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INFLUENCE OF TEMPERATURE DEPENDENT VISCOSITY, VISCOUS DISSIPATION AND JOULE HEATING ON MHD NATURAL CONVECTION FLOW: A SEMI ANALYTICAL APPROACH

***Yusuf, A. B. and Abdullahi, F.**

Department of Mathematics, Federal University Dutsin-ma, Katsina State, Nigeria.

*Corresponding authors' email: ayusuf@fudutsinma.edu.ng Phone: +2349130824575

ABSTRACT

This paper investigates influences of temperature dependent viscosity, viscous dissipation and Joule heating on method MHD natural convection flow through a vertical porous channel. The equations representing the flow formation are of highest complexity as such their solutions are difficult to obtain through any analytical means. To achieve the solution, the use of Adomian decomposition of solution (ADM) is therefore deployed. The method of ADM is a semi-analytic method which is a powerful tool capable of decoupling the complexity into series form upon which a computer algebra package can be used for the final solution. This investigation may have application in the context of refining of crude oil as its components are separated under changing temperatures.

Keywords: Temperature-dependent viscosity, Viscous dissipation, Joule heating, MHD fluid, Adomian decomposition method

INTRODUCTION

Magneto-hydrodynamic (MHD) flow is the flow of electrically conducting fluids like plasma, liquid metals and electrolytes. This flow is built on the concept that when a magnetic field passes through a fluid it induces an electric current and this polarizes the fluid which in return changes the magnetic field through a force called Lorent force. The pioneering work on MHD flow was done by Alfven (1942) who described the class of MHD waves as Alfven waves. Sinha (2015) acknowledged that MHD has applications in the field of therapies where it is used in relieving pains during surgery, accelerating healing of fracture in bones, controlling blood flow during surgery and treatment of health disorder in human arthritis.

Joule heating effect on MHD flow has applications in technology as in electric heating devices such as electric heater, boiler, electric iron, bread toaster and other devices which use high temperature to operate. MHD flow over a stretching porous due to Joule heating and viscous dissipation on sheet subjected to the power law heat flux was investigated by Jaber (2016) where he concluded that increasing the magnetic parameter lead to the rise in temperature and the local Nusselt number and decreasing the velocity. The work of Mohammad and Nazma (2021) show that, the temperature within the boundary layer increases for increasing value of Joule heating index.

The ability of fluids to deform continuously under shear stress is termed viscosity. Temperature variation affects viscosity of fluids. Reynold (1883) was the first to state that the viscosity of liquid decreases exponentially with increase in temperature while that of gas increases exponentially with increase in temperature. Effects of variable viscosity, thermal conductivity and magnetic field on free convective flow through porous medium with constant suction and heat flux was studied by Borah and Hazaria (2013). Their finding is that the temperature of the fluid decreases with increase in both viscosity and thermal conductivity. Correlated studies such as that of Sigh and Shwefa (2013), Vajravelu *et al.* (2013) and Isaac and Anselm (2014) reflect that the fluid velocity decreases with increasing viscosity while the temperature increase with intensification in viscosity.

The present investigation is motivated by the study of Ajibade *et al.* (2021) in which the effects of Joule heating and MHD flow are coopted into their model while neglecting some parameters. The conquered equations modelling our problem are of high complexity which are difficult to solve using any analytical means and we therefore resort to the use of Adomian decomposition method of solution (ADM) to handle these equations.

Mathematical Formulation of the Problem

Fig. 1 below shows the flow configuration of the problem. It consists of a vertical channel formed by two infinite parallel vertical porous plate stationed h distance apart. It is assumed that a magnetic fluid of strength B_0 flow in the channel under the influence of gravity g. It is also assumed that all the properties of the fluid are constant except for the viscosity which is temperature dependent. The *x*-axis is taken along the channel in the vertically upward direction, which is the direction of the flow while the *y*-axis is taken normal to it. The temperature of the plate kept at $y = 0$ is raised to T_w and thereafter remained constant while the other plate at $y = h$ is fixed and maintained at temperature T_0.

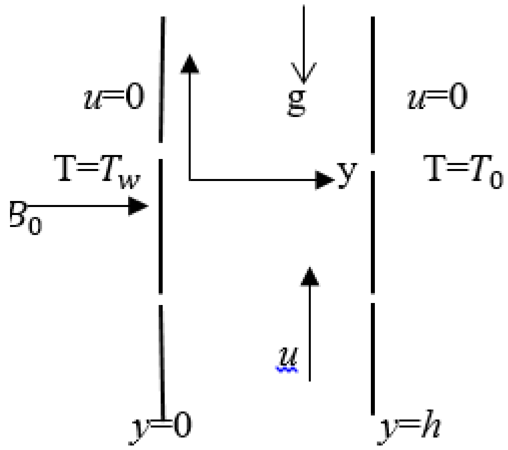

Figure 1: Flow diagram

Under these assumptions, the equations representing the flow are thus:

$$V_0\frac{du'}{dy'} - \frac{1}{\rho}\frac{d}{dy'}\left(\mu\frac{du'}{dy'}\right) - g\beta\,(T' - T_0) - \frac{\mu}{\rho}\,\sigma B_0^2 u' = 0 \quad (1)$$

$$V_0\frac{dT'}{dy'} - \frac{k}{\rho Cp}\frac{d^2T'}{dy'^2} - \frac{\mu}{\rho Cp}\left(\frac{du'}{dy'}\right)^2 - \frac{\delta B_0^2 u'^2}{\rho Cp} = 0 \quad (2)$$

with the boundary conditions:

$$u' = 0, T' = T_w \quad at\ \ y' = 0 \quad (3)$$

$$u' = 0, T' = T_0 \quad at\ \ y' = h \quad (4)$$

The varying viscosity (μ) is assumed to adopt the one given by Carey and Mollendorf (1978) which has the form:

$$\mu = \mu_0\left(1 - \lambda\left(\frac{T'-T_0}{T_w-T_0}\right)\right) \text{ for } \lambda \in (0,1) \quad (5)$$

The meaning of the physical quantities through equations (1-5) can be referred to the table of nomenclature. Equation (1) represents the momentum equation with the first, second, third and fourth terms representing the fluid velocity due to suction, variable viscosity, force of gravity and magnetic effect accordingly. Also Equation (2) represents energy equation having first, second, third and fourth terms donating heat due to fluid suction, thermal conduction, viscous dissipation and Joule heating effect respectively.

MATERIALS AND METHODS

Non-dimensionalization of the Problem

Equations (1 - 5) includes quantities in dimensional form and these equations are to be transformed into dimensionless. To realize this aim, we utilise the quantities adopted in Ajibade *et al.* (2021) which are:

$$u = \frac{u'}{g\beta(T'-T_0)h^2},\ y = \frac{y'}{h},\ \theta(y) = \frac{T'-T_0}{T_w-T_0} \quad (6)$$

using equations (3) and (6) into equations (1) and (2) we achieve the below equations:

$$u'' = c(1+\lambda\theta)u' + \lambda(1+\lambda)\theta' u' - \left(\theta(1+\lambda\theta)\right) - Mu \quad (7)$$

$$\theta'' = c\theta' - Bru'^2 - Ju^2 \quad (8)$$

Also using equation (6) in equations (4) and (5), the boundary conditions become:

$u(0) = 0$, $\theta(0) = 1$ at y = 0 (9)

$u(1) = 0$, $\theta(1) = 0$ at y = 1 (10)

Where single prime ($'$) denotes the first derivative with respect to y and double primes ($''$) indicate the second derivative with respect to y.

$$c = \frac{V_0\, h}{\nu},\ M = \sigma\beta_0^2 h^2,\ J = \frac{\sigma\beta_0^2\,\nu C_p}{k},\ Br = P_r E_c,$$

$$P_r = \frac{\nu}{\alpha}\ \ and\ E_c = \frac{h^4 g^2\beta^2\,(T_w - T_0)}{\nu C_p}$$

Description of Adomian Decomposition Method (ADM):

ADM is a semi-analytical method of solution which involves splitting a given differential equation into linear and nonlinear parts. The nonlinear is then decomposed into a recursive relation using Adomian polynomial. Assume the differential equation

$Fy = q$ (11)

Where F is a general nonlinear ordinary differential equation containing both linear and nonlinear terms with q as a given function. F*y* is decomposed into L*y* +R*y* where L is the highest order derivative which is also invertible and R is the remainder of the linear operator.

So, equation (11) can now be written in the form:

L*y* +R*y* +N*y* = r , where N*y* indicates the nonlinear terms in F*y* (12)

$\Rightarrow Ly = r - Ry - Ny$ (13)

Since L is invertible, we take $L^{-1}$to the both sides of equation (13) to have:

$L^{-1}Ly = L^{-1}r - L^{-1}Ry - L^{-1}Ny$ (14)

Which is equivalent to

$y = \varphi - L^{-1}r - L^{-1}Ry - L^{-1}Ny$ (15)

Where φ is a constant achieved from integration of r(x).

Next, due to ADM we assume,

$y = \sum_{n=0}^{\infty} y_n$, and $N_y = \sum_{n=0}^{\infty} A_n$ (16)

Using equations (16) into equation (15) we get:

$$\sum_{n=0}^{\infty} y_n = \varphi + L^{-1}r - L^{-1}R\sum_{n=0}^{\infty} y_n - L^{-1}N\sum_{n=0}^{\infty} A_n \quad (17)$$

The A_n are calculated via the Adomian polynomial

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n} f\left(\sum_{i=0}^{\infty}\lambda^i y_i\right)_{\lambda=0},\ n = 0, 1, 2\ldots \quad (18)$$

To get the solution, we set:

$y_0 = \varphi + L^{-1} r$, (19)

$y_{n+1} = -L^{-1}Ry_n - L^{-1}A_n, n \geq 0$ (20)

Since we have an infinite series and all the terms cannot be computed, we assume the final solution to be $y = \sum_{n=0}^{K} y_n$, where K is a truncation point for which the ADM solution converges.

ADM Solution of the Problem

Equation (7−8) are now solved using ADM as follow:

Let $L = \frac{d^2}{dy^2}$ and $L^{-1} = \iint_0^y (.)dydy$ so that equation (7) and equation (8) can be written as:

$Lu = c(1+\lambda\theta)u' + \lambda(1+\lambda\theta)\theta' u' - (\theta\,(1+\lambda\theta)) - Mu$ (21)

$L = c\theta' - Bru'^2 - Ju^2$ (22)

Taking $L^{-'}$ to both sides of the equations (21) and (22)

$L^{-\prime}\mathrm{L}u = \mathrm{c}L^{-\prime}((1+\lambda\theta))u' + \lambda L^{-\prime}((1+\lambda\theta)\theta' u') - L^{-\prime}(\theta(1+\lambda\theta)) - \mathrm{M}L^{-\prime}u$ (23)

$L^{-\prime}\mathrm{L}u = \mathrm{c}L^{-\prime}\theta' - BrL^{-'}(u'^2) - JL^{-'}(u^2)$ (24)

But $L^{-\prime}Lu = \iint_0^y u'' dydy = u(y) - yu'(0) - u(0)$ (25)

and $L^{-\prime}L\theta = \iint_0^y \theta'' dydy = \theta(y) - y\theta'(0) - \theta(0)$ (26)

Substituting equation (25) into equation (23) and rearranging:

$u(y) = u(0) + yu'(0) + \mathrm{c}\iint_0^y u'\, dydy + c\lambda \iint_0^y \theta u'\, dydy + \lambda \iint_0^y \theta' u' + \lambda^2 \iint_0^y \theta\theta' u'\, dydy - \lambda \iint_0^y \theta\, dydy - \mathrm{M}\iint_0^y u\, dydy - \iint_0^y \theta\, dydy$ - - (27)

But $u(0) = 0$, so that equation (27) is now:

$u(y) = \mathrm{A}y + \mathrm{c}\iint_0^y B_z\, dydy + c\lambda \iint_0^y C_z\, dydy + \lambda \iint_0^y D_z + \lambda^2 \iint_0^y E_z\, dydy - \lambda \iint_0^y F_z\, dydy - \mathrm{M}\iint_0^y G_z\, dydy - \iint_0^y K_z\, dydy$ (28)

Similarly, by substituting equation (26) into equation (24) and rearranging:

$\theta(y) = \theta(0) + y\theta'(0) + \mathrm{c}\iint_0^y \theta' dydy' - Br \iint_0^y u'^2\, dydy - J \iint_0^y u^2\, dydy$ (29)

But $\theta(0) = 1$, so equation (29) becomes:

$\theta(y = yH + 1) + c \iint_0^y L_z' dydy' - Br \iint_0^y P_z\, dydy - J \iint_0^y Q_z\, dydy$ (30)

Where $\mathrm{A} = u'(0)$, $H = \theta'(0)$ are assumed values to be obtained using equation (10).

Now assume $u = \sum_{n=0}^{\infty} u_n$, $\theta = \sum_{n=0}^{\infty} \theta_n$ and substitute into equations (29) and (30) to have the recurrence relation:

$u_0 = \mathrm{y}A$ (31)

$u_{z+1} = \mathrm{c}\iint_0^y B_z\, dydy + c\lambda \iint_0^y C_z\, dydy + \lambda \iint_0^y D_z + \lambda^2 \iint_0^y E_z\, dydy - \lambda \iint_0^y F_z\, dydy - \mathrm{M}\iint_0^y G_z\, dydy$, for $z \geq 0$ (32)

$\theta_0 = \mathrm{y}H + 1$ - (33)

$\theta_{z+1} = \mathrm{c}\iint_0^y L_z' dydy' - Br \iint_0^y P_z\, dydy - J \iint_0^y Q_z\, dydy$, for $z \geq 0$ - (34)

The final solution is:

$u(y) = u_0 + u_1 + u_2 + \ldots + u_s$ - (35)

$\theta(y) = \theta_0 + \theta_1 + \theta_2 + \ldots + \theta_s$ - (36)

Where S is a truncation point for which the ADM solution converges.

Convergence of the ADM Solution of the Problem

ADM solution has been assured to be rapidly convergent Adomian (1994) and Cherruault (1990).

RESULTS AND DISCUSSION

The paper investigates influence of temperature dependent viscosity, viscous dissipation and Joule heating on MHD flow in a vertical porous channel. The parameters of interest are; varying viscosity index (λ), suction index (c), Brinkman number (Br), Joule heating term (J) and magnetic parameter (M). Using a computer algebra package of Maple, a simulation is carried out when the values of λ, Br, J, M and c are chosen respectively as: (0.1, 0.3 , 0.6), (-1, 1 , 1.5), (1, 5 , 10), (0.1, 0.5 , 0.8) and (-1, 0.1 , 1) with results displayed on graphs and tables.

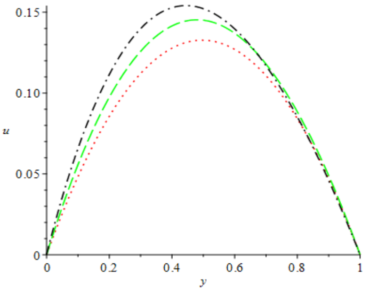

Figure 2: Velocity profiles for different λ
Br = M = c = 0.1 and J = 1
λ = 0.1, λ = 0.3 λ = 0.6

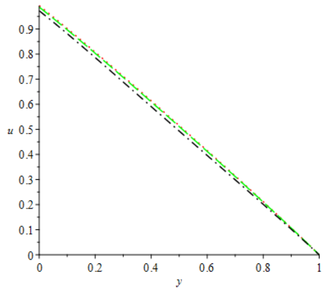

Figure 3: Temperature profiles for different λ
Br = M = c = 0.1 and J = 1
λ = 0.1, λ = 0.3 λ = 0.6

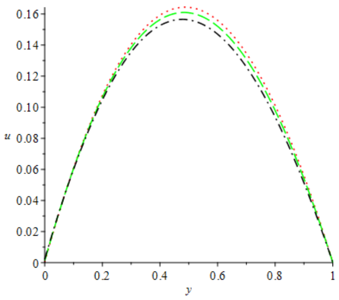

Figure 4: Velocity profiles for different J
λ = c = M = Br = 0.5
J = 1, J = 5J = 10

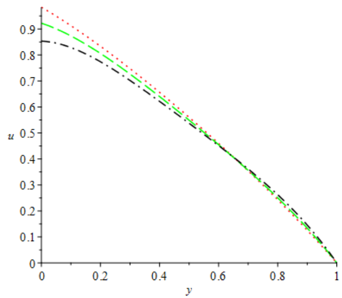

Figure 5: Temperature profiles for different J
λ = c = M = Br = 0.5
J = 1, J = 5J = 10

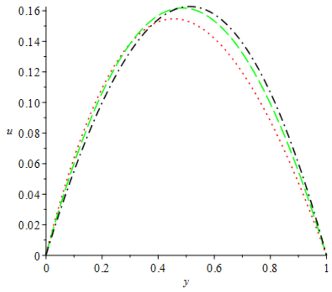

Figure 6: Velocity profiles for different c
λ = M = 0.5 = Br = 1.5 and J = 2
c = 0.1, c = 0.5 c = 0.9

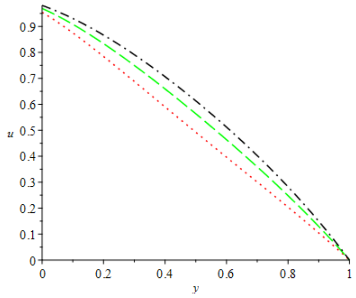

Figure 7: Temperature profiles for different c
λ = M = 0.5 = Br = 1.5 and J = 2
c = 0.1, c = 0.5 c = 0.9

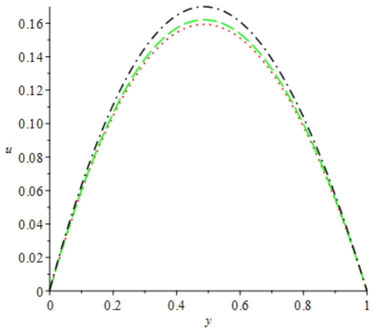

Figure 8: Velocity profiles for different M
λ = c = Br = 0.5 and J = 2
M = 0.2, M = 0.4 M = 0.9

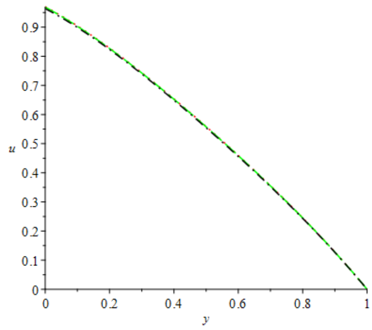

Figure 9: Temperature profiles for different M
λ = c = Br = 0.5 and J = 2
M = 0.2, M = 0.4 M = 0.9

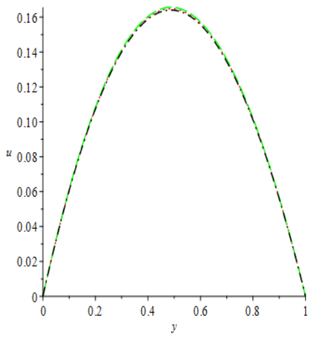

Figure 10: Velocity profiles for different Br
λ = c = M = J = 0.5
Br = 0.1, Br = 0.5 Br = 0.9

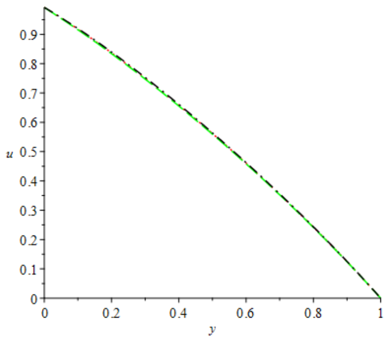

Figure 11: Temperature profiles for different Br
λ = c = M = J = 0.5
Br = 0.1, Br = 0.5 Br = 0.9

Figure 2 and 3 mirrored the response of escalating viscosity on the velocity and temperature within the channel. It is understood that a reduction in viscosity causes increase in the velocity and a decrease in the temperature of the fluid within the channel. The effects of varying Joule heating is depicted in figure 4 and 5 where it show that both the velocity and temperature of the fluid within the channel increase with increase in J. These fashions are as a result of the decrease of thermal conduction of the fluid which in return upsurge the kinematic viscosity of the fluid. It is observed form figure 6 and 7 that intensifying fluid suction (c) on both the velocity and temperature within the channel marks to the decreasing in both the temperature and velocity of the fluid in the channel. These tendencies are the momentous effect of the lessening in the kinematic viscosity of the fluid which attributed to the decrease of the force of attraction between the fluid molecules. Responses of magnetic effects (M) on the flow are displayed in figure 8 and 9. The figures show that with increase in magnetic parameter, the velocity and temperature of the fluid within the channel were found to decrease. The influence of growing Brinkman number (Br) is signified in figure 10 and 11 where it clearly indicate that both the velocity and temperature in the channel increase.

Validation of the results

Equation (7-8) are highly nonlinear in nature and are also coupled differential equations whose solution are difficult to obtain. In order to authenticate the validity of our results we therefore their complexity by neglecting the effects of variable viscosity (i.e.$\lambda = 0, Br = 0$). In the cause of this the emerging equations are thus:

$$u'' - cu' + Mu + \theta = 0 \quad (37)$$

$$\theta'' - c\theta' - Ju^2 = 0 \quad (38)$$

The reduced equations are then solved using Differential transformation method of solution (DTM) for the purpose of comparison and the results are displayed in the table below.

Table 1: omparison between ADM and DTM Solution

y	ADM	DTM	Error (ADM – DTM)
0.2	u = 0.00099134	u = 0.00099979	u = 0.00000845
	θ = 0.00098139	θ = 0.00099997	θ = 0.00003690
0.4	u = 0.00098222	u = 0.00099919	u = 0.00001697
	θ = 0.00096301	θ = 0.00099991	θ = 0.00003690
0.6	u = 0.00097265	u = 0.00099819	u = 0.00002554
	θ = 0.94459	θ = 0.00099981	θ = 0.00005522
0.8	u = 0.00096261	u = 0.00099679	u = 0.00003418
	θ = 0.00092612	θ = 0.00099967	θ = 0.00007355

Table 1 shows that the Adomian decomposition method (ADM) and differential transformation method (DTM) of solution are of good agreement.

Table 2: Nomenclature and Greek Symbols

Symbols	Interpretation	Unit
y'	Dimensional length	m
y	Dimensionless length	
g	Gravitational acceleration	ms^{-2}
k	Thermal conductivity	W/mK
T	Dimensional temperature	K
h	Dimensional channel width	m
T_w	Wall temperature	K
T_0	Ambient temperature	K
u'	Dimensional velocity	ms^{-1}
u	Dimensionless velocity	
ν	Kinematic viscosity of the fluid	m^2s^{-1}
α	Thermal diffusivity	m^2s^{-1}
β	Volumetric expansion coefficient	K^{-1}
μ_0	Variable fluid viscosity	kg$m^{-1}s^{-1}$
μ	Dynamic fluid viscosity	kg$m^{-1}s^{-1}$
φ,C_p	Temperature difference parameter	K
θ	Dimensionless temperature	
σ	Stefan-Boltzman constant	JK^{-1}
ε	Thermal conductivity variation parameter	
λ	Viscosity variation parameter	
V_0	Dimensional suction velocity	
M	Magnetic parameter	
Ec	Eckert number	
Pr	Prandtl number	
Br	Brinkman number	

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