FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645-2944
Vol. 9 No. 4, April 2025, pp 347 - 356
DOI: https://doi.org/10.33003/fjs-2025-0904-3491

AN IN-DEPTH STUDY ON THE APPLICATION OF THE DIFFERENTIAL TRANSFORM METHOD FOR EFFECTIVELY SOLVING VARIOUS-ORDER ORDINARY DIFFERENTIAL EQUATIONS

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Abstract

This study presents a comprehensive analysis of the Differential Transform Method (DTM) as an effective tool for solving ordinary differential equations (ODEs) of various orders. Emphasis is placed on the method's ability to handle both linear and nonlinear ODEs without the need for common simplification techniques such as linearization, discretization, or perturbation, which often introduce additional complexities or reduce accuracy. By systematically applying DTM to different classes of ODEs, the study highlights its versatility and accuracy in handling initial value problems across a range of complexities with the solution of the first, second, third and fourth-orders ODEs. Comparative analyses with analytical methods demonstrate the superiority of DTM in terms of computational efficiency and solution accuracy. Additionally, graphical representations of both the analytical solutions and the approximate results obtained using DTM were plotted across various orders to showcase the robustness of the DTM. This article also includes detailed examples illustrating the step-by-step application of DTM, providing insights into its potential for broader applications in engineering, physics, and applied mathematics. The increasing complexity of systems modeled by ODEs in scientific and engineering fields necessitates efficient and accurate methods for obtaining reliable solutions, thereby justifying the need to explore alternative approaches like DTM. The findings underscore the relevance of DTM as a powerful method for solving ODEs of various orders, making it a valuable addition to the toolbox of researchers and practitioners in the field. Conclusively, the results show that DTM is an efficient, accurate, and reliable method.

Keywords: DTM, Ordinary Differential Equations, Various-order ODEs, Initial Value Problems, Initial Conditions, Series Solution.

INTRODUCTION

The Differential Transform Method (DTM) involves transforming a differential equation into a series solution that is subsequently solved through an iterative process. This method is fundamentally based on the Taylor series expansion, which allows it to be highly effective in deriving analytical solutions to complex differential equations. The DTM was first introduced by Zhou (1986), specifically to address both linear and nonlinear initial value problems encountered in the analysis of electrical circuits. The significance of this method extends beyond its original application, as it has proven to be a powerful tool for solving differential equations of various orders, which frequently arise in the fields of physics, engineering, and applied mathematics. These higher-order differential equations are integral to modeling a wide range of phenomena, making DTM a valuable technique for researchers and practitioners working within these disciplines. One of the significant strengths of the Differential Transform Method (DTM) is its robust ability to handle a diverse array of differential equations. This includes both linear and nonlinear equations, whether homogeneous or non-homogeneous, and equations with either constant or variable coefficients. The method's flexibility makes it particularly useful in solving complex differential equations that are often encountered in various scien-

tific and engineering problems. Furthermore, the solutions obtained through DTM tend to exhibit rapid convergence towards the exact solutions, often requiring fewer computational steps compared to other numerical methods. This efficiency not only reduces computational cost but also enhances the accuracy of the results. Additionally, DTM can be easily applied to both initial value and boundary value problems, making it a versatile tool for researchers. The method's capability to simplify the analytical process while maintaining high accuracy underscores its growing importance in modern computational mathematics and engineering applications.
Ogwumu *et al.,* (2020) effectively applied the Differential Transform Method (DTM) to solve first, second, and third-order linear differential equations, demonstrating the method's versatility and reliability across different levels of complexity. Their work highlighted DTM's ability to efficiently handle equations of varying orders while maintaining high accuracy in the solutions. In a more recent study, Brociek and Pleszczyński (2024) innovatively combined the DTM with physics-informed neural networks (PINNs) to tackle complex variational calculus problems that involve differential equations. This hybrid approach capitalized on the strengths of both methodologies, enhancing the solution process for intricate mathematical models. A comparative analysis of the results from these studies underscored the superior accuracy of solutions obtained through DTM, further establishing it as a powerful tool in both classical and modern computational techniques. This version provides additional context and emphasizes the contributions of the studies, as well as the importance of DTM in modern research.

The effectiveness of the Differential Transform Method (DTM) is notably demonstrated in its application to solving singular integral equations, where its infinite series expansion achieves rapid convergence to the exact solution, as evidenced by Mondal et al., (2023). This capability underscores DTM's precision and efficiency in handling complex integral equations. Additionally, Ogunrinde and Ojo (2018) utilized DTM to address higher-order boundary value problems, specifically those of the seventh and eighth orders. Their application of the method showcased its robustness and reliability, as DTM successfully converged to solutions with high accuracy, closely aligning with exact results. This further highlights DTM's utility in numerically analyzing and solving challenging higher-order boundary value problems.

Oke (2017) conducted an in-depth investigation into the convergence properties of the Differential Transform Method (DTM) for ordinary differential equations. His research established that if a solution to a differential equation can be expressed as a Taylor series expansion, DTM is a viable and effective tool for deriving that solution. This finding highlights DTM's theoretical foundation and practical applicability. Furthermore, extensive research has been devoted to exploring the diverse applications of DTM in solving differential equations across various contexts. Studies by Sutkar (2017) and Devi and Jakhar (2022) have emphasized DTM's notable strengths, including its capacity to handle a broad range of differential equations with simplicity, reliability, and robustness. These investigations collectively underscore the method's versatility and effectiveness, making it a valuable asset in computational mathematics and applied sciences. Additional research continues to validate and expand on these findings, demonstrating DTM's enduring relevance and efficacy in solving complex differential problems.
Aris and Azlisa (2023) employed the Differential Transform Method (DTM) to address Lane-Emden equations, which are fundamental in astrophysics and other fields. Their study involved a comprehensive comparison of the approximate solutions generated by DTM with those obtained using several other advanced methods, including the Adomian Decomposition Method (ADM), Legendre wavelet techniques, and the Homotopy Perturbation Method in conjunction with Laplace transformations. This comparative analysis aimed to evaluate the accuracy of the solutions produced by DTM in relation to the exact values. By assessing the performance of DTM against these established methods, the study provided valuable insights into the relative effectiveness and precision of DTM, highlighting its strengths and potential advantages in solving complex differential equations.
Alahmad et al. (2022) presented an innovative numerical approach known as the Modified Differential Transform Method (MDTM), which they applied to obtain accurate approximate solutions for a diverse range of nonlinear differential equations. This new scheme builds upon the traditional Differential Transform Method by incorporating modifications that enhance its functionality and applicability. The results from their study highlighted that MDTM is not only effective in delivering precise solutions but also offers a clear, straightforward, and user-friendly framework for solving complex nonlinear differential equations. The successful application of MDTM across various problem classes underscores its potential as a valuable tool in computational mathematics, offering both practicality and robustness in achieving reliable results.

DIFFERENTIAL TRANSFORM METHOD AND ITS APPLICATION TO VARIOUS-ORDER ODEs

The Differential Transform Method is a semi-analytical technique that transforms differential equations into algebraic equations, simplifying the process of finding solutions. Developed by P. W. L. (1978), DTM leverages the concept of differential transformation to convert differential operators into algebraic operations. The method then utilizes the transformed series to derive approximate solutions, which can be refined to approach exact results.

Motivation and Significance in Solving Various-order ODEs

First-Order Differential Equations: First-order differential equations are the simplest form of ODEs and frequently model real-world phenomena such as population growth, chemical reactions, and cooling processes. Solving these equations accurately is crucial for understanding fundamental processes. DTM's application to first-order differential equations offers several advantages. The method's ability to transform the equation into a series expansion simplifies the process of finding exact or approximate solutions. By leveraging DTM, researchers can efficiently tackle first-order equations without resorting to more complex numerical methods, thus saving time and computational resources.

Second-Order Differential Equations: Second-order differential equations appear in a wide range of applications, including mechanical vibrations, electrical circuits, and wave propagation. They often describe systems with acceleration or second derivatives, making them central to dynamic modeling. For second-order ODEs, DTM provides a systematic approach to deriving solutions. The method's ability to handle equations with variable coefficients and complex boundary conditions makes it a valuable tool. DTM's series expansion offers an effective way to approximate solutions and analyze system behavior, enhancing both theoretical and practical insights.

Third-Order Differential Equations: Third-order differential equations arise in advanced applications such as structural analysis, control systems, and fluid dynamics. These equations often model complex systems with multiple interacting components. Applying DTM to third-order differential equations allows researchers to tackle these more complex problems with a clear and structured approach. The method's capability to provide approximate solutions through series expansion helps in understanding and solving intricate systems. DTM's efficiency in dealing with third-order equations can significantly aid in the design and analysis of engineering systems.

Fourth-Order Differential Equations: Fourth-order differential equations are essential in fields like beam theory, elasticity, and advanced mechanical systems. These equations model systems with even higher complexity and multiple boundary conditions. The DTM's application to fourth-order differential equations demonstrates its robustness and versatility. The method's ability to handle high-order equations effectively makes it a powerful tool for solving complex boundary value problems. By converting the differential operators into algebraic forms, DTM simplifies the solution process, offering accurate and reliable results that are crucial for advanced research and engineering applications.

The Differential Transform Method offers a significant advantage in solving ordinary differential equations of various orders. Its ability to transform complex differential equations into manageable algebraic forms simplifies the solution process, making it an invaluable tool for researchers and practitioners. From first-order equations to more complex fourth-order cases, DTM's versatility, efficiency, and effectiveness underscore its importance in both theoretical and applied contexts. There are also fractional-order differential equations, Odetunde and Ibrahim (2025) and various methods have been employed to solve them. As research and technology continue to advance, DTM remains a relevant and powerful method for addressing the diverse challenges posed by differential equations across multiple domains.

DIFFERENTIAL TRANSFORM METHOD

The Differential Transform Method (DTM) is a highly effective technique for solving both ordinary and partial differential equations. It achieves this by converting these complex equations into a series of algebraic equations, which facilitates the derivation of semi-analytical solutions. This transformation simplifies the process of finding solutions, making DTM particularly valuable for addressing initial value problems. Furthermore, DTM is versatile in its application, capable of handling a wide range of equations, including both linear and nonlinear types. Its ability to streamline the solution process and manage complex differential equations underscores its significance and utility in various mathematical and engineering contexts.
The transformation of the n^{th} derivative of a function with one variable is given below:

$$W(n) = \frac{1}{n!}\left[\frac{d^n w(q)}{dq^n}\right]_{q=q_0} \tag{1}$$

where $w(q)$ is the original function and $W(n)$ is the transformed function.
The differential inverse transformation $W(n)$ is defined by

$$w(q) = \sum_{n=0}^{\infty} W(n)(q-q_0)^n \tag{2}$$

When $q_0 = 0$, the function $w(q)$ defined by equation (2) is expressed as

$$w(q) = \sum_{n=0}^{\infty} \frac{q^n}{n!}\left[\frac{d^n w(q)}{dq^n}\right]_{q_0=0} \tag{3}$$

Equation (3) implies that the concept of differential transform is derived from Taylor Series expansion. If $g(q)$ and $h(q)$ are two unrelated functions with variable q, $G(q)$ and $H(q)$ are the transformed functions corresponding to $g(q)$ and $h(q)$ respectively. The following fundamental theorems on differential transform method are used in this work:

Foundamental Theorems on Differential Transform Methods

THEOREM 1

$$If \quad w(q) = \mu g(q) \pm \gamma h(q),$$
$$then \quad W(n) = \mu G(n) \pm \gamma H(n)$$

where μ and γ are scalars.
THEOREM 2

$$If \quad w(q) = \frac{d^m g(q)}{dq^m},$$
$$then \quad W(n) = (n+1)(n+2)...(n+m)G(n+m)$$

THEOREM 3

$$If \quad w(q) = \frac{dg(q)}{dq},$$
$$then \quad W(n) = (n+1)G(n+1)$$

THEOREM 4

$$If w(q) = cg(q),$$
$$then \quad W(n) = cG(n)$$

where c is a constant.
THEOREM 5

$$If \quad w(q) = g(q) \pm h(q),$$
$$then \quad W(n) = G(n) \pm H(n)$$

THEOREM 6

$$If \quad w(q) = e^q,$$
$$then \quad W(n) = \frac{1}{n!}$$

THEOREM 7

$$If \quad w(q) = e^{\lambda q},$$
$$then \quad W(n) = \frac{\lambda^n}{n!}$$

where λ is a constant.
THEOREM 8

$$If \quad w(q) = sin(\mu q + \gamma),$$
$$then \quad W(n) = \frac{\mu^n}{n!} sin\left(\frac{n\pi}{2} + \gamma\right)$$

where μ and γ are constants.
THEOREM 9

$$If \quad w(q) = cos(\mu q + \gamma),$$
$$then \quad W(n) = \frac{\mu^n}{n!} cos\left(\frac{n\pi}{2} + \gamma\right)$$

where μ and γ are constants.
THEOREM 10

$$If \quad w(q) = g(q).h(q),$$
$$then \quad W(n) = \sum_{r=0}^{n} G(r)H(n-r)$$

THEOREM 11

$$If \quad w(q) = q,$$
$$then \quad W(n) = \delta(n-1)$$

THEOREM 12

$$If \quad w(q) = 1,$$
$$then \quad W(n) = \delta(n)$$

THEOREM 13

$$If \quad w(q) = q^a,$$
$$then \quad W(n) = \delta(n-a) = \begin{cases} 1 & \textbf{if} \quad n = a \\ 0 & \textbf{if} \quad n \neq a \end{cases}$$

THEOREM 14

$$If \quad w(q) = cq^a,$$
$$then \quad W(n) = c\delta(n-a) = \begin{cases} c & \textbf{if} \quad n = a \\ 0 & \textbf{if} \quad n \neq a \end{cases}$$

THEOREM 15

$$If \quad w(q) = (1+q)^a,$$
$$then \quad W(n) = a(a-1)(a-2)...(a-n+1)$$

NUMERICAL APPLICATIONS

Example 1
First-order homogeneous differential equation

$$\frac{dw}{dq} - 3w = 0, \quad w(0) = 2 \tag{4}$$

with exact solution

$$w(q) = 2e^{3q} \tag{5}$$

Solution
The differential transform of equation (4), as derived using the aforementioned theorem, is expressed as follows

$$(n+1)W(n+1) - 3W(n) = 0$$

$$W(n+1) = \frac{3W(n)}{n+1}$$

with the initial condition $w(0) = 2$.
When $n = 0$,

$$W(1) = \frac{3W(0)}{1}$$
$$W(1) = 6$$

When $n = 1$,

$$W(2) = \frac{3W(1)}{2}$$
$$W(2) = 9$$

When $n = 2$,

$$W(3) = \frac{3W(2)}{3}$$
$$W(3) = 9$$

When $n = 3$,

$$Y(4) = \frac{3W(3)}{4}$$
$$W(4) = 6.75$$

When $n = 4$,

$$W(5) = \frac{3W(4)}{5}$$
$$W(5) = 4.05$$

When $n = 5$,

$$W(6) = \frac{3W(5)}{6}$$
$$W(6) = 2.025$$

When $n = 6$,

$$W(7) = \frac{3W(6)}{7}$$
$$W(7) = 0.867857$$

When $n = 7$,

$$W(8) = \frac{3W(7)}{8}$$
$$W(8) = 0.325446$$

When $n = 8$,

$$W(9) = \frac{3W(8)}{9}$$
$$W(9) = 0.108482$$

When $n = 9$,

$$W(10) = \frac{3W(9)}{10}$$
$$W(10) = 0.032545$$

Thus, the solution is written as

$$w(q) = \sum_{n=0}^{\infty} q^n W(n)$$
$$w(q) = q^0 W(0) + q^1 W(1) + q^2 W(2) + q^3 W(3) + q^4 W(4) + ...$$
$$w(q) = 2 + 6\,q + 9\,q^2 + 9\,q^3 + 6.75q^4 + ...$$

Table 1: Numerical Comparison for Example 1

q	Exact	DTM	Error
0	0.00000000	0.00000000	0.00000000
0.1	2.69971762	2.69971762	0.00000000
0.2	3.64423760	3.64423760	0.00000000
0.3	4.91920622	4.91920622	0.00000000
0.4	6.64023385	6.64023385	0.00000000
0.5	8.96337814	8.96337814	0.00000000
0.6	12.09929493	12.09929494	1×10^{-8}
0.7	16.33233983	16.33233981	2×10^{-8}
0.8	22.04635276	22.04635279	3×10^{-8}
0.9	29.75946344	29.75946344	0.00000000
1.0	40.17107384	40.17107384	0.00000000

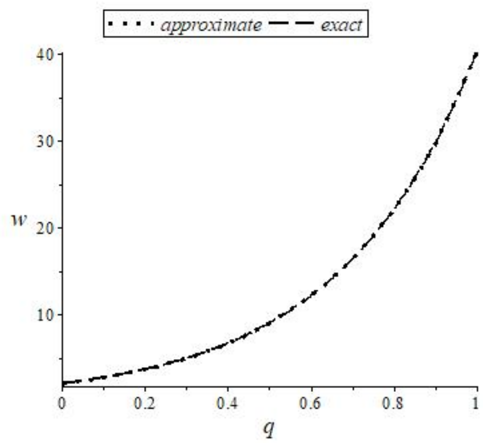

Figure 1: The plot of Exact and DTM Solutions

Example 2
Second-order non homogeneous differential equation

$$\frac{d^2w}{dq^2} - 6\frac{dw}{dw} + 8w = e^{3q} \quad (6)$$

$$w(0) = 0, \quad w^{'}(0) = 2$$

$$with \quad exact \quad solution$$

$$w(x) = \frac{3}{2}e^{4q} - \frac{1}{2}e^{2q} - e^{3q} \quad (7)$$

Solution
The differential transform of equation (6), obtained using the theorem previously described, is provided by

$$(n+1)(n+2)W(n+2) - 6(n+1)W(n+1) + 8W(n) = \frac{3^n}{n!}$$

$$W(n+2) = \frac{3^n}{n!\,(n+1)(n+2)} + \frac{6(n+1)W(n+1) - 8W(n)}{(n+1)(n+2)} \quad \mathbf{(8)}$$

with the initial condition $w(0) = 0, \quad w^{'}(0) = 2.$
When $n = 0,$

$$W(2) = \frac{3^0}{0!\,(1)(2)} + \frac{6(1)W(1) - 8W(0)}{(1)(2)}$$
$$W(2) = \frac{13}{2}$$

When $n = 1,$

$$W(3) = \frac{3^1}{1!\,(2)(3)} + \frac{6(2)W(2) - 8W(1)}{(2)(3)}$$
$$W(3) = \frac{65}{6}$$

When $n = 2,$

$$W(4) = \frac{3^2}{2!\,(3)(4)} + \frac{6(3)W(3) - 8W(2)}{(3)(4)}$$
$$W(4) = \frac{295}{24}$$

When $n = 3,$

$$W(5) = \frac{3^3}{3!\,(4)(5)} + \frac{6(4)W(4) - 8W(3)}{(4)(5)}$$
$$W(5) = \frac{1277}{120}$$

When $n = 4,$

$$W(6) = \frac{3^4}{4!\,(5)(6)} + \frac{6(5)W(5) - 8W(4)}{(5)(6)}$$
$$W(6) = \frac{5383}{720}$$

When $n = 5,$

$$W(7) = \frac{3^5}{5!\,(6)(7)} + \frac{6(6)W(6) - 8W(5)}{(6)(7)}$$
$$W(7) = \frac{4465}{1008}$$

When $n = 6,$

$$W(8) = \frac{3^6}{6!\,(7)(8)} + \frac{6(7)W(7) - 8W(6)}{(7)(8)}$$
$$W(8) = \frac{18323}{8064}$$

When $n = 7,$

$$W(9) = \frac{3^7}{7!\,(8)(9)} + \frac{6(8)W(8) - 8W(7)}{(8)(9)}$$
$$W(9) = 1.0286513$$

When $n = 8,$

$$W(10) = \frac{3^8}{8!\,(9)(10)} + \frac{6(9)W(9) - 8W(8)}{(9)(10)}$$
$$W(10) = 0.41702574$$

Thus, the solution is written as

$$w(q) = \sum_{n=0}^{\infty} q^n W(n)$$
$$w(q) = q^0W(0) + q^1W(1) + q^2W(2) + q^3W(3) + q^4W(4) + ...$$
$$w(q) = 2q + \frac{13}{2}q^2 + \frac{65}{6}q^3 + \frac{295}{24}q^4 + ...$$

Table 2: Numerical Comparison for Example 2

q	Exact	DTM	Error
0	**0.00000000**	**0.00000000**	**0.00000000**
0.1	**0.27717686**	**0.27717686**	**0.00000000**
0.2	**0.77028024**	**0.77028024**	**0.00000000**
0.3	**1.60951287**	**1.60951287**	**0.00000000**
0.4	**2.99666125**	**2.99666125**	**0.00000000**
0.5	**5.24275417**	**5.24275417**	**0.00000000**
0.6	**8.82505865**	**8.82505865**	**0.00000000**
0.7	**14.47320027**	**14.47320027**	**0.000000000**
0.8	**23.29910271**	**23.29910271**	**0.00000000**
0.9	**36.99279621**	**36.99279621**	**0.00000000**
1.0	**58.11716007**	**58.11716007**	**0.00000000**

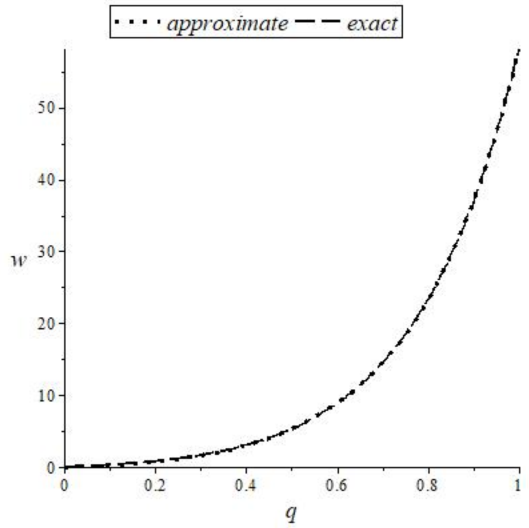

Figure 2: The plot of Exact and DTM Solutions

Example 3 [Kreyszig et al.,(2011)]. **Third-order homogeneous differential equation**

$$\frac{d^3w}{dq^3} - \frac{d^2w}{dq^2} + 100\frac{dw}{dq} - 100w = 0 \qquad (9)$$

$$w(0) = 4, \quad w^{'}(0) = 11, \quad w^{''}(0) = -299$$

with exact solution

$$w(q) = e^q + 3\cos 10q + \sin 10q \qquad (10)$$

Solution

Applying the aforementioned theorem to equation (8) **yields its differential transform as follows**

$$(n+1)(n+2)(n+3)W(n+3) - (n+1)(n+2)W(n+2) + 100(n+1)W(n+1) - 100W(n) = 0$$

$$W(n+3) = \frac{(n+1)(n+2)W(n+2) - 100(n+1)W(n+1) + 100W(n)}{(n+1)(n+2)(n+3)} \qquad \textbf{(11)}$$

with the initial condition $w(0) = 4,\ w^{'}(0) = 11,\ w^{''}(0) = -299.$

When $n = 0$,

$$W(3) = \frac{(1)(2)W(2) - 100(1)W(1) + 100W(0)}{(1)(2)(3)}$$

$$W(3) = -166.5$$

When $n = 1$,

$$W(4) = \frac{(2)(3)W(3) - 100(2)W(2) + 100W(1)}{(2)(3)(4)}$$

$$W(4) = 1250.041667$$

When $n = 2$,

$$W(5) = \frac{(3)(4)W(4) - 100(3)W(3) + 100W(2)}{(3)(4)(5)}$$

$$W(5) = 833.3416664$$

When $n = 3$,

$$W(6) = \frac{(4)(5)W(5) - 100(4)W(4) + 100W(3)}{(4)(5)(6)}$$

$$W(6) = -4166.665280$$

When $n = 4$,

$$W(7) = \frac{(5)(6)W(6) - 100(5)W(5) + 100W(4)}{(5)(6)(7)}$$

$$W(7) = -1984.126786$$

When $n = 5$,

$$W(8) = \frac{(6)(7)W(7) - 100(6)W(6) + 100W(5)}{(6)(7)(8)}$$

$$W(8) = 7440.476220$$

When $n = 6$,

$$W(9) = \frac{(7)(8)W(8) - 100(7)W(7) + 100W(6)}{(7)(8)(9)}$$

$$W(9) = 2755.731924$$

When $n = 7$,

$$W(10) = \frac{(8)(9)W(9) - 100(8)W(8) + 100W(7)}{(8)(9)(10)}$$

$$W(10) = -8267.195773$$

Thus, the solution is written as

$$w(q) = \sum_{n=0}^{\infty} q^n W(n)$$

$$w(q) = q^0W(0) + q^1W(1) + q^2W(2) + q^3W(3) + q^4W(4) + ... \qquad \textbf{(12)}$$

$$w(q) = 4 + 11\,q - 149.5\,q^2 - 166.5\,q^3 + 1250.041667q^4 + ...$$

Table 3: Numerical Comparison for Example 3

q	Exact	DTM	Error
0	4.00000000	4.00000000	0.00000000
0.1	3.56754882	3.56754882	0.00000000
0.2	0.88225967	0.88225967	0.00000000
0.3	-1.47899867	-1.47899868	1×10^{-8}
0.4	-1.22590866	-1.22590867	1×10^{-8}
0.5	1.54078355	1.54078355	0.00000000
0.6	4.42321416	4.42321540	1.24×10^{-6}
0.7	4.93244607	4.93251713	7.106×10^{-5}
0.8	2.77839907	2.78067581	0.00227674
0.9	0.13833081	0.18653640	0.04820559
1.0	-0.34295387	0.39442700	0.73738087

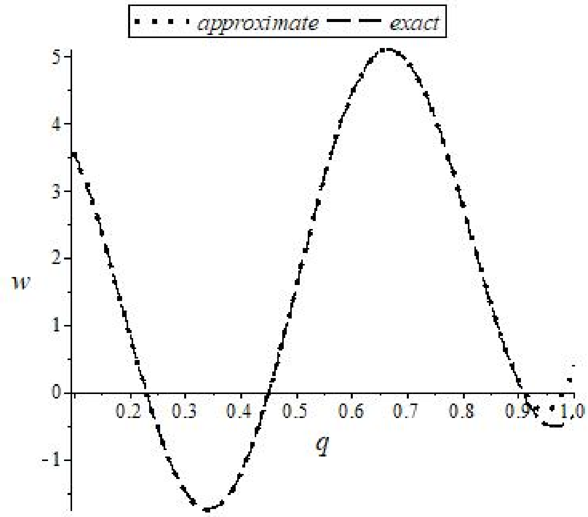

Figure 3: The plot of Exact and DTM Solutions

Example 4 [Ganesh and Balasubramanian (2009)]. Fourth-order homogeneous differential equation

$$\frac{d^4w}{dq^4} - w = 0 \tag{13}$$

$w(0) = 1,\ w^{'}(0) = 0,\ w^{''}(0) = 0,\ w^{'''}(0) = 0$
with exact solution

$$w(q) = \frac{1}{2}\cosh q + \frac{1}{2}\cos q \tag{14}$$

Solution

The differential transform of equation (10) **using the above mentioned theorem is given by**

$$(n+1)(n+2)(n+3)(n+4)W(n+4) - W(n) = 0$$

$$W(n+4) = \frac{W(n)}{(n+1)(n+2)(n+3)(n+4)}$$

with the initial condition $w(0) = 1,\ w^{'}(0) = 0,\ w^{''}(0) = 0,\ w^{'''}(0) = 0$**.**

When $n = 0$,

$$W(4) = \frac{W(0)}{(1)(2)(3)(4)}$$
$$W(4) = 0.0416667$$

When $n = 1$,

$$W(5) = \frac{W(1)}{(2)(3)(4)(5)}$$
$$W(5) = 0$$

When $n = 2$,

$$W(6) = \frac{W(2)}{(3)(4)(5)(6)}$$
$$W(6) = 0$$

When $n = 3$,

$$W(7) = \frac{W(3)}{(4)(5)(6)(7)}$$
$$W(7) = 0$$

When $n = 4$,

$$W(8) = \frac{W(4)}{(5)(6)(7)(8)}$$
$$W(8) = 0.0000248$$

When $n = 5$,

$$W(9) = \frac{W(5)}{(6)(7)(8)(9)}$$
$$W(9) = 0$$

When $n = 6$,

$$W(10) = \frac{W(6)}{(7)(8)(9)(10)}$$
$$W(10) = 0$$

Thus, the solution is written as

$$w(q) = \sum_{n=0}^{\infty} q^n W(n)$$
$$w(q) = q^0W(0) + q^1W(1) + q^2W(2) + q^3W(3) + q^4W(4) + ...$$
$$w(q) = 1 + 0.0416667\,q^4 + 0.0000248016\,q^8 + ...$$

Table 4: Numerical Comparison for Example 4

q	Exact	DTM	Error
0	1.00000000	1.00000000	0.00000000
0.1	1.00000417	1.00000417	0.00000000
0.2	1.00006667	1.00006667	0.00000000
0.3	1.00033750	1.00033750	0.00000000
0.4	1.00106668	1.00106668	0.00000000
0.5	1.00260426	1.00260426	0.00000000
0.6	1.00540042	1.00540042	0.00000000
0.7	1.01000560	1.01000560	0.00000000
0.8	1.01707083	1.01707083	0.00000000
0.9	1.02734818	1.02734818	0.00000000
1.0	1.04169147	1.04169147	0.00000000

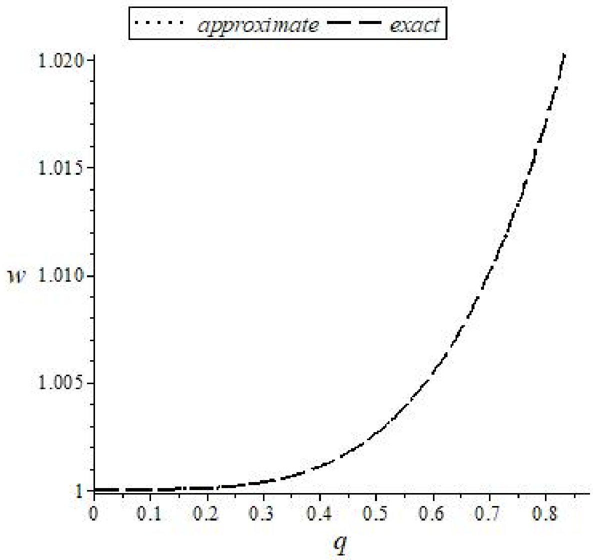

Figure 4: The plot of Exact and DTM Solutions

CONCLUSION

In this study, the Differential Transform Method (DTM) has been effectively employed to solve first, second, third, and fourth-order linear ordinary differential equations, providing accurate approximate solutions. Initially, the differential transform of each problem was computed using the relevant DTM theorems. Following this, recurrence relations were derived from the transformed equations, and solutions were obtained iteratively. These iterative solutions were then organized into Taylor series expansions. To assess the precision of the solutions, both error tables and graphical representations were generated. It was observed that increasing the number of terms in the iterations significantly enhanced the accuracy of the results. A comparative analysis of the exact solutions and those obtained via DTM revealed that the method delivered excellent results, demonstrating its effectiveness. The problems addressed showcased DTM's robustness, reliability, and high convergence rate, affirming its capability to provide precise solutions for linear differential equations. The study underscores DTM's value as a powerful tool for solving complex differential equations with a high degree of accuracy and efficiency. Finally, this work could be extended in the future and apply to non-linear, coupled systems, and fractional version of ordinary differential equations with computational tools, enhancing accuracy and convergence, applying to real-world problems, and advancing educational and theoretical understanding. DTM can continue to evolve as a vital tool in mathematical and engineering research. Embracing these opportunities will enable researchers and practitioners to tackle increasingly complex problems with greater precision and efficiency.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests concerning the publication of this article.

DATA AVAILABILITY STATEMENT

Data sharing is only applicable to this article on reqeuest.

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