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A MATHEMATICAL MODEL OF THE EFFECTS OF IRRIGATION AND NITROGEN FOR UPLANDRICE DURING DRY SEASON IN BADEGGI ENVIRONMENT

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ABSTRACT

Rice is one of the major stable foods in Nigeria. It is also a source of energy, income and employment opportunity in Nigeria. The demand for rice in Nigeria has exceeded the production, due to ever increasing population and there is hike in the price of rice. The aim of this work is to apply a Mathematical model of Quadratic Response Surface for Two Factors to determine the nitrogen fertilizer and irrigation levels that would maximize rice production and to analyze the effects of nitrogen fertilizer and irrigation on upland rice yield. Analysis of variance (ANOVA) was used for the data collected form National Cereal Research Institute Badeggi, Niger State. The method of least square, R^2 (coefficient of determination) and root mean square error (RMSE) was used to evaluate the rate of nitrogen (30, 60, 90, 120 $kgNha^{-1}$) and (7, 14, 21days). The result showed that the models were adequate and significant at 5%. The results also indicated that application of 90-120$kgNha^{-1}$and 7days interval of irrigation enhanced upland rice yield. It was also known that there was an increase in yield when there is frequent irrigation combined with application of nitrogen fertilizer (90-120$kgNha^{-1}$) and computation of the data was adequate with R^2 above 50% and root mean square error (RMSE) was very small.

Keywords: Irrigation, Least Square, Nitrogen fertilizer, Root Mean Square Error (RMSE), Significant at 5%

INTRODUCTION

Background of the Study

Rice is an important staple in Nigeria and accounts for the primary food source for 50% of the global populace (Zewdined et al., 2020). Furthermore, it is a great source of energy worldwide, coming second after maize in terms of production (Manjapa & Shailaja, 2014). In Nigeria, growing rice helps improve food security and provides income and jobs (Zewdined et al., 2020). The production of rice is influenced by different agronomic and managerial factors such as climate, soil and its fertility, water, weed management, fertilizer, temperature, and rice seed quality.

Water is needed to maximize rice yield, because it is one of the most important factors for rice production in the world since it consumes the highest amount of water than any other crop in the agricultural sector (Gbetondji, 2017). According to Suredra *et al.* (2021) about 34 to 43% of the total world's available fresh water is used for irrigation. About three thousand to five thousand liters of water is required to produce one kilogram of rice, it is therefore, necessary to plant rice in a flooded area, therefore, irrigation is needed during dry season to maximize rice yield.

Irrigation is the act of artificially providing water to soil through various systems of tubes, pumps and sprays based on the requirement of the planted crops throughout the growing season to enhance the complete nourishment of the crop, which increases productivity. The productivity on irrigated land is higher as compared to the un-irrigated soil. It shows that water is highly needed to maximize rice yield (Maduri *et al.,* 2023).

Traditionally, rice is cultivated under continuously flooded condition in irrigated areas, which resulted in high amount of water used (Dominic *et al*., 2017). Previous studies have shown that alternate wet and dry (AWD) irrigation management is the best method to reduce water input as well as increase water production of rice, to alternate wet and dry reduced water used and increase the percentage of water saved when compared to the continuous submergence treatment. This agrees with Abdul-Ganiyu and Chu (2015) who reported that continuous submergence increased water use and reduced water productivity. Another factor which affects the yield of upland rice is nitrogen fertilizer.

Nitrogen is also one of the vital factors which affect the yield of rice. It has been shown that increasing Nitrogen fertilizer would reduce yield and bring negative effect on soil, (Gewail, 2018), therefore, before making recommendations for the nitrogen fertilizer dose for any crop one should evaluate the efficiency and optimum rate for different application level for better growth and yield performance of each released rice variety. According to (Gewaily *et al*., 2018) Excessive use of nitrogen fertilizer has negative impact on soil and environment through residual effects.

Determination of appropriate dosage of Nitrogen fertilizer application will improve the productivity and consequent profit of the grower. Application of Nitrogen fertilizer either in excess or less than optimum rate affects the yield and quality of rice to remarkable extent (Zewdineh *et al.*, 2020). Nitrogen promotes rapid plant growth and improves grain yield and grain quality through higher filling and protein synthesis. Nitrogen is also a component of chlorophyll molecule which enables the plant to capture sunlight energy, through photosynthesis which plants need for growth and grain yield. Mathematical models are needed to determine the level of nitrogen fertilizer and irrigation needed to maximize upland rice yield.

According to Shehu (2016), Mathematical model are useful experiment tools for building, testing theories, answering specific questions and estimating specific parameters from data observed or collected. Application of mathematics to the production of crops has becomes a global issue today. Mathematical models are numerical or quantitative representation of a real word system. There are different types of mathematical models, and they are commonly constants and variables.

Musa (2023) stated that Mathematical model is a triplet [S,Q,M] where S is a system, Q is a question relating to S and M is a set of mathematical statements which can be used to answer Q. There are three main types of mathematical models which can be used to display rice yield which includes: equations, graphs, and computer modeling and each of these can be used to learn about different aspects of the rice yield and it has the ability to analyze and optimize rice production (Kaltenine et al., 2023). The aim of this work is to apply a Mathematical model of Quadratic Response Surface for Two Factors and statistical approach to determine the Nitrogen fertilizer and irrigation levels that will maximize rice production and to analyze the effects of Nitrogen fertilizer and irrigation on upland rice yield.

MATERIALS AND METHODS

The data used for this research work was obtained from Rice Research Program of the National Cereal Research Institute (NCRI), located in Baddegi, Niger state, Nigeria. Field trials were conducted during the third quarter of the 2016 rainy seasons in Niger State. The treatments that were applied in the course of experimentation consisted of three (3) irrigation intervals (7, 14 and 21 days) and four(4) fertilizer rates (30, 60, 90 and 120kg/ha^{-1}) to check their combinational effects on the yield on the rice. Hence, the experiment was based on a mixed level complete factorial design.

Model Equation

The quadratic model for the two factors used in this research work is given as:

$$Y = b_0 + b_1x_1 + b_2x_2 + b_{11}x_1^2 + b_{22}x_2^2 + b_{12}x_1x_2 + e \quad (1)$$

Where

Y= yield response, b_1 = estimated parameter (I = 0, 1, 2 and 3), x_1 = irrigation system

x_2 = Nitrogen fertilizer applied, e = random error (i.e. constant)

Differentiating equation (1) partially and equating to zero, with respect to x_1 and x_2 we have

$$b_1 + 2b_1x_1 + b_{12}x_2 = 0 \quad (2)$$

$$b_2 + b_{1_2}x_1 + 2b_{22}x_2 = 0 \quad (3)$$

$$2b_{11}x_1 + b_{1_2}x_2 = -b_1 \quad (4)$$

$$b_{12}x_1 + 2b_{2_2}x_2 = -b_2 \quad (5)$$

Equaction (4 and 5) were changed into matrix form

$$\begin{bmatrix} 2b_{11} & b_{12} \\ b_{12} & 2b_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -b_1 \\ -b_2 \end{bmatrix} \quad (6)$$

By using Crammar's Rule

$$\begin{bmatrix} 2b_{11} & b_{12} \\ b_{12} & 2b_{22} \end{bmatrix} \quad (7)$$

$$det(\Delta_1) = 4b_{11}b_{22} - b_{12}b_{12}$$

$$\Delta_2 = \begin{bmatrix} b_{12} & -b_1 \\ 2b_{22} & -b_2 \end{bmatrix}$$

$$det(\Delta_2) = -2b_1b_{22} + b_2b_{12}$$

$$\Delta_3 = \begin{bmatrix} 2b_{11} & -b_1 \\ b_{12} & -b_2 \end{bmatrix} \quad (8)$$

$$det(\Delta_3) = -2b_{11}b_2 + b_1b_{12}$$

$$x_1 = \frac{\Delta_2}{det(\Delta_1)} = \frac{-2b_1b_{22}+b_2b_{12}}{4b_{11}b_{22}-b_{12}b_{12}} \quad (9)$$

The value of x_2 is:

$$x_2 = \frac{\Delta_3}{det(\Delta_1)} = \frac{-2b_{11}b_2+b_1b_{12}}{4b_{11}b_{22}-b_{12}b_{12}} \quad (10)$$

Step 3

To determine the characteristics equation, divide R_1 and R_2 of equation (7) by 2

$$\begin{bmatrix} b_{11} - \lambda & \frac{b_{12}}{2} \\ \frac{b_{12}}{2} & b_{22} - \lambda \end{bmatrix} = 0$$

$$(b_{11} - \lambda)(b_{22} - \lambda) - \frac{b_{12}b_{12}}{4} = 0$$

Multiply through by 4

$$4(b_{11} - \lambda)(b_{22} - \lambda) - b_{12}b_{12} = 0$$

$$4(b_{11}b_{12} - \lambda b_{11} - \lambda b_{22} + \lambda^2) - b_{12}b_{12} = 0$$

$$4b_{11}b_{12} - 4\lambda b_{11} - 4\lambda b_{22} + 4\lambda^2 - b_{12}b_{12}$$

$$4b_{11}b_{22} - (4b_{11} + 4b_{22})\lambda + 4\lambda^2 - b_{12}b_{12} = 0$$

$$4\lambda^2 - (4b_{11} + 4b_{22})\lambda + (4b_{11}b_{22} - b_{12}b_{12}) = 0 \quad (11)$$

This step was computed by maple software;

$$y - y_p = b_{11}x_1^2 + b_{22}x_2^2$$

$$y_p = y - b_{11}x_1^2 - b_{22}x_2^2 \quad (12)$$

where

y= observed value

y_p = maximum value

b_{11} and b_{22} = unknown parameter

$$b_{11} = \frac{[(\sum x_2^2)(\sum x_1y)] - (\sum x_1x_2)(\sum x_2y)}{(\sum x_1^2)(\sum x_2^2) - (\sum x_1x_2)^2} \quad (13)$$

$$b_{22} = \frac{[(\sum x_1^2)(\sum x_2y)] - (\sum x_1x_2)(\sum x_1y)}{(\sum x_1^2)(\sum x_2^2) - (\sum x_1x_2)^2} \quad (14)$$

RESULTS AND DISCUSSION

From table 1 the fitted model is given as:

$$Y = 45.904 + 0.696x_1 + 0.087x_2 + 0.021x_1^2 + 0.000161x_2^2 - 0.0064x_1x_2$$

Table 1: The Quadratic Model for the Two Factors

Term	Coefficient
CONSTANT	45.9042
IRRIGATION	-0.695982
NITROGEN	0.0874069
IRRIGATION*IRRIGATION	0.0214605
NITROGEN*NITROGEN	0.000160880
IRRIGATION*NITROGEN	-0.00638988

The sign of the coefficients indicates the direction of the relationship while the coefficient value represents the strength of the relationship, the linear coefficient value of the irrigation is -0.696, which indicates that if all other terms are held constant in the model, the rice will decrease by 0.696. The quadratic coefficient of the irrigation is 0.021 which indicates that, if other terms are held constant in the model, the rice will increase by 0.021

Table 2: Analysis of Multiple Regressions

Source of variation	df	SS	MS	F-value	Prob
Model	2	543.541	271.7702	17.72093	2.11E-06
Residual	45	690.1259	15.33613		
Total	47	1233.667			

Test of Hypothesis
Ho: $\beta = 0$
H1: $\beta \neq 0$

Table 3: Analysis of Variance

Source	Df	Seq SS	Adj SS	Adj MS	F-Value	P-value
Model	5	628.37	628.367	125.673	8.72	0.000
Linear	2	543.54	543.541	271.770	18.86	0.000
Irrigation	1	517.21	517.213	517.213	35.89	0.000
Nitrogen	1	26.33	26.328	26.328	1.83	0.184
Square	2	12.80	12.801	6.401	0.44	0.644
Irrigation *Irrigation	1	11.80	11.795	11.795	0.82	0.371
Nitrogen*Nitrogen	1	1.01	1.006	1.006	0.07	0.793
2-way Interaction	1	72.03	72.025	72.025	5.00	0.031
Irrigation*Nitrogen	1	72.03	72.025	72.025	5.00	0.031
Residual Error	42	605.30	605.299	14.412		
Lack-of-Fit	6	146.90	146.902	24.484	1.92	0.104
Pure Error	36	458.40	458.398	12.733		
Total	47	1233.67				

Table 4: The Minimum and the Maximum point of interaction of irrigation and Nitrogen fertilizer

N/S	x_1	x_2	x_1^2	x_2^2	$x_1 * y$	$x_2 * y$	$x_1 * x_2$	Y
1	7	30	49	900	307.475	1317.75	210	42.751
2	7	60	49	3600	318.85	2733	420	43.941
3	7	90	49	8100	341.425	4389.75	630	46.442
4	7	120	49	14400	330.6275	5667.9	840	43.885
5	14	30	196	900	548.1	1174.5	420	34.889
6	14	60	196	3600	529.97	2271.3	840	33.159
7	14	90	196	8100	621.95	3998.25	1260	39.005
8	14	120	196	14400	612.71	5251.8	1680	37.331
9	21	30	441	900	863.625	1233.75	630	31.719
10	21	60	441	3600	817.425	2335.5	1260	28.834
11	21	90	441	8100	747.6	3204	1890	25.035
12	21	120	441	14400	791.07	4520.4	2520	24.671

Table 2 showed that the probability value is less than level of significant (0.05) so we reject the null hypothesis (H_0) and conclude that rice is significantly dependent on irrigation and nitrogen.

Table 3 showed the statistical significance is checked using the analysis of variance (ANOVA) the overall model p-value (0.000) is less than the level of significance (0.05). Therefore, the null hypothesis (H_o) of no relationship between the dependent (rice yield) and the independent variables (irrigation and nitrogen fertilizer) is rejected. Therefore, the full quadratic model of the irrigation and the nitrogen factors (independent variables) significantly affect the response rice (dependent variable). The p-value (0.000) for the linear terms which are the irrigation and the nitrogen are also lower than the level of significance (5%). Therefore, the linear terms significantly affect rice. The p-value (0.644) for the quadratic terms for both factors is observed to be higher than the level of significance. Therefore, the quadratic terms for the irrigation and the nitrogen were not significant.

The interaction between the irrigation and the nitrogen is observed to be significant with respect to rice. The p-value (0.104) for lack-of-fit is larger than the level of significance (0.05). Therefore, the quadratic model with the predictor variable irrigation is significant.

Table 4 showed the minimum and maximum point of the interaction of irrigation and nitrogen fertilizer to determine the interaction that will optimize upland rice production

The maximum rice yield was obtained from 90kgN/ha^{-1} at irrigation interval of seven (7) days, followed by 120kgN/ha^{-1} at seven (7) days. The performance of rice yield significantly varied according to nitrogen application with irrigation levels, this indicated that the increased nitrogen rates up to 90kgN/ha^{-1} at seven days significantly enhanced the maximum yield. While, the minimum rice yield was obtained at irrigation level of 21days at 120kgN/ha^{-1} application.

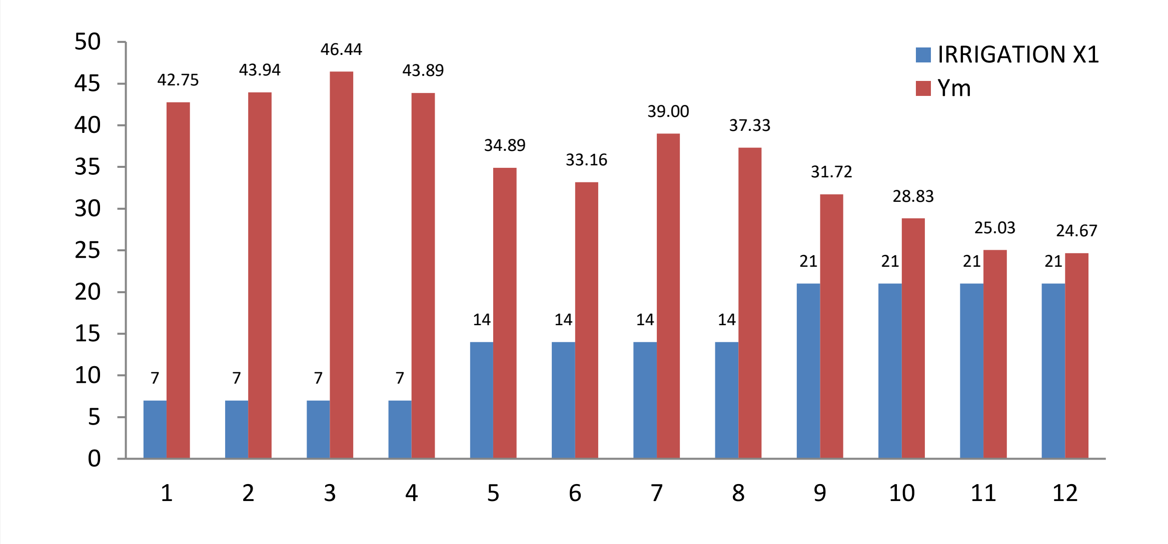

Figure 1: Variation on rice yield with three different levels of irrigation

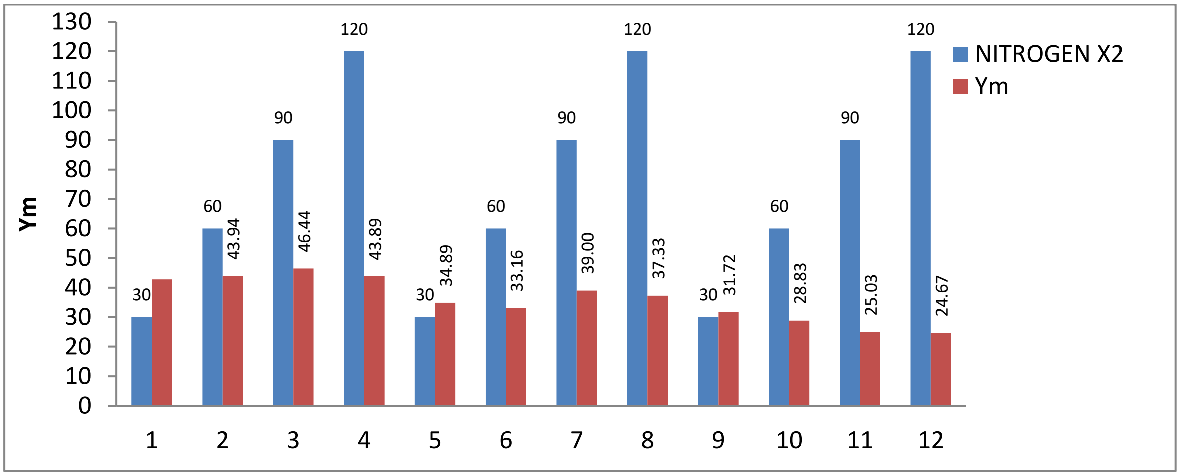

Figure 2: Variation on rice yield with four different levels of Nitrogen

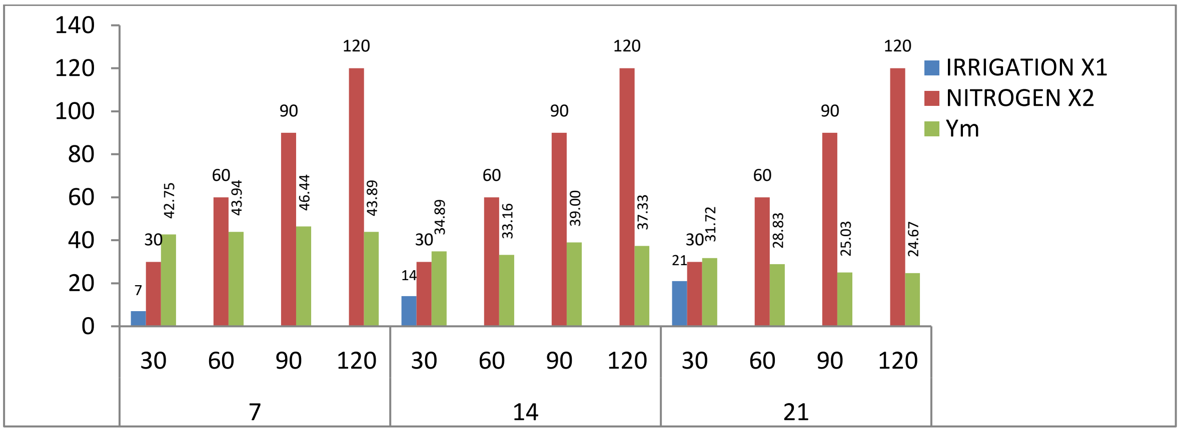

Figure 3: Interaction between irrigation and nitrogen levels

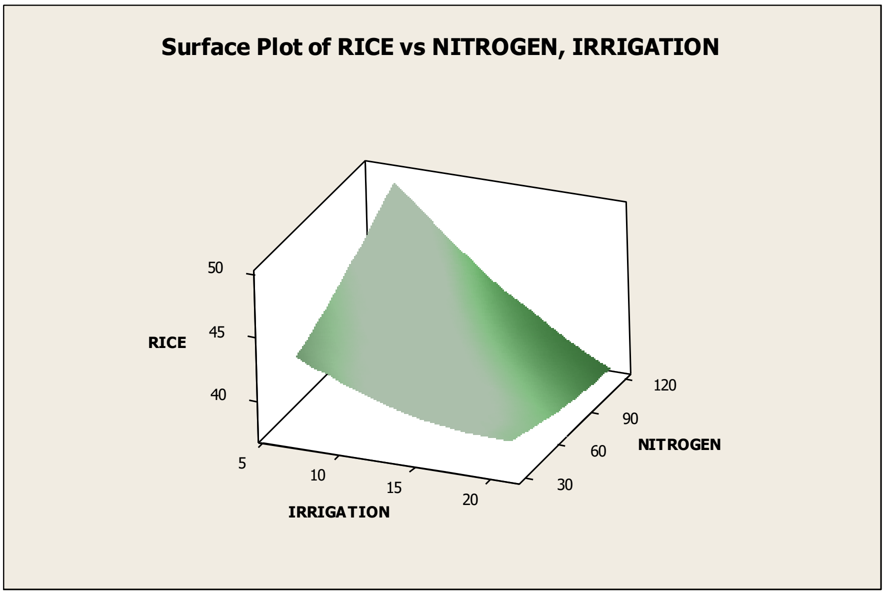

Figure 4: Response Surface Plot of Rice vs irrigation and nitrogen

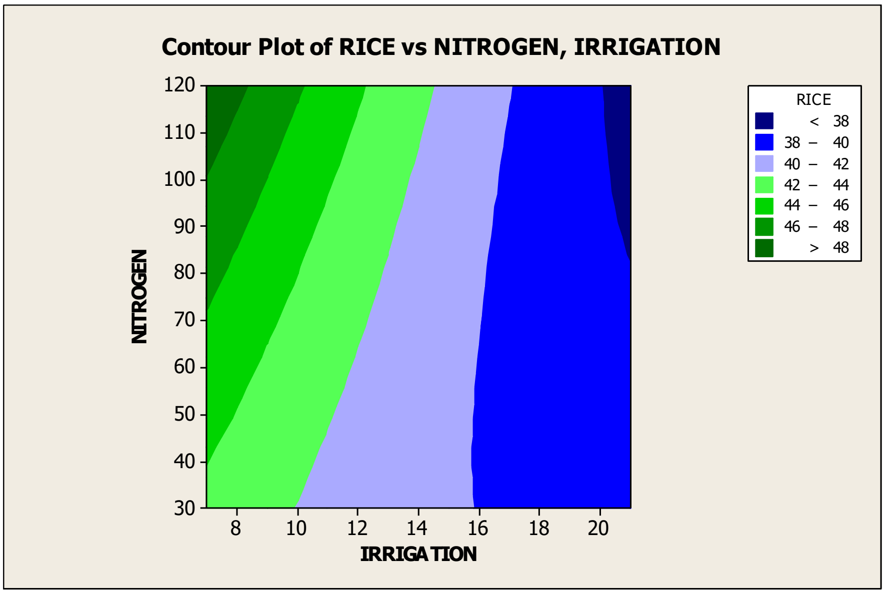

Figure 5: Contour Plot of Saddle Point of Rice vs Nitrogen and Irrigation

Figure 1 showed irrigation levels (7, 14 and 21days) of rice yield. The highest rice yield was found from irrigation of seven (7) days followed by fourteen (14) days, while the lowest was obtained from irrigation of twenty one (21days). Rice yield increases with irrigation level of 7 days but it does not increase further at the rate of 14 and 21days.

Figure 2 shows nitrogen fertilizer rate (30, 60, 90 and 120) of rice yield. Rice yield was increased with the increasing rates of nitrogen fertilizer at 90 and it was found significantly higher than yield from the other levels of nitrogen.

Figure 3 shows the Interaction between irrigation and nitrogen levels, nitrogen at 90kgN/ha^{-1} showed the highest value of rice yield followed by 120kgN/ha^{-1} at irrigation level of seven (7) days, while, the minimum rice yield was obtained at irrigation level of 21days at 120kgN/ha^{-1} application, it illustrates that the maximum rice yield was obtained from 90kgN/ha^{-1} at irrigation level of seven (7) days. The performance of rice yield significantly varied according to nitrogen application with irrigation levels, this indicated that the increased nitrogen rates up to 90kgN/ha^{-1} at seven days significantly enhanced the maximum yield.

Figure 4 shows the three-dimensional response surface plot for the rice response in terms of the irrigation and nitrogen, to optimize irrigation and nitrogen for the rice production, rice is measured on a scale between 40 to 50, where 50 is the most comfortable.

Figure 5 showed that the contour plot is a two-dimensional (2D) representation of the response (rice yield) plotted against combinations of factors (irrigation and nitrogen fertilizer) which show the relationship between the response and the factors, the saddle contour plot represents the rice level over 48 can be achieved for the irrigation and nitrogen fertilizer

CONCLUSION

From the data collected, the interaction between nitrogen fertilizer and irrigation have significant effect on upland rice yield. The results showed that, there was an increase in yield when irrigation and nitrogen fertilizer are combined at quadratic and linear level respectively. From the analysis of the data collected, we found that irrigation in an interval of seven days (7days) gave the highest value with application of nitrogen fertilizer level of 90kgN/ha^{-1} while the minimum rice yield was obtained at irrigation level of 21days at 120kgN/ha^{-1} application, it showed that application of nitrogen without frequent irrigation do not have significant effects on upland rice yield. From the finding of the research, it is recommended that Farmers should apply seven days (7days) irrigation interval and 90kgN/ha for optimum upland rice yield. The current government should use the model to diversify the nation's economy into agricultural production. Government should subsidize the cost of irrigation and nitrogen fertilizer so that farmers can be encouraged to go into upland rice farming so that there will be food security and availability all through the year.

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