



ADVANCED SURVIVAL MODELING OF TUBERCULOSIS PATIENTS: INSIGHTS FROM EXPONENTIAL AND WEIBULL AFT MODELS

*¹Augustina Akor, †²Ibrahim Abubakar Sadiq, ‡²Abubakar Usman,
 §²Sani Ibrahim Doguwa and §³Lawrence Ocheme Akor

¹Doctrine Curriculum and Development Department, Armed Forces Command and Staff College Jaji, Kaduna State Nigeria.

²Department of Statistics, Faculty of Physical Sciences, Ahmadu Bello University, Zaria

³Department of Medical Laboratory Science, Nigerian Defence Academy Hospital, Kaduna State Nigeria.

*Corresponding authors' email: augustina.akor@gmail.com Phone: +2347067016940

ORCID iD: * <https://orcid.org/0009-0004-2231-0832> † <https://orcid.org/0000-0002-2122-9344> ‡ <https://orcid.org/0000-0002-9104-1937> § <https://orcid.org/0000-0002-5779-2358> § <https://orcid.org/0009-0005-6038-5218>

ABSTRACT

Tuberculosis is a significant public health issue in high-burden countries like Nigeria, causing increased disability and claiming many lives. The Cox Proportional Hazards model is commonly used in survival studies, but it fails to define the distribution of survival time. This study uses data from the National Tuberculosis and Leprosy Center (NTLC), Zaria, Kaduna State, Nigeria, to determine Tuberculosis survival and compare alternative parametric survival models. The objectives include determining predictors of TB mortality, evaluating the effect of these predictors on survival probability, and comparing Exponential and Weibull AFT models on NTLC Zaria TB survival data. The results show that the Weibull AFT model is most effective in modelling TB survival rates, with the lowest AIC score of 485.1 and the highest log likelihood of -228.6. Major factors associated with mortality include age over 55 years, pulmonary tuberculosis, family history of tuberculosis, alcohol and smoking history, and BMI less than 18.5 kgs/m². The study emphasizes the need for region-specific survival models to reveal major directions for successful interventions and TB policies. Future studies should consider translating the highly-parametric approach into next-generation non-parametric models/machine learning for more accurate prognoses for implementing state-of-the-art public health interventions.

Keywords: Tuberculosis, Survival Models, AFT model, Weibull Distribution, Exponential Distribution, Proportional Hazard Models

INTRODUCTION

Tuberculosis (TB), caused by *Mycobacterium tuberculosis*, remains a global health challenge despite advances in medical research. In 2021 alone, TB caused an estimated 1.6 million deaths globally, making it one of the leading causes of mortality from infectious diseases (WHO, 2022). TB disproportionately affects developing countries, including Nigeria, where social determinants such as poverty, malnutrition, and inadequate healthcare services contribute to the high burden of the disease (Rexy *et al.*, 2024; WHO, 2024).

In Nigeria, TB incidence remains alarmingly high, with significant mortality rates among vulnerable populations such as the elderly, smokers, and malnourished individuals (Ogunniyi *et al.*, 2024). The National Tuberculosis and Leprosy Control Program (NTLCP) has implemented several interventions, including free TB treatment and community-based diagnostic strategies. Despite these efforts, gaps remain in understanding the survival patterns and risk factors associated with TB mortality.

Survival analysis is a critical tool for exploring time-to-event data in public health research. The Cox proportional hazards (Cox PH) model has been widely used to identify risk factors associated with TB mortality. However, the Cox PH model assumes proportional hazards, which may not always hold in real-world datasets (Kleinbaum and Klein, 2012). Parametric survival models, such as the Weibull and log-logistic models, offer an alternative by allowing for the estimation of specific survival distributions, which can provide deeper insights into TB progression and mortality risk (Collett, 2023). The NOF-G (Sadiq *et al.*, 2022), NGOF-G (Sadiq *et al.*, 2023a), NGOF-

Et-G (Sadiq *et al.*, 2023b), NGOF-OE-G (Sadiq *et al.*, 2023c), NETD using the generalized logarithmic function (Obafemi *et al.*, 2024), and extension of the T-L distribution (Habu *et al.*, 2024), the regression model for diabetes risk factors (Sadiq & Komali, 2020), the general linear model for epilepsy (Sadiq *et al.*, 2020), The Odd Rayleigh-G Family of Distribution: Properties, Applications, and Performance Comparisons (Sadiq *et al.*, 2024); Exploring Accelerated Failure Time Models for Tuberculosis Survival: Loglogistic and Weibull Survival Regression Model (Usman *et al.*, 2025); Modified Inverted Kumaraswamy Distribution Using Inverse Power Function: Properties And Applications (Yusuf *et al.*, 2025), Machine Learning Models in Predicting Failure Times Data Using a Novel Version of the Maxwell Model (Panitanarak *et al.*, 2025) are among the other contributions to parametric survival distribution.

While studies such as Bajehson *et al.* (2019) have explored TB mortality in Nigeria using the Cox PH model, there is limited research employing parametric survival models to evaluate TB data comprehensively. Moreover, the dynamic nature of TB risk factors, influenced by recent health interventions and socio-economic changes, necessitates an updated analysis. Addressing this gap is critical for accurately assessing the effectiveness of interventions and tailoring public health strategies to current realities.

Tuberculosis (TB) continues to be a global health challenge in 2024, with significant advancements in diagnostic techniques, treatment regimens, and public health interventions. Despite these strides, the burden of TB remains substantial, particularly in low- and middle-income countries, where social determinants of health exacerbate its impact

(WHO, 2024). Nigeria, with one of the highest TB burdens globally, faces unique challenges due to socioeconomic disparities, weak healthcare infrastructure, and limited access to diagnostic and treatment facilities (Ogunniyet al., 2024). The advancements in TB research and survival analysis, recent studies emphasize the importance of survival analysis in understanding the progression and outcomes of TB. Traditional approaches, such as the Cox proportional hazards (Cox PH) model, have been instrumental in identifying risk factors associated with TB mortality. However, the assumption of proportional hazards often limits the Cox PH model's applicability to datasets where hazard ratios change over time (Collett, 2023). This limitation has led to increased interest in parametric survival models, which provide greater flexibility and precision in estimating survival probabilities and modelling time-to-event data.

Parametric models, including the Weibull, log-logistic, and exponential models, have shown promise in TB research. For instance, Daniel et al. (2020) applied the Weibull survival model to a cohort of TB patients in Asia, identifying significant risk factors such as age, HIV co-infection, and malnutrition. Their findings highlight the ability of parametric models to capture time-dependent risk factors, offering deeper insights into TB progression. Similarly, Adewale et al. (2024) compared parametric models to the Cox PH model in a study on TB patients in sub-Saharan Africa, concluding that the log-logistic model provided a better fit for data with heavy tails. Recent studies continue to explore the risk factors influencing TB mortality and patient outcomes. Age, gender, comorbidities (especially HIV/AIDS), malnutrition, and socioeconomic status consistently emerge as significant predictors of TB mortality (Bajehsonet al., 2019; WHO, 2024). Additionally, lifestyle factors such as smoking, alcohol use, and previous TB history have been linked to poorer outcomes (Jinet al., 2024). However, recent interventions, including community-based TB treatment programs and nutritional support, have shown the potential to mitigate these risks (Ogunniyet al., 2024).

In Nigeria, studies have focused on understanding the unique socio-cultural and healthcare-related factors influencing TB outcomes. For example, Ogunniyet al. (2024) investigated TB mortality in rural and urban populations, finding disparities driven by healthcare access and diagnostic delays. Moreover, Bajehsonet al. (2019) highlighted the high mortality rates among drug-resistant TB patients, emphasizing the need for robust treatment strategies. These findings underscore the importance of tailoring interventions to Nigeria's diverse population and health system challenges. Despite the growing body of literature, gaps remain in understanding the dynamic risk factors influencing TB survival, particularly in regions like Nigeria. The reliance on traditional Cox PH models in many studies limits their ability to capture non-proportional hazards and varying survival patterns. As such, recent research calls for the application of parametric survival models to address these limitations and provide a more nuanced understanding of TB mortality (Collett, 2023; Ogunniyet al., 2024).

MATERIALS AND METHODS

Method of Data Collection

Survival analysis is a valuable tool for investigating survival time, which refers to the duration until a specific event occurs. This study adopts a retrospective cohort design because it examines the same subjects over four years. The study relies on secondary data obtained from the medical records of HIV/TB patients. The retrospective nature of the studies will disclose the effect of the risk factors identified, and also

follow some clinical factors and the extent to which they influenced the death of the patients. An advantage of a retrospective study is that it reduces the extent to which data collectors influence some of the risk factors and it is based on evidence of what has happened and not what is yet to come. Throughout this study, secondary data from the patient's records is used.

This research adopts a retrospective cohort design, examining the medical records of patients over four years and some necessary information related to tuberculosis was retrieved from the National TB and Leprosy Center Hospital in Zaria, Kaduna State. The patients' folders from the years of interest (2020-2023) were thoroughly examined, and relevant variables such as the survival time were recorded the dependent variable is continuous; it is the waiting time until the occurrence of the event which is the death of a patient. (Dead: 1, alive or censored: 0). Survival time was months from the start of the study year to death while an individual is on TB treatment, in the case of individuals who did not die, Observations are censored, in the sense that, for some subjects, the event of interest has not occurred at the time the data are analyzed. Additionally, several independent variables that may influence patient survival were recorded, including gender, marital status, patients' age, comorbidity, types of TB, smoking history, alcohol and history of patients were all obtained from each patient selected for the study.

Data Analysis Framework

The description of survival data utilizes non-parametric methods to compare the survival functions of two or more groups and kaplan-meier plot(s) would be employed for this purpose (Kaplan & Meier, 1958). A frequency distribution table was used to summarize the data obtained based on the study variables in the National TB and Leprosy Center Hospital in Zaria, Kaduna State. Additionally, various survival models including parametric proportional hazards (PH) models, accelerated failure time (AFT) models, and semi-parametric models were employed to identify risk factors associated with tuberculosis patient survival times. The optimal model was selected based on the lowest information criterion.

Kaplan-Meier (K-M) Estimator (1958) of the Survival Function

The Kaplan-Meier (KM) estimator is a non-parametric method used in survival analysis to estimate the survival function from time-to-event data. It is particularly valuable when dealing with censored data, where the event of interest has not occurred for all subjects within the study period. The Kaplan-Meier estimator also known as the product-limit estimator is the most widely used non-parametric method for estimating the survival function. The Kaplan-Meier estimator provides an estimate of the survival function $S(t)$, which represents the probability that an individual survives beyond time t :

$$S(t) = P(T > t) \quad (1)$$

where T is the random variable representing the time-to-event.

Then the K-M estimator of $S(t)$ is defined:

The KM estimator is defined as:

$$\hat{S}(t) = \prod_{t_i \leq t} \left(1 - \frac{d_i}{n_i}\right) \quad (2)$$

where: t_i : Time of the i^{th} event, d_i : Number of events (e.g., deaths, treatment completion) at t_i , n_i : Number of individuals at risk just before t_i , and \prod : Product overall event times up to t .

The estimator adjusts for censored data by ensuring that only those individuals still at risk at time t are included in the calculation of the survival probability. It produces a step-function survival curve, which is widely used in practice.

Accelerated Failure Time Model (AFT)

The Accelerated Failure Time (AFT) model is a parametric survival analysis approach that directly models the time-to-event (e.g., failure or death). Unlike the proportional hazards model, which focuses on the hazard function, the AFT model describes how covariates accelerate or decelerate the time to an event.

The AFT model assumes that the survival time T is multiplied by a scaling factor due to the effect of covariates. This scaling factor accelerates or decelerates the event's occurrence, depending on the covariate values.

Model Formulation Accelerated Failure Time Model (AFT)

$$\ln(T_i) = \beta^T x_i + \sigma \epsilon_i \tag{3}$$

where: T_i : Survival time for individual i , x_i : Vector of covariates for individual i , β : Coefficient vector quantifying the effect of covariates, σ : Scale parameter, ϵ_i : Random error term with a specified distribution (e.g., exponential, Weibull, Log-logistic etc.).

Alternatively, the model in equation (3) can be written as:

$$T_i = T_0 \exp\{\beta^T x_i\} \tag{4}$$

where $T_0 = \exp\{\sigma \epsilon_i\}$ represents the baseline survival time.

Therefore, The survival functional form of the AFT model is:

$$S(t|x_i) = S_0(t \exp\{-\beta^T x_i\}) \tag{5}$$

Exponential Accelerated Failure Time Model (AFT)

The Accelerated Failure Time (AFT) model with an exponential distribution assumes that survival times follow an exponential distribution, and covariates act to accelerate or decelerate the time to the event. Suppose the baseline survival time S_0 in equation (5) follows an exponential distribution. Therefore the PDF and the baseline survival function for the exponential distribution are given as:

$$f(t) = \lambda \exp\{-\lambda t\}, \quad t \geq 0 \tag{6}$$

where $\lambda > 0$ is the rate parameter (hazard rate).

$$S_0(t) = \exp\{-\lambda t\} \tag{7}$$

However, by substituting equation (5) into equation (7), the exponential AFT model is as:

$$S(t|x_i) = \exp\{-\lambda t \exp\{-\beta^T x_i\}\} \tag{8}$$

Likelihood Function of the Exponential AFT model

The likelihood function for the exponential AFT model is based on the parametric form of the exponential distribution.

For n observations, the likelihood is:

$$L(\lambda, \beta) = \prod_{i=1}^n \lambda \exp\{-\lambda t_i \exp\{-\beta^T x_i\}\} \tag{9}$$

Taking the log-likelihood:

$$\ln L(\lambda, \beta) = \sum_{i=1}^n [\ln \lambda - \lambda t_i \exp\{-\beta^T x_i\}] \tag{10}$$

This function presented in equation (10) can be maximized using MLE to estimate λ and β .

Weibull Accelerated Failure Time Model (AFT)

The Weibull Accelerated Failure Time (AFT) model is an extension of the exponential AFT model, where survival times follow a Weibull distribution. Unlike the exponential distribution, the Weibull distribution allows for non-constant hazard rates, making it more flexible for modelling survival data. The Weibull AFT model assumes that the survival time T_0 in equation (4) follows a Weibull distribution with two parameters:

$$f(t) = \alpha \lambda t^{\alpha-1} \exp\{-(\lambda t)^\alpha\}, \quad t > 0 \tag{11}$$

where:

$\alpha > 0$: Shape parameter, controlling the hazard rate.

$\lambda > 0$: Scale parameter (related to the hazard rate).

Therefore, the baseline survival function for the Weibull distribution is given as:

$$S_0(t) = \exp\{-(\lambda t)^\alpha\} \tag{12}$$

With covariates, however, by substituting equation (5) into equation (12), the Weibull AFT model is as:

$$S(t|x_i) = S_0(t \exp\{-\beta^T x_i\}) = \exp\{-(\lambda t \exp\{-\beta^T x_i\})^\alpha\} \tag{13}$$

Likelihood Function of the Weibull AFT Model

The likelihood function for the Weibull AFT model is derived from the Weibull distribution for n observations:

$$L(\alpha, \lambda, \beta) = \prod_{i=1}^n \alpha \lambda^\alpha t_i^{\alpha-1} \exp\{-(\lambda t_i \exp\{-\beta^T x_i\})^\alpha\} \tag{14}$$

The log-likelihood function is:

$$\ln L(\alpha, \lambda, \beta) = \sum_{i=1}^n [\ln \alpha + \alpha \ln \lambda + (\alpha - 1) \ln t_i - (\lambda t_i \exp\{-\beta^T x_i\})^\alpha] \tag{15}$$

Parameters α , λ , and β in equation (15) can be estimated using maximum likelihood estimation (MLE).

RESULTS AND DISCUSSION

Table 1: Description and Categorization of the covariates

Variable	Description	Categories
Age	The age group of patients	0 = <35 years 1 = 35-55 years 2 = > 55 years
BMI (Kg/m2)	Body Mass Index of Patients	0 = Normal Weight 1= Under Weight 2=Overweight and Obesity
Gender	Sex of patients	0 = Female 1=Male
Marital Status	Patient's marital status	0 = Single 1=Married
Site of Tuberculosis	Type of TB associated with patients	1=Yes 0=No
Family history	History of TB cases in the family of patients	1=Yes 0=No
Alcohol History	Alcohol History	1=Yes 0= No
Smoking History	Smoking History	1=Yes 0=No

Comorbidity	Types of Comorbidities	0 = No Comorbidity 1 = HIV-AIDS 2= Hepatitis
Initial TB Treatment History	If patients had received initial treatment	0 = No 1 =Yes
Life Status	Status of TB	0 = Alive (Censored) 1 = Dead

Table 1 presents a description and categorization of the covariates for better comprehension. The study's response variable was the survival time of tuberculosis in months. The independent variables considered in the study included patient

age, gender, marital status, site of tuberculosis, family history, alcohol history, smoking history, comorbidities, BMI, and initial TB treatment history. These variables were categorized accordingly to facilitate analysis.

Table 2: Distribution of Covariates in the Dataset

		Frequency	Percentage
Gender	Female	115	36%
	Male	208	64%
Age	<35 years	89	28%
	35-55 years	191	59%
	> 55 years	43	13%
Site tuberculosis	Pulmonary	271	84%
	Extra Pulmonary	52	16%
Family history of TB	No	238	74%
	Yes	85	26%
Alcohol history	No	78	24%
	Yes	245	76%
Smoking history	No	170	53%
	Yes	153	47%
Comorbidity	No comorbidity	93	28%
	Hepatitis	115	36%
	HIV-AIDS	115	36%
Initial tb treatment	No	120	37%
	Yes	203	63%
BMI	Normal Weight	112	35%
	Under Weight	134	41%
	Overweight and Obesity	77	24%
Status	Censored	259	80%
	Dead	64	20%

Table 2 above provides an overview of the demographic, clinical, and behavioural characteristics of tuberculosis (TB) patients, along with their treatment history and outcomes. The demographic data indicate a male-dominated population, with 64% being male and 36% female. Most patients (59%) fall within the 35-55 age group, while 28% are younger than 35 years, and only 13% are older than 55 years. This distribution suggests that TB predominantly affects individuals in their prime working years, emphasizing the disease's socio-economic impact.

Clinically, the majority of TB cases are pulmonary (84%), with only 16% categorized as extra-pulmonary. Notably, 26% of patients reported a family history of TB and 74% did not, highlighting a significant hereditary or environmental exposure factor. Alcohol consumption is prevalent among these patients, with 76% admitting to a history of alcohol use and 24% not, smoking history is also significant, with 47% having a history of smoking and 53% not. These behavioural factors are likely contributing to disease susceptibility and progression.

Regarding comorbidities, the population is equally affected by hepatitis (36%) and HIV/AIDS (36%), while 28% reported no additional health conditions. This high prevalence of comorbidities emphasizes the vulnerability of TB patients to other infectious and chronic diseases, which can complicate treatment and outcomes. Moreover, a significant proportion of the patients were underweight (41%), with 24% overweight or obese, and 35% having a normal BMI. Malnutrition, as indicated by the underweight category, could be a contributing factor to weakened immunity and poorer prognosis.

In terms of treatment history and outcomes, 63% of the patients had received prior TB treatment, while 37% were new to treatment. The response variable represents the survival time to death (in months) with the censoring indicator showing that 80% of them were censored (either successfully treated or still undergoing treatment), whereas 20% resulted in death. This mortality rate emphasizes the seriousness of TB, particularly in the context of associated risk factors such as comorbidities, behavioural influences, and undernutrition. These findings highlight the need for integrated care

approaches addressing both TB and its coexisting conditions to improve patient outcomes.

Kaplan-Meier Survival Analysis for the Tuberculosis Patients For a more elaborate descriptive analysis, the study uses a non-parametric Kaplan-Meier approach to provide a summary

of the distribution of the variables. The mean overall survival time of tuberculosis patients is 18.21 months. The maximum and minimum survival times for tuberculosis patients were 0 to 20 months. Patients who survive beyond 18 months after diagnosis are considered long-term survivors in this study.

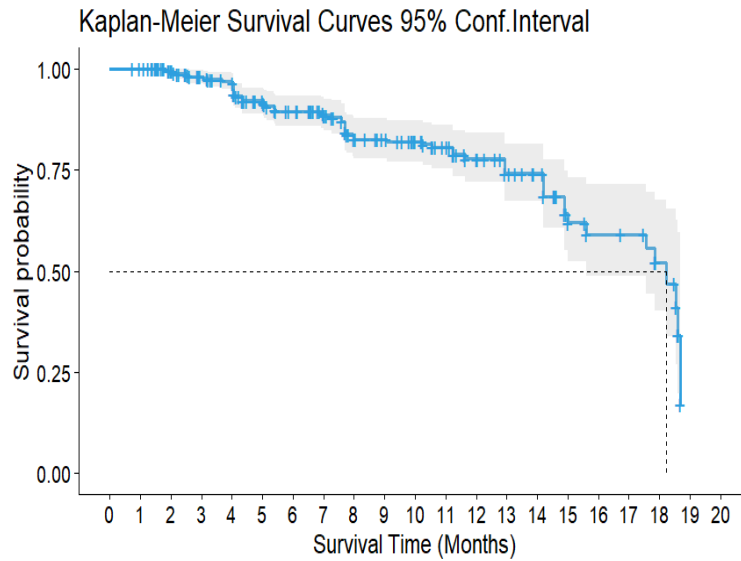


Figure 1: Kaplan-Meier probability of the survival time of Patients with Tuberculosis

Figure 1 depicts the Kaplan-Meier probability of the survival time of tuberculosis patients sampled with a 95 per cent confidence bound. The Kaplan-Meier method estimates survival probabilities over time based on observed survival durations.

Table 3: Kaplan-Meier Survival Probability of the TB Patients

Time	n_i	d_i	Survival Probability	Standard Error	95% Lower CI	95% Upper CI
1.840	302	2	0.993	0.005	0.984	1.000
2.100	296	1	0.990	0.006	0.979	1.000
2.140	293	1	0.987	0.007	0.974	1.000
2.530	286	1	0.983	0.007	0.969	0.998
2.600	284	1	0.980	0.008	0.964	0.996
3.060	276	1	0.976	0.009	0.959	0.994
3.220	274	1	0.973	0.010	0.954	0.992
3.680	267	1	0.969	0.010	0.949	0.989
3.980	266	1	0.965	0.011	0.944	0.987
4.040	262	8	0.936	0.015	0.908	0.965
4.110	238	1	0.932	0.015	0.903	0.962
4.340	235	3	0.920	0.016	0.888	0.953
5.030	214	1	0.916	0.017	0.883	0.949
5.060	212	1	0.911	0.017	0.878	0.946
5.130	209	1	0.907	0.018	0.873	0.943
5.360	207	1	0.903	0.018	0.868	0.939
5.390	206	1	0.898	0.019	0.862	0.936
5.420	203	1	0.894	0.019	0.857	0.932
6.940	185	1	0.889	0.020	0.851	0.928
7.000	182	1	0.884	0.020	0.846	0.924
7.270	178	1	0.879	0.021	0.840	0.920
7.590	174	2	0.869	0.022	0.828	0.912
7.690	167	5	0.843	0.024	0.798	0.891
7.760	149	1	0.837	0.024	0.791	0.886
7.960	145	1	0.832	0.025	0.784	0.882
7.990	144	1	0.826	0.025	0.778	0.877
9.070	131	1	0.820	0.026	0.770	0.872
10.260	117	1	0.813	0.027	0.762	0.866
10.550	115	1	0.805	0.027	0.754	0.861
11.240	91	2	0.788	0.029	0.732	0.848

11.640	72	1	0.777	0.031	0.718	0.840
12.920	64	3	0.740	0.036	0.673	0.814
14.200	53	4	0.685	0.043	0.606	0.774
14.890	32	2	0.642	0.050	0.552	0.747
14.990	28	1	0.619	0.053	0.523	0.732
15.620	22	1	0.591	0.057	0.488	0.715
17.560	17	1	0.556	0.064	0.444	0.696
17.850	16	1	0.521	0.069	0.403	0.675
18.210	10	1	0.469	0.079	0.337	0.653
18.540	8	1	0.410	0.088	0.269	0.626
18.610	6	1	0.342	0.097	0.197	0.595
18.670	2	1	0.171	0.130	0.039	0.760

Table 3 provides insights into the survival probabilities of tuberculosis (TB) patients over time using Kaplan-Meier estimates. At the earliest recorded time (1.840 months), the survival probability is high at 0.993 (99.3%), indicating that most patients are still alive or event-free at this point. Over time, the survival probability decreases steadily, reflecting the occurrence of deaths or events among the patients.

At 4.04 months, there is a noticeable decline in survival probability from 0.965 to 0.936, likely due to the occurrence of 8 events. This sharp drop indicates a critical point where many patients experienced the event of interest. Significant decreases in survival probabilities are also observed at 7.69 months (5 events) and 14.2 months (4 events), suggesting clusters of adverse outcomes.

At 18.67 months the survival probability drops to 0.17 (17.1%), indicating that only a small fraction of patients remain event-free by this time. The wide confidence intervals (95% CI of 0.039 to 0.760) at later times suggest uncertainty due to a smaller number of patients remaining under observation.

At the beginning of the study, the 95% confidence intervals (e.g., 0.984 to 1.000 at 1.840 months) are narrow, reflecting high precision due to a larger sample size. As time progresses and fewer patients remain at risk, the confidence intervals widen (e.g., 0.039 to 0.760 at 18.670 months), indicating reduced precision in survival probability estimates.

The Kaplan-Meier survival curve shows a gradual decline in survival probability, highlighting the progressive nature of TB's impact on patient survival. Critical points in time (e.g., 4.04, 7.69, and 14.2 months) suggest periods where interventions or enhanced monitoring might be necessary to improve patient outcomes. The early and steep declines suggest the importance of timely diagnosis and treatment to improve survival outcomes. This analysis emphasizes the practical use of Kaplan-Meier estimates in understanding survival trends and identifying critical periods for intervention in TB management.

Kaplan Meier curves are plotted for 9 of the categorical variables with time recorded in months. The categorical variables considered for the study are gender, age group, site of tuberculosis, family history, alcohol history, smoking history, comorbidity, initial tuberculosis treatment and BMI. A formal test was carried out using the Log-rank test to compare the difference between each categorical variable. The general hypothesis states that there is no difference between the groups regarding their survival time of tuberculosis. Thus we wish to test that:

Hypothesis

H_0 : The survival times of tuberculosis patients between the groups are not different

H_1 : The survival times of tuberculosis patients between the groups are different

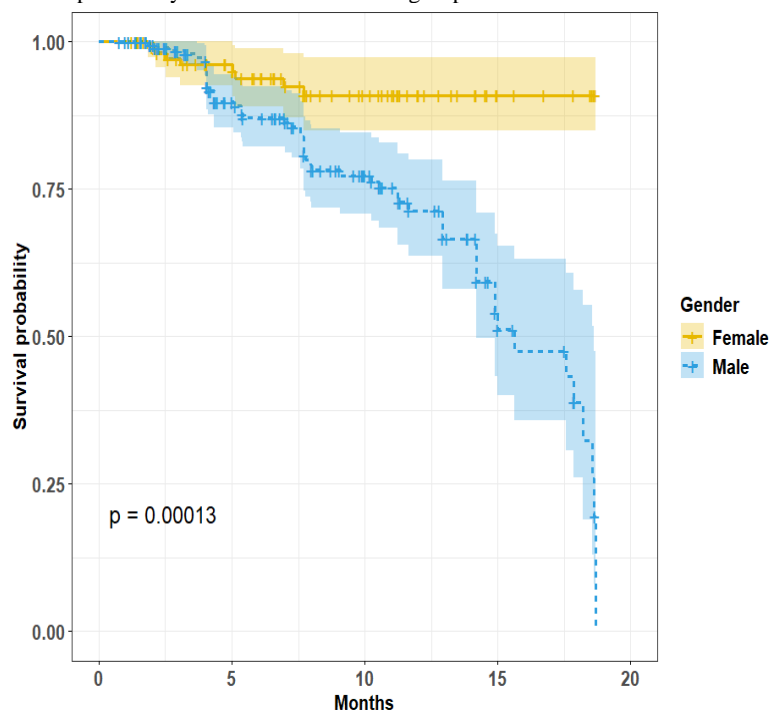


Figure 2: The Kaplan-Meier Plot for Gender

Figure 2 shows the Kaplan-Meier plot for the gender of the TB patients. The yellow curve (females) shows a higher survival probability compared to the blue curve (males) over 20 months. The survival probability for females remains relatively high and stable throughout the period. The survival probability for males decreases more sharply over time, indicating a higher risk of events (e.g., death) compared to females. The p-value of 0.00013 suggests that the difference

in survival probabilities between males and females is statistically significant, meaning that gender has a significant impact on survival outcomes. The Kaplan-Meier survival curve indicates that females have a better survival outcome compared to males over the 20 months, with a statistically significant difference in survival probabilities between the two genders. This suggests that gender plays a crucial role in influencing survival rates.

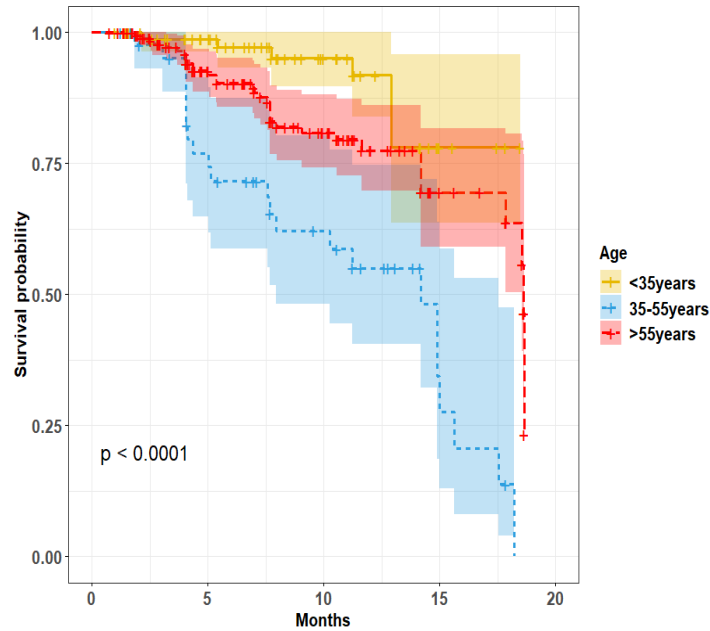


Figure 3: The Kaplan-Meier Plot for Age Group

Figure 3 depicts the Kaplan-Meier plot for the age group of TB patients. The yellow curve (Age < 35 years) shows the highest survival probability, followed by the blue curve (Age 35-55 years), and the red curve (Age > 55 years) shows the lowest survival probability. The survival probability decreases more rapidly for older age groups, indicating a higher risk of events (e.g., death) compared to younger age groups. The p-value of less than 0.0001 suggests that the

differences in survival probabilities among the age groups are statistically significant, meaning that age has a significant impact on survival outcomes. The Kaplan-Meier survival curve indicates that younger individuals have better survival outcomes compared to older individuals over the 20 months, with a statistically significant difference in survival probabilities among the age groups. This suggests that age plays a crucial role in influencing survival rates.

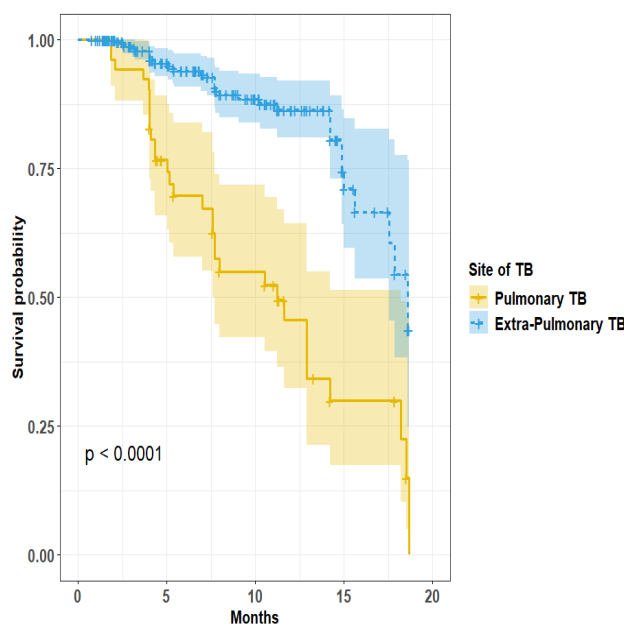


Figure 4: The Kaplan-Meier Plot for the Site of the TB

Figure 4 depicts the Kaplan-Meier plot for the site of TB patients. The yellow curve (Pulmonary TB) shows a higher survival probability compared to the blue curve (Extra-Pulmonary TB) over the 20 months. The survival probability for Pulmonary TB remains relatively high and stable throughout the period. The survival probability for Extra-Pulmonary TB decreases more sharply over time, indicating a higher risk of events (e.g., death) compared to Pulmonary TB. The p-value of less than 0.0001 suggests that the difference in

survival probabilities between Pulmonary TB and Extra-Pulmonary TB is statistically significant, meaning that the site of TB has a significant impact on survival outcomes. The Kaplan-Meier survival curve indicates that patients with Pulmonary TB have better survival outcomes compared to those with Extra-Pulmonary TB over the 20 months, with a statistically significant difference in survival probabilities between the two groups. This suggests that the site of TB plays a crucial role in influencing survival rates.

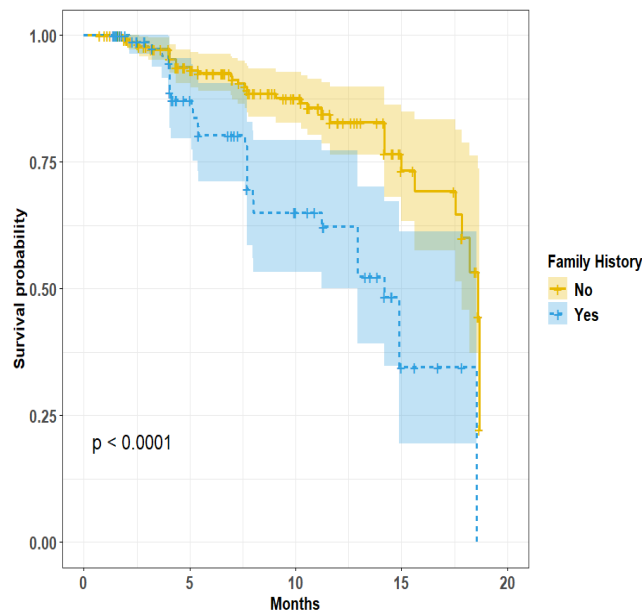


Figure 5: The Kaplan-Meier Plot for the Family History of the TB

Figure 5 depicts the Kaplan-Meier plot for the family history of TB patients. The yellow curve (No Family History of TB) shows a higher survival probability compared to the blue curve (Family History of TB) over 20 months. The survival probability for individuals with a family history of TB decreases more sharply over time, indicating a higher risk of events (e.g., death) compared to those with no family history of TB. The p-value of less than 0.0001 suggests that the difference in survival probabilities between the two groups is

statistically significant, meaning that a family history of TB has a significant impact on survival outcomes. The Kaplan-Meier survival curve indicates that individuals with no family history of TB have better survival outcomes compared to those with a family history of TB over 20 months, with a statistically significant difference in survival probabilities between the two groups. This suggests that a family history of TB plays a crucial role in influencing survival rates.

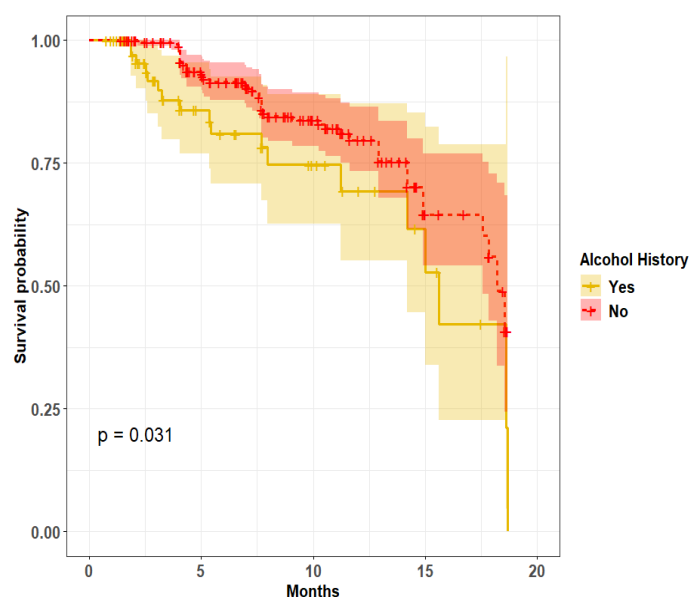


Figure 6: The Kaplan-Meier Plot for the Alcohol History of the TB

Figure 6 depicts the Kaplan-Meier plot for the alcohol history of TB patients. The red curve (No Alcohol History) shows a higher survival probability compared to the yellow curve (Alcohol History) over 20 months. The survival probability for patients with a history of alcohol use decreases more sharply over time, indicating a higher risk of events (e.g., death) compared to those without a history of alcohol use. The p-value of 0.031 suggests that the difference in survival probabilities between the two groups is statistically

significant, meaning that alcohol history has a significant impact on survival outcomes. The Kaplan-Meier survival curve indicates that TB patients without a history of alcohol use have better survival outcomes compared to those with a history of alcohol use over the 20 months, with a statistically significant difference in survival probabilities between the two groups. This suggests that alcohol history plays a crucial role in influencing survival rates.

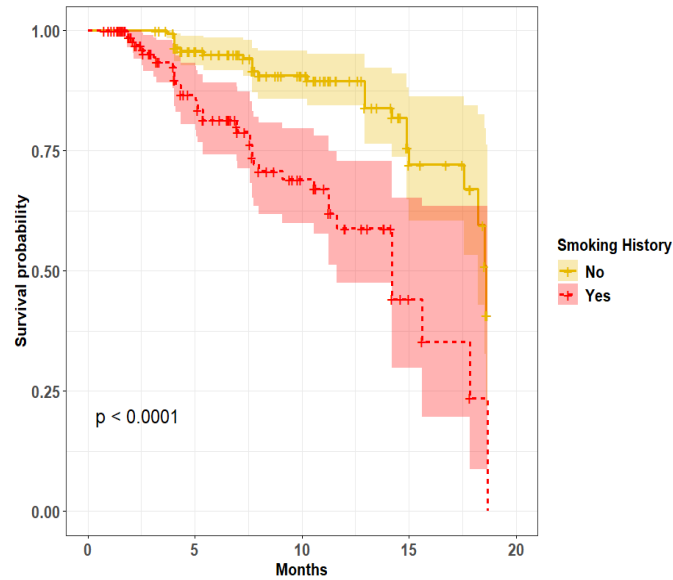


Figure 7: The Kaplan-Meier Plot for the Smoking History of the TB

Figure 7 depicts the Kaplan-Meier plot for the smoking history of TB patients. The yellow curve (No Smoking History) shows a higher survival probability compared to the red curve (Smoking History) over the 20 months. The survival probability for patients with a history of smoking decreases more sharply over time, indicating a higher risk of events (e.g., death) compared to those without a history of smoking. The p-value of less than 0.0001 suggests that the difference in survival probabilities between the two groups is statistically

significant, meaning that smoking history has a significant impact on survival outcomes. The Kaplan-Meier survival curve indicates that TB patients without a history of smoking have better survival outcomes compared to those with a history of smoking over the 20 months, with a statistically significant difference in survival probabilities between the two groups. This suggests that smoking history plays a crucial role in influencing survival rates.

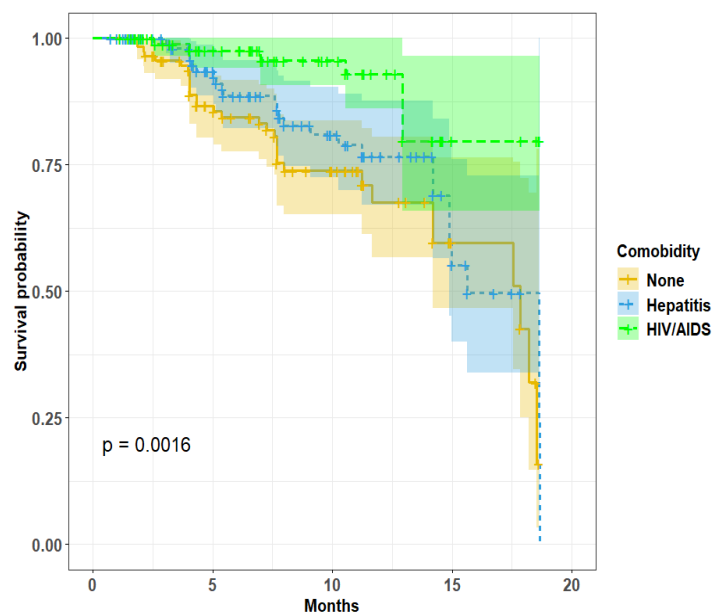


Figure 8: The Kaplan-Meier Plot for the Comorbidity of the TB

Figure 8 depicts the Kaplan-Meier plot for the comorbidity of TB patients. The yellow curve (No Comorbidity) shows the highest survival probability compared to the other groups over the 20 months. The blue curve (Hepatitis) shows a lower survival probability than the no comorbidity group but higher than the HIV/AIDS group. The green curve (HIV/AIDS) shows the lowest survival probability, indicating a higher risk of events (e.g., death) compared to the other groups. The p-value of 0.0016 suggests that the differences in survival probabilities among the three groups are statistically

significant, meaning that comorbidity status has a significant impact on survival outcomes. The Kaplan-Meier survival curve indicates that TB patients with no comorbidities have the best survival outcomes, followed by those with Hepatitis, and those with HIV/AIDS have the worst survival outcomes over the 20 months. The statistically significant difference in survival probabilities among the groups suggests that comorbidity status plays a crucial role in influencing survival rates.

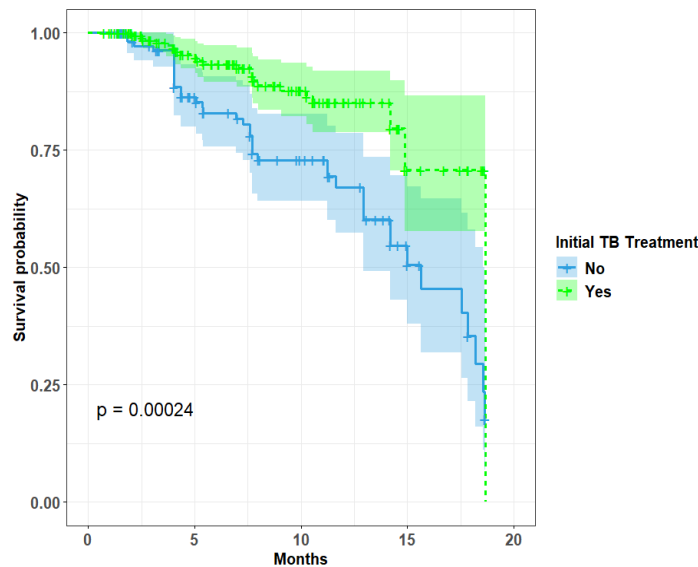


Figure 9: The Kaplan-Meier Plot for the Initial Treatment of the TB

Figure 9 depicts the Kaplan-Meier plot for the initial treatment of TB patients. The green curve (Initial TB Treatment) shows a higher survival probability compared to the blue curve (No Initial TB Treatment) over the 20 months. The survival probability for patients who received initial TB treatment remains relatively high and stable throughout the period. The survival probability for patients who did not receive initial TB treatment decreases more sharply over time, indicating a higher risk of events (e.g., death) compared to those who received initial TB treatment. The p-value of

0.00024 suggests that the difference in survival probabilities between the two groups is statistically significant, meaning that initial TB treatment has a significant impact on survival outcomes. The Kaplan-Meier survival curve indicates that TB patients who received initial TB treatment have better survival outcomes compared to those who did not receive initial TB treatment over the 20 months, with a statistically significant difference in survival probabilities between the two groups. This suggests that initial TB treatment plays a crucial role in influencing survival rates.

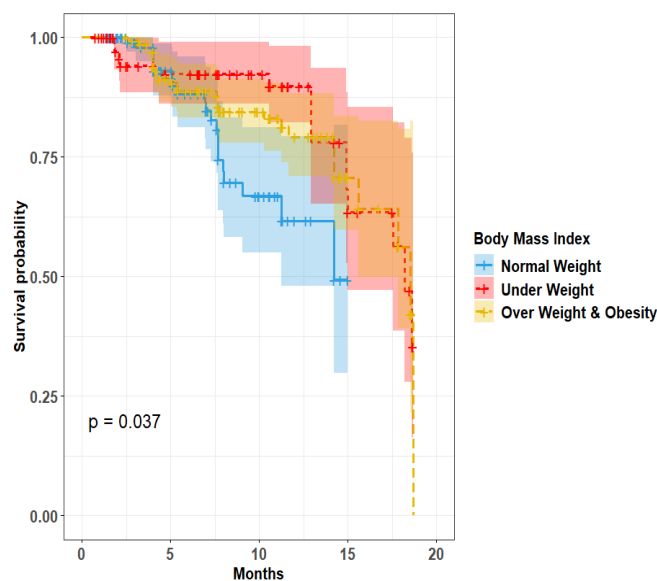


Figure 10: The Kaplan-Meier Plot for the BMI of the TB Patients

Figure 10 depicts the Kaplan-Meier plot for the initial treatment of TB patients. The blue solid line (Normal Weight) shows the highest survival probability compared to the other BMI categories over the 20 months. The yellow dotted line (Overweight & Obesity) shows a lower survival probability than the normal weight group but higher than the underweight group. The red dashed line (Underweight) shows the lowest survival probability, indicating a higher risk of events (e.g., death) compared to the other BMI categories. The p-value of 0.037 suggests that the differences in survival probabilities

among the BMI categories are statistically significant, meaning that BMI has a significant impact on survival outcomes. The Kaplan-Meier survival curve indicates that TB patients with a normal BMI have the best survival outcomes, followed by those who are overweight or obese, and those who are underweight have the worst survival outcomes over the 20 months. The statistically significant difference in survival probabilities among the BMI categories suggests that BMI plays a crucial role in influencing survival rates.

Table 4: Exponential AFT Model fitted to the Tuberculosis (TB) Dataset

Covariates	Coefficients	Standard Error	z	P
(Intercept)	3.7183	0.7758	4.79	1.60e-06
(Age) > 55 years	-1.5950	0.4721	-3.38	0.00073
(Age) 35-55 years	-0.9790	0.4322	-2.27	0.0235
(Gender) Male	-0.7839	0.3939	-1.99	0.04657
(SiteTB) Pulmonary	1.25080	0.2678	4.67	3.00e-06
(FamilyHist) Yes	-0.7268	0.2823	-2.57	0.01004
(AlcoholHist) Yes	0.6676	0.3371	1.98	0.04762
(SmokingHist) Yes	-0.7052	0.2864	-2.46	0.0138
(Comorbidity) HIV-AIDS	0.1738	0.3010	0.58	0.56362
(Comorbidity) No comorbidity	0.7310	0.4567	1.6	0.10944
(TB treatment) Yes	0.7057	0.2811	2.51	0.01206
(BMI) Overweight and Obesity	0.0522	0.3883	0.13	0.89299
(BMI) Under Weight	0.4719	0.3146	1.5	0.13356
	χ^2	p-value	Loglik	AIC
Exponential AFT	99.45	7.1e-16	-252.6	531.2722

Table 4 presents the results of an exponential AFT model fitted to the tuberculosis (TB) dataset. The model log-likelihood is -252.6, while the intercept-only model is -302.4. This indicates an improvement in fit when covariates are included. Then ($\chi^2 = 99.45, df = 12, p = 7.1 \times 10^{-16}$), indicating that the covariates significantly improve the model fit. The Akaike Information Criterion (AIC) is 531.27, useful for comparing model fits.

Each covariate represents a coefficient from the exponential AFT model. Positive coefficients indicate an increase in survival time (hazard decreases), while negative coefficients indicate a decrease in survival time (hazard increases). Patients aged >55 years ($z = -3.38, p = 0.00073$) and 35-55 years ($z = -2.27, p = 0.02350$) have significantly shorter survival times compared to those aged <35 years. Male patients ($z = -1.99, p = 0.04657$) have significantly shorter survival times compared to females.

Pulmonary TB ($z = 4.67, p = 3.0 \times 10^{-6}$) is associated with longer survival compared to extrapulmonary TB. Having a family history of TB ($z = -2.57, p = 0.01004$) is associated

with shorter survival. A history of alcohol consumption ($z = 1.98, p = 0.04762$) is associated with longer survival. A history of smoking ($z = -2.46, p = 0.01380$) is associated with shorter survival. Neither HIV/AIDS ($p = 0.56362$) nor "No comorbidity" ($p = 0.10944$) show significant effects on survival. Patients who received TB treatment ($z = 2.51, p = 0.01206$) have significantly longer survival. Neither underweight ($p = 0.13356$) nor overweight/obesity ($p = 0.89299$) show significant effects on survival.

The model identifies significant predictors of survival for TB patients, including age, gender, TB site, family history, alcohol history, smoking history, and treatment status. Non-significant covariates, such as comorbidity and BMI, may not substantially affect survival time in this context. The exponential distribution assumes a constant hazard rate over time, which may need to be assessed for appropriateness with diagnostic tools.

Table 5: Weibull AFT Model fitted to the Tuberculosis (TB) Dataset

Covariates	Coefficients	Standard Error	z	p
(Intercept)	2.8114	0.3739	7.5200	5.50e-14
(Age) > 55 years	-0.7649	0.2294	-3.3300	0.00086
(Age) 35-55 years	-0.5082	0.2029	-2.500	0.01228
(Gender) Male	-0.2853	0.1843	-1.5500	0.12166
(SiteTB) Pulmonary	0.5852	0.1315	4.4500	8.60e-06
(FamilyHist) Yes	-0.4787	0.1324	-3.6100	0.00030
(AlcoholHist) Yes	0.4207	0.1667	2.5200	0.01161
(SmokingHist) Yes	-0.4981	0.1402	-3.5500	0.00038
(Comorbidity) HIV-AIDS	0.0857	0.1475	0.5800	0.56131
(Comorbidity) No comorbidity	0.2547	0.2129	1.2000	0.23141
(TB treatment) Yes	0.3544	0.1314	2.7000	0.00700

(BMI) Overweight and Obesity	0.2272	0.1832	1.2400	0.21483
(BMI) Under Weight	0.5061	0.1446	3.5000	0.00046
Log(scale)	-0.8047	0.0981	-8.2100	2.30e-16
	χ^2	p-value	Loglik	AIC
Weibull AFT	117.46	2.0e-19	-228.6	485.1086

Table 5 presents the results of the Weibull AFT model fitted to the tuberculosis (TB) dataset. The model's log-likelihood is -228.6, significantly improved from the intercept-only model's -287.3, indicating that the inclusion of covariates improves the model's fit. Then ($\chi^2 = 117.46, df = 12, p = 2 \times 10^{-19}$), demonstrates that the covariates collectively contribute significantly to the model. The Akaike Information Criterion (AIC) is 485.11, useful for model comparison. A Weibull scale parameter less than 1 suggests decreasing hazard rates over time.

Patients aged >55 years ($z = -3.33, p = 0.00086$) and 35-55 years ($z = -2.50, p = 0.01228$) have significantly shorter survival times compared to those aged <35 years. Male patients ($p = 0.12166$) show no significant difference in survival compared to females. Pulmonary TB ($z = 4.45, p = 8.6 \times 10^{-6}$) is associated with longer survival compared to extrapulmonary TB. Having a family history of TB ($z = -3.61, p = 0.00030$) is associated with shorter survival. Patients with a history of alcohol consumption

($z = 2.52, p = 0.01161$) have significantly longer survival. A history of smoking ($z = -3.55, p = 0.00038$) is associated with shorter survival. Neither HIV/AIDS ($p = 0.56131$) nor "No comorbidity" ($p = 0.23141$) are significantly associated with survival. Patients receiving TB treatment ($z = 2.70, p = 0.00700$) have significantly longer survival. Being underweight ($z = 3.50, p = 0.00046$) is associated with longer survival, but being overweight or obese ($p = 0.21483$) shows no significant effect.

Age, TB site, family history, alcohol history, smoking history, TB treatment, and being underweight are significant predictors of survival time. Gender, comorbidities, and being overweight/obese do not significantly affect survival time. The decreasing hazard rate of the Weibull AFT model suggests that the risk of death declines as survival time increases, which aligns with the disease progression in treated TB patients. These findings emphasize the importance of targeted interventions for older patients, smokers, and those with a family history of TB to improve survival outcomes.

Table 6: AFT Models Performance Comparison fitted to the Tuberculosis (TB) Dataset

Models	χ^2	p-value	Loglikelihood	AIC
Exponential AFT	99.45	7.1e-16	-252.6	531.2722
Weibull AFT	117.46	2.0e-19	-228.6	485.1086

Table 6 presents the comparative performance of three parametric Accelerated Failure Time (AFT) models, Exponential, and Weibull fitted to the tuberculosis (TB) dataset. The performance metrics include the chi-square statistic, p-value, log-likelihood, and Akaike Information Criterion (AIC). All models have statistically significant chi-square values ($p < 0.05$), indicating that the covariates contribute meaningfully to explaining the survival times. Among the models, the Weibull AFT model has the highest chi-square statistic (117.46), suggesting the strongest explanatory power for survival time variability. The Weibull AFT model has the highest log-likelihood (-228.6), indicating the best fit to the data among the three models. The AIC measures model quality, balancing fit and complexity. Lower AIC values indicate better models. The Weibull AFT model has the lowest AIC (485.1086), demonstrating the best trade-off between model fit and parsimony.

The exponential AFT model is simplistic with constant hazard assumption and has the lowest chi-square value (99.45) and highest AIC (531.2722), indicating it performs poorly compared to the Weibull model. The Weibull AFT model is flexible with varying hazard rates. Best overall performance, as it has the highest chi-square value, highest log-likelihood, and lowest AIC. This model is most suitable for the TB dataset. The Weibull AFT model is the best-fitting model for the tuberculosis dataset, as it provides the best balance of explanatory power and model parsimony. It is recommended for use in further analysis and interpretation of TB survival data.

CONCLUSION

A convenience sample of data covering a total of 324 patients was considered in this study, consisting of tuberculosis patients at the respiratory ward and TB treatment units of the

National TB and Leprosy Center (NTLC) in Zaria, Kaduna State. The study focused on patients who have ever suffered from tuberculosis and are on referral to the respiratory unit and general ward of the hospital. Since the hospital serves as the major referral centre for most Kaduna TB patients and even other Neighboring states in Nigeria, limits the coverage area and general conclusion. This study highlights the importance of parametric survival models in understanding the survival patterns and risk factors influencing tuberculosis (TB) outcomes. The Weibull AFT model emerged as the most suitable for analyzing TB survival data from the National Tuberculosis and Leprosy Center (NTLC) in Zaria, Nigeria, outperforming other models in capturing the time-to-event dynamics. Key risk factors, including age, TB site, smoking history, and body mass index, were identified as significant determinants of mortality. The findings underscore the need for tailored interventions targeting these risk factors to improve patient survival. Additionally, the study emphasizes the value of applying diverse survival models to ensure robust and accurate analyses, contributing to evidence-based strategies for TB management in Nigeria. Future studies should consider translating the highly-parametric approach into next-generation non-parametric models/machine learning for more accurate prognoses for implementing state-of-the-art public health interventions. Based on the findings, the following recommendations are made: for the policy and intervention strategies, targeted interventions should focus on addressing significant risk factors such as smoking, underweight status, and advanced age to improve TB survival outcomes; for patient-centred care, efforts should be made to enhance nutritional support for TB patients, especially those underweight, and provide additional care for older patients and those with pulmonary TB; for capacity building, healthcare professionals should receive training on the

application of parametric survival models to ensure accurate and insightful analysis of TB survival data; for surveillance and monitoring, continuous collection and analysis of TB data at NTLC and other centres are recommended to monitor trends, assess interventions' effectiveness, and refine predictive models; for broader model applications, researchers are encouraged to explore diverse survival models beyond Cox PH to gain deeper insights into the survival distributions of TB patients in various contexts; for public health awareness, strengthen awareness campaigns focusing on the risks associated with smoking and poor nutritional status to mitigate their impact on TB progression and mortality.

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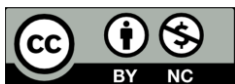
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