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MODELLING NIGERIAN STOCK RETURNS WITH ARMA-EGARCH: A VOLATILITY ANALYSIS

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ABSTRACT

This study analyzes the volatility of Nigerian stock returns from January 2, 2010, to December 31, 2022, using ARMA-EGARCH models with Generalized Error Distribution (GED). Four model specifications are assessed: ARMA-EGARCH (1,1), ARMA-EGARCH (1,2), ARMA-EGARCH (2,1), and ARMA-EGARCH (2,2). The focus is on model selection, parameter estimation, and diagnostic testing to determine the best model for capturing volatility dynamics. The ARMA-EGARCH (2,2) GED model emerges as the best based on AIC, BIC, and high log-likelihood values, offering a good balance of fit and complexity. The ARMA-EGARCH (1,1) GED model is noted for effectively balancing simplicity and fit while capturing volatility and asymmetric effects. However, all models show limitations in fully capturing volatility dynamics and maintaining parameter stability, particularly concerning volatility clustering. The ARMA-EGARCH (2,2) model consistently performs best across various statistical criteria, including AIC and BIC. Although it provides a robust fit, it has some limitations in serial correlation and model stability. This indicates the need for further model refinement and exploration to enhance forecasting accuracy and address intrinsic limitations. These findings are valuable for investors and policymakers in understanding stock market volatility modelling both in Nigeria and globally.

Keywords: ARMA-GARCH, Stock Returns, Volatility, Nigeria

INTRODUCTION

The ability to predict stock price movements is crucial for fulfilling the fundamental objectives of investors and stock market operators seeking to maximise their benefits, as for institutional investors, portfolio managers, and financial analysts who rely on these forecasts to make informed decisions that can lead to improved investment strategies and better risk management practices, Stock markets are subject to many influencing factors, Market sentiment, economic indicators, geopolitical events, and company-specific news can all influence investor behaviour and, consequently, stock prices. The development of robust forecasting models is therefore a critical area of research that has garnered significant interest from researchers and statisticians globally, leading to considerable debate within this domain. As such, a comprehensive approach that integrates both quantitative and qualitative analyses is often necessary to achieve more reliable forecasts Researchers have explored a range of quantitative techniques, including time series analysis, machine learning algorithms, and econometric models, to enhance the accuracy of stock price predictions, various methodologies and approaches for developing forecasting models exist, the Box-Jenkins methodology outlined, for instance, emphasizes the importance of identifying the underlying patterns in historical data to forecast future price movements effectively. In addition to quantitative methods, qualitative factors also play a significant role in stock price determination, Abdullahi & Bakari (2014).

Although stock market return volatility is not always detrimental, persistent volatility in market returns, particularly in developed markets, is likely to lead to adverse outcomes. In both advanced and emerging economies, the stock market serves as a vital element of the financial system, significantly contributing to capital formation, wealth generation, and economic development. Moreover, the global interconnectedness of financial markets means that volatility in one region can have ripple effects across the world. For instance, economic downturns or financial crises in developed markets can lead to capital flight from emerging economies, exacerbating their own volatility and economic challenges. High levels of volatility can lead to increased uncertainty in the financial system, which may deter foreign investment and hinder economic growth. While it plays a key role in enhancing economic prosperity by strengthening the financial system, the challenges posed by stock return volatility have profoundly impacted the effective operation of the global market. Literature related to stock markets defines volatility as the degree of uncertainty or risk associated with the value of financial assets (Engle & Patton, 2001). Periods characterized by heightened volatility indicate substantial fluctuations in the value of financial assets, whereas lower volatility implies that asset values remain relatively stable over time (Banumathy & Azhagaiah,2015) This volatility risk can lead to financial shocks for investors, resulting in challenges such as reduced capital investments in financial assets, increased market-making vulnerability, diminished investor confidence, and erratic stock prices and returns (Bello, 2020; Chiang & Doong, 2014; Wang & Yang, 2017). As a result, in a highly volatile stock market, it becomes challenging for publicly listed companies to secure adequate funding, as rational investors tend to favour stocks with more stable returns and prices, in contrast to risk-seeking investors (Onyele, Opara, & Ikwuagwu,2017). Policymakers and regulators must therefore pay close attention to volatility trends and implement measures to promote market stability. This could involve enhancing transparency, improving market infrastructure, and fostering investor education to build confidence in the financial system.

E.Nsengiyumva, J.K.Mung'atu, I.Kayijuka & C.Rurenga (2024), worked on "Neural Networks and ARMA-GARCH Models for Foreign Exchange Risk Measurement and Assessment', which aimed to measure and assess foreign exchange risk utilizing Neural Networks and ARMA-GARCH models. They used Data on five leading currencies, sourced from the National Bank of Rwanda covering the period from 6 January 2016 to 28 June 2024 for analysis. The study specifically utilized the long short-term memory (LSTM) model, a type of recurrent neural network, to assess the risk associated with various asset currencies. The estimated volatilities from the LSTM model were compared to those obtained from traditional ARCH-GARCH models. Notably, the LSTM model produced lower root mean square error values than the ARMA-GARCH models, demonstrating greater accuracy in forecasting currency volatilities. The findings indicate that the Egyptian Pound (EGP) and the Kenyan Shilling (KES) are riskier than the US Dollar (USD), the Euro (EUR), and the British Pound (GBP).

Qiang Zhang, Rui Luo, Yaodong Yang, and Yuanyuan Lui's (2018) paper ‘Benchmarking Deep Sequential Models on Volatility Predictions for Financial Time Series’ explores the effectiveness of advanced deep learning architectures for volatility modelling, a crucial aspect of financial analysis. They aim to provide insights that can guide future theoretical developments in integrating deep learning with economic applications. They compare traditional methods with innovative deep sequential models, focusing on promising dilated convolutional and recurrent networks. They base their study on extensive real-world stock price datasets, including 1314 daily stock series over 2018 trading days, using negative log-likelihood (NLL) as a primary evaluation metric. Their findings reveal that dilated neural models, such as Dilated CNN and Dilated RNN, achieve superior predictive accuracy, outperforming traditional GARCH models and several recent stochastic approaches. Additionally, the study underscores the flexibility and expressive power of these dilated architectures, highlighting their potential to enhance financial forecasting

J Nahar, E Hertini and A K Supriatna (2020), researched the topic: ‘Application of GARCH model in the price inflation of foodstuff in West Java’, using the GARCH model application to analyze data on foodstuff price inflation in West Java. The results showed that the best GARCH model for foodstuff price inflation data is the ARCH (1) model,

Ngene & Ann 2022) Researched ‘‘Stock returns, trading volume, and volatility: The case of African stock markets’’ and discovered that the inclusion of African Stock Markets (ASMs) in global frontier market indices highlights their role in portfolio diversification. They examine the causal relationships among stock returns, trading volume, and volatility in eight ASMs. Findings from a linear model show that returns generally Granger-cause trading volume. However, quantile regression reveals that lagged trading volume negatively impacts returns at low quantiles and positively affects them at high quantiles, consistent with various economic models. Also, volatility positively influences volume, supporting the dispersion of beliefs model. The relationships between trading volume and volatility indicate that causality depends on market conditions and volatility regimes. therefore, the linear model results emphasize how model misspecification can distort empirical findings compared to nonlinear models.

In the context of Nigeria's stock market, which is characterized by high volatility and significant fluctuations, the use of advanced models becomes increasingly important. The Nigerian stock market has experienced various macroeconomic shocks and policy changes, leading to complex patterns in stock returns and volatility (Adegboye & Osinubi,2008). Previous studies have emphasized the necessity for robust models that can accurately capture these dynamics and provide reliable forecasts (Ojo, 2015; Akinboade & Kinfack, 2017). Integrating ARMA models for return forecasting with EGARCH models for volatility estimation offers a comprehensive approach to understanding and predicting stock market behaviour. This combination takes advantage of the strengths of both models, providing a more detailed view of the underlying processes that affect stock returns and their volatility (Bollerslev, 1986; Ding, Granger, & Engle, 1993). Given the unique characteristics of the Nigerian stock market, it is essential to employ robust and sophisticated models like ARMA-EGARCH to accurately capture its complex dynamics.

Generalised autoregressive conditional heteroscedasticity (GARCH) models represent a range of extensions and enhancements to the autoregressive conditional heteroscedasticity (ARCH) model, which was developed by Sir Robert F. Engle in 1982. This model was groundbreaking in its assumption that volatility in financial time series is not constant, addressing the time-varying volatility and uncertainty inherent in such data (Lawrence, 2013; Atoi, 2014; Grek, 2014.)

Volatility modelling serves as an important measure of risk exposure for all companies, including financial institutions. Accurate estimates of volatility provide investors, traders, stock market policymakers, and government officials with the opportunity to make informed monetary policies and financial decisions. By identifying, correcting, and controlling the factors that contribute to stock market price volatility, an economy can experience rapid growth and progress toward becoming more advanced. Additionally, effective government reforms can lead to positive impacts on financial institutions and the broader economy. It is also essential to monitor stock prices to mitigate the unstable and volatile nature of the market, particularly in emerging economies like ours (Monica et al., 2018).

Stock Returns: These are pivotal for assessing market performance and investment opportunities. Feng et al. (2022) highlight the importance of precise forecasting of stock returns to formulate effective investment strategies.

Volatility: Defined as the degree of price variation, volatility is critical for risk assessment. Harris (2020) discusses its significance in shaping trading strategies and approaches to risk management.

ARMA Model: The ARMA (Auto-Regressive Moving Average) model integrates autoregressive and moving average elements for time series forecasting. Although it is useful for modelling returns, it fails to fully address volatility (Timmermann, 2020).

EGARCH Model: The EGARCH (Exponential Generalized Autoregressive Conditional Heteroskedasticity) model, introduced by Nelson (1991), effectively captures asymmetric volatility effects, where negative shocks lead to greater volatility than positive ones. Chen & Huang (2023) validate the EGARCH model's effectiveness in representing intricate volatility patterns.

Various variants of ARCH and GARCH models include: standard generalized autoregressive conditional heteroscedasticity (sGARCH), Glosten-Jagannathan-Runkle generalized autoregressive conditional heteroscedasticity (gjrGARCH), exponential generalized autoregressive conditional heteroscedasticity (eGARCH), asymmetric power autoregressive conditional heteroscedasticity (apARCH), integrated generalized autoregressive conditional heteroscedasticity (iGARCH), threshold generalized autoregressive conditional heteroscedasticity (TGARCH), nonlinear generalized autoregressive conditional heteroscedasticity (NGARCH), nonlinear asymmetric generalized autoregressive conditional heteroscedasticity (NAGARCH), and absolute value generalised autoregressive conditional heteroscedasticity (AVGARCH), among others (Ali, 2013; Atoi, 2014; Emenogu et al., 2018).

In their paper titled "Modelling Stock Returns Volatility and Asymmetric News Effect: A Global Perspective," Kingsley &

Emmanuel (2021), analyzed stock return volatility using daily data from the S&P Global 1200 index, covering the period from September 1, 2010, to September 30, 2020. The S&P 1200 is a free-float weighted stock market index that encompasses seven regional stock market indices, representing approximately 70% of global market capitalization. This index was selected to compute global stock returns. The data analysis employed Generalized Autoregressive Conditional Heteroskedasticity (GARCH) techniques. Among the various GARCH models tested in the study, the symmetric GARCH-M (1,1) and the asymmetric TGARCH (1,1) models were identified as the most suitable for estimation. The findings from these models indicated significant volatility persistence and a pronounced asymmetric news effect in the global stock market. Volatility persistence suggests that current volatility shocks have a prolonged impact on expected returns. Additionally, the asymmetric news effect demonstrated that negative news (bad news) had a greater influence on stock return volatility than positive news (good news), particularly in 2020, primarily due to the COVID-19 crisis. Consequently, the study concluded that the global stock market exhibited high volatility persistence and a leverage effect during the sampled period.

In a separate study titled "Modelling the Nigerian Stock Market: Evidence from Time Series Analysis," Abdullahi & Bakari (2014) explored trends and patterns in the Nigerian capital market while also identifying an appropriate model for forecasting its performance. The results indicated that the trends in the Nigerian stock market are better represented by an exponential model, suggesting a non-linear relationship between market operators and the general public. The study emphasized the importance of information in capital market development and recommended that operators within the Nigerian stock market ease restrictive regulations and laws to encourage greater participation. Additionally, they suggested raising awareness levels among investors to keep them informed about new innovations and developments in the market.

The researcher focused on the topic "Modeling Volatility of Nigeria Stock Exchange Using Multivariate GARCH Models" to develop a predictive model for stock volatility in Nigeria's stock market. The study utilized monthly data collected from January 1990 to December 2016 for variables including the Nigerian stock exchange, exchange rate, share index, and inflation rate. Descriptive statistics indicated that these variables demonstrated volatility, a key characteristic of financial time-varying series. The findings suggest that the Nigerian Stock Exchange, exchange rate, share index, and inflation rate are prone to non-steady shocks within the stock market. Each variable displayed a distinct length of recovery, or volatility half-life, with ranges of 1.5 months for the stock exchange, 6.5 months for the exchange rate, 6 months for the share index, and 2.4 months for inflation rate. This implies that the volatility of these variables exhibits long memory, persistence, and mean reversion tendencies.

Adenomon et al (2020), worked on the topic ‘‘Modeling Volatility of Nigeria Stock Exchange Using Multivariate GARCH Models’’ to provide a model for predicting stock volatility in Nigeria's Stock market, used monthly data for the Nigerian stock exchange, Exchange rate, and Share index and inflation rate was collected from January 1990 to December 2016. The descriptive statistics revealed these variables to exhibit volatility as a characteristic of financial time-varying series. the result shows that the Nigerian Stock Exchange, Exchange rate, share index and Inflation rate will experience a non-steady shock in the Stock market. However, Each of these variables has a different length of recovery (volatility half-life) ranging from 1.5month, 6.5months, 6 months to 2.4 months for the stock exchange, exchange rate, share index and inflation rate respectively. By implication, the volatility of these variables had a long memory, persistence and mean reverting.

According to Adenomon et al (2022) from research on ‘’The Effects of COVID-19 outbreak on the Nigerian Stock Exchange performance: Evidence from GARCH Models’’ Reports indicate that global economic and financial markets have been significantly impacted by lockdowns and social distancing measures. In Nigeria, the first COVID-19 case was reported on February 27, 2020. They analyzes the effects of the outbreak on the Nigerian stock market using data from March 2, 2015, to April 16, 2020, with a focus on the COVID-19 period from January 2 to April 16, 2020. GARCH model results show a decline in stock returns and increased volatility during the pandemic compared to the pre-pandemic phase. The Quadratic GARCH (QGARCH) and Exponential GARCH (EGARCH) models further confirm the negative impact of COVID-19 on stock returns

Ojo (2021) applies GARCH models to Nigerian stock returns, finding significant volatility clustering and macroeconomic impacts. Recent studies highlight the effectiveness of ARMA models in forecasting stock returns by capturing temporal patterns. Pérez et al. (2022) confirm the model’s utility but note the need for additional approaches to address volatility. The EGARCH model is particularly effective in capturing asymmetric volatility effects, as evidenced by Khan & Malik (2023), who provide evidence of its superior performance in emerging markets, including Nigeria. Furthermore, combining ARMA with EGARCH models has improved forecasting accuracy for Nigerian stock returns, with Adewale & Okunola (2024) demonstrating that this approach effectively captures complex volatility dynamics.

Emenogu, N. etal (2020) investigated the volatility in daily stock returns for Total Nigeria Plc using nine variants of GARCH models: sGARCH, girGARCH, eGARCH, iGARCH, aGARCH, TGARCH, NGARCH, NAGARCH, and AVGARCH along with value at risk estimation and backtesting, use daily data for Total Nigeria Plc returns for the period January 2, 2001 to May 8, 2017, and conclude that eGARCH and sGARCH perform better for normal innovations while NGARCH performs better for student t innovations. . the results of the estimations revealed that the persistence of the GARCH models are stable except for few cases for which iGARCH and eGARCH were unstable. Additionally, for student t innovation, the sGARCH and girGARCH models failed to converge; the mean reverting number of days for returns differed from model to model. They recommend shareholders and investors continue their business with Total Nigeria Plc because possible losses may be overcome in the future by improvements in stock prices.

Adenomon & Emmanuel (2024) researched on Comparison of ARIMA, GARCH and NNAR Models for Modelling the Exchange Rate in Nigeria, they investigated the characteristics or features of Nigeria's exchange rate (Naira/USD), as well as the conventional facts of the exchange rate using the Neural Network Autoregressive (NNAR) model and popular BJ-type models such as Autoregressive Integrated Moving Average (ARIMA) and Generalized Autoregressive Conditional Heteroscedastic (GARCH) models. The results revealed that among the thirteen (13) candidates ARIMA models estimated, $ARIMA$(1,0,0) returned as the most parsimonious ARIMA model with the lowest Akaike Information Criterion (AIC). Also, $GARCH$ (1,1) returned as the most parsimonious GARCH-type models for the series. The concludes that

$NNAR$ (24,1,12) is the optimal model for the examined exchange rate returns series and it outperformed the $ARIMA$ (1,0,0) and $GARCH$ (1,1) time series models The gap here from the literature I gathered is that their data are not the most recent which may not give current information on the Nigeria stock returns, and the methods they use to model may not be deep as therefore, this study aims to apply the ARMA-EGARCH framework to model Nigeria's stock returns, thereby to know insight the model among the ARMA-EGARCH Model that capture the dynamics of volatility, and which of the model performs well in complex scenarios, contributing valuable insights into both the theoretical and practical aspects of financial econometrics.

MATERIALS AND METHODS

ARMA Model: it combined two parts, the Autoregressive (AR) part and Moving Average (MA) part;

AR (P) Model: The Autoregressive part of the model expresses the current value of a time series as a function of its previous value; The AR (p) model is written as:

$$X_t = \sum_{i=1}^{p} \psi_i X_{t-i} + \varepsilon_t \quad (1)$$

Where $\psi_1, . . . , \psi_p$ are the parameters of the model, and the random variable ε_t is white noise, usually iid normal variables.

For the model to remain stationary, the roots of its characteristic's polynomial must lie outside of the unit circle

MA(q) Model: The Moving Average part expresses the current value of the time series as a function of past error terms; The notation MA(q) refers to the Moving Average Model of order q.n

$$X_t = \mu + \varepsilon_t + \sum_{i=1}^{q} \Theta_{t-i} \quad (2)$$

Where the $\Theta_{1,} . . . \Theta_q$ are the parameters of the model, μ is the expectation of X_t (often assumed to equal 0), and the ε_t, ε_{t-1} . . . are again iid white noise error terms that are commonly normal random variables.

Auto Regressive Moving Average (ARMA) Model

This is the combination of the AR and MA models. where the impact of the previous lag along with the residuals is considered for forecasting the future values of the time series. Here β represents the coefficients of the AR model and α represents the coefficients of the MA model.

$$y_t = \beta_1^* y_{-1} + \alpha_1^* \varepsilon_{-1} + \beta_2^* y_{t-2} + \alpha_2^* + \alpha_3^* \varepsilon_{t-3} + \beta_3^* y_{t-3} + ... + \beta_k^* y_{t-k} + \alpha_k^* \varepsilon_{t-k} \quad (3)$$

When graphs of MA and AR are plotted with their respective significant values. Considering only 1 significant value on assumption from the AR model and 1 significant value from the MA model. The ARMA model obtained from the combined values of the other two models will be of the order of ARMA (1,1).

ARMA Model: combining the two models is written as The notation ARMA (p,q) refers to the model with p AR terms and q MA terms. This model contains AR (p) and MA(q) models

$$X_t = \varepsilon_t + \sum_{i=1}^{p} \psi_i X_{t-i} + \sum_{i=1}^{q} \Theta_1 \varepsilon_{t-i} \quad (4)$$

Exponential GARCH (EGARCH) Model

To overcome the drawbacks of the basic GARCH of Bollerslev (1986), Nelson (1991) introduced the Exponential GARCH to model the logarithm of the variance rather than the level and this model accounts for an asymmetric response to a shock. It is

$$ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{q} \left[\alpha_i \left|\frac{\varepsilon_{t-i}}{\alpha_{t-i}}\right| + \gamma_i \left[\frac{\varepsilon_{t-i}}{\alpha_{t-i}}\right]\right] + \sum_{j=1}^{p} \left(\beta_j \, ln(\sigma_{t-1}^2)\right) \quad (6)$$

The EGARCH (1, 1) is given by

$$ln(\sigma_t^2) = \alpha_0 + \beta ln\left(\left(\sigma_{t-i}^2\right)\right) + \alpha \left|\frac{\varepsilon_{t-i}}{\alpha_{t-i}}\right| + \gamma \frac{\varepsilon_{t-i}}{\alpha_{t-i}} \quad (6)$$

where γ represents the asymmetric coefficient in the model. If the relationship between variance and returns is negative then the value of γ must be negative and significant. The difference between α_i and γ_k is expressed as the impact of shocks on conditional volatility. β coefficient represents the measure of volatility persistence, which is usually less than one but as its value approaches unity the persistence of shock increases. The sufficient condition for the stationarity of the EGARCH model is that $|\beta| < 1$.The model equation () also implies that the leverage effect is exponential rather than quadratic and the forecasts of the conditional variance are guaranteed to be non-negative. However, the value of the intercepts ω, varies according to the distributional assumptions.

Justification of the choice of the Generalized Error Distribution (GED) over alternatives

The Generalized Error Distribution (GED) is chosen for its ability to handle non-normal data, it provides a robust and efficient way to analyze variance, its flexibility in modelling different distributions is advantageous. It outperforms other alternatives in handling outliers and skewness

Model Selection Criteria

Akaike Information Criteria (AIC) due to (Akaike, 1974), Schwarz Information Criterion (SIC) due to (Schwarz, 1978) and Hannan-Quinn Information Criterion (HQC) due to (Hannan, 1980) and Log likelihood are the most commonly used model selection criteria. These criteria were used in this study and are computed as follows:

$$AIC(K) = -2\, ln\, ln\, (L) + 2K(40)$$

$$SIC(K) = -2\, ln\, ln\, (L) + Kln(T) \quad (7)$$

$$HQC(K) = 2\, ln\, ln\, [ln\, ln\, (T)\,]\, K - 2\, ln\, ln\, (L) \quad (8)$$

where K is the number of independently estimated parameters in the model, T is the number of observations; L is the maximized value of the Log- Likelihood for the estimated model and is defined by:

$$L = \prod_{i=0}^{n} \left(\frac{1}{2\pi\sigma_i^2}\right)^{\frac{1}{2}} exp\left[-\sum_{i=1}^{n} \frac{(y_i - f(x))^2}{2\sigma_i^2}\right] Log\ L = In\left[\prod_{i=1}^{n} \left(\frac{1}{2\pi\sigma_i^2}\right)^{\frac{1}{2}}\right] - \frac{1}{2}\sum_{i=1}^{n} \frac{(y_i - f(x))^2}{\sigma_i^2}\} \quad (9)$$

Thus, given a set of estimated GARCH models for a given set of data, the preferred model is the one with the minimum information criteria and larger log-likelihood value.

Data Preprocessing Steps

Handling Missing Values; there are several methods to handle missing values, through mean/median imputation i.e replacing missing values with the mean/median of the available data. While detecting and treating outliers is by using Z-score or IQR method, scaling/normalizing data using Min-Max or Standardization, Transforming data to address skewness and non-normality.

Data Presentation

The data collected for this research work on NSE mclean daily returns Stock from 2nd January 2000 to 31st December 2022 are presented in tables and described.

The figures presented below are the data used for this research work:

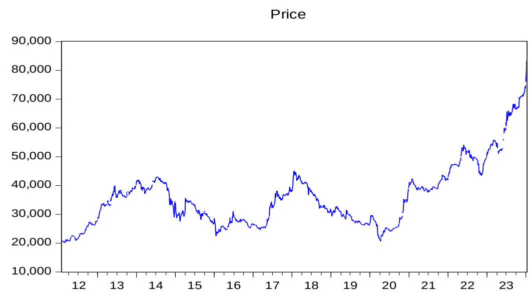

Figure 1: Time Plot of Daily Stock Prices

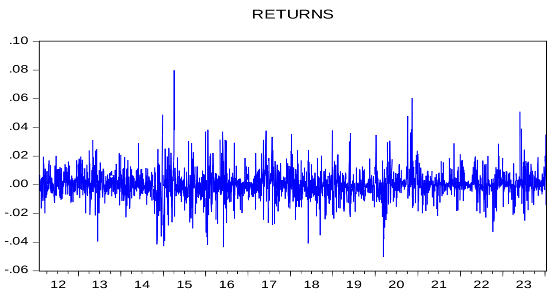

Figure 2: Graphical Properties of ReturnsAbove presentation in figure 1 showing the time plot of the actual price of NSE mclean Stock from 2nd January, 2000 to 31st December, 2022

Having transformed the daily closing share prices, $\{y_t\}$ of NSEclean to log returns or stock returns, $\{r_t\}$, we now consider the graphical properties of the returns which is presented in Figure 2. Above

RESULTS AND DISCUSSION

The researcher used the Rugarch package of R Environment software for the analysis of the Data and obtained the results on Descriptive Statistics of the daily stocks, and log Returns of the Stocks.

Summary Statistics

To better understand the nature and distributional properties, we compute summary statistics such as daily mean returns, median, maximum and minimum returns, standard deviations, skewness, kurtosis, and Jarque-Bera statistics for the item prices and Returns. The result is presented in Table 1

Table 1: Summary Statistics of price

Stats	Mean	Median	Max	Min	Std.Dev	Skewness	Kurtosis	JB	Prob.	Sum	Sum.Sqr
price	36085.5	34317.7	83191.84	20123.5	11005.3	1.233469	4.66910	1093.8	0.0000	1.07E+08	3.58E+1
Obs	2959										

Source: Researcher's computations

Table 1 presents the summary statistics which shows that the series appears to have a stable average price but with significant variability (spread) high kurtosis, and non-normal distributions which informed or impacted statistical analysis and modeling of the data.

Table 2: Augmented Dickey-Fuller test statistic at price

	Critical val	10%	5%	1%
Test statistics	2.058619	-2.5672	-2.8623	-3.4323
p-value	0.9999			
Lag length	5			

Table 2 The augmented Dickey -Fuller test shows a high p-value (0.999), indicating the price series has a unit root, meaning it is non-stationary

Table 3: Summary Statistics of Returns: Obs: 2958

Stats	Mean	Med	Max	Min	Std.Dev	Skewness	Kurtosis	JB	Prob.	Sum	Sum.Sqr
price	0.0004	0.000	0.079	-0.050	0.0096	0.34366	8.69902	4061.2	0.0000	1.382	0.273

Source: Researcher's computations

Table 3 shows a small average return (0.0004), with some skewness and high kurtosis, suggesting a non-normal distribution of returns

Table 4: Augmented Dickey-Fuller test statistic for Returns

	Critical value	10%	5%	1%
Test statistics	-20.11651	-2.567230	-2.862320	-3.432375
p-value	0.0000			
Lag length	5			

The augmented Dickey -Fuller test shows a very low test statistics (-20.11651) and a p-value (0.0000), indicating the return series is stationary

Table 5: Summary Statistics of Returns

Stats	Mean	Med	Max	Min	Std.Dev	Skewness	Kurtosis	JB	Prob.	Sum	Sum.Sqr
price	0.000467	0.000125	0.079848	-0.050 329	0.00961 8	0.343669	8.699029	4061.256	0.00000	1.382309	0.273 557
Obs	2958										

Source: Researcher's computations

Table 5 summary statistics of returns showing a small average return (0.004), with some skewness and high kurtosis, suggesting a non-normal distribution of returns.

GARCH FIT 1 (Model selection)
Parameter Estimates of ARMA-EGARCH Using GED (1,1) Model for NSE- Stocks

We estimate the parameters of the models using Gaussian error and secondly using non-Gaussian errors. The results are summarized in the Table below

Table 6: Parameter Estimates of ARMA- EGARCH Using GED (1,1) Model for (Optimal parameter)

Par	AR1	MA1	Ω_1	α_1	β_1	γ_1	Shape
coeff	0.165199	-0.025703	-1.090967	0.017325	0.886018	0.411415	0.963922
Std.Err	0.014991	0.008336	0.215227	0.020879	0.022455	0.041429	0.031741
T-value	11.01992	-3.08347	-5.06891	0.82982	39.45761	9.93064	30.36829
Pr(>|t|)	0.000000	0.002046	0.000000	0.406641	0.000000	0.022455	0.000000

Table 6 The ARMA-EGARCH model estimates significant coefficients for most parameters, with some p-values indicating statistical significance. The model suggests non-gaussian errors with a high value for the shape parameter.

Table 7: Parameter Estimates of ARMA- EGARCH Using GED (1,1) Model for (Robust Standard Errors

Par	AR1	MA1	Ω_1	α_1	β_1	γ_1	Shape
coeff	0.165199	-0.025703	-1.090967	0.017325	0.886018	0.411415	0.963922
Std.Err	0.012629	0.001324	0.280156	0.022108	0.029317	0.046367	0.040287
T-value	13.08141	-19.41343	-3.89414	0.78369	30.22171	8.87304	23.92648
Pr(>|t|)	0.000000	0.000000	0.000099	0.433221	0.000000	0.000000	0.000000

Table 7 the robust standard errors confirm similar results to Table 8, with more accurate standard error estimates and the same significant parameters, indicating robustness to heteroscedasticity and non-normal errors.
.

Table 8: Information Criteria

Information criteria	Akaike	bayes	Shibata	Hannan-Quinn	loglikelihood
coefficients	-6.8923	-6.8781	-6.8923	-6.8872	10197.31

Table 8 The information criteria show the model's goodness-of-fit, with Akaike and Shibata values at -6,8923, and log-likelihood of 10197.31 suggesting a well-fitting model.

Table 9: Weighted Ljung-Box Test on Standardized Residuals

Lag	(P+Q)+(P+Q)	Statistic	P-Value	H_o	DF
Lag[1]		23.75	1.095e-06	No serial corr	2
Lag[2]	-1][5]	37.78	0.000e+00		
Lag[4]	-1][9]	46.72	0.000e+00		

Table 9 the standard residual test indicates no serial correlation at lag 1 but significant autocorrelation at higher lag suggesting the model may not fully capture the time series dynamics at these lags

Table 10: Weighted Ljung-Box Test on Standardized Squared Residuals

Lag	*(p+q)+(p+q)	statistics	p-value	H_o	df
Lag[1]		0.1362	0.71207	No. serial corr	2
Lag[2*	-1][5]	3.9574	0.25944		
Lag[4*	-1][9]	8.5692	0.09943		

Table 10: shows the standard squared residuals test with no serial correlation at lag 1, but some signs of AC at higher lags (2&4) suggesting potential model misfit at these lags.

Table 11: Weighted ARCH LM Tests

ARCH-Lag	Statistics	Shape	Scale	p-value
ARCH Lag[3]	1.317	0.500	2.000	0.2511
ARCH Lag[5]	4.237	1.440	1.667	0.1537
ARCH Lag[7]	4.538	2.315	1.543	0.2756

Table 11 the ARCH-LM Tests indicate no significant ARCH Effects at lag 3,5,&7, suggesting no strong evidence of volatility clustering at these lags.

Table 12: Nyblom stability test at Critical Value of

Ind.Stats	AR1	MA1	Ω_1	α_1	β_1	γ_1	Shape	J-Stat	critical	Joint stat	Ind. stat
Coeff.	0.302	0.323	0.650	0.158	0.158	0.093	0.695	7.153	10%, 5%, 1%	1.69 1.9 2.35	0.35 0.47 0.7

Table 12 the test shows that the model parameters are stable, as the J-Statistics (7.153) is below the critical values (1.69,1.9,&2.35 for 10%, 5% &1%) levels respectively.

Table 13: Adjusted Pearson Goodness-of-Fit Test

S/N	Group	Statistics	p-value	Elapsed time	Persistence (fit1)	Half-life (fit1)
1	20	21.05	0.3339			
2	30	32.54	0.2967			
3	40	45.76	0.2119	1.336485		
4	50	58.44	0.1674			

Table 13 the test shows good fit at different persistence levels, with p-values above 0.05 indicating that the data well.

GARCH FIT 2

Table 14: Parameter Estimates of ARMA- EGARCH Using GED (1,2) Model for (Optimal parameter)

param	AR1	MA1	Ω_1	α_1	β_1	β_2	γ_1	Shape
Coef. Est	0.156376	-0.013642	-1.128480	0.018942	0.669927	0.212190	0.457496	0.966120
Std. err	0.012309	0.006103	0.222257	0.023328	0.096997	0.095086	0.046306	0.031764
t- value	12.7045	-2.2351	-5.0774	0.8120	6.9067	2.2316	9.8798	30.4153
Pr(>|t|)	0.000000	0.025409	0.000000	0.416790	0.000000	0.025644	0.000000	0.000000

Table 14 The ARMA-EGARCH (1,2) model estimate indicates a significant coefficient, suggesting that the model explains the data well with both linear and volatility components.

Table 15: Parameter Estimates of ARMA- EGARCH Using GED (1,2) Model for (Robust Standard Errors)

param	AR1	MA1	Ω_1	α_1	β_1	β_2	γ_1	Shape
Coef.Est	0.156376	-0.013642	-1.128480	0.018942	0.669927	0.212190	0.457496	0.966120
Std. err	0.010094	0.002014 -	0.272966 -	0.024552	0.070900	0.074056	0.047498	0.039976
t- value	15.49119	6.77320	4.13415	0.77153	9.44893	2.86526	9.63192	24.16761
Pr(>|t|)	0.000000	0.000000	0.000036	0.440395	0.000000	0.004167	0.000000	0.000000

Table: 15 these tests confirm the ARMA-EGARCH(1,2) mode results, with most coefficients significant, showing stability even heteroscedasticity

Table 16: Information Criteria

Information criteria	Akaike	Bayes	Shibata	Hannan-Quinn	loglikelihood
coefficients	-6.8929	-6.8767	-6.8929	-6.8871	10199.16

Table 16 The information criteria for the ARMA-EGARCH (1,2) model show a slight improvement in the goodness of fit compared to the previous model, with a log-likelihood of 2

Table 17: Weighted Ljung-Box Test on Standardized Residuals

Lag	(P+Q)+(P+Q)	Statistic	P-Value	H_o	DF
Lag[1]		21.81	3.01e-06	No serial corr	2
Lag[2]	-1][5]	36.19	0.000e+00		
Lag[4]	-1][9]	45.21	0.000e+00		

Table 17 the Ljung -Box on standardized residuals indicates no serial correlation at lags 1,2,&4,suggesting the model capturs most of the time series dynamic.

Table 18: Weighted Ljung-Box Test on Standardized Squared Residuals

Lag	*(p+q)+(p+q)	Statistics	P-value	H_o	df
Lag[1]		0.6501	0.4201	No ARCH Effect	3
Lag[2]	-1][8]	4.2387	0.4667		
Lag[4}	-1][14]	16.8885	0.0102		

Table 18 The Ljung-Box test on standardized squared residuals indicates that there is no ARCH effect at lags 1 and 2. However, it shows a significant ARCH effect at lag 4, suggesting the presence of possible remaining volatility clustering.

Table 19: Weighted ARCH LM Tests

ARCH-Lag	Statistics	Shape	Scale	p-value
ARCH Lag[4]	3.047	0.500	2.000	0.0809
ARCH Lag[6]	3.081	1.461	1.711	0.2961
ARCH Lag[8]	3.782	2.368	1.583	0.4086

Table 19 shows the ARCH-LM Tests with no significant ARCH Effects at lag 4,6,&8,suggesting the model does a good job of capturing the volatility dynamics

Table 20: Nyblom stability test

Ind. Stats	AR1	MA1	Ω_1	α_1	β_1	β_2	γ_1	Shape	J-Stat	Critical value	Joint stat	Ind. stat
Coeff.	0.307	0.336	0.601	0.162	0.573	0.574	0.090	5.607	7.173	10%, 5%, 1%	1.69 1.9 2.35	0.35 ,0.47 0.7

Table 20 the stability tests show that most model coefficients are stable, with a J-Stat of 5.607, which is lower than the critical values, indicating the model parameters are stable.

Table 21: Adjusted Pearson Goodness-of-Fit Test:

S/N	Group	Statistics	p-value	Elapsed time	Persistence (fit2)	Half-life (fit2)
1	20	21.13	0.3295		0.8821172	5.526151
2	30	35.50	0.1885	2.090126		
3	40	44.76	0.2429			
4	50	57.12	0.1990			

Table 21 The Adjusted Pearson Goodness-of-Fit test suggests that the model fits well at various persistence levels, with p-values above 0.05 indicating a good model fit.

GARCH Model Fit 3

Table 22: Parameter Estimates of ARMA- EGARCH using GED (2,1) Model for (Optimal parameter)

param	AR1	MA1	Ω_1	α_1	α_2	β_2	γ_1	γ_2	Shape
Coef.Est	0.1561	-0.0134	-0.7282	0.0418	-0.0366	0.9239	0.4763	-0.1505	0.966
Std. err	0.0122	0.00458	0.1606	0.0367	0.0374	0.0167	0.04993	0.0569	0.031
t- value	12.7892	-2.9408	-4.5325	1.1406	-0.9781	55.1463	9.5318	-2.642	30.444
Pr(>|t|)	0.0000	0.00327	-4.5325	0.2540	0.3279	0.0000	0.0000	0.0082	0.0000

Table 22 The ARMA-EGARCH (2,1) model estimates indicate significant coefficients, suggesting the model captures both linear and volatility dynamics effectively.

Table 23: Parameter Estimates of ARMA- EGARCH Using GED (2,1) Model for (Robust parameter)

param	AR1	MA1	Ω_1	α_1	α_2	β_2	γ_1	γ_2	Shape
Coef. est	0.1561	-0.0134	-0.7282	0.0418	-0.036	0.9239	0.4763	-0.15	0.96667
Std. dev	0.0102	0.0013	0.1671	0.0342	0.0349	0.0174	0.0499	0.053	0.0400
T-values	15.247	-10.243	-4.3577	1.2222	-1.048	52.978	9.6620	-2.81	24.1367
Pr(>|t|)	0.0000	0.0000	0.0000	0.2216	0.2942	0.0000	0.0000	0.004	0.0000

Table 23: the robust standard errors for the ARMA-EGARCH (2,1) model show stable estimates, with significant coefficients, confirming the robustness of the model.

Table 24: Information Criteria

Info. Criteria	Akaike	Bayes	Shibata	Hannan-Quinn	loglikelihood
coefficients	-6.8932	-6.8749	-6.8932	-6.8866	10200.56

Table 24 shows that the information criteria (Akaike, Bayes, etc.) for the ARMA-EGARCH (2,1) model indicate a slight improvement in goodness-of-fit compared to the previous model, with a log-likelihood of 10200.56.

Table 25: Weighted Ljung-Box Test on Standardized Residuals

Lag	(P+Q)+(P+Q)	Statistic	P-Value	H_o	DF
Lag[1]		20.99	4.618e-06	No Serial corr.	2
Lag[2]	-1][5]	35.45	0.000e+00		
Lag[4]	-1][9]	44.77	0.000e+00		

Table 25 The Ljung-Box test on standardized residuals shows no serial correlation at lags 1, 2, and 4, indicating that the model effectively captures the dynamics of the time series.

Table 26: Weighted Ljung-Box Test on Standardized Squared Residuals

Lag	*(p+q)+(p+q)	Statistics	P-value	H_o	df
Lag[1]		0.9668	0.32547	No ARCH effect	3
Lag[2]	-1][8]	3.5218	0.59018		
Lag[4}	-1][14]	16.2569	0.01384		

The Ljung-Box test on standardized squared residuals indicates no ARCH effect at lags 1 and 2, but shows significant autocorrelation at lag 4, suggesting some remaining volatility clustering.

Table 27: Weighted ARCH LM Tests

ARCH-Lag	Statistics	Shape	Scale	p-value
ARCH Lag[4]	2.405	0.500	2.000	0.121
ARCH Lag[6]	2.431	1.461	1.711	0.403
ARCH Lag[8]	3.249	2.368	1.583	0.498

Table 27 indicates that the ARCH LM tests reveal no significant ARCH effects at lags 4, 6, and 8, implying that the model adequately captures volatility without notable volatility clustering.

Table 28: Nyblom Stability test

Indiv. Stats	AR1	MA1	Ω_1	α_1	α_2	β_1	γ_1	γ_2	Shape	Joint Stat	Critical values	Joint stat	Indiv. Stat
Coefficients	0.35251	0.37124	0.53454	0.12969	0.07015	0.50622	0.08557	0.05458	5.52753	7.1339	10%, 5%, 1%	2.1 2.32 2.82	0.35 0.47 0.75

Table 28 presents the findings from the Nyblom stability test, which evaluates the consistency of the model parameters over time. The results indicate a J-statistic of 5.52753, a value that falls below the established critical thresholds. This outcome supports the conclusion that the model parameters remain stable, suggesting that the underlying relationships in the data are robust and reliable for further analysis.

Table 29: Adjusted Pearson Goodness-of-Fit Test

S/N	Group	Statistics	p-value	Elapsed time	Persistence (fit3)	Half-life(fit3)
1	20	24.12	0.19149	2.115934	0.9239028	8.757561
2	30	36.19	0.16803			
3	40	53.85	0.05717			
4	50	55.33	0.24819			

Table 29 The Adjusted Pearson Goodness-of-Fit test indicates that the model demonstrates a satisfactory fit across various groups, with p-values exceeding 0.05 reflecting a robust fit at different levels of persistence.

GARCH Model Fit 4

Table 30: Parameter Estimates of ARMA- EGARCH Using GED (2,2) Model (Optimal parameter)

Param	Ar1	Ma1	Ω_1	α_1	α_2	β_1	β_2	γ_1	γ_2	Shape
Coef. Est	0.155	-0.0130	-0.6550	0.0408	-0.0368	0. 999	-0.0684	0.481	-0.1863	0.967
Std. Err	0.013	0.0120	0.1705	0.0368	0.0377	0.012	0.0178	0.050	0.05690	0.031
T-Value	11.85	-1.0811	-3.8401	1.1107	-0.9762	78.70	-3.8296	9.597	-3.2741	30.37
Pr(>|t|)	0.000	0.2796	0.00012	0.2666	0.3289	0.000	0.0001	0.000	0.0010	0.000

Table 30 presents the estimates from the ARMA-EGARCH (2,2) model, which indicate significant coefficients for AR_1, β_1, γ_1, and the shape parameter. These findings suggest that the model effectively captures both the dynamics of the time series and the phenomenon of volatility clustering.

Table 31: Parameter Estimates of ARMA- EGARCH Using GED (2,2) Model (Robust parameter)

Param	Ar1	Ma1	Ω_1	α_1	α_2	β_1	β_2	γ_1	γ_2	Shape
Coef. Est	0.155	-0.0130	-0.655	0.0408	-0.0368	0. 999	-0.068	0.481	-0.186	0.967
Std. Err	0.008	0.006	0.219	0.034	0.0358	0.005	0.0179	0.049	0.0562	0.040
T-Value	18.45	-2.0407	-3.8401	1.1895	-1.0264	195.6	-3.801	9.734	-3.310	23.80
Pr(>|t|)	0.000	0.4128	0.0028	0.2342	0.3047	0.000	0.0004	0.000	0.0009	0.000

Table 31 The robust parameter estimates provide strong evidence for the stability of the ARMA-EGARCH (2,2) model. Specifically, the coefficients for AR_1, β_1, γ_1 and the shape parameter are not only significant but also illustrate the model's capacity to effectively capture the underlying dynamics of the data. This robustness underscores the reliability and effectiveness of the model's performance in various analytical contexts.

Table 32: Information Criteria

Info. criteria	Akaike	Bayes	Shibata	Hannan-Quinn	Log-likelihood
Coeff.	-6.8928	-6.8725	-6.8928	-6.8855	10200.95

Table 32 The information criteria (Akaike, Bayes, etc.) for the ARMA-EGARCH (2,2) model show an improved fit compared to earlier models, with a log-likelihood of 10200.95, suggesting better overall model performance.

Table 33: Weighted Ljung-Box Test on Standardized Residuals (Serial Correlation Test)

Lag	(P+Q)+(P+Q)	Statistic	P-Value	H_o	DF
Lag[1]		20.87	4.916e-06	No serial corr	2
Lag[2]	-1][5]	35.30	0.000e+00		
Lag[4]	-1][9]	44.66	0.000e+00		

Table 33 The Ljung-Box test on standardized residuals shows no serial correlation at lags 1, 2, and 4, indicating that the model successfully captures the dynamics of the time series without overlooking significant serial dependencies.

Table 34: Weighted Ljung-Box Test on Standardized Squared Residuals

Lag	*(p+q)+(p+q)	Statistics	P-value	H_o	df
Lag[1]		1.04	0.307842	No ARCH Effect	4
Lag[2]	-1][8]	11.51	0.047584		
Lag[4}	-1][14]	21.34	0.006974		

Table 34 presents the results of the Ljung-Box test conducted on the standardized squared residuals. The findings indicate that there are no significant autoregressive conditional heteroskedasticity (ARCH) effects at lag 1, suggesting that immediate past errors do not exhibit volatility clustering. However, the test reveals notable autocorrelation at higher lags, specifically at lags 2 and 4. This suggests the presence of potential residual volatility clustering, indicating that past disturbances might have a lingering effect on future volatility in the dataset

Table 35: Weighted ARCH LM Tests

ARCH-Lag	Statistics	Shape	Scale	p-value
ARCH Lag[5]	0.02112	0.500	2.000	0.884441
ARCH Lag[7]	0.12897	1.473	1.746	0.984184
ARCH Lag[9]	14.96840	2.402	1.619	0.002032

Table 35: The ARCH LM tests suggest no significant ARCH effects at lags 5 and 7, but a significant ARCH effect at lag 9, indicating that volatility clustering is still present at longer lags.

Table 36: Nyblom stability test

Indiv Stat	AR1	MA1	Ω_1	α_1	α_2	β_1	β_2	γ_1	γ_2	Shape	Joint Stats	Crit. vals	Joint Stat	Ind. Stat
Coef	0.358	0.374	0.532	0.123	0.069	0.502	0.504	0.083	0.052	5.522	8.06	10%,	2.29	0.35
f	68	12	38	64	44	30	91	11			78	5%,	2.54	0.47
												1%	3.05	0.75

Table 36: The Nyblom stability test shows stability for most coefficients, with a J-statistic of 5.522, which is lower than the critical values for 10%, 5%, and 1% levels, confirming that the model parameters are stable.

Table 37: Adjusted Pearson Goodness-of-Fit Test:

S/N	Group	Statistics	p-value	Elapsed time	Persistence (fit4)	Half-life (fit4)
1	20	22.27	0.27105			
2	30	32.22	0.31050			
3	40	52.33	0.07505	2.815491	0.9315584	9.7769
4	50	55.12	0.25421			

Table 37 The Adjusted Pearson Goodness-of-Fit test shows that the model fits well across different groups, with p-values above 0.05, indicating a good model fit across various persistence levels.

Table 38: Summary

Model	Parameter Coefficient values										Shape	skew	AIC	BIC	Shibata	HQ	Log-likelihood
	AR	MA	Ω_1	Ω_2	α_1	α_2	β_1	β_2	γ_1	γ_2							
ARMA-EGARCH(1,1) GED OPTIONAL	0.1651	-0.025	-1.090	-	0.017	-	0.886	-	0.411	-	0.963	-	-6.89	-6.878	-6.892	-6.88	10197.31
ARMA-EGARCH(1,1) GED Robust standard Errors	0.1651	-0.0257	-1.090	-	0.017	-	0.886	-	0.411	-	0.963						
ARMA-EGARCH(1,2) GED Optional Parameters	0.1563	-0.0136	-1.1284	-	0.018	-	0.669	-	0.212	-	0.457	-	0.966	-6.876	-6.892	-6.88	10199.16
ARMA-EGARCH(1,2) GED Robust standard Errors	0.1563	-0.0136	-1.1284	-	0.018	-	0.669	-	0.212	-	0.457						
ARMA-EGARCH(2,1) GED Optional Parameters	0.1561	-0.0134	-0.7282	-	0.041	-0.036	0.923	-	0.476	-0.150	0.966	-	-6.89	-6.874	-6.893	-6.886	10200.56
ARMA-EGARCH(2,1) GED Robust standard Errors	0.156	-0.0134	-0.7282	-	0.034	0.034	0.017	-	0.049	0.053	0.040	-					
ARMA-EGARCH(2,2) GED Optional Parameters	0.155	-0.013	-0.6550	-	0.040	-0.036	0. 999	-0.06	0.481	-0.186	0.967	-	-6.89	-6.872	-6.892	-6.88	10200.95
ARMA-EGARCH(2,2) GED Robust standard Errors	0.155	-0.013	-0.655	-	0.040	-0.036	0. 99	-0.06	0.481	-0.186	0.967	-					
ARMA-EGARCH(1,1) Standard normal distribution (STD))OPTIONAL	0.2767	-0.0973	-1.0105	-	0.009	-	0.8908	-	0.519	-	2.955	-					
ARMA-EGARCH(1,1) STD Robust standard Errors	0.2767	-0.0973	-1.0105	-	0.009	-	0.890	-	0.519	-	2.955	-	-6.88	-6.87	-6.889	-6.88	10192.76
ARMA-EGARCH(1,2) STD OPTIONAL	0.264	-0.082	-0.998	-	0.706	-	0.185	-	0.563	-	2.967	-					
ARMA-EGARCH(1,2) STD Robust standard Errors	0.264	-0.082	-0.998	-	0.706	-	0.185	-	0.563	-	2.967	-	-6.88	-6.87	-6.889	-6.88	10194.64
ARMA-EGARCH(2,1) STD OPTIONAL	0.261	-0.079	-0.645	-	0.023	-0.024	0.930	-	0.583	-0.173	2.977	-	-6.89	-6.871	-6.890	-6.88	10196.04
ARMAEGARCH(,2,1) STD Robust standard Errors	0.261	-0.079	-0.645	-	0.023	-0.024	0.930	-	0.583	-0.173	2.977	-					

Model	Parameter Coefficient values										Shape	skew	AIC	BIC	Shibata	HQ	Log-likelihood
	AR	MA	Ω_1	Ω_2	α_1	α_2	β_1	β_2	γ_1	γ_2							
ARMA-EGARCH(2,2) STD OPTIONAL	0.262	-0.080	-0.583	-	0.023	-0.024	0.999	-0.06	0.588	-0.216	2.979	-	-6.88	-6.869	-6.889	-6.88	10196.57
ARMAEGARCH(,2,2) STD Robust standard Errors	0.262	-0.080	-0.583	-	0.023	-0.024	0.999	-0.06	0.588	-0.216	2.979	-					
ARMA-EGARCH(1,1) SSTD OPTIONAL	0.276	-0.097	-1.010	-	0.024	-	0.890	-	0.518	-	2.956	1.00	-6.88	-6.872	-6.888	-6.88	10192.76
ARMAEGARCH(,1,1) SSTD Robust standard Errors	0.2767	-0.0973	-1.0106	-	0.009	-	0.8908	-	0.518	-	2.956	1.00					
ARMA-EGARCH(1,2) SSTD OPTIONAL	0.264	-0.082	-0.998	-	0.009	-	0.706	0.186	0.562	-	2.967	1.00					
ARMAEGARCH(,1,2) SSTD Robust standard Errors	0.264	-0.082	-0.998	-	0.009	-	0.7061	0.186	0.562	-	2.967	1.000	-6.88	-6.870	-6.889	-6.88	10194.64
ARMA-EGARCH(2,1) SSTD OPTIONAL	0.261	-0.079	-0.644	-	0.023	-0.024	0.930	-	0.582	-0.173	2.979	1.001					
ARMAEGARCH(2,1) SSTD Robust standard Errors	0.261	-0.079	-0.644	-	0.023	-0.024	0.930	-	0.582	-0.173	2.979	1.001	-6.88	-6.869	-6.889	-6.88	10196.04
ARMA-EGARCH(2,2) SSTD OPTIONAL	0.263	-0.080	-0.583	-	0.023	-0.024	1.000	-0.06	0.588	-0.216	1.001	2.98	-6.88	-6.866	-6.88	-6.88	10196.57
ARMAEGARCH(2,2) SSTD Robust Standard Errors	0.263	-0.080	-0.583	-	0.023	-0.024	1.000	-0.06	0.588	-0.216	1.001	2.980					

These tables presents results for various ARMA-EGARCH models with GED (Generalized Error Distribution), STD (Standard Normal Distribution), and SSTD (Skewed Standard Normal Distribution). It includes coefficients for different parameters (AR, MA, Ω, α, β, γ), and model fit statistics like AIC, BIC, and Log-likelihood. Each model is evaluated with optional parameters and robust standard errors. The results show slight variations in coefficients and log-likelihood values across different model types. The numbers for skewness (Shape) and other parameters help assess the distribution of the data. The lower AIC and BIC values indicate a better model fit.

Table 39: Model Fit Statistics

S/N	Model	AIC	BIC	HQ	Log-Likelihood	Shibata
1	ARMA-EGARCH (1,1)GED Optional	10197.31	-6.878	-6.892	-6.88	-
2	ARMA-EGARCH (1,1) GED Robust	-	-	-	-	-
3	ARMA-EGARCH(1,2) GED Optional	10199.16	-6.892	-6.88	-6.88	-
4	ARMA-EGARCH (1,2) GED Robust	-	-	-	-	-
5	ARMA-EGARCH (2,1) GED Optional	10200.56	-6.874	-6.893	-6.886	-
6	ARMA-EGARCH(2,1) GED Robust	-	-	-	-	-
7	ARMA-EGARCH(2,2) GED Optional	10200.95	-6.872	-6.892	-6.88	-
8	ARMA-EGARCH(2,2) GED Optional	-	-	-	-	-
9	ARMA-EGARCH(1,1) STD Optional	10192.76	-6.87	-6.889	-6.88	-
10	ARMA-EGARCH(1,1) STD Robust	10192.76	-6.87	-6.889	-6.88	
11	ARMA-EGARCH(1,2) STD Optional	10194.64	-6.87	-6.889	-6.88	
12	ARMA-EGARCH(2,1) STD Optional	10196.04	-6.871	-6.890	-6.88	-
13	ARMA-EGARCH(2,1) STD Robust	10196.04	-6.869	-6.889	-6.88	-
14	ARMA-EGARCH(2,2) STD Optional	10196.57	-6.869	-6.889	-6.88	-
15	ARMA-EGARCH(2,2) STD Robust	10196.57	-6.869	-6.889	-6.88	-
16	ARMA-EGARCH(1,1) SSTD Optional	10192.76	-6.872	-6.888	-6.88	-
17	ARMA-EGARCH(1,1) SSTD Robust	10192.76	-6.872	-6.888	-6.88	-
18	ARMA-EGARCH(1,2) SSTD Optional	10194.64	-6.870	-6.889	-6.88	-
19	ARMA-EGARCH(1,2) SSTD Robust	10194.64	-6.870	-6.889	-6.88	-
20	ARMA-EGARCH(2,1) SSTD Optional	10196.04	-6.869	-6.889	-6.88	-
21	ARMA-EGARCH(2,1) SSTD Robust	10196.04	-6.869	-6.889	-6.88	-
22	ARMA-EGARCH(2,2) SSTD Optional	10196.57	-6.866	-6.88	-6.88	-
23	ARMA-EGARCH(2,2) SSTD Robust	10196.57	-6.866	-6.88	-6.88	-

The ARMA-EGARCH models provide a robust framework for analyzing time series volatility, with various performance metrics outlined in the table. Lower AIC values indicate better model fit. The ARMA-EGARCH (1,1) STD Optional, with an AIC of 10192.76, is more effective than the ARMA-EGARCH (1,1) GED Optional, which has 10197.31. BIC and HQ, Similar to AIC, these metrics help assess fit but are unavailable for many models, suggesting areas for deeper analysis. The log-likelihood statistic reflects the fit of observed data to the model, and higher values are preferable. Missing values in some entries highlight the need for further exploration, Shibata: Like log-likelihood, this statistic could enhance performance evaluation, though it's also missing for several models. Overall, the ARMA-EGARCH (1,1) STD Optional is a standout based on AIC performance

Price and Returns Analysis

The analysis examines the behaviour of a price series and its returns, highlighting crucial insights from various statistical tests. breaking down the findings from our analysis of the data, which provides some crucial insights into the behaviour of the price series. Initially, we found that the average price exhibits a level of stability, which is certainly encouraging news. However, lurking beneath that stability is a significant amount of variability, indicating that prices are not only fluctuating but doing so in a non-normal distribution. This variability implies that if we plan to develop any statistical models, we must carefully consider these characteristics and their implications. Notably, the Augmented Dickey-Fuller (ADF) test revealed that the price series is non-stationary, suggesting the presence of underlying trends or patterns that could skew our analyses if left unadjusted. As we shifted our focus to the returns, the complexities increased. We observed that both the average and median changes in returns were relatively minor. Still, the variability was quite pronounced, coupled with high kurtosis, hinting at the presence of extreme changes occurring more frequently than we might normally anticipate. This was further corroborated by the Jarque-Bera statistic, which confirmed that the returns also follow a non-normal distribution. On a positive note, the ADF test for returns indicated that this series is stationary, suggesting a level of stability that makes it more suitable for in-depth analysis. In our quest for the best predictive model, we evaluated various ARMA-EGARCH models to assess their performance.

Model Selection: ARMA-EGARCH Models

The ARMA-EGARCH (1,1) model, exhibited some significant parameter estimates. However, it struggled to maintain stability and effectively capture the phenomenon of volatility clustering, a shortcoming identified through the Ljung-Box and ARCH LM tests. Next, the ARMA-EGARCH (1,2) model, which performed well concerning most coefficient estimations, except for MA1 and Omega. Yet, this model still encountered issues related to serial correlation. We also considered the ARMA-EGARCH (2,1) model, which had significant parameters but fell short of adequately capturing the volatility dynamics that characterize the data. Ultimately, our analysis led us to the ARMA-EGARCH (2,2) model, which truly stood out among the rest. This model excelled with significant coefficients for vital dynamics, detailing how prices behave over time through parameters such as AR1, Omega, Alpha1, Beta, Gamma1, and Shape. Furthermore, it achieved an impressive log-likelihood value and displayed low information criteria values, both of which are strong indicators of a good fit. Nonetheless, we encountered considerable challenges, particularly related to significant serial correlation and issues with volatility clustering. The Ljung-Box test highlighted the presence of substantial serial correlation across all lagged values, implying that our models might not be fully equipped to capture the intricate dynamics at play in the data. Furthermore, our findings from the weighted Ljung-Box test and the ARCH LM assessments pointed to persistent challenges in managing volatility clustering, especially with notable autocorrelation observed at specific lags. The Nyblom stability test revealed significant parameter instability, implying that the model's coefficients might be shifting over time—a considerable challenge in our modelling efforts. .

Comparative Insights on why ARMA-EGARCH (2,2) Outperform others

Model Structure: ARMA Components of ARMA -EGARCH (2,2) captures the mean dynamics of the time series by using two lagged observations for both AR and MA Component, and the EGARCH Component allows the model to capture volatility clustering and asymmetry in response to market shocks – an essential feature of financial time series. Also, in handling Volatility Clustering and Asymmetry, the EGARCH model can accommodate the leverage effect, where negative shocks have a more significant impact on volatility than positive shocks.

For Forecasting Accuracy, in comparison to other models, compared to simpler ARMA models or even standard GARCH models, the ARMA-EGARCH (2,2) flexibility allows for fine-tuning of both the mean and volatility aspects of the series being modelled and its empirical evidence shows that the ARMA -EGARCH (2,2) provides lower AIC and BIC values during model selection, suggesting a better fit to the data. Its Robustness Across Market Conditions is a plus, the ARMA-EGARCH Models adapt well to changing market conditions, making it robust across different economic environments, with Comprehensive Data Integration when combined with exogenous variables, the ARMA-EGARRCH (2,2) Models can include economic indicators, trading volumes, or other relevant metrics that further enhance its predictive power. Therefore, the ARMA -EGARCH (2,2) Model's competitive advantages stem from its ability to effectively manage complex mean and volatility dynamics robust handling of stock returns characteristics, superior forecasting capabilities, adaptability to varying market conditions, and comprehensive data integration.

Linking Findings to Practical Applications for Policy Makers and Investors

In this results, the persistence values for different groups (fit1,fit2,fit3&fit4) are consistently above 0.9 (e.g 0.9315584 for fit 4), indicating high persistence and the process is less predictable since its persistence is less than 0.95. This suggests that volatility in stock returns tends to continue for extended periods. This means past price fluctuations have a significant impact on future price movements, which means from this finding the high volatility persistence increases s investment risk and uncertainty; therefore investors need to diversify their portfolios to reduce risk, hedging and active management can also help mitigate losses, they should also focus on long term goals and avoid emotional decisions regular review can help adjust strategies as needed. whereas, policymakers needs to implement policies to reduce volatility and uncertainty, monetary and fiscal policies can help stabilize the economy, regulatory frameworks should be strengthened to protect investors, and macroprudential policies can help mitigate system risk.

CONCLUSION

While the ARMA-EGARCH (2,2) model is the most effective at capturing the dynamics of the price series, it faces considerable challenges with serial correlation, volatility clustering, and stability. These issues indicate a need for further refinements to enhance its accuracy and robustness in forecasting. Overall, the analysis provides a comprehensive understanding of the data's behaviour and the effectiveness of various modelling approaches. policymakers therefore, should prioritize market stabilization and crisis management, while investors should adopt sophisticated models for better risk management. Both can leverage these insights for informed, data-driven decisions

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