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MONTE CARLO EVALUATION OF WHITE'S TEST FOR DETECTING HETEROSCEDASTICITY IN GENERALIZED LINEAR MODELS

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ABSTRACT

Heteroscedasticity in regression analysis occurs when the variance of the error term changes across different levels of the independent variable(s), leading to inefficient estimates and incorrect inference. In Generalized Linear Models (GLMs), heteroscedasticity significantly impacts prediction and inference accuracy. This study evaluates White's test for detecting heteroscedasticity in GLMs through Monte Carlo simulations. We investigate the test's power, Type II errors, and Type I errors at different sample sizes (100, 250, and 500). Our findings reveal that White test performs well in detecting strong heteroscedasticity, particularly for exponential heteroscedasticity structures (EHS), but poorly for weaker forms like linear heteroscedasticity structures (LHS) and square root heteroscedasticity structures (SQRTHS). While increased sample size enhances performance, the test remains susceptible to over-rejection of homoscedasticity. We recommend cautious use, especially with weaker heteroscedasticity or specific structures. For improved performance, use the test with moderate to high sample sizes (e.g., n = 500), particularly for EHS and quadratic heteroscedasticity structures (QHS). Alternative tests may be considered for researchers with limited sample sizes or dealing with LHS and SQRTHS. Finally, we emphasize the importance of assessing the underlying structure of heteroscedasticity in the dataset to choose the most suitable test and interpretation.

Keywords: White test, Heteroscedasticity, Monte Carlo simulations, Statistical inference, Econometrics, Regression analysis

INTRODUCTION

Heteroscedasticity, a common issue in regression analysis, occurs when the variance of the error term changes across different levels of the independent variable(s), leading to inefficient estimates and incorrect inference (Muhammad et al., 2023). In Generalized Linear Models (GLMs), heteroscedasticity can have a significant impact on the accuracy of predictions and inference. The White test is a widely used diagnostic tool for detecting heteroscedasticity in GLMs. Furthermore, the test's performance can vary significantly across different structures and variations (Akewugberu et al., 2024). However, its performance across different levels of heteroscedasticity and structures in GLMs has not been thoroughly investigated.

Previous studies have investigated the performance of various tests for heteroscedasticity in GLMs, including the White test (Onifade & Olanrewaju, 2020). Onifade and Olanrewaju (2020) conducted a Monte Carlo simulation study to investigate the performance of some statistical tests for heteroscedasticity assumption in GLMs, including the White test. However, there is still a need for a comprehensive evaluation of the White test's performance in detecting heteroscedasticity in GLMs across various structures and levels using other metrics.

Existing literature has identified various tests for heteroscedasticity in GLMs, including the Breusch-Pagan, Bartlett's, Goldfeld-Quandt, White, and Koenker-Bassett tests (Wiedermann et al., 2017). The Breusch-Pagan test has been recommended for its robustness and sensitivity in detecting heteroscedasticity in GLMs (Harvey, 1976; Zeileis, 2004; Hayes & Cai, 2007).

This study aims to fill the gap in the literature by investigating the White test's performance in detecting heteroscedasticity in GLMs across different levels of heteroscedasticity and structures. Specifically, this study evaluates the White test's power, frequency of Type II errors (when σ = 0), Type I errors (σ ≠ 0) and Power of the test in confirming homoscedasticity assumptions at different sample sizes.

MATERIALS AND METHODS

Forms of heteroscedasticity

This study examines four distinct heteroscedasticity structures, which are variations of additive and multiplicative models. For this research, we focus on a specific heteroscedastic structure where the error term's variance is directly proportional to the response variable's mean.

There are two primary forms of heteroscedasticity:

i. Exponential Form: Var(a,a) = $\sigma^2 e^{E(y_i)}$ Exponential form.
ii. Linear Form: Var(a,a) = $\sigma^2 E(y_i)^g$, where g > 0 Linear form.

We explore four heteroscedasticity structures, specifically exponential, linear, square-rooted (with g = 0.5), and quadratic (with g = 2) forms.

To confirm the homoscedasticity assumption, we use the White test. We inject different levels of heteroscedasticity into generalized linear models with varying sample sizes (100, 250, and 500) and standard deviations (σ = 0, 0.1, 0.3, 0.5, 0.7, and 0.9).

Procedure for the Monte Carlo Simulation Experiment

To investigate the finite sample properties of the test statistics of the presence of heteroscedasticity in any given dataset, Monte Carlo experiment is adopted. The simulation consisted of 1000 iterations, each with varying sample sizes of 100, 250 and 500 data points.

In each iteration, we generated two data sets, i.e Heteroscedastic data and Homoscedastic data. For the heteroscedastic data, We simulated x from a standard normal distribution x ~N(0,1) and y as a linear function of x with added noise, where the variance of the noise increased with x.

$$y = \beta_1 x + \varepsilon \quad (1)$$

However, for the homoscedastic data, We simulated x and y from same standard normal distribution, x,y ~ N(0,1), with no relationship between x and the variance of y.

The structure was then formulated thus:

Linear Form: The variance of the dependent variable increases linearly with the independent variable.

$$\varepsilon \sim N(0, \sigma^2(x)) = N(0, \beta_1 x) \quad (2)$$

Exponential Form: The variance of the dependent variable increases exponentially with the independent variable.

$$\varepsilon \sim N(0, \sigma^2(x)) = N(0, exp(\beta_1 x)) \quad (3)$$

Quadratic Form: Quadratic Form: The variance of the dependent variable changes quadratically with the independent variable.

$$\varepsilon \sim N(0, \sigma^2(x)) = N(0, \beta_1 x^2) \quad (4)$$

Square root Form: The variance of the dependent variable increases with the square root of the independent variable.

$$\varepsilon \sim N(0, \sigma^2(x)) = N(0, \beta_1 \sqrt{x}) \quad (5)$$

Which were then set to produce the three metrics used for the analysis i.e, power, Type I error and Type II error.

White Test

Another widely used test for homoscedasticity, which doesn't require knowledge of the heteroscedasticity form, was proposed by White (1980). This test compares the variance of OLS estimates under homoscedasticity and heteroscedasticity. A key advantage of this test is that it doesn't rely on the normality assumption, making it straightforward to implement.

Consider a simple three-variable regression model:

$$y_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon_i \quad (6)$$

The White test is a straightforward procedure to detect heteroscedasticity:

i. Calculate the residual $\varepsilon_i$from the original regression model (equation 6).
ii. Run an auxiliary regression using the squared residual as the dependent variable and the original regressors, their squares, and cross-products as independent variables

$$\varepsilon_i^2 = \alpha_0 + \alpha_1 y_i + \alpha_2 y_2 + \alpha_3 y_i^2 + \alpha_4 y_2^2 + \alpha_5 y_1 y_2 + v_i \quad (7)$$

iii. Obtain the R-squared (R^2) value from the auxiliary regression;
iv. Under the null hypothesis of homoscedasticity, the product of the sample size (n) and R^2 follows a chi-squared distribution with k degrees of freedom, where k is the number of regressors in the auxiliary regression:

$$nR^2 \sim {\chi^2}_{k:a} \quad (8)$$

v. Reject the null hypothesis if nR^2 exceeds the critical value from the chi-squared distribution at a given significance level.

A note of caution: when dealing with multiple regressors, introducing all possible terms and interactions can quickly deplete degrees of freedom. Therefore, it's essential to exercise caution when applying the White test.

RESULTS AND DISCUSSION

As stated earlier, White test for detecting heteroscedasticity is used in this study, with the use of three metrices, that is the number of time (frequency) each test commits type II error and type I error as the case may be and the power of the test.

The null hypothesis assumes that the data exhibits homoscedasticity, meaning that the variance of the error term is constant.

Table 1: Performance of White test when n=100 across different heteroscedasticity structures

		Variations					
		σ = 0	**σ = 0.1**	**σ = 0.3**	**σ = 0.5**	**σ = 0.7**	**σ = 0.9**
Performance of the test when error follows LHS at 5% level of significance							
N=100	Power	0.044	0.053	0.046	0.051	0.065	0.051
	Type I Error	0.074	0.048	0.050	0.051	0.050	0.044
	Type II Error	0.956	0.947	0.954	0.949	0.935	0.949
Performance of the test when error follows EHS at 5% level of significance							
N=100	Power	0.404	0.435	0.466	0.444	0.434	0.430
	Type I Error	0.919	0.914	0.915	0.880	0.793	0.768
	Type II Error	0.596	0.565	0.534	0.556	0.566	0.570
Performance of the test when error follows QHS at 5% level of significance							
N=100	Power	0.128	0.123	0.123	0.137	0.134	0.132
	Type I Error	0.201	0.179	0.179	0.190	0.189	0.183
	Type II Error	0.872	0.877	0.877	0.863	0.866	0.868
Performance of the test when error follows SQRTHS at 5% level of significance							
N=100	Power	0.042	0.042	0.134	0.041	0.044	0.051
	Type I Error	0.054	0.054	0.181	0.044	0.039	0.046
	Type II Error	0.958	0.958	0.866	0.959	0.956	0.949

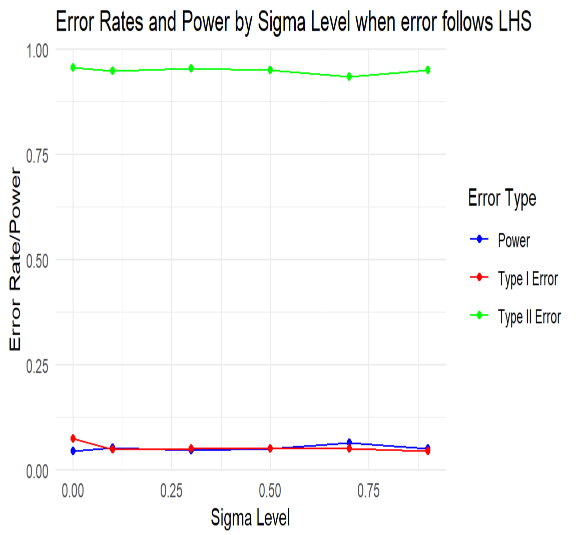

Figure 1: Error Rates and Power by Sigma Level when error follows LHS

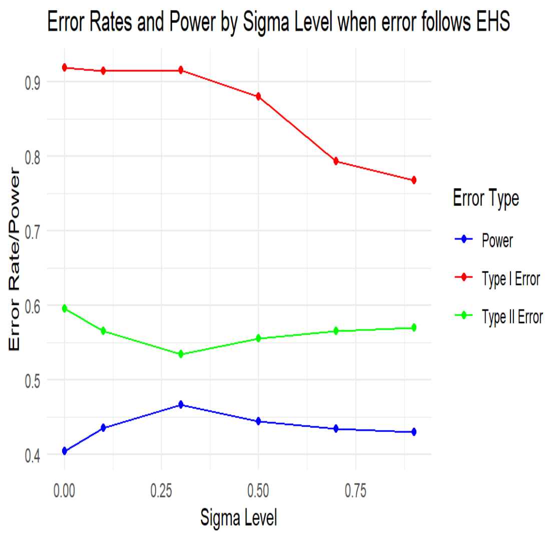

Figure 2: Error Rates and Power by Sigma Level when error follows EHS

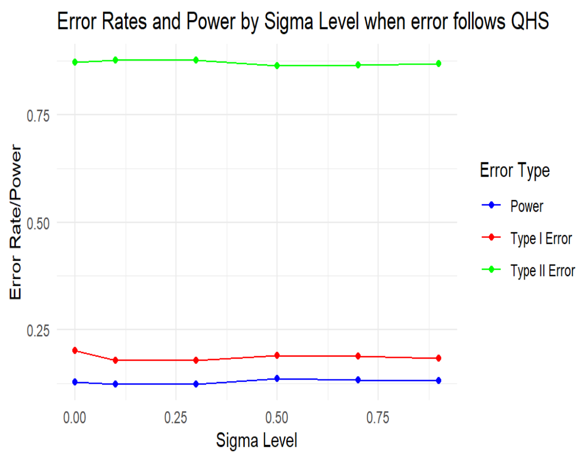

Figure 3: Error Rates and Power by Sigma Level when error follows QHS

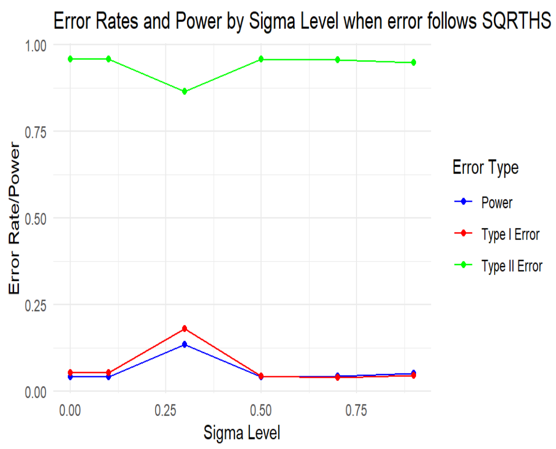

Figure 4: Error Rates and Power by Sigma Level when error follows SQRTHS

The above Figures shows the performance of White test when n=100 across different heteroscedasticity structures.

Table 2: Performance of White test when n=250 across different heteroscedasticity structures

		Variations					
		σ = 0	**σ = 0.1**	**σ = 0.3**	**σ = 0.5**	**σ = 0.7**	**σ = 0.9**
Performance of the test when error follows LHS at 5% level of significance							
N=250	Power	0.043	0.046	0.052	0.044	0.061	0.061
	Type I Error	0.069	0.057	0.052	0.046	0.040	0.040
	Type II Error	0.957	0.954	0.948	0.956	0.939	0.939
Performance of the test when error follows EHS at 5% level of significance							
N=250	Power	0.755	0.753	0.781	0.777	0.762	0.773
	Type I Error	0.999	0.999	1.000	0.997	0.990	0.970
	Type II Error	0.245	0.247	0.219	0.223	0.238	0.227
Performance of the test when error follows QHS at 5% level of significance							
N=250	Power	0.153	0.191	0.180	0.187	0.186	0.177
	Type I Error	0.211	0.246	0.238	0.264	0.264	0.255
	Type II Error	0.847	0.809	0.820	0.813	0.814	0.823
Performance of the test when error follows SQRTHS at 5% level of significance							
N=250	Power	0.037	0.041	0.041	0.051	0.054	0.043
	Type I Error	0.118	0.098	0.098	0.040	0.051	0.039
	Type II Error	0.963	0.959	0.959	0.949	0.946	0.957

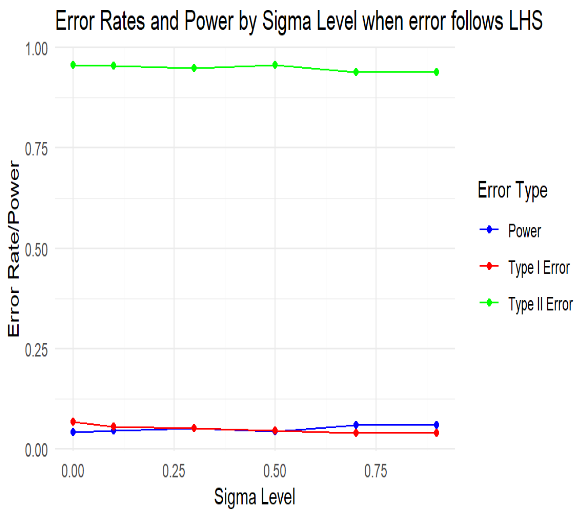

Figure 5: Error Rates and Power by Sigma Level when error follows LHS

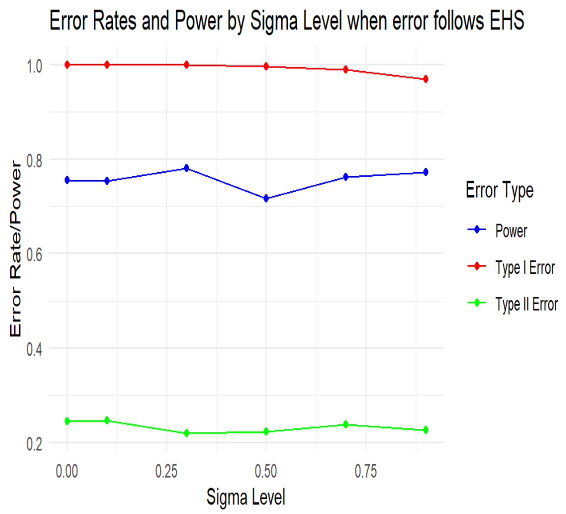

Figure 6: Error Rates and Power by Sigma Level when error follows EHS

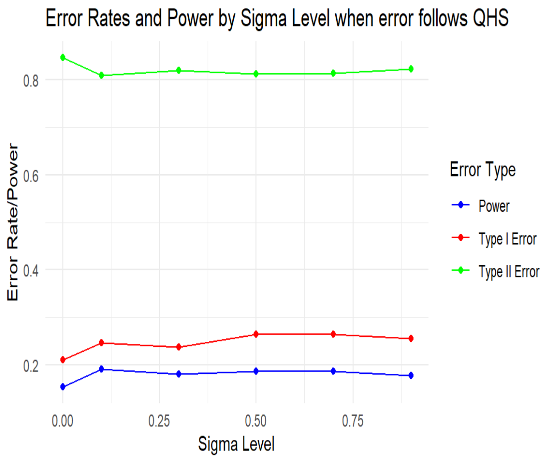

Figure 7: Error Rates and Power by Sigma Level when error follows QHS

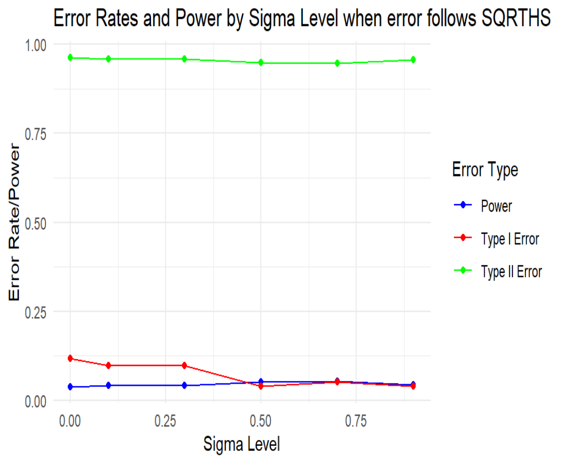

Figure 8: Error Rates and Power by Sigma Level when error follows SQRTHS

Figures 5,6,7 and 8 above shows the performance of White test when n=250 across different heteroscedasticity structures.

Table 3: Performance of White test when n=500 across different heteroscedasticity structures

		Variations					
		σ = 0	**σ = 0.1**	**σ = 0.3**	**σ = 0.5**	**σ = 0.7**	**σ = 0.9**
Performance of the test when error follows LHS at 5% level of significance							
N=500	Power	0.048	0.047	0.045	0.048	0.050	0.047
	Type I Error	0.054	0.058	0.048	0.061	0.052	0.051
	Type II Error	0.952	0.953	0.955	0.952	0.950	0.953
Performance of the test when error follows EHS at 5% level of significance							
N=500	Power	0.924	0.958	0.952	0.959	0.945	0.945
	Type I Error	1.000	1.000	1.000	1.000	0.999	1.000
	Type II Error	0.076	0.042	0.048	0.041	0.055	0.055
Performance of the test when error follows QHS at 5% level of significance							
N=500	Power	0.205	0.188	0.192	0.213	0.205	0.201
	Type I Error	0.264	0.253	0.245	0.247	0.243	0.282
	Type II Error	0.795	0.812	0.808	0.787	0.795	0.799
Performance of the test when error follows SQRTHS at 5% level of significance							
N=500	Power	0.047	0.041	0.044	0.051	0.051	0.043
	Type I Error	0.146	0.129	0.060	0.043	0.061	0.050
	Type II Error	0.953	0.959	0.956	0.949	0.949	0.957

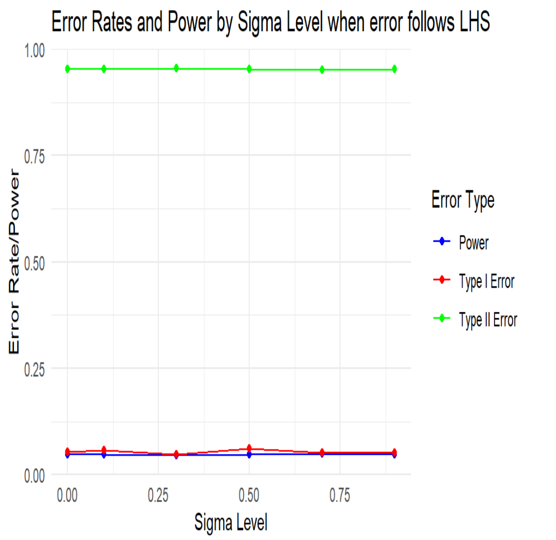

Figure 9: Error Rates and Power by Sigma Level when error follows LHS

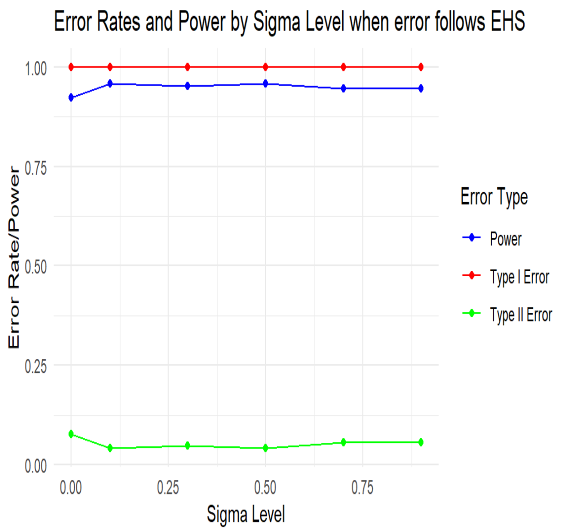

Figure 10: Error Rates and Power by Sigma Level when error follows EHS

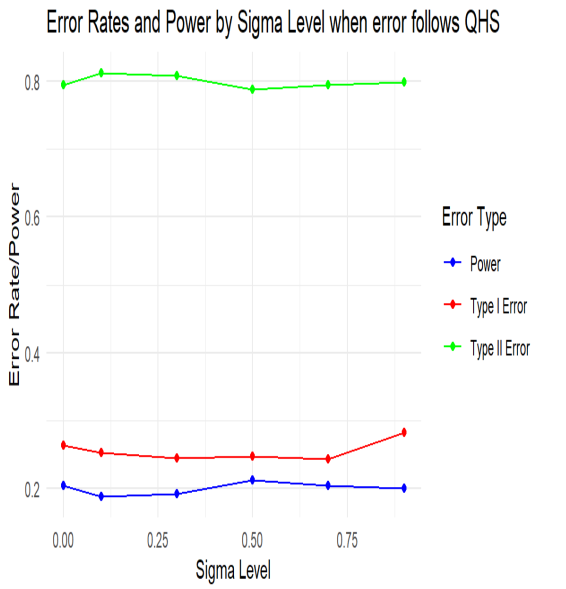

Figure 11: Error Rates and Power by Sigma Level when error follows QHS

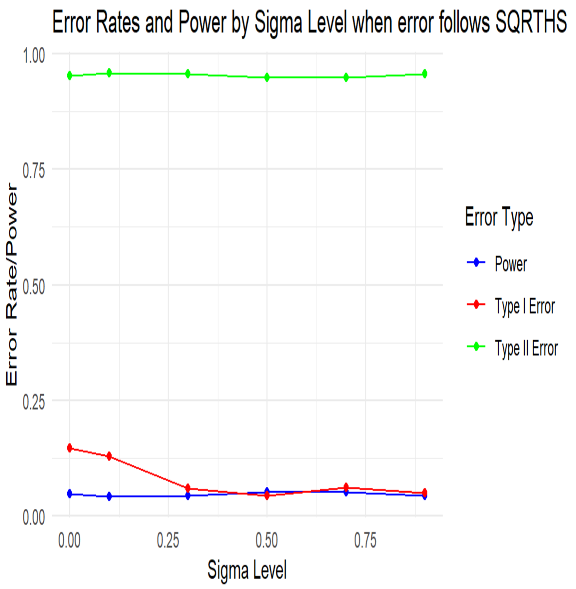

Figure 12: Error Rates and Power by Sigma Level when error follows SQRTHS

Figures 9,10,11 and 12 shows the performance of White test when n=500 across different heteroscedasticity structures.

Table 1 above shows the performance of White test when evaluated across different heteroscedasticity structures (LHS, EHS, QHS, SQRTHS) at 5% significance with varying error standard deviation (σ) levels, using a sample size of 100. When σ=0 (no heteroscedasticity), the test exhibits low power (<0.05) and high Type II error (>0.94) across all structures. As σ increases (introducing heteroscedasticity), power improves substantially for EHS (up to 46.6%), moderately for QHS (up to 13.7%), but remains low for LHS and SQRTHS (<6.5%). Notably, EHS shows high Type I error rates (up to 91.9%), indicating over-rejection of homoscedasticity. As heteroscedasticity increases (σ=0.3 to σ=0.9), the test's power generally plateaus or decreases for LHS, QHS, and SQRTHS, while EHS's power remains relatively stable.

Table 2 shows the performance of White test when evaluated across different heteroscedasticity structures (LHS, EHS, QHS, SQRTHS) at 5% significance with varying error standard deviation (σ) levels, using a sample size of 250. When σ=0 (no heteroscedasticity), the test exhibits low power (<0.06) and high Type II error (>0.94) across all structures. As σ increases (introducing heteroscedasticity), power improves substantially for EHS (up to 78.1%), moderately for QHS (up to 19.1%), but remains low for LHS and SQRTHS (<6.1%). Notably, EHS shows high Type I error rates (up to 1.000), indicating over-rejection of homoscedasticity. As heteroscedasticity increases (σ=0.3 to σ=0.9), the test's power generally stabilizes for EHS and QHS, while LHS and SQRTHS remain challenging to detect.

Table 3 shows the performance of white test when evaluated across different heteroscedasticity structures (LHS, EHS, QHS, SQRTHS) at 5% significance with varying error standard deviation (σ) levels, using a sample size of 500. When σ=0 (no heteroscedasticity), the test exhibits low power (<0.05) and high Type II error (>0.95) across all structures. As σ increases (introducing heteroscedasticity), power improves substantially for EHS (up to 95.9%), moderately for

QHS (up to 21.3%), but remains low for LHS and SQRTHS (<0.05). Notably, EHS shows high Type I error rates (up to 1.000), indicating over-rejection of homoscedasticity. As heteroscedasticity increases (σ=0.1 to σ=0.9), EHS power stabilizes, QHS power fluctuates slightly, and LHS and SQRTHS remain challenging to detect.

CONCLUSION

The Monte Carlo simulation results demonstrate that the White test's performance varies significantly across different heteroscedasticity structures and sample sizes. While it excels at detecting strong heteroscedasticity (EHS), its power and Type I error rates are less desirable for weaker forms (LHS, QHS, SQRTHS). Increasing sample size improves detection ability, especially for EHS. However, the test's tendency to over-reject homoscedasticity and struggle with weaker heteroscedasticity structures warrants caution. These findings have important implications for researchers using generalized linear models, highlighting the need for careful consideration of heteroscedasticity structure and sample size when employing the White test.

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