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DEVELOPMENT OF MATHEMATICAL MODEL FOR OPTIMAL RICE PRODUCTION IN NIGER STATE, NIGERIA

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ABSTRACT

Rice is a staple food and a critical crop for food security and economic stability in Niger State, Nigeria. However, achieving optimal production levels is challenged by various factors, including environmental variability, land use inefficiency, and rising production costs. Mathematical modeling offers a systematic approach to understanding and optimizing these factors to enhance yields and promote sustainable agricultural practices. A mathematical model to optimize rice production by integrating key agronomic, environmental, and economic factors were formulated. This research paper aims to predict optimal rice yields based on input variables such as rainfall, temperature, humidity, land area use and production cost using a multivariate linear regression (MLR) method. The developed model is validated with real-world data from agricultural research stations. It was observed from the analysis that predicted values were not significantly different from the observed values. The results show that R-square, Mean Square Error (MSE) and Root Mean Square Errors (RMSE) values were 0.96345, 0.0249 and 0.1578 respectively; indicating that approximately 96.35% of the variance in rice production can be explained by the independent variables. Due to its high level of accuracy in predicting rice yield; it can be concluded that the model can be used to determine optimum rice production in Niger state, Nigeria and provide a decision-support tool for farmers and policymakers.

Keywords: Mathematical model, Rice yield, Food security, Optimization, Niger State

INTRODUCTION

Rice is a staple food for billions of people worldwide, making its efficient production a critical agricultural goal (Bin Rahman & Zhang, 2023). The increase in population worldwide requires a corresponding increase in the production of food to support the growing population (Peter, 2021). With this growing global demand and limited arable land, optimizing rice yield through scientific methods has become a necessity (Hussain *et al.*, 2020).

Traditional farming practices often rely on experience and intuition, which may not always yield the best results (Saiz-Rubio & Rovira-Más, 2020). In Nigeria, rice consumption far exceeds production with a yearly average production deficit of about 2.4 million tonnes recorded between 2007 and 2018 (Dangora *et al*., 2023).

A systematic approach using mathematical models will offer insights to optimize rice production strategies and promote sustainable and efficient agricultural practice.

The production of rice is influenced by a complex interplay of factors, including climatic conditions, soil properties, water management, and agronomic practices (Islam *et al.*, 2020). Understanding these relationships through a mathematical framework allows for more accurate yield predictions and informed decision-making. Moreover, such models can help policymakers formulate strategies to ensure food security while minimizing environmental impacts (Wang *et al.*, 2021). The integration of technology into agriculture not only enhances productivity but also promotes sustainability, reducing waste and conserving essential resources like water and fertilizers (Saikanth *et al.*, 2023).

The aim of this research paper is to develop a mathematical model that predict optimal rice yield base on various input variables.

MATERIALS AND METHODS

Data collection

This study mainly use secondary source to obtained data of rice production. The data set is a statistical records comprising of dependent variable (rice yield) and independent variables (rainfall, temperature, humidity, land size and production cost).

The Data were collected from Niger State Ministry of Agriculture, Federal Ministry of Agriculture and Rural Development and Nigerian Meteorological Agency (NIMET) bulletins. The data collected is for a period of 21 years.

Development of Mathematical Model

In this research paper, the development of mathematical model will follow and extend the existing work of Hakimi *et al.,* (2017) and Rania, (2020). Meanwhile, the developed Mathematical Model contains five (5) variables and will represent the relationship between Crop production and the variables (rainfall, temperature, humidity, production cost, and land size and incorporates the selected variables into the model. However, in other to produce reliable and accurate results, the following assumptions are made in the development of the model;

Crop production depends on the combined effects of rainfall, temperature, relative humidity and land size

The relationship between these variables and rice yield is assumed to be linear and non-linear interaction.

Rice yield is affected directly by the variables.

Model Formulation

A multivariate linear regression (MLR) with first and second order terms of independent variable for rice yield prediction is used to formulate the mathematical model for rice yield prediction consequently, to formulate the problem mathematically; the following variables and parameters are denoted as follows:

Variables
$Y: Riceyield(dependent\text{variable})$
$X_1: \rightleftarrows\ \ Rain\ fall\ (mm)$
$X_2:\ Temprature\ (\ ^0C)$
$X_3:\ Re\ l\ ativeHumidity(\%)$
$X_4:\ Landsize(Ha)$
$X_5: Pr\ o\ ductionCost(Naira/Ha)$
$\varepsilon = Error\ term$
Let the rice yields (output) be represented as follows
$Y_{rice}:\ Rice\ yield\ (in\ tons\ per\ hectare)$
Then by assuming both linear and non-liner relationship between the variables and rice yields; produce the model:

$$Y_{rice} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + f(X_1, X_2, X_3, X_4, X_5) + \varepsilon \quad (1)$$

Where:

$$f(X_1, X_2, X_3, X_4, X_5) = \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5 + \beta_{34}X_3X_4 + \beta_{35}X_3X_5 + \beta_{45}X_4X_5 \quad (2)$$

Equation (3.2) represents the non linear function that model the complex relationship between the variables. By adding equations (3.1) and (3.2), we have a hybrid model equation affected by both linear and non-linear terms as follows:

$$Y_{rice} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5 + \beta_{34}X_3X_4 + \beta_{35}X_3X_5 + \beta_{45}X_4X_5 + \varepsilon \quad (3)$$

Where:
$Y_{rice}\ is\ the\ yield\ of\ rice$
β_0 Represents the intercept, the expected yield when all independent variables are zero, which is the base yield without any inputs.
$\beta_1,\ \beta_2,\ \beta_3,\ \beta_4, \beta_5\ are\ coefficients\ for\ the\ lineareffects\ of\ X_1, X_2, X_3, X_4, X_5.$
$\beta_{12},\ \beta_{13},\ \beta_{14},\ \beta_{15}, \beta_{23}, \beta_{24}, \beta_{25}, \beta_{34}, \beta_{35}, \beta_{45} are\ coefficients\ for\ the$ interaction $effects\ between X_1, X_2, X_3, X_4, X_5.$
$\varepsilon\ is\ the\ errorr\ term\ accounting\ for\ thefactors\ not\ included\ in\ the$ model

Development of Rice Fitting Model

The values of the model coefficients can be obtained by using the normal equation that provides the least squares estimate of the model coefficients.

$$\beta = (X^T X)^{-1} X^T y \quad (4)$$

Where:
$X$is the design matrix (input variables)
$X^T$is the transpose of X
$(X^T X)^{-1}$is the inverse of $X^T X$
$y$is the vector of observed crop yield
$\beta$is the vector coefficient to be determined.

$$x = \left[\begin{array}{cccccccccccccccc}
1 & 1249.5 & 33.6 & 49 & 205.42 & 23225 & 41983.2 & 61225.5 & 256672.3 & 29019637.5 & 1646.4 & 6902.112 & 780360 & 10065.58 & 1138025 & 4770880 \\
1 & 1274.5 & 32.7 & 46 & 184.92 & 33500 & 41676.15 & 58627 & 235680.5 & 42695750 & 1504.2 & 6046.884 & 1095450 & 8506.32 & 1541000 & 6194820 \\
1 & 1233 & 33.8 & 48 & 200.76 & 43770 & 41675.4 & 59184 & 247537.1 & 53968410 & 1622.4 & 6785.688 & 1479426 & 9636.48 & 2100960 & 8787265 \\
1 & 1294 & 32.6 & 48.8 & 152.74 & 54045 & 42184.4 & 63147.2 & 197645.6 & 69934230 & 1590.88 & 4979.324 & 1761867 & 7453.712 & 2637396 & 8254833 \\
1 & 1108.7 & 34 & 49.6 & 160.28 & 64320 & 37695.8 & 54991.52 & 177702.4 & 71311584 & 1686.4 & 5449.52 & 2186880 & 7949.888 & 3190272 & 10309210 \\
1 & 1423.2 & 34.2 & 46.8 & 278 & 74590 & 48673.44 & 66605.76 & 395649.6 & 106156488 & 1600.56 & 9507.6 & 2550978 & 13010.4 & 3490812 & 20736020 \\
1 & 1423.3 & 38.3 & 48.2 & 150.8 & 84865 & 54512.39 & 68603.06 & 214633.6 & 120788355 & 1846.06 & 5775.64 & 3250330 & 7268.56 & 4090493 & 12797642 \\
1 & 1269.2 & 28.7 & 49.7 & 164.07 & 95135 & 36426.04 & 63079.24 & 208237.6 & 120745342 & 1426.39 & 4708.809 & 2730375 & 8154.279 & 4728210 & 15608799 \\
1 & 1421.6 & 34.3 & 46.8 & 168.12 & 105410 & 48760.88 & 66530.88 & 238999.4 & 149850856 & 1605.24 & 5766.516 & 3615563 & 7868.016 & 4933188 & 17721529 \\
1 & 1221.6 & 33.8 & 50.1 & 178.82 & 94180 & 41290.08 & 61202.16 & 218446.5 & 115050288 & 1693.38 & 6044.116 & 3183284 & 8958.882 & 4718418 & 16841268 \\
1 & 942.8 & 33 & 46.8 & 371 & 108000 & 31112.4 & 44123.04 & 349778.8 & 101822400 & 1544.4 & 12243 & 3564000 & 17362.8 & 5054400 & 40068000 \\
1 & 1423 & 34.2 & 46.8 & 945 & 150000 & 48666.6 & 66596.4 & 1344735 & 213450000 & 1600.56 & 32319 & 5130000 & 44226 & 7020000 & 1.42E+08 \\
1 & 1269 & 38.3 & 48.2 & 264 & 155000 & 48602.7 & 61165.8 & 335016 & 196695000 & 1846.06 & 10111.2 & 5936500 & 12724.8 & 7471000 & 40920000 \\
1 & 1185 & 28.7 & 49.7 & 447 & 172000 & 34009.5 & 58894.5 & 529695 & 203820000 & 1426.39 & 12828.9 & 4936400 & 22215.9 & 8548400 & 76884000 \\
1 & 1157 & 34.3 & 46.8 & 447 & 180000 & 39685.1 & 54147.6 & 517179 & 208260000 & 1605.24 & 15332.1 & 6174000 & 20919.6 & 8424000 & 80460000 \\
1 & 1422 & 36.1 & 50.1 & 514 & 197000 & 51334.2 & 71242.2 & 730908 & 280134000 & 1808.61 & 18555.4 & 7111700 & 25751.4 & 9869700 & 1.01E+08 \\
1 & 1179 & 34.2 & 46.8 & 224.41 & 180000 & 40321.8 & 55177.2 & 264579.4 & 212220000 & 1600.56 & 7674.822 & 6156000 & 10502.39 & 8424000 & 40393800 \\
1 & 1196 & 38.3 & 48.2 & 227.14 & 200000 & 45806.8 & 57647.2 & 271659.4 & 239200000 & 1846.06 & 8699.462 & 7660000 & 10948.15 & 9640000 & 45428000 \\
1 & 1247 & 38.3 & 48.2 & 260.19 & 200000 & 47760.1 & 60105.4 & 324456.9 & 249400000 & 1846.06 & 9965.277 & 7660000 & 12541.16 & 9640000 & 52038000 \\
1 & 1157 & 36.7 & 56.7 & 258.06 & 200000 & 42461.9 & 65601.9 & 298575.4 & 231400000 & 2080.89 & 9470.802 & 7340000 & 14632 & 11340000 & 51612000 \\
1 & 872.1 & 34.2 & 46.8 & 255.91 & 230000 & 29825.82 & 40814.28 & 223179.1 & 200583000 & 1600.56 & 8752.122 & 7866000 & 11976.59 & 10764000 & 58859300
\end{array}\right] \quad (5)$$

$$Y = \begin{bmatrix} 1.98 \\ 2.2 \\ 2.86 \\ 2.78 \\ 2.11 \\ 2.71 \\ 3 \\ 3.4 \\ 3.22 \\ 3.24 \\ 3 \\ 2 \\ 3 \\ 4 \\ 5 \\ 5 \\ 2.41 \\ 2.59 \\ 2.41 \\ 2.41 \\ 2.46 \end{bmatrix} \qquad (6)$$

Solving the normal equation above; we obtained equation the coefficients of the model as:

$$\beta = \begin{bmatrix} -299.8009872 \\ 0.000873741 \\ 13.86879939 \\ 2.545350826 \\ 0.002891391 \\ -8.19691E-05 \\ -0.003458026 \\ 0.002563288 \\ -3.23639E-05 \\ 2.88845E-08 \\ -0.181938799 \\ 0.001777448 \\ -7.85361E-06 \\ -0.000675343 \\ 6.18228E-06 \\ 8.58944E-08 \end{bmatrix} \qquad (7)$$

By substituting the values of the coefficients obtained in (7) into the rice model equation (3) gives the required rice model as follow:

$$Y_{rice} = -299.8009872 + 0.000873741X_1 + 13.86879939X_2 + 2.545350826X_3 + 0.002891391X_4 - 0.00000819691X_5 - 0.003458026X_1X_2 + 0.002563288X_1X_3 - 0.0000323639X_1X_4 - 0.0000000288845X_1X_5 - 0.181938799X_2X_3 + 0.001777448X_2X_4 - 0.00000785361X_2X_5 - 0.000675343X_3X_4 - 0.00000618228X_3X_5 - 0.0000000858944X_4X_5 + \varepsilon \qquad (8)$$

RESULTS AND DISCUSSION

The accuracy of this prediction model is measured using three (3) performance metrics to test and assess the validity and reliability of the developed rice model with empirical data.

Coefficient of Determination (R-squared)

Mean Square Error (MSE)

Root Mean Square Errors(RMSE)

The R^2 is a statistical measure of how close the data are to the fitted regression line. It is given by the following equation

$$R^2 = 1 - \left(\frac{SSE}{SST}\right)$$

$$R^2 = 1 - \left(\frac{\sum(Y - \hat{Y})^2}{\sum(Y - \bar{Y})^2}\right)$$

$$R^2 = 1 - \left(\frac{0.52285}{14.32372}\right) \qquad (9)$$

$$R^2 = 1 - 0.03650$$

$$R^2 = 0.9635$$

If the value of R is closer to 1, it indicates that large proportion of variance is explained by the model and the better the model, while a value of R close to 0 indicates that the model explains very little of the variance.

Similarly, Mean Square Error (MSE) measures the average squared difference between the predicted and the actual values. MSE value closer to 0 indicates better model performance. It is computed as follows:

$$MSE = \left(\frac{1}{n}\right)\sum\left(Y - \hat{Y}\right)^2$$

$$MSE = \left(\frac{1}{21}\right)(0.52285)$$

$$MSE = 0.024 \qquad (10)$$

Lastly, Root Mean Square Error (RMSE) is the square root of Mean Square Error calculated by the equation $RMSE = \sqrt{MSE}$

$$RMSE = \sqrt{MSE}$$

$$RMSE = \sqrt{0.0249}$$

$$RMSE = 0.1578 \qquad (11)$$

The above performance metrics are computed using the residual analysis and prediction of rice yield table shown below:

Table 1: Residual analysis and rice yield prediction using the developed model

Y Actual Yield	$\bar{Y}$ Y Mean	$\hat{Y}$ Predicted	$(Y-\bar{Y})$	$(Y-\bar{Y})^2$	$(Y-\hat{Y})$	$(Y-\hat{Y})^2$	$\varepsilon = y-\hat{y}$ Error
1.98	2.941905	2.228808	-0.961905	0.925261	-0.2488082	0.06191	-0.24881
2.2	2.941905	2.250286	-0.741905	0.550423	-0.0502864	0.00253	-0.05029
2.86	2.941905	2.850292	-0.081905	0.006708	0.0097085	9.4E-05	0.009708
2.78	2.941905	2.841361	-0.161905	0.026213	-0.0613609	0.00377	-0.06136
2.11	2.941905	1.991661	-0.831905	0.692066	0.1183386	0.014	0.118339
2.71	2.941905	2.481523	-0.231905	0.05378	0.2284767	0.0522	0.228477
3	2.941905	3.018059	0.058095	0.003375	-0.0180594	0.00033	-0.01806
3.4	2.941905	3.474589	0.458095	0.209851	-0.0745889	0.00556	-0.07459
3.22	2.941905	3.129826	0.278095	0.077337	0.0901737	0.00813	0.090174
3.24	2.941905	2.919142	0.298095	0.088861	0.320858	0.10295	0.320858
3	2.941905	2.917278	0.058095	0.003375	0.0827219	0.00684	0.082722
2	2.941905	2.034343	-0.941905	0.887185	-0.0343434	0.00118	-0.03434
3	2.941905	3.370202	0.058095	0.003375	-0.3702023	0.13705	-0.3702
4	2.941905	4.036284	1.058095	1.119566	-0.0362842	0.00132	-0.03628
5	2.941905	4.982549	2.058095	4.235756	0.0174506	0.0003	0.017451
5	2.941905	4.897312	2.058095	4.235756	0.1026881	0.01054	0.102688
2.41	2.941905	2.674376	-0.531905	0.282923	-0.2643757	0.06989	-0.26438
2.59	2.941905	2.389979	-0.351905	0.123837	0.2000212	0.04001	0.200021
2.41	2.941905	2.404355	-0.531905	0.282923	0.0056454	3.2E-05	0.005645
2.41	2.941905	2.463906	-0.531905	0.282923	-0.0539055	0.00291	-0.05391
2.46	2.941905	2.423868	-0.481905	0.232232	0.0361321	0.00131	0.036132
61.78				**14.32372**		**0.52285**	

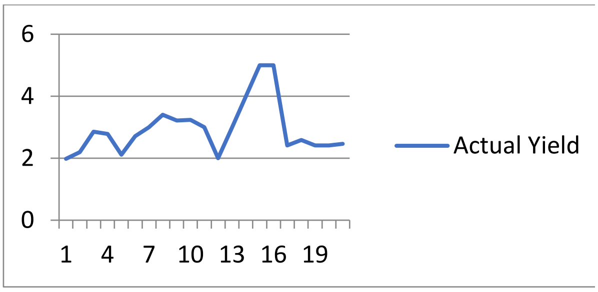

Figure 1: Accuracy graph for actual rice yield

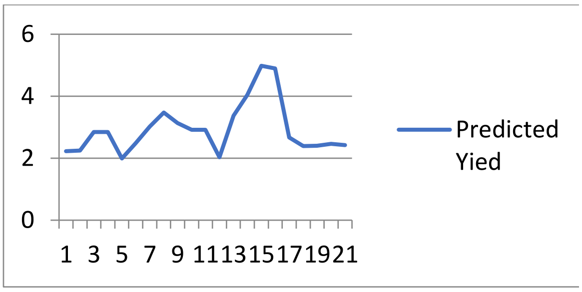

Figure 2: Accuracy graph for predicted rice yield

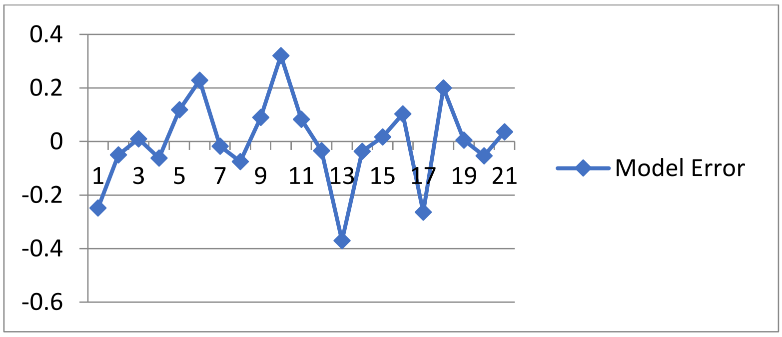

Figure 3: Differences between the actual and predicted rice yield

The results presented in Table 1.0 emphasize the high predictive accuracy of the developed mathematical model for optimizing rice production. The close alignment between the predicted rice yield values and the observed data, as reflected by the lower root mean square error (RMSE) value of 0.1578, indicates the model's reliability in estimating yields. The residual errors, which quantify the differences between observed and predicted yields, are minimal across most data points, which further confirmed the accuracy of the developed model.

The R-squared value of 0.9635 suggests that the model accounts for approximately 96.35% of the variability in rice yield data, demonstrating a strong correlation between input variables (rainfall, temperature, humidity, land area, and production costs) and rice yield outcomes. This high explanatory power underscores the model's ability to effectively capture the complex relationships influencing rice production.

Figures 1 and 2 validate these observations by showing no significant differences between observed and predicted values. Figure 3 illustrates the distribution of residuals, reinforcing the model's accuracy through minimal and unbiased prediction errors. Such consistency establishes the model as a valuable decision-support tool that can guide farmers and policymakers in optimizing resource allocation and improving overall rice productivity.

CONCLUSION

The developed model is suitable for forecasting rice yield production since it has a lower root mean square (RMS) and prediction error. The model offers valuable insights for farmers and policymakers, enabling informed decision-making. Future research should explore incorporating advanced technologies like remote sensing and precision agriculture to further refine the model's accuracy and practical applications.

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