{r=b} + \beta_2\varphi|_{r=b} = g_2(t) \tag{7}$$

where $F(r,t,\varphi)$ are the other terms in equation (4), $n=0$ for a slab, $n=1$ for cylinder, $n=2$ for sphere, $f(r), g_1(t), g_2(t)$ are three known functions, $n, k, \alpha_1, \alpha_2, \beta_1, \beta_2$ are constants, and $a\le r\le b$ is the boundary of Ω.

Method of Solution

Now consider equation (4) and compare with equation equation (5) - (7), we have

$$k=\varepsilon, n=0, \phi=\theta_0, f(r)=1, r=x,$$
$$f(r,t,\phi) = (\gamma-\theta_0)\,m_0\cos(\pi t) + \left(s_0+\frac{E-s_0}{1+e^{-\beta(t-t_{si})}}\right),$$
$$\alpha_1=1, \beta_1=-\gamma_1, g_1(t)=\gamma_2 LE, \alpha_2=0, \beta_2=1, g_2(t)=1, a=0, b=1. \tag{8}$$

Then,

$$A = \left(\frac{2}{3}+\frac{1}{6}\gamma_1\right), B=\frac{1}{6}, C=\frac{1}{6} \tag{9}$$

Where A, B and C are arbitrary constants obtained from the method of solution

The assume Olayiwola's Generalized Polynomial Solution of the form equation (5) is

$$\theta_0(x,t) = \theta_0|_{x=0} + (\gamma_1\theta_0|_{x=0}+\gamma_2 LE)x + (1-\gamma_2 LE-(1+\gamma_1)\theta_0|_{x=0})x^2 \tag{10}$$

Then,

$$\int_0^1 \left(\begin{array}{l} m_0 \cos(\pi t)\left(\gamma - \theta_0\big|_{x=0} - \left(\gamma_1\theta_0\big|_{x=0} + \gamma_2 LE\right)x - \left(1-\gamma_2 LE - (1+\gamma_1)\theta_0\big|_{x=0}\right)x^2\right) + \\ \left(\left(\left(s_0 + \frac{1}{2}(E-s_0)\right)\right) + \frac{\beta(E-s_0)}{4}(t-t_{si})\right) \end{array} \right) dx$$

$$= m_0 \cos\pi t\left(\gamma - \theta_0\big|_{x=0}\right)x\Big|_0^1 - \frac{1}{2}m_0\cos\pi t\left(\gamma_1\theta_0\big|_{x=0} + \gamma_2 LE\right)x^2\Big|_0^1 - \frac{1}{3}m_0\cos\pi t\left(1-\gamma_2 LE - (1+\gamma_1)\theta_0\big|_{x=0}\right)x^3\Big|_0^1 +$$

$$\left(\left(s_0 + \frac{1}{2}(E-s_0) - \frac{1}{4}\beta(E-s_0)t_{si}\right) + \frac{1}{4}\beta(E-s_0)t\right)x\Big|_0^1$$

$$= -\left(\frac{2}{3} + \frac{1}{6}\gamma_1\right)m_0\cos\pi t\theta_0\big|_{x=0} + \left(\gamma - \frac{1}{3} + \frac{1}{6}\gamma_2 LE\right)m_0\cos\pi t + \left(s_0 + \frac{1}{2}(E-s_0) - \frac{1}{4}\beta(E-s_0)t_{si}\right) +$$

$$\frac{1}{4}\beta(E-s_0)t.$$

This implies,

$$p_1 = -\left(\frac{2}{3} + \frac{1}{6}\gamma_1\right)m_0 \cos\pi t \tag{11}$$

$$q_1(t) = \left(\gamma - \frac{1}{3} + \frac{1}{6}\gamma_2 LE\right)m_0\cos\pi t + \left(s_0 + \frac{1}{2}(E-s_0) - \frac{1}{4}\beta(E-s_0)t_{si}\right) + \frac{1}{4}\beta(E-s_0)t. \tag{12}$$

Then,

$$p(t) = -\frac{1}{A}\left(\varepsilon\gamma_1 - \left(\frac{2}{3} + \frac{1}{6}\gamma_1\right)m_0\cos\pi t\right)$$

$$q_1(t) = \frac{1}{A}(2-\gamma_2 LE)\varepsilon + \left(\gamma - \frac{1}{3} - \frac{1}{6}\gamma_2 LE\right)m_0\cos\pi t + \left(s_0 + \frac{1}{2}(E-s_0) - \frac{1}{4}\beta(E-s_0)t_{si}\right) + \frac{1}{4}\beta(E-s_0)t. \tag{13}$$

So,

$$\theta_0|_{x=0} = e^{\int(B_4 + B_5\cos\pi t)dt}\int_0^t (B_6 + B_8 y + B_7\cos\pi y)e^{-\left(B_4 y + \frac{B_5}{\pi}\sin\pi y\right)dy} + f(x)\,e^{B_4 t + \frac{B_5}{\pi}\sin\pi t}$$

$$= e^{B_4 t + \frac{B_5}{\pi}\sin\pi t}\int_0^t (B_6 + B_8 y + B_7\cos\pi y +)e^{-\left(B_4 y + \frac{B_5}{\pi}\sin\pi y\right)dy} + f(x)\,e^{B_4 t + \frac{B_5}{\pi}\sin\pi t}$$

$$= \left(1 + \frac{B_5}{\pi}\sin\pi t\right)e^{B_4 t}\int_0^t \left(\begin{array}{l} B_6 + B_8 y + B_7\cos\pi y - \frac{B_5 B_6}{\pi}\sin\pi y - \frac{B_5 B_8}{\pi}y\sin\pi y - \\ \frac{B_5 B_7}{\pi}\cos\pi y\sin\pi y \end{array} \right) e^{-B_4 y}\,dy +$$

$$\left(1 + \frac{B_5}{\pi}\sin\pi t\right)e^{B_4 t}$$

$$= \left(1 + \frac{B_5}{\pi}\sin\pi t\right)e^{B_4 t}\int_0^t \left(\begin{array}{l} B_6 e^{-B_4 y} + B_7\cos\pi y e^{-B_4 y} - \frac{B_5 B_6}{\pi}\sin\pi y e^{-B_4 y} + B_8 y e^{-B_4 y} - \frac{B_5 B_8}{\pi}y\sin\pi y e^{-B_4 y} \\ -\frac{B_5 B_7}{\pi}\cos\pi y\sin\pi y e^{-B_4 y} \end{array} \right) dy +$$

$$\left(1 + \frac{B_5}{\pi}\sin\pi t\right)e^{B_4 t}$$

$$= \left(1 + \frac{B_5}{\pi}\sin\pi t\right)e^{B_4 t} \left(\begin{array}{l} -\frac{B_6}{B_4}e^{-B_4 y}\Big|_t^0 + B_7\left(\frac{-B_4 e^{-B_4 y}\cos\pi y}{\pi^2 + B_4^2}\Big|_t^0 + \frac{\pi e^{-B_4 y}\sin\pi y}{\pi^2 + B_4^2}\Big|_t^0\right) - \\ \frac{(B_4 y + 1)B_8 e^{-B_4 y}}{B_4^2}\Big|_t^0 + \\ \frac{B_5 B_6}{\pi}\left(\frac{-\pi e^{-B_4 y}\cos\pi y}{\pi^2 + B_4^2}\Big|_t^0 - \frac{B_4 e^{-B_4 y}\sin\pi y}{\pi^2 + B_4^2}\Big|_t^0\right) - \\ \frac{B_5 B_7}{2\pi}\left(-\frac{2\pi e^{-B_4 y}\cos 2\pi y}{4\pi^2 + B_4^2}\Big|_t^0 - \frac{B_4 e^{-B_4 y}\sin 2\pi y}{4\pi^2 + B_4^2}\Big|_t^0\right) - \\ \frac{B_5 B_8}{\pi}\left(\begin{array}{l} \left(-\frac{-\pi y}{\pi^2 + B_4^2} - \frac{-2B_4\pi}{(\pi^2 + B_4^2)^2}\right)e^{-B_4 y}\cos\pi y\Big|_t^0 + \\ \left(-\frac{B_4 y}{\pi^2 + B_4^2} - \frac{\pi^2 + B_4^2}{(\pi^2 + B_4^2)^2}\right)e^{-B_4 y}\sin\pi y\Big|_t^0 \end{array} \right) + \\ \left(1 + \frac{B_5}{\pi}\sin\pi t\right)e^{B_4 t} \end{array} \right)$$

$$= \left(1+\frac{B_5}{\pi}\sin\pi\, t\right)\begin{pmatrix} \frac{B_6}{B_4}\left(e^{B_4 t}-1\right)+B_7\left(-\frac{B_4\left(\cos\pi\, t-e^{B_4 t}\right)}{\pi^2+{B_4}^2}+\frac{\pi\sin\pi\, t}{\pi^2+{B_4}^2}\right)- \\ \frac{B_8}{{B_4}^2}\left(\left(B_4 t+1\right)-e^{B_4 t}\right)-\frac{B_5B_6}{\pi}\left(\frac{-\pi\left(\cos\pi\, t-e^{B_4 t}\right)}{\pi^2+{B_4}^2}-\frac{B_4\sin\pi\, t}{\pi^2+{B_4}^2}\right)- \\ \frac{B_5B_7}{2\pi}\left(-\frac{2\pi\left(\cos 2\,\pi t-e^{B_4 t}\right)}{4\pi^2+{B_4}^2}-\frac{B_4\sin 2\,\pi t}{4\pi^2+{B_4}^2}\right)- \\ \frac{B_5B_8}{\pi}\begin{pmatrix}\left(-\frac{\pi t}{\pi^2+{B_4}^2}-\frac{-2B_4\pi}{\left(\pi^2+{B_4}^2\right)^2}\right)\cos\pi\, t+\frac{2B_4\pi e^{B_4 t}}{\pi^2+{B_4}^2}+ \\ \left(-\frac{B_4 t}{\pi^2+{B_4}^2}-\frac{{B_4}^2-\pi^2}{\left(\pi^2+{B_4}^2\right)^2}\right)\sin\pi\, t\end{pmatrix}+ \\ \left(1+\frac{B_5}{\pi}\sin\pi\, t\right)e^{B_4 t} \end{pmatrix}$$

$$= \left(1+\frac{B_5}{\pi}\sin\pi\, t\right)\begin{pmatrix} \frac{B_6}{B_4}\left(e^{B_4 t}-1\right)+B_7\left(-\frac{B_4\left(\cos\pi\, t-e^{B_4 t}\right)}{\pi^2+{B_4}^2}+\frac{\pi\sin\pi\, t}{\pi^2+{B_4}^2}\right)- \\ \frac{B_8}{{B_4}^2}\left(\left(B_4 t+1\right)-e^{B_4 t}\right)-\frac{B_5B_6}{\pi}\left(\frac{-\pi\left(\cos\pi\, t-e^{B_4 t}\right)}{\pi^2+{B_4}^2}-\frac{B_4\sin\pi\, t}{\pi^2+{B_4}^2}\right)- \\ \frac{B_5B_7}{2\pi}\left(-\frac{2\pi\left(\cos 2\,\pi t-e^{B_4 t}\right)}{4\pi^2+{B_4}^2}-\frac{B_4\sin 2\,\pi t}{4\pi^2+{B_4}^2}\right)- \\ \frac{B_5B_8}{\pi}\begin{pmatrix}\left(-\frac{\pi t}{\pi^2+{B_4}^2}-\frac{-2B_4\pi}{\left(\pi^2+{B_4}^2\right)^2}\right)\cos\pi\, t+\frac{2B_4\pi e^{B_4 t}}{\pi^2+{B_4}^2}+ \\ \left(-\frac{B_4 t}{\pi^2+{B_4}^2}-\frac{{B_4}^2-\pi^2}{\left(\pi^2+{B_4}^2\right)^2}\right)\sin\pi\, t\end{pmatrix}+ \\ \left(1+\frac{B_5}{\pi}\sin\pi\, t\right)e^{B_4 t} \end{pmatrix}$$

$$= \left(1+\frac{B_5}{\pi}\sin\pi\, t\right)e^{B_4 t}\begin{pmatrix} -\frac{B_6}{B_4}\left(e^{-B_4 t}-1\right)+B_7\left(-\frac{B_4\left(e^{-B_4 t}\cos\pi t-1\right)}{\pi^2+{B_4}^2}+\frac{\pi e^{-B_4 t}\sin\pi t}{\pi^2+{B_4}^2}\right)- \\ \frac{B_8\left(\left(B_4 t+1\right)e^{-B_4 t}-1\right)}{{B_4}^2}- \\ \frac{B_5B_6}{\pi}\left(\frac{-\pi\left(e^{-B_4 t}\cos\pi t-1\right)}{\pi^2+{B_4}^2}-\frac{B_4 e^{-B_4 t}\sin\pi t}{\pi^2+{B_4}^2}\right)- \\ \frac{B_5B_7}{2\pi}\left(-\frac{2\pi\left(e^{-B_4 t}\cos 2\pi t-1\right)}{4\pi^2+{B_4}^2}-\frac{B_4 e^{-B_4 t}\sin 2\pi t}{4\pi^2+{B_4}^2}\right)- \\ \frac{B_5B_8}{\pi}\begin{pmatrix}\left(-\frac{-\pi t}{\pi^2+{B_4}^2}-\frac{-2B_4\pi}{\left(\pi^2+{B_4}^2\right)^2}\right)e^{-B_4 t}\cos\pi\, t+\frac{2B_4\pi}{\pi^2+{B_4}^2}+ \\ \left(-\frac{B_4 t}{\pi^2+{B_4}^2}-\frac{\pi^2+{B_4}^2}{\left(\pi^2+{B_4}^2\right)^2}\right)e^{-B_4 t}\sin\pi\, t\end{pmatrix}+ \\ \left(1+\frac{B_5}{\pi}\sin\pi\, t\right)e^{B_4 t} \end{pmatrix},$$

$$= \left(1+\frac{B_5}{\pi}\sin\pi t\right)\begin{pmatrix} -\left(\frac{B_6}{B_4}+\frac{B_8}{{B_4}^2}\right)-\frac{B_8}{B_4}t+\left(\frac{B_5B_6\pi}{\pi^2+{B_4}^2}-\frac{B_4B_7}{\pi^2+{B_4}^2}+\frac{2B_4B_5B_7}{\left(\pi^2+{B_4}^2\right)^2}\right)\cos\pi t+ \\ \left(\frac{B_7\pi}{\pi^2+{B_4}^2}-\frac{B_4B_5B_6}{\pi^2+{B_4}^2}+\frac{B_5B_8\left({B_4}^2-\pi^2\right)}{\pi\left(\pi^2+{B_4}^2\right)^2}\right)\sin\pi t+ \\ \begin{pmatrix} \frac{B_6}{B_4}+\frac{B_4B_7}{\pi^2+{B_4}^2}+\frac{B_8}{{B_4}^2}- \\ \frac{B_5B_6}{\pi^2+{B_4}^2}-\frac{B_5B_7}{4\pi^2+{B_4}^2}+ \\ \frac{2B_4B_5B_8}{\pi^2+{B_4}^2}+1 \end{pmatrix} e^{B_4t}+\frac{B_5B_7}{4\pi^2+{B_4}^2}\cos 2\pi t \\ +\frac{B_4B_5B_7}{2\pi\left(4\pi^2+{B_4}^2\right)}\sin 2\pi t+\frac{B_5B_8}{\pi^2+{B_4}^2}t\cos 2\pi t-\frac{B_4B_5B_8}{\pi\left(4\pi^2+{B_4}^2\right)}t\sin 2\pi t \end{pmatrix}$$

$$= \left(1+\frac{B_5}{\pi}\sin\pi t\right)\begin{pmatrix} -\left(\frac{B_6}{B_4}+\frac{B_8}{{B_4}^2}\right)-\frac{B_8}{B_4}t+\left(\frac{B_5B_6\pi}{\pi^2+{B_4}^2}-\frac{B_4B_7}{\pi^2+{B_4}^2}+\frac{2B_4B_5B_7}{\left(\pi^2+{B_4}^2\right)^2}\right)\cos\pi t+ \\ \left(\frac{B_7\pi}{\pi^2+{B_4}^2}-\frac{B_4B_5B_6}{\pi^2+{B_4}^2}+\frac{B_5B_8\left({B_4}^2-\pi^2\right)}{\pi\left(\pi^2+{B_4}^2\right)^2}\right)\sin\pi t+ \\ \begin{pmatrix} \frac{B_6}{B_4}+\frac{B_4B_7}{\pi^2+{B_4}^2}+\frac{B_8}{{B_4}^2}- \\ \frac{B_5B_6}{\pi^2+{B_4}^2}-\frac{B_5B_7}{4\pi^2+{B_4}^2}+ \\ \frac{2B_4B_5B_8}{\pi^2+{B_4}^2}+1 \end{pmatrix} e^{B_4t}+ \\ \frac{B_5B_7}{4\pi^2+{B_4}^2}\cos 2\pi t+\frac{B_4B_5B_7}{2\pi\left(4\pi^2+{B_4}^2\right)}\sin 2\pi t+\frac{B_5B_8}{\pi^2+{B_4}^2}t\cos 2\pi t \\ -\frac{B_4B_5B_8}{\pi\left(4\pi^2+{B_4}^2\right)}t\sin 2\pi t \end{pmatrix},$$

Thus, the solution of the propose model equation (4) together with its initial and boundary conditions is of the form:

$$\theta(x,t)=\gamma_2 LEx+(1-\gamma_2 LE)x^2+(1+\gamma_1 x-(1+\gamma_1)x^2)\theta_0|_{x=0}. \tag{14}$$

Where,

$$\theta_0|_{x=0}=c_{24}+c_{25}t+c_{26}\cos\pi t+c_{27}\sin\pi t+c_{28}e^{B_4t}+c_{29}\cos\pi t+c_{30}\sin2\pi t+c_{31}t\cos\pi t+c_{32}t\sin\pi t+ c_{33}\cos\pi t\sin\pi t+c_{34}\sin\pi t\sin\pi t+c_{35}e^{B_4t}\sin\pi t+c_{36}\cos2\pi t\sin\pi t+c_{37}\sin2\pi t\sin\pi t+c_{38}\cos\pi t\sin\pi t+c_{39}t\sin\pi t\sin\pi t. \tag{15}$$

And,

$$B_4=\frac{\varepsilon\gamma_1}{A}, B_5=-\frac{1}{A}\left(\frac{2}{3}+\frac{2}{3}\gamma_1\right)m_0, B_6=\frac{1}{A}\left((2-\gamma_2 LE)\varepsilon+S_0+\frac{1}{2}(E-S_0)-\frac{1}{4}\beta(E-S_0)\,t_{si}\right),$$

$$B_7=\frac{1}{A}\left(\gamma_1-\frac{1}{3}-\frac{1}{6}\gamma_2 LE\right)m_0, B_8=\left(\frac{\beta(E-S_0)}{4A}\right), C_{24}=-\left(\frac{B_6}{B_4}+\frac{B_8}{{B_4}^2}\right), C_{25}=-\frac{B_8}{B_4},$$

$$C_{26}=\left(\frac{B_5B_6}{\pi^2+{B_4}^2}-\frac{B_4B_7}{\pi^2+{B_4}^2}+\frac{2B_4B_5B_7}{\left(\pi^2+{B_4}^2\right)^2}\right), C_{27}=\left(\frac{B_7\pi}{\pi^2+{B_4}^2}-\frac{B_4B_5B_6}{\pi^2+{B_4}^2}+\frac{B_5B_8\left({B_4}^2-\pi^2\right)}{\pi\left(\pi^2+{B_4}^2\right)^2}+\frac{B_5}{\pi}C_{24}\right),$$

$$C_{28}=\begin{pmatrix} \frac{B_6}{B_4}+\frac{B_4B_7}{\pi^2+{B_4}^2}+\frac{B_8}{{B_4}^2}- \\ \frac{B_5B_6}{\pi^2+{B_4}^2}-\frac{B_5B_7}{4\pi^2+{B_4}^2}+ \\ \frac{2B_4B_5B_8}{\pi^2+{B_4}^2}+1 \end{pmatrix}, C_{29}=\frac{B_5B_7}{4\pi^2+{B_4}^2}, C_{30}=\frac{B_4B_5B_7}{2\pi\left(4\pi^2+{B_4}^2\right)}, C_{31}=\frac{B_5B_8}{\pi^2+{B_4}^2},$$

$$C_{32}=-\left(\frac{B_4B_5B_8}{\pi\left(\pi^2+{B_4}^2\right)}+\frac{B_5B_8}{\pi B_4}\right), C_{33}=\frac{B_5}{\pi}C_{26}, C_{34}=\frac{B_5}{\pi}C_{27}, C_{35}=\frac{B_5}{\pi}C_{28}, C_{36}=\frac{B_5}{\pi}C_{29},$$

$$C_{37}=\frac{B_5}{\pi}C_{30}, C_{38}=\frac{B_5}{\pi}C_{31}, C_{39}=-\frac{B_5}{\pi}\left(\frac{B_4B_5B_8}{\pi\left(\pi^2+{B_4}^2\right)}\right). \tag{16}$$

Table 1: Numerical Data Collected

S/No.	Meaning	Symbol	Figure	Reference
1	Blood Mass Flow Rate	m_0	1	Saidu M.Tech Thesis (2017)
2	Metabolic Rate	s_0	0.2	Saidu et al.(2024)
3	Diffusion Term	ε	0.2	Saidu et al.(2024)
4	Metabolic Control Parameter	α	0.5	Saidu et al.(2024)
5	Thermal Conductivity	σ	0.4	Saidu M.Tech Thesis (2017)
6	Latent Heat	L	579	Gokul *et al.*, 2015
7	Basal Metabolic Rate	BMR	0.3165cal/cm^2 min	Brashaw et al. (2006)
8	Sigmoid Mid-Point of the Curve	t_{si}	1	Saidu et al. (2025)
9	Radius of The Skin (Dermis)	γ	1	Saidu et al. (2025)
10	Radius of The Skin (Epidermis)	γ_1	0.5	Saidu et al. (2025)
11	Radius of The Skin (Subscutaneous Tissue)	γ_2	0.3	Saidu et al. (2025)
12	Rate of Sweat Evaporation in Tissue	E	0.00048g/cm^2 min	Shrestha *et al.*, 2017

Model Simulation

Using the data gathered above, Maple 16 software versions was used to simulate the outcome and provide graphical representations and interpretations of the body's reactions to exercise when values were allocated to certain parameters of interest.

RESULTS AND DISCUSSION

Results

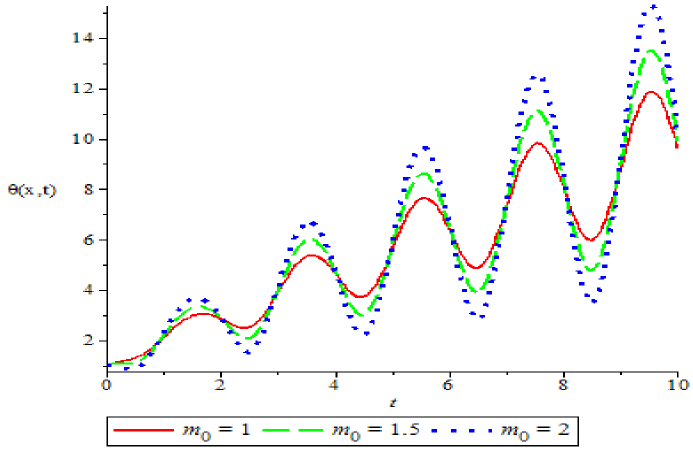

Figure 1: Variation of blood mass flow rate m_0 in human body during undergoing periodic physical exercise at time t

This graph illustrates how the blood mass flow rate progressively rises and falls during a periodic exercise session until the exercise reaches its stop time. Therefore, a rise or fall in the blood mass flow rate during intermittent exercise also lowers the tissue temperature profile $\theta(x,t)$ At time t.

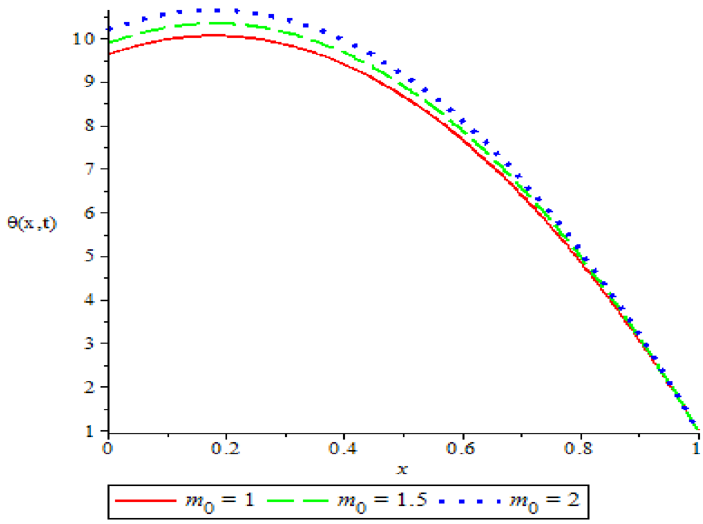

Figure 2: Variation of blood mass flow rate m_0 in human body during physical exercise along distance x

This graph shows that their is a decreases in blood mass flow rate during a physique session untill the exercise gets to the stopage time. This inturn decreases the tissue temperature profile $\theta(x,t)$ along distance x

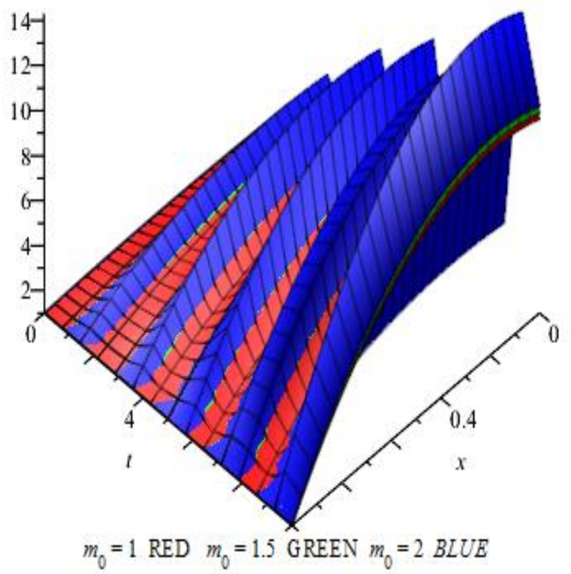

Figure 3: A 3-dimensional graph showing variation of blood mass flow rate m_0 during extensive exercise at time t and along the distance x

The graph of figure 3 clearly describe a 3-D graph of a human subject ungergoing an exercise with a variation of blood mass flow rate increasing and decreasing against time until the exercise gets to its stopage time but decreases along the given distance x. It is seen here that an increase/decrease in the blood mass flow rate against time and also decrease in the blood mass flow rate along the given distance during exercise, also decreases the human tissue temperature profile $\theta(x,t)$ at time t and along the given distance x.

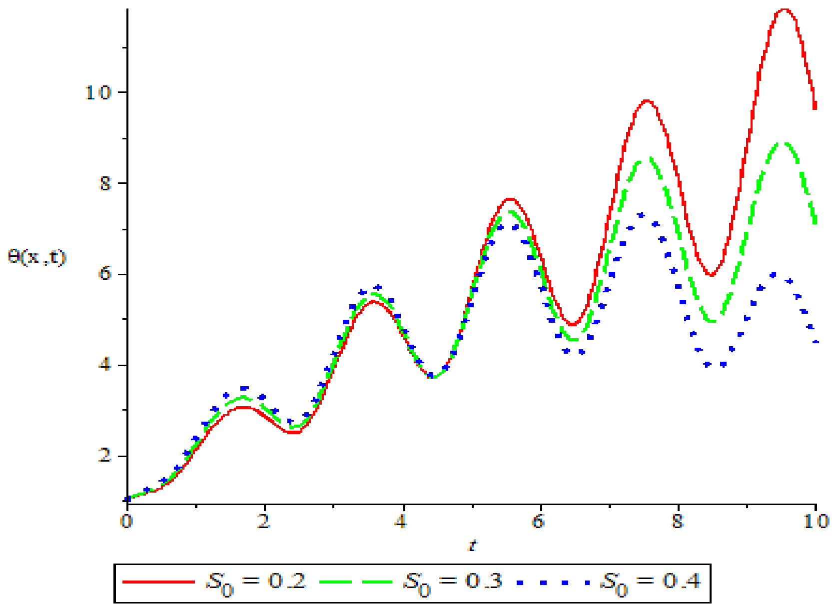

Figure 4: Variation of metabolic rate s_0 in human body during physical exercise at time t

The graph of figure 4 shows that during a periodic exercise, the metabolic rate gradually increases and decerases until the exercise get to the stopage time. Thus, an increases/decerase in the rate of metabolism during periodic exercise also increases the tissue temperature profile $\theta(x,t)$ At time t.

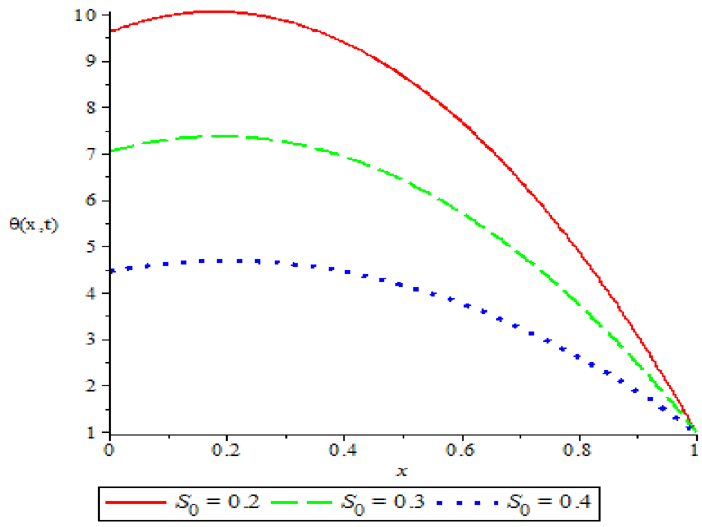

Figure 5: Variation of metabolic rate s_0 in human body during physical exercise along distance x

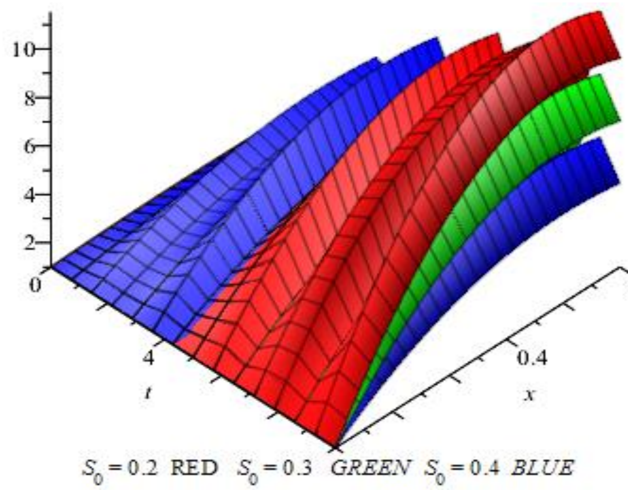

Figure 6: A 3-dimensional graph showing variation of metabolic rate S_0 during extensive exercise at time t and along the distance x

This graph shows a significant increase in metabolic rate along the given distance as the period of exercise continue until it gets to the stopage time.Thus, an increase in the rate of metabolism increases the tissue temperature profile $\theta(x,t)$ along the distance x.

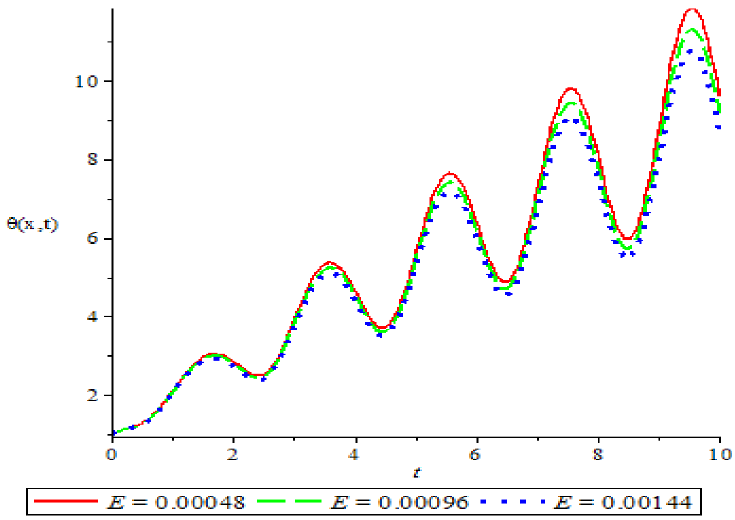

Figure 7: Variation of rate of sweat evaporation in tissue E during extensive exercise at time t

The graph of figure 7 shows that, during a periodic exercise, the amount of sweat evaporation in tissue gradually increases and decerases until the exercise get to the stopage time. Thus, an increases/decerase in the rate of sweat evaporation in tissue during periodic exercise also increases the tissue temperature profile $\theta(x,t)$ at time t .

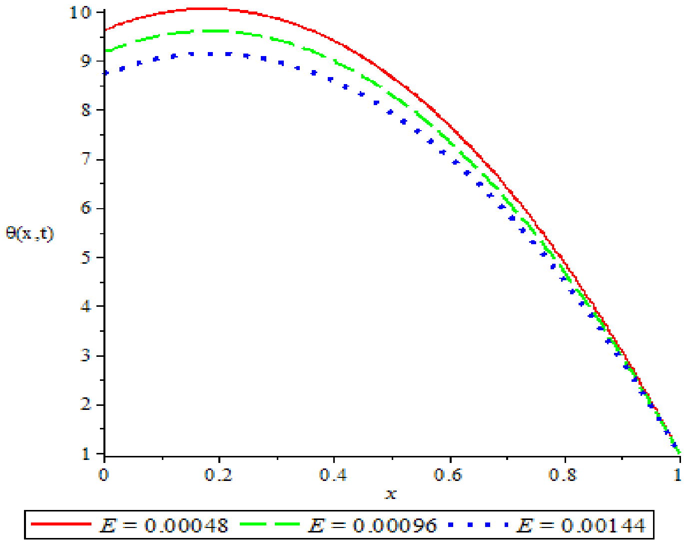

Figure 8: Variation of rate of sweat evaporation in tissue E during extensive exercise along the distance x

The graph of figure 8 show that during physique the rate of sweat evaporation in tissue increases logistically along the given distance. This scenerio continued until the exercise gets to the stopage time. It can be interpreted here that an increase in sweat evaporation in tissue during an active physical exercise also increases the tissue temperature profile $\theta(x,t)$ along the given distance x.

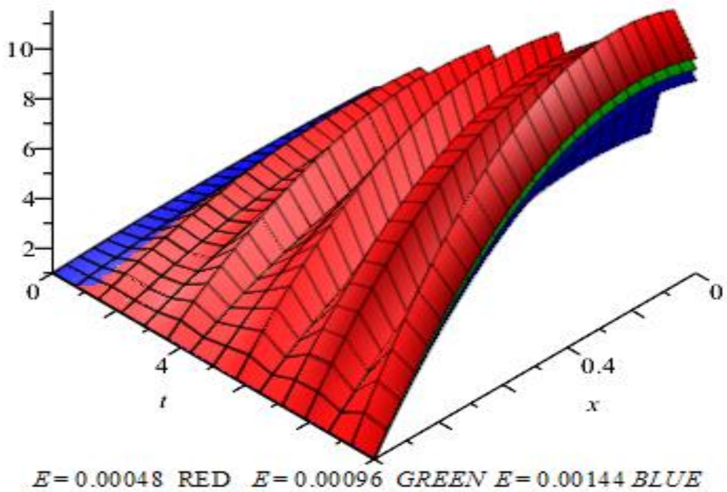

Figure 9: A 3-dimensional graph showing variation of rate of sweat evaporation in tissue E during extensive exercise at time t and along the distance x.

The graph of figure 9 show that during the period of exercise, the rate of sweat evaporation in tissue increases and decreases periodically after some time and this scenerio continue until the exercise get to the stopage time but it increases logistically along the given distance. It can be interpreted here that, an increase in the metabolic rate during an active physical exercise also increases the tissue temperature profile $\theta(x,t)$ at time t and along the given distance x.

CONCLUSION

The physiology role played by the blood mass flow rate and metabolic rate during a periodic physical exercise has been described. Results shows that during periodic exercise, blood mass flow rate increases and decrease at time t and a long distance x thus decreasing the tissue temperature profile $\theta(x,t)$. In the same vain it was observed that during periodic exercise, metabolic rate increases at time t and a long distance x .

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