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AN ECONOMIC ORDER QUANTITY MODEL FOR AMELIORATING ITEMS WITH QUADRATIC DEMAND

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ABSTRACT

It is evidently recognized that in a real life, the volume of stocked items has motivational effect on customers. Stores with large amount of displayed items attract buyers more than stores with scanty items. The EOQ model for Quadratic Demand items that undergo amelioration stage was developed. The poultry, fishery, and so on, where the stocked item increases in weight and/or value within a shortest period of time. The model determines the best cycle length so as to minimize the overall cost. Numerical examples are given to illustrate the model and a sensitivity analysis carried out to see the effect of changes to some model parameters on the decision variables.

Keywords: Economic Order Quantity, Amelioration, Quadratic Demand

INTRODUCTION

An Economic Order Quantity (EOQ) model was developed by Ford W. Harris in 1913, which marked 112th birthday this year, 2025. Surprisingly, the EOQ model continues to be the central model in the inventory, supply chain and logistics management. Gwanda *et al*. (2023) explained that Amelioration on the other hand, refers to a situation where stocked items increase in quantity and/or quality while in stock. Some items have the property of incurring amelioration and deterioration though simultaneously as they are kept in warehouse. Jaber *et al.,* (2013) defined (EOQ) as the order quantity that minimizes the total holding costs and ordering costs in the inventory management. EOQ model can be seen as one of the famous inventory control systems widely investigated and developed by many researchers. Based on the (EOQ) model, many different models have been introduced with various assumptions. The (EOQ) is the number of units that a company should add to inventory with each order to minimize the Total Costs of Inventory such as holding costs, order costs, and shortage costs. Okwabi (2014) presented an EOQ model such that the demand was constant, and that inventory is depleted at a fixed rate until it reaches zero. At that point, a specific number of items arrive to return the inventory to its beginning level. Since the model assumes instantaneous replenishment, there are no inventory shortages or associated costs. Gwanda *et al.* (2019a) developed an EOQ model for ameliorating items to describe the three stages and finally evolved a comprehensive model that merged all the models together to determine the optimal ordering quantity under the cost minimization. Numerical examples were given to illustrate the developed model and sensitivity analysis was carried out on the results obtained from one of the examples in order to see the effect of parameter changes on the decision variables. Gwanda *et al.* (2019b) reported that when unripe items like fruits were taken to warehouse; they stay dormant for sometimes without showing any sign of change. This period of full persisted for some time until when they became ripe and thus incurred amelioration in value and utility. An EOQ model for such items was formatted where the demand was constant throughout the cycle period. Zhang *et al.* (2022) proved the existence and uniqueness of the optimal order-up-to level, the reorder point and the preservation technology investment under any given two cases and presented an algorithm to search for decision variables such that the total profit per unit time is maximized. The linearity or otherwise of stock dependent demand was also a subject of intensive research. Cárdenas-Barrón (2020) for instance, studied an EOQ model with nonlinear stock dependent holding cost, nonlinear stock dependent demand and trade credit. The authors developed the model from retailers' point of view where the supplier offers a trade credit period to the retailer. The model relaxed the traditional assumption of zero ending inventory level to a non-zero ending which was consequently obtained as positive, zero or negative. Gwanda and Sani (2011) stated that demand for items that ameliorate would be expected to change over time, their expectations leads to many researches with different demand patterns including this research. Begum *et al.* (2012) reported that the scope of the model lies in its applicability in the management inventories of time-quadratic demand. It is also seen that large pile of goods displayed in a supermarket would motivate the customer to buy more. So the presence of inventory has a motivational effect on the people around it. There might be also occasional shortages in inventory due to many reasons. Therefore, they developed an EOQ model for the inventory of deteriorating items, taking demand rate and allowing shortages in inventory. In their research, they presented an inventory model for deteriorating items with quadratic demand shortages were allowed and partially backordered. The backlogging rate is a variable and dependent on the waiting time for the next replenishment. Khanra *et al.* (2010) considered an EOQ model with stock and price dependent demand rate. Abbasi *et al.* (2022) reported that, the main aim of the inventory control and production planning problems was to optimize the economic quantity of the order or determine the size of the production batch according to the capacities and limitations to minimize the total costs related to the order, purchase, maintenance, and delivery. Sana and Chaudhuri (2004) developed production policy for a deteriorating item with time-dependent demand and shortages. Gwanda and Sani (2012) extended their earlier model of Gwanda and Sani (2011) and considered for linear trended in demand. In this research work, it is intended to further extend the work of Gwanda and Sani (2012) to model the situation where the demand rate is a continuous function of time and items ameliorate at a constant rate, were shortages are not allowed. In addition, a new approach to the inventory was introduced by taking the demand rate to be the quadratic function of time. Quadratic function of time was seen to be the best representation of accelerated growth in the demand. Ten solutions of the developed model were discussed and it was illustrated with the help of numerical examples. Sensitivity analysis of the optimal solution with respect to changes in different parameter values was also carried out.

Assumptions

The EOQ model for ameliorating items with quadratic demand was developed based on the following assumptions:

i. The inventory system involves only one single item and one stocking point
ii. Amelioration occurs when the items are effectively in stock.
iii. Shortages are not allowed
iv. Demand rate is a Quadratic function of time
v. The unit cost of an item is constant
vi. The replenishment cost is constant per each replenishment

Notations

In this developed model, the following notations were used:

T: Cycle length
v_0: Initial inventory level
C_h: Inventory holding cost per cycle
D_R: Total demand rate within the cycle
β: Constant rate of Amelioration
A_T: Total Ameliorated amount within the cycle
T_Q: Total on Hand inventory within the cycle
c_0: Unit cost of an item
O_c: Ordering Cost

MATERIALS AND METHODS

Model Formulation

In this study, the integrating factor method was employed to solve the developed model. This was accomplished by analysing the on Hand Inventory Items and its solutions using relevant boundary conditions within the interval ($0 \leq t \leq T$) were obtained. The on Hand Inventory could then be solved to obtain each of the following totals: On Hand Inventory, Holding Cost, and Ameliorated amount within the cycle period. The concept of Maxima and Minima was also employed to minimize the Total Variable Cost (TVC) to obtain the cost function, which could be solved using Microsoft Excel spreadsheet.

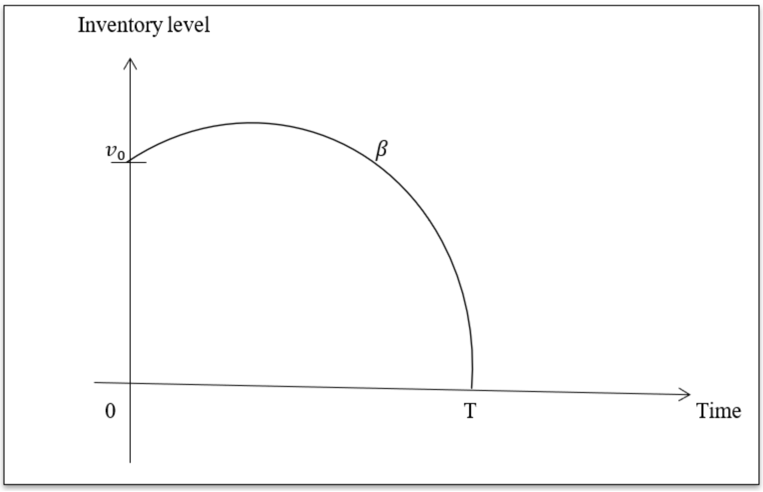

Figure 1: The graphical representation for the inventory system

In the initial stage, within the interval $0 \leq t \leq T$, amelioration occurs at a constant rate of β and the demand follows the Quadratic demand pattern. The differential equation that describes the state of inventory level $V(t)$ is given by:

$$\frac{dV(t)}{dt} - \beta V(t) = -(at^2 + bt + c), \qquad 0 \leq t \leq T \quad (1)$$

at $t = 0,\ V(0) = v_0$ at $t = T,\ V(T) = 0$

By applying the integrating factor method to equation (1), $V(t)$ is obtained as:

$$V(t) = \frac{1}{\beta^3}[at^2\beta^2 + 2at\beta + 2a + bt\beta^2 + b\beta + c\beta^2] + ke^{\beta t} \quad (2)$$

Applying the first boundary condition: at $at\ t = 0; V(0) = v_0$, the value of constant k is:

$$v_0 = \frac{1}{\beta^3}[2a + b\beta + c\beta^2] + k \quad (3)$$

Equation (3) becomes:

$$k = v_0 - \frac{1}{\beta^3}[2a + b\beta + c\beta^2] \quad (4)$$

Substituting equation (4) into equation (2) to obtain:

$$V(t) = \frac{1}{\beta^3}[at^2\beta^2 + 2at\beta + 2a + bt\beta^2 + b\beta + c\beta^2] + \left[v_0 - \frac{1}{\beta^3}[2a + b\beta + c\beta^2]\right]e^{\beta t} \quad (5)$$

$$V(t) = \frac{1}{\beta^3}[at^2\beta^2 + 2at\beta + 2a + bt\beta^2 + b\beta + c\beta^2] - \frac{1}{\beta^3}[2a + b\beta + c\beta^2]e^{\beta t} + v_0 e^{\beta t} \quad (6)$$

Applying the second boundary condition, $at\ t = T;\ V(T) = 0$ to obtain equation (7)

$$0 = \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2] - \frac{1}{\beta^3}[2a + b\beta + c\beta^2]e^{\beta T} + v_0 e^{\beta T} \quad (7)$$

Equation (6) becomes:

$$v_0 = \frac{1}{\beta^3}[2a + b\beta + c\beta^2] - \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]e^{-\beta T} \quad (8)$$

Substituting equation (8) into equation (6) to obtain:

$$V(t) = \frac{1}{\beta^3}[at^2\beta^2 + 2at\beta + 2a + bt\beta^2 + b\beta + c\beta^2] - \frac{1}{\beta^3}[2a + b\beta + c\beta^2]e^{\beta t} + \left[\frac{1}{\beta^3}[2a + b\beta + c\beta^2] - \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]e^{-\beta T}\right]e^{\beta t} \quad (9)$$

Simplifying equation (9) we obtain:

$V(t) = \frac{1}{\beta^3}[at^2\beta^2 + 2at\beta + 2a + bt\beta^2 + b\beta + c\beta^2] - \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2](e^{\beta(t-T})$ (10)

Hence equation (10) is the expression for the on-hand inventory within the interval $(0, T)$.

The total demand rate within the interval $\mathbf{0 \leq t \leq T}$ is given by:

$D_R = \int_0^T (at^2 + bt + c)\, dt$

$D_R = \frac{1}{3}aT^3 + \frac{1}{2}bT^2 + cT$ (11)

The total on Hand inventory within the interval $\mathbf{0 \leq t \leq T}$ is given by:

$T_Q = \int_0^T V(t)\, dt$

$T_Q = \int_0^T \left[\frac{1}{\beta^3}[at^2\beta^2 + 2at\beta + 2a + bt\beta^2 + b\beta + c\beta^2] - \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2](e^{\beta(t-T})\right] dt$ (12)

Simplifying equation (11) we obtain:

$T_Q = \frac{1}{\beta^3}\left[\frac{1}{3}aT^3\beta^2 + aT^2\beta + 2aT + \frac{1}{2}bT^2\beta^2 + bT\beta + cT\beta^2\right] - \frac{1}{\beta^4}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2](1 - e^{-\beta T})$ (13)

The Ameliorated amount over the interval $\mathbf{0 \leq t \leq T}$ is given by

$A_T = D_R - v_0$

$A_T = \frac{1}{3}aT^3 + \frac{1}{2}bT^2 + cT - \left[\frac{1}{\beta^3}[2a + b\beta + c\beta^2] - \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]e^{-\beta T}\right]$ (14)

$A_T = \frac{1}{3}aT^3 + \frac{1}{2}bT^2 + cT + \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]e^{-\beta T} - \frac{1}{\beta^3}[2a + b\beta + c\beta^2]$ (15)

The holding inventory cost within the interval $\mathbf{0 \leq t \leq T}$ is obtained as:

$C_h = iCT_Q$

$C_h = \frac{iC}{\beta^3}\left[\frac{1}{3}aT^3\beta^2 + aT^2\beta + 2aT + \frac{1}{2}bT^2\beta^2 + bT\beta + cT\beta^2\right] - \frac{iC}{\beta^4}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2](1 - e^{-\beta T})$ (16)

Total variable cost within the cycle is calculated as follows:

$TVC(T) = O_c + C_h - c_0 A_T$ (17)

Total variable cost per unit time will be calculated as follows:

$TVC(T) = \frac{1}{T}$[Ordering cost + Inventory holding cost per cycle − Amelioration cost per cycle]

$TVC(T) = \frac{1}{T}\left[O_c + \frac{iC}{\beta^3}\left[\frac{1}{3}aT^3\beta^2 + aT^2\beta + 2aT + \frac{1}{2}bT^2\beta^2 + bT\beta + cT\beta^2\right] - \frac{iC}{\beta^4}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2](1 - e^{-\beta T}) - c_o\left[\frac{1}{3}aT^3 + \frac{1}{2}bT^2 + cT + \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]e^{-\beta T} - \frac{1}{\beta^3}[2a + b\beta + c\beta^2]\right]\right]$ (18)

$TVC(T) = \frac{O_c}{T} + \frac{iC}{T\beta^3}\left[\frac{1}{3}aT^3\beta^2 + aT^2\beta + 2aT + \frac{1}{2}bT^2\beta^2 + bT\beta + cT\beta^2\right] - \frac{iC}{T\beta^4}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2](1 - e^{-\beta T}) - \frac{c_o}{T}(\frac{1}{3}aT^3 + \frac{1}{2}bT^2 + cT) - \frac{c_o}{T\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]e^{-\beta T} + \frac{c_o}{T\beta^3}[2a + b\beta + c\beta^2]$ (19)

To obtain the value of T which minimizes the total variable cost per unit time, equation (19) will differentiated with respect to T as follows:

$\frac{d}{dT}(TVC(T)) = 0$ (20)

Differentiating equation (19) with respect to T, applying quotient rule the result follows:

$\frac{dTVC(T)}{dT} = -\frac{O_C}{T^2} + \frac{1}{T^2\beta^3}[TiC(aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2] - iC\left[\frac{1}{3}aT^3\beta^2 + aT^2\beta + 2aT + \frac{1}{2}bT^2\beta^2 + bT\beta + cT\beta^2\right] - \frac{1}{T^2\beta^4}\left[[TiC(aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2)]\beta e^{-\beta T} - iC(2aT\beta^2 + 2a\beta + b\beta^2)(1 - e^{-\beta T}) + iC(aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2)(1 - e^{-\beta T})\right] - \frac{1}{T^2}\left[c_oT(aT^2 + bT + c) - c_o\left(\frac{1}{3}aT^3 + \frac{1}{2}bT^2 + cT\right)\right] - \frac{1}{T^2\beta^3}[-c_oT[(aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]\beta e^{-\beta T} - c_o(2aT\beta^2 + 2a\beta + b\beta^2)e^{-\beta T} + c_o(aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2 e^{-\beta T}] - \frac{1}{T^2\beta^3}[c_o(2a + 2a\beta + b\beta^2)]$ (21)

Applying equation (20), to obtain the optimal T which minimizes the total variable cost per unit time, hence we obtain equation (22).

$\left[-\beta^4 O_C + \beta[TiC(aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2] - iC\beta(\frac{1}{3}aT^3\beta^2 + aT^2\beta + 2aT + \frac{1}{2}bT^2\beta^2 + bT\beta + cT\beta^2) - TiC(aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2)\beta e^{-\beta T} - iC(2aT\beta^2 + 2a\beta + b\beta^2)(1 - e^{-\beta T}) + iC(aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2)(1 - e^{-\beta T}) - \beta^4 c_oT(aT^2 + bT + c) + c_o\beta^4\left(\frac{1}{3}aT^3 + \frac{1}{2}bT^2 + cT\right) + c_oT[(aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]\beta^2 e^{-\beta T} - c_o\beta(2aT\beta^2 + 2a\beta + b\beta^2)e^{-\beta T} + c_o\beta(aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2 e^{-\beta T} - \beta c_o(2a + 2a\beta + b\beta^2) = 0\right]$ (22)

The Economic Order Quantity was derived from equation (8) for:

$EOQ = v_0 = D_R - A_T$

EOQ $= \frac{1}{3}aT^3 + \frac{1}{2}bT^2 + cT - \left[\frac{1}{3}aT^3 + \frac{1}{2}bT^2 + cT + \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]e^{-\beta T} - \frac{1}{\beta^3}[2a + b\beta + c\beta^2]\right]$ (23)

$= \frac{1}{\beta^3}[2a + b\beta + c\beta^2] - \frac{1}{\beta^3}[aT^2\beta^2 + 2aT\beta + 2a + bT\beta^2 + b\beta + c\beta^2]e^{-\beta T}$ (24)

Equation (22) can then be solved to obtain the optimum values T^* of T using any suitable numerical method provided that, $\frac{d^2}{dT^2}(TVC(T^*)) > 0$

RESULTS AND DISCUSSION

Numerical Examples

The table 1 below shows the solutions of ten different numerical examples with varying parameters. The obtained values presented in the last column of table 1, titled EOQ the solutions computed from equation (24), while the remaining parameters value was arbitrarily chosen. In example 1, for instance, the assigned parameters' values are: $a = 500, b = 250$, $c = 300$, $\beta = 0.3$, $c_0 = 300$, $iC = 2000$, $O_c = 1000$ and T=363$days$, has shown that with the values assigned to the decision parameters, the optimal ordering quantity is obtained as 64units within an optimal cycle period of 363days.

Table 1: Computed EOQ obtained numerical examples with varying parameters

S/N	a	b	c	β	c_0	iC	O_c	T	EOQ
1	500	250	300	0.3	300	2000	1000	363days	64units
2	100	110	150	0.2	400	4000	1500	362 days	26units
3	2000	1500	400	0.3	300	3500	1000	362 days	320units
4	1000	500	400	0.2	300	3500	1000	360 days	869units
5	3000	600	500	0.15	400	20000	10000	360 days	1631units
6	11000	1000	500	0.25	250	25000	40000	360 days	3912units
7	15000	1500	500	0.5	250	30000	15000	364 days	443units
8	100000	15000	500	0.3	1000	40000	30000	361 days	6248units
9	80000	17000	1000	0.3	200	300	5000	361 days	7382units
10	70000	16000	2000	0.3	300	100	15000	361 days	7844units

Analysis of the Result

As "β" decreases EOQ increases and the days to which it is achieved has increased.
As "a" increases EOQ increases and the days to which it is achieved has increased.
As "O_c" " increases EOQ increases and the days to which it is achieved has increased.
As "b" increases EOQ increases and the days to which it is achieved has increased.
As "iC" increases EOQ increases and the days to which it is achieved has increased
As "c" increases EOQ increases and the days to which it is achieved has increased.
As "c_0" decreases EOQ increases and the days to which it is achieved has increased.

Discussion

In the case of this research, as the rate of Amelioration "β" decreased EOQ increased and the days to which it is achieved has also increased. However in the case of Gwanda and Sani (2017) research article, as the rate of Amelioration "A" increases, the EOQ increases, and the days to which it was achieved was decreased. In the case of this research as parameter "a" increased EOQ increased and the days to which it is achieved has also increased, the same result was obtained from that of Gwanda and Sani (2017). In this research work as the remaining parameters, O_c, $b, iC, and\ C$ have similar result pattern with a, except the unit cost parameter c_0 which has similar result pattern with that of parameter β. Based on the analysis, it can be observed that in all cases, as the parameter increased or decreased, EOQ increased, suggesting that the system generally responds to changes in costs or demand factors by adjusting the order quantity upwards to optimize operations. However, the time to achieve EOQ increased consistently across all parameters, indicating that larger order quantities tend to slow down the speed at which the EOQ is reached, due to supply chain adjustments. This implies that increasing various cost or parameters could leads to larger EOQ values.

CONCLUSION

In this article, EOQ model was developed, where the larger EOQ values resulted from changes in cost or demand parameters, the changes introduces operational challenges that delay the system's ability to achieve the optimal order quantity. Therefore, the businesses need to strike a balance between cost optimization through larger EOQ and the practical limitations of implementing those changes efficiently in the supply chain. Accepting these dynamics could aid the organizations to make more informed decisions that consider both short-term adjustments and long-term supply chain strategies.

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