FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645 - 2944
Vol. 8 No. 6, December, 2024, pp 299 - 305
DOI: https://doi.org/10.33003/fjs-2024-0806-2986

PERFORMANCE EVALUATION OF IMPUTATION-BASED ESTIMATORS FOR NON-RESPONSE AND MEASUREMENT ERROR CHALLENGES

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ABSTRACT

This study aims to develop a robust class of estimators designed to address non-response and measurement errors, which frequently complicate data collection in medical and social science research. By employing call-back and imputation schemes, the proposed estimators enhance efficiency and accuracy. We derived properties such as bias and mean squared error using Taylor's series expansion and tested their consistency. An empirical study with simulated data from various distributions revealed that the proposed estimators outperform existing ones. Thus, these modified classes are recommended for practical application in data analysis, especially in the presence of non-response and measurement errors.

Keywords: Efficiency, Non-response, Simulation, Estimation, Imputation

INTRODUCTION

Non-response is a significant challenge faced by statisticians and researchers in medical and social sciences, impacting data compilation, computation, and estimation. It occurs when respondents are absent, refuse to participate, or are inaccessible. Hansen and Hurwitz (1946) were the pioneers in addressing non-response, typically resolved by returning to the field to collect missing information (call-back). This method incurs additional costs and logistical challenges. Alternatively, imputation schemes can address missing data by substituting it with specific values, allowing for standard analysis methods. Common imputation techniques include mean imputation, hot deck imputation, nearest neighbour imputation, worm deck imputation, and mean-cum-nearest neighbour imputation. Many researchers have advocated for mean imputation techniques to mitigate non-response issues, including Singh and Horn (2000), Singh and Deo (2003), Wang and Wang (2006), Kadilar and Cingi (2008), Toutenburg et al. (2008), Singh (2009), Singh et al. (2010), Daina and Perri (2010), Al-omari et al. (2013), Gira (2015), Singh et al. (2016), Bhushan and Pandey (2016), Singh and Gogoi (2017), and Audu et al. (2020).

Measurement errors represent another type of non-random error in data collection, which contradicts the assumption in survey sampling that observations are free from such errors. In reality, data often becomes contaminated, leading to significant mean square errors and potentially misleading results. Cochran (1968) discussed sources of measurement error in survey data, and various authors, including Shallabh and Tsai (2017), Maneesha and Singh (2002), Allen et al. (2003), Singh and Vishwakarma (2021), have examined the effects of measurement errors on estimation methods like ratio, product, and regression across different sampling schemes. This study aims to modify the estimators proposed by Zaman et al. (2021) to account for non-response and measurement errors affecting both study and auxiliary variables.

Some Existing Related Estimators in the Presence of Non-Response and Measurement Error

Consider a finite population $\left(H = H_{1,}H_{2,}\ldots H_N\right)$ of size N such that Y be a study variable and X be any auxiliary variable and draw a sample of size n from a population by using simple random sampling without a replacement scheme. Suppose that N_1units respond to the survey questions and N_2units do not respond. Then by Hansen Hurwitz (1946) sampling plan, a sub-sample of size $h = \frac{u_2}{t}(t > 1)$from N_2non-respondents is selected at random and re-contacted for their direct interview. Here, it is assumed that r units respond to the survey.

Let $\left(x_r^*, y_r^*\right)$be the observed values and $\left(X_r^*, Y_r^*\right)$be the true values of the study Y and auxiliary variable X, where (r=1,2,….n) units in the sample. Then measurement error is given by:

$aa_r = y_r^* - Y_r^*$ and $bb_r = x_r^* - X_r^*$

Where (aa_r, bb_r) are random and both are uncorrelated with mean zero and variance S_{aa}^2 and S_{bb}^2 are associated with measurement error in Y and X respectively for the responding part of the population. $S_{aa(2)}^2$ and $S_{bb(2)}^2$ are the variances associated with measurement error in Y and X respectively for the non-responding part of the population.

S_y^2 and S_x^2 are the population variances of Y and X respectively for the responding part of the population. $S_{Y(2)}^2$ and $S_{X(2)}^2$are the variances of X and Y respectively for the non-responding part of the population.

$\rho$and $\rho_{(2)}$ are the population correlation coefficients between X and Y for the responding and non-responding parts of the population respectively. $T_y$and $T_{y(2)}$are the coefficients of variation of Y for the responding and non-responding part of the population respectively. Similarly, $T_X$and $T_{X(2)}$ are the coefficients of variation of X for responding and non-responding parts of the population respectively.

Let $q_Y^* = \sum_{r=1}^n\left(Y_r^* - \bar{Y}\right)$ and $q_X^* = \sum_{r=1}^n\left(X_r^* - \bar{X}\right)$, then

$q_{aa}^* = \sum_{r=1}^n aa_r$and $q_{bb}^* = \sum_{h=1}^n bb_r$

Therefore,

$$\frac{1}{n}(\phi_{bb} + \phi_{aa}) = \frac{1}{n}\sum_{r=1}^n(Y_r^* - \bar{Y}) + \frac{1}{n}\sum_{r=1}^n(y_r^* - \bar{Y}_r^*)$$

Singh*et al.* (2018) consider the effect of Measurement error and Non-response on an estimation of population mean and suggested estimator defined in (2.59)

$$\theta_1 = \left[\frac{1}{2}\left\{\bar{y}^* e^{\left(\frac{\bar{X}-\bar{x}^*}{\bar{X}+\bar{x}^*}\right)} + \bar{y}^* e^{\left(\frac{\bar{x}^*-\bar{X}}{\bar{x}^*+\bar{X}}\right)}\right\} + \omega_1\left(\bar{X} - \bar{x}^*\right) + \omega_2\bar{y}^*\right] e^{\left[\frac{\bar{X}^*-\bar{x}^{**}}{\bar{X}^*+\bar{x}^{**}}\right]} \quad (1)$$

where:
$\bar{X}^* = \bar{X}\rho,\ \bar{x}^{**} = \bar{x}^* + \bar{X}(p-1),\ \ p = \frac{\rho_{xy}+1}{4},$
$\omega_1 = \dfrac{UC - LF}{BC - L^2}$
$\omega_2 = \dfrac{BF - LC}{BU - L^2}$
$Bias(\theta_1) = \omega_2\bar{Y} + \left(\frac{\bar{Y}}{8\bar{X}^2} + \frac{\omega_1}{2\bar{X}\rho} + \frac{3}{8}\frac{\bar{Y}}{\bar{X}^2\rho^2} + \frac{3}{8}\frac{\omega_2\bar{Y}}{\bar{X}^2\rho^2}\right)\Delta_1^2 - \left(\frac{1}{2\bar{X}\rho} + \frac{\omega_2}{2\bar{X}\rho}\right)\Delta_0\Delta_1$ (2)

where:
$\Delta_0^2 = E(\Psi_0^2) = h_1(S_Y^2 + S_{aa}^2) + h_2(S_{Y(2)}^2 + S_{aa(2)}^2)$
$\Delta_1^2 = E(\Psi_1^2) = h_1(S_X^2 + S_{bb}^2) + h_2(S_{X(2)}^2 + S_{bb(2)}^2)$
$\Delta_0\Delta_1 = E(\Psi_0\Psi_1) = h_1\rho_{YX}S_YS_X + h_2\rho_{YX(2)}S_{Y(2)}S_{X(2)}$
$MSE(\theta_1) = \omega_1^2B + \delta_2^2U - 2\omega_1\omega_2L - 2\omega_1C + 2\omega_2F + Z$ (3)

where $B = \Delta_1^2,\ \ U = \left(\bar{Y}^2 + \Delta_0^2 + \frac{\bar{Y}^2}{\bar{X}^2\rho^2}\Delta_1^2 - 2\frac{\bar{Y}}{\bar{X}\rho}\Delta_0\Delta_1\right)$,
$L = \left(\Delta_0\Delta_1 - \frac{\bar{Y}}{\bar{X}\rho}\Delta_1^2\right),\ C = \left(\Delta_0\Delta_1 - \frac{\bar{Y}}{2\bar{X}\rho}\Delta_1^2\right),$
$F = \left(\frac{\bar{Y}}{8\bar{X}}\Delta_1^2 - \frac{3}{2}\frac{\bar{Y}}{\bar{X}\rho}\Delta_0\Delta_1 + \Delta_1^2 + \frac{5}{8}\frac{\bar{Y}^2}{\bar{X}^2}\frac{\Delta_1^2}{\rho^2}\right)$
$Z = \Delta_0^2 - \frac{\bar{Y}}{\bar{X}\rho}\Delta_0\Delta_1 + \frac{\bar{Y}^2}{4\bar{X}^2}\frac{\Delta_1^2}{\rho^2},\ \omega_1 = \frac{UC-LF}{BU-L^2},\ \omega_2 = \frac{BF-LC}{BU-L^2}$ (4)

Munneer et al. (2018) proposed exponential-type estimators of population mean in the presence of non-response as in (5) – (7)

$$\theta_2 = \bar{y}^{*\left(\frac{(\bar{X}-\bar{x}^*)}{(\bar{X}+\bar{x}^*)}\right)^{\left(\frac{(\bar{Z}-\bar{z}^*)}{\bar{Z}+\bar{z}^*}\right)}}_{(exp())} \quad (5)$$
$$\theta_3 = \bar{y}^{*\left(\frac{(\bar{X}-\bar{x}^*)}{(\bar{X}+\bar{x}^*)}\right)^{\left(\frac{(\bar{z}^*-\bar{Z})}{\bar{z}^*+\bar{Z}}\right)}}_{(exp())} \quad (6)$$
$$\theta_4 = \bar{y}^{*\left(\frac{(\bar{X}-\bar{x}^*)}{(\bar{X}+\bar{x}^*)}\right)^{\left(\frac{n(\bar{Z}-\bar{z}^*)}{2N\bar{Z}-n(\bar{z}^*+\bar{Z})}\right)}}_{(exp())} \quad (7)$$
$$\theta_5 = \bar{y}^{*\left(\frac{(\bar{x}^*-\bar{X})}{(\bar{x}^*+\bar{X})}\right)^{\left(\frac{(\bar{Z}-\bar{z}^*)}{(\bar{Z}+\bar{z}^*)}\right)}}_{(exp())} \quad (8)$$
$$\theta_6 = \bar{y}^{*\left(\frac{\bar{x}^*-\bar{X}}{\bar{x}^*+\bar{X}}\right)^{\left(\frac{\bar{z}^*-\bar{Z}}{\bar{z}^*+\bar{Z}}\right)}}_{(exp())} \quad (9)$$
$$\theta_7 = \bar{y}^{*\left(\frac{(\bar{x}^*-\bar{X})}{\bar{x}^*+\bar{X}}\right)^{\left(\frac{n(\bar{Z}-\bar{z}^*)}{2N\bar{Z}-n(\bar{z}^*+\bar{Z})}\right)}}_{(exp())} \quad (10)$$
$$\theta_8 = \bar{y}^{*\left(\frac{n(\bar{X}-\bar{x}^*)}{2N\bar{X}-n(\bar{x}^*+\bar{X})}\right)^{\left(\frac{(\bar{Z}-\bar{z}^*)}{\bar{Z}+\bar{z}^*}\right)}}_{(exp())} \quad (11)$$
$$\theta_9 = \bar{y}^{*\left(\frac{n(\bar{X}-\bar{x}^*)}{2NZ\bar{X}-n(\bar{x}^*+\bar{X})}\right)^{\left(\frac{(\bar{z}^*-\bar{Z})}{\bar{z}^*+\bar{Z}}\right)}}_{(exp())} \quad (12)$$
$$\theta_{10} = \bar{y}^{*\left(\frac{n(\bar{X}-\bar{x}^*)}{2N\bar{X}-n(\bar{x}^*+\bar{X})}\right)^{\left(\frac{(\bar{Z}-\bar{z}^*)}{\bar{z}^*+\bar{Z}}\right)}}_{(exp())} \quad (13)$$

The estimators $\theta_r, r = 1,2,\ldots\ldots\ldots,9$ can be generally written as in

$$\theta = \bar{y}^*\left[e^{\left(\frac{\varpi_1 - C_1}{\varpi_1 + C_1}\right)}\right]e^{\left(\frac{\varpi_2 - C_2}{\varpi_2 + C_2}\right)} \quad (14)$$

$\varpi_1 = (B_1 + L_1)\bar{X} + jU_1\bar{x}^*$ $\varpi_2 = (B_2 + L_2)\bar{Z} + jU_2\bar{z}^*$ $C_1 = (B_1 + jU_1)\bar{X} + L_1\bar{x}^*$
$C_2 = (B_2 + jU_2)\bar{Z} + L_2\bar{z}^*$ $B_h = (h_r - 1)(h_r - 2)$ $U_r = (h_r - 1)(h_r - 4)$
$L_r = (h_r - 2)(h_r - 3)(h_r - 4)$
$r = 1,2.3,4,\ \ldots\ldots 9$ r= 1,2,…, 9 (15)
$T_y^* = \tau L_y^2 + \gamma L_{y(2)}^2, T_x^* = \tau L_x^2 + \gamma L_{x(2)}^2$ $T_z^* = \tau L_z^2 + \gamma L_{z(2)}^2$
$T_{yx}^* = \tau L_{yx} + \gamma L_{yx(2)}, T_{yz}^* = \tau L_{yz} + \gamma L_{yz(2)}$ $T_{yz}^* = \tau L_{xz} + \gamma L_{xz(2)}$
$T_{yz}^* = \tau L_{yz} + \gamma L_{yz(2)}, L_{yx} = \rho_{yx}L_yL_x, L_{yz} = \rho_{yz}L_yL_{z,}$
$L_{xz} = \rho_{xz}L_xL_{z,} L_{y(2)}^2 = \dfrac{S_{y(2)}^2}{\bar{Y}^2}, L_{x(2)}^2 = \dfrac{S_{x(2)}^2}{\bar{X}^2}, L_{z(2)}^2 = \dfrac{S_{z(2)}^2}{\bar{Z}^2}$
$L_{yx(2)} = \rho_{yx(2)}L_{y(2)}L_{x(2),} L_{yz(2)} = \rho_{yz(2)}L_{y(2)}L_{z(2),} L_{xz(2)} = \rho_{xz(2)}L_{x(2)}L_{z(2),}$
$\tau = \left(\dfrac{1-j}{n}\right), j = \dfrac{n}{N}, \gamma = C_2\left(\dfrac{i-1}{n}\right), C_2 = \dfrac{N_2}{N}$

Salim and Onyango (2022) proposed a modified estimator for single phase sampling in the presence of observational errors as in

$$\lambda = \left[\left(\bar{y}_e^* + \psi(\bar{X} - \bar{x}_e^*)\right)\left(\delta e^{\left(\frac{\bar{Z}-\bar{z}_e^*}{\bar{Z}+\bar{z}_e^*}\right)} + (1-\delta)e^{\left(\frac{\bar{Z}-\bar{z}_e^*}{\bar{Z}+\bar{z}_e^*}\right)}\right)\right] \quad (16)$$

$MSE(\lambda) = \gamma\bar{Y}^2(L_Y^2 + L_\varpi^2) + \bar{Y}\left(\frac{1}{2} - \delta\right)\frac{1}{2}\gamma\bar{Y}\bar{Z}\rho YZL_YL_Z - \beta\gamma\bar{Y}\bar{X}\rho XYL_YL_X$ (17)
$\delta = \dfrac{1}{2}, \psi = \dfrac{\rho S_Y}{S_X}$

Proposed Estimators

In this study, four classes of robust estimators of population mean were proposed in the presence of non-response and measurement error using an imputation scheme as defined in (18) – (21)

$$G_1 = \bar{y}_{u(e)}\frac{u}{n} + \left(1 - \frac{u}{n}\right)\left[\begin{array}{l}\bar{y}_{u(e)}\left(\frac{\bar{X}_1}{\bar{x}_{1u(e)}}\right)^{\tau_{11}}\left(\frac{\bar{X}_2}{\bar{x}_{2u(e)}}\right)^{\tau_{21}} + g_{1(LTS)}(\bar{X}_1 - \bar{x}_{1u(e)}) \\ +g_{2(LTS)}(\bar{X}_2 - \bar{x}_{2u(e)})\end{array}\right] \quad (18)$$

$$G_2 = \bar{y}_{u(e)}\frac{u}{n} + \left(1 - \frac{u}{n}\right)\left[\begin{array}{l}\bar{y}_{u(e)}\left(\frac{\bar{X}_1}{\bar{x}_{1u(e)}}\right)^{\tau_{21}}\left(\frac{\bar{X}_2}{\bar{x}_{2u(e)}}\right)^{\tau_{22}} + g_{1(S)}(\bar{X}_1 - \bar{x}_{1u(e)}) \\ +g_{2(S)}(\bar{X}_2 - \bar{x}_{2u(e)})\end{array}\right] \quad (19)$$

$$G_3 = \bar{y}_{u(e)}\frac{u}{n} + \left(1 - \frac{u}{n}\right)\left[\begin{array}{l}\bar{y}_{u(e)}\left(\frac{\bar{X}_1}{\bar{x}_{1u(e)}}\right)^{\tau_{31}}\left(\frac{\bar{X}_2}{\bar{x}_{2u(e)}}\right)^{\tau_{32}} + g_{1(LMS)}(\bar{X}_1 - \bar{x}_{1u(e)}) \\ +q_{2(LMS)}(\bar{X}_2 - \bar{x}_{2u(e)})\end{array}\right] \quad (20)$$

$$G_4 = \bar{y}_{u(e)}\frac{u}{n} + \left(1 - \frac{u}{n}\right)\left[\begin{array}{l}\bar{y}_{u(e)}\left(\frac{\bar{X}_1}{\bar{x}_{1u(e)}}\right)^{\tau_{41}}\left(\frac{\bar{X}_2}{\bar{x}_{2u(e)}}\right)^{\tau_{42}} + g_{1(HUBM)}(\bar{X}_1 - \bar{x}_{1u(e)}) + \\ q_{2(HUBM)}(\bar{X}_2 - \bar{x}_{2u(e)})\end{array}\right] \quad (21)$$

where $\bar{y}_e^* = \frac{n_1\bar{y}_{1(e)}+n_2\bar{y}_{i_{2(e)}}}{n_1+n_2}$, $\bar{x}_{1(e)}^* = \frac{n_1\bar{x}_{1(e)}+n_2\bar{x}_{1i_2(e)}}{n_1+n_2}$, $\bar{x}_{2(e)}^* = \frac{n_1\bar{x}_{2(e)}+n_2\bar{x}_{2i_2(e)}}{n_1+n_2}$

Bias and MSEs of the proposed estimators

$G_r, r = 1,2,3,4$

$$Bias(G_r) = \left(1-\frac{u}{n}\right)\left(\frac{1}{u}+\frac{1}{N}\right)\begin{bmatrix}\frac{\tau_{r2}(\Delta_{r2}-1)}{2}L_{x_2}^2 + \frac{\Delta_{r1}(\Delta_{r1}-1)}{2}L_{x_1}^2 + \tau_{r1}\tau_{r2}\rho_{x_1x_2} - \tau_{r2}\rho_{yx_2}L_yL_{x_2} \\ -\tau_{r1}\rho_{yx_1}L_yL_{x_1}\end{bmatrix} \quad (22)$$

$$MSE(G_r)00_{r1(opt)}{}_{r2(opt)}{}_{r1}^{\ 2}{}_{(opt)}{}_{r2}^{\ 2}{}_{(opt)}{}_{r1(opt)}{}_{r2(opt)}{}_{(min())} \quad (23)$$

where $\tau_{r1} = \frac{BC-UF}{LC-F^2}$, $\tau_{r2} = \frac{UL-BF}{LC-F^2}$

$B = \left(1-\frac{u}{n}\right)\bar{Y}[A_{01}-g_1A_{11}-g_2A_{12}]$, $U = \left(1-\frac{u}{n}\right)\bar{Y}[A_{02}-g_2A_{22}-g_1A_{12}]$, $L = \left(1-\frac{1}{n}\right)\bar{Y}^2A_{11}$

$C = \left(1-\frac{1}{n}\right)\bar{Y}^2A_{22}$, $F = \bar{Y}^2\left(1-\frac{u}{n}\right)A_{12}$, $Z = g_1^2A_{11}+g_2^2A_{22}-2g_1A_{01}-2g_2A_{02}+2g_1g_2A_{12}$

Efficiency comparisons of the proposed Estimators G_r

In this subsection, efficiency conditions of the estimators of the proposed ratiotype over some existing estimators were established.

$$MSE(G_r) - MSE(\gamma) < 0 \quad (24)$$

where $\gamma$are the existing estimators

$$MSE(G_r) - MSE(\lambda) < 0$$

$$\begin{pmatrix}A_{00}-2B\tau_{r1}-2U\tau_{r2}+2L\tau_{r1}^2+2C\tau_{r2}^2+ \\ 2E\Delta_{r1}\Delta_{r2}+Z\end{pmatrix} - \begin{bmatrix}\gamma\bar{Y}^2(L_Y^2+L_Y^2)\bar{Y}\left(\frac{1}{2}-\delta\right)\gamma\bar{Y}\bar{Z}\rho_{YZ}L_YL_Z \\ -\psi\gamma\bar{Y}\bar{X}\rho_{XY}L_YL_X\end{bmatrix} < 0 \quad (25)$$

$$Z < 2\begin{pmatrix}-A_{00}+B\tau_{r1}+U\tau_{r2}-L\tau_{r1}^2 \\ -C\tau_{r2}^2-F\tau_{r1}\tau_{r2}\end{pmatrix} + \begin{bmatrix}\gamma\bar{Y}^2(L_Y^2+L_Y^2)+\bar{Y}\left(\frac{1}{2}-\delta\right)\gamma\bar{Y}\bar{Z}\rho_{YZ}L_YL_Z \\ -\psi\gamma\bar{Y}\bar{X}\rho_{XY}L_YL_X\end{bmatrix} \quad (26)$$

$$MSE(G_r) - MSE(\theta) < 0$$

$$\begin{pmatrix}A_{00}-2B\tau_{r1}-2U\tau_{r2}+2L\tau_{r1}{}^2+2C\tau_{r2}{}^2+ \\ 2F\tau_{r1}\tau_{r2}+Z\end{pmatrix} - \left[\bar{Y}^2\left(T_y^* - \frac{T_{yx}^{*2}T_x^*+T_{yz}^*T_x^*-T_{yx}^*T_{yz}^*T_{xz}^*}{T_x^*T_z^*-T_{xz}^{*2}}\right)\right] < 0 \quad (27)$$

$$Z < 2\begin{pmatrix}-A_{00}+B\tau_{r1}+U\Delta\tau_{r2}-L\tau_{r1}^2 \\ -C\tau_{r2}^2-F\tau_{r1}\tau_{r2}\end{pmatrix} + \left[\bar{Y}^2\left(T_y^* - \frac{T_{yx}^{*2}T_x^*+T_{yz}^*T_x^*-T_{yx}^*T_{yz}^*T_{xz}^*}{T_x^*T_z^*-T_{xz}^{*2}}\right)\right] \quad (28)$$

RESULTS AND DISCUSSION

This section evaluates the performance of the proposed estimators against existing ones through empirical studies, calculating biases, mean square errors (MSEs), and percent relative errors (PREs) using equations (29), (30), and (31).

$$Bias(\hat{y}_r) = \frac{1}{1000}\sum_{r=1}^{1000}(\hat{y}_r - \bar{Y}), \hat{y}_r = \bar{y}_r \quad (29)$$

$$MSE(\hat{y}_r) = \frac{1}{1000}\sum_{r=1}^{1000}(\hat{y}_r - \bar{Y})^2, \hat{y}_r = \bar{y}_r \quad (30)$$

$$PRE(\hat{y}_r) = \frac{MSE(G_r)}{MSE(\hat{y}_r)} \times 100\%, \hat{y}_r = \bar{y}_r, G_r, r = 1,2,3,4 \quad (31)$$

Table 1: Population used for Simulation Study

Population	Auxiliary Variable (X)	Study Variable (Y)
1	$X_1 = r\,exp(N, 2)$	$Y = 0.3{*}X_1 + 5{*}X_2 + e_i$
	$X_2 = r\,exp(N, 1)$	Where, e ~ Normal (0, 1)
2	$X_1 = rchisq(N, 2)$	
	$X_2 = rchisq(N, 3)$	U~ Normal (0, 25)
3	$X_1 = rgamma(N, 0.5, 2.5)$	V_1~ Normal (0, 16)
	$X_2 = rgamma(N, 1.5, 3.5)$	V_2~ Normal (0, 9)
4	$X_1 = rpois(N, 3)$	
	$X_2 = rpois(N, 1.5)$	

Table 2: Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from Exponential Distribution

Estimator		Bias	MSE	PRE
Sample mean		-0.009989655	3.670668	100
	θ_1	3.023097e+111	1.606164e+224	2.285364e-222
SO (2022)λ		0.1601833	1.593409	378.4095
	θ_2	0.4332807	10.93393	230.3658
	θ_3	-0.009989655	3.670668	33.57135
	θ_4	0.1738815	3.833255	95.75853
	θ_5	-0.1675946	1.865367	196.7799

θ_6	72.44146	6090.717	0.0602666
θ_7	-0.1404735	4.09145	89.71559
θ_8	-0.06705114	0.9670699	379.566
θ_9	0.2097258	9.696611	37.85517
θ_{10}	-0.06705114	0.9670699	379.566
Proposed Estimators under imputation schemes			
G_1	-0.0123064	0.4026394	911.6515
G_2	-0.008408729	0.4209914	871.9106
G_3	-0.03844668	0.2309159	1589.612
G_4	-0.1847394	4.459426	864.6427

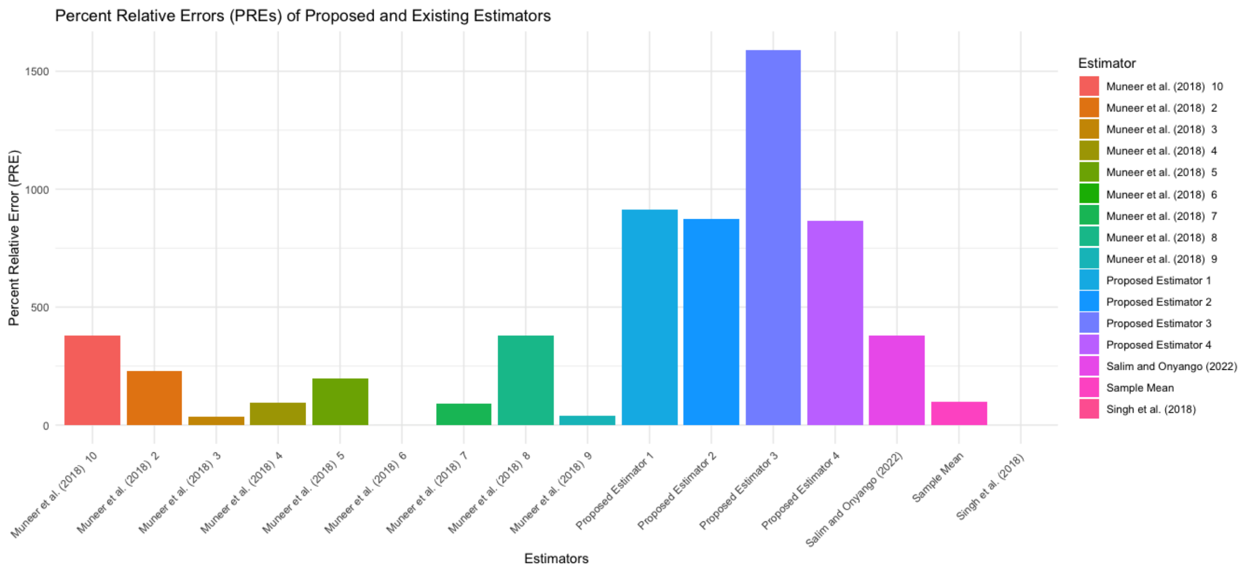


Table 3: Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from Chi-square Distribution

Estimator	Bias	MSE	PRE
Sample mean	-0.08410128	21.12014	100
θ_1	-6.712408e+113	1.103315e+229	1.914243e-226
SO (2022)λ	0.4067392	10.83238	449.5515
θ_2	0.9687119	68.01479	194.9722
θ_3	-0.08410128	21.12014	31.05228
θ_4	0.4246086	25.7296	82.085
θ_5	-0.2983239	13.9069	151.868
θ_6	217.3016	52068.63	0.04056212
θ_7	-0.293289	27.66242	76.34957
θ_8	-0.1910325	4.67781	451.4963
θ_9	0.3349565	54.84342	38.50989
θ_{10}	-0.1910325	4.67781	451.4963
Proposed Estimators under imputation schemes			
G_1	-0.05648113	2.141613	986.179
G_2	-0.05570801	2.175715	970.7216
G_3	-0.2050515	3.528707	598.5234
G_4	-1.994144	29.3358	380.305

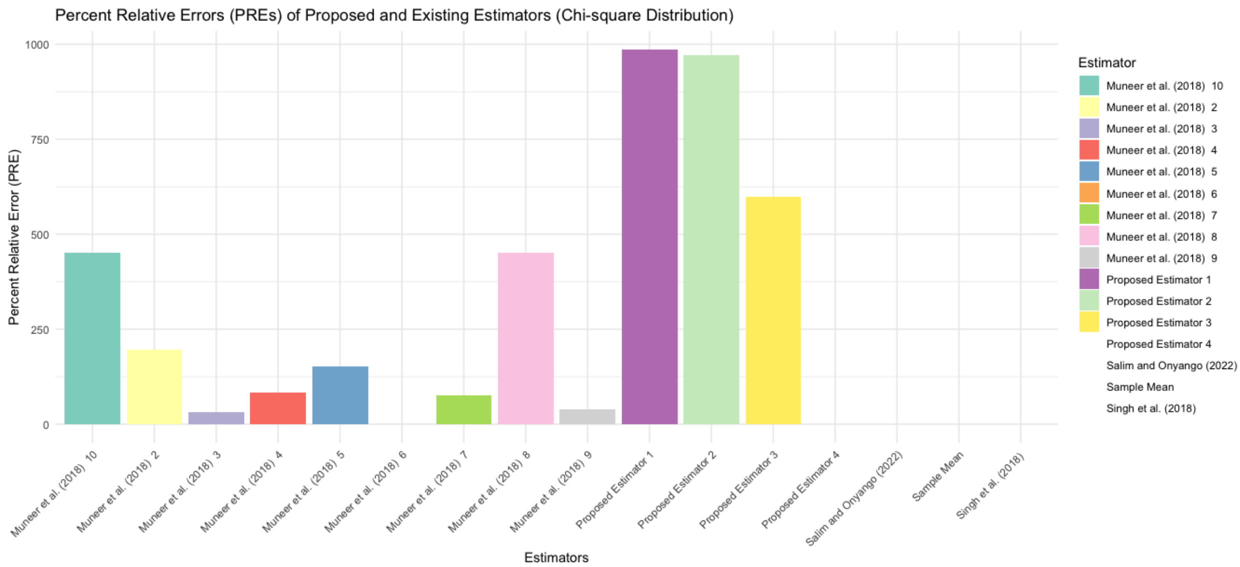


Table 4: Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from gamma Distribution

Estimator		Bias	MSE	PRE
Sample mean		-0.003964409	0.6811538	100
	θ_1	1.41007e+111	7.553066e+222	9.018242e-222
SO (2022)λ		0.1328968	0.55617	221.8051
	θ_2	0.2428026	2.080598	122.4722
	θ_3	-0.003964409	0.6811538	32.73836
	θ_4	0.1419581	0.920482	73.99968
	θ_5	-0.09282761	0.5294192	128.6606
	θ_6	30.5277	1088.365	0.06258505
	θ_7	-0.08356495	0.7941444	85.77203
	θ_8	-0.02729135	0.3115916	218.6047
	θ_9	0.07586364	1.525341	44.65584
	θ_{10}	-0.02729135	0.3115916	218.6047
Proposed Estimators under imputation schemes				
	G_1	-0.05599152	0.08630086	789.2781
	G_2	-0.05498659	0.08482839	802.9786
	G_3	-0.05425017	0.0593816	1147.079
	G_4	-0.09695563	1.022151	768.975

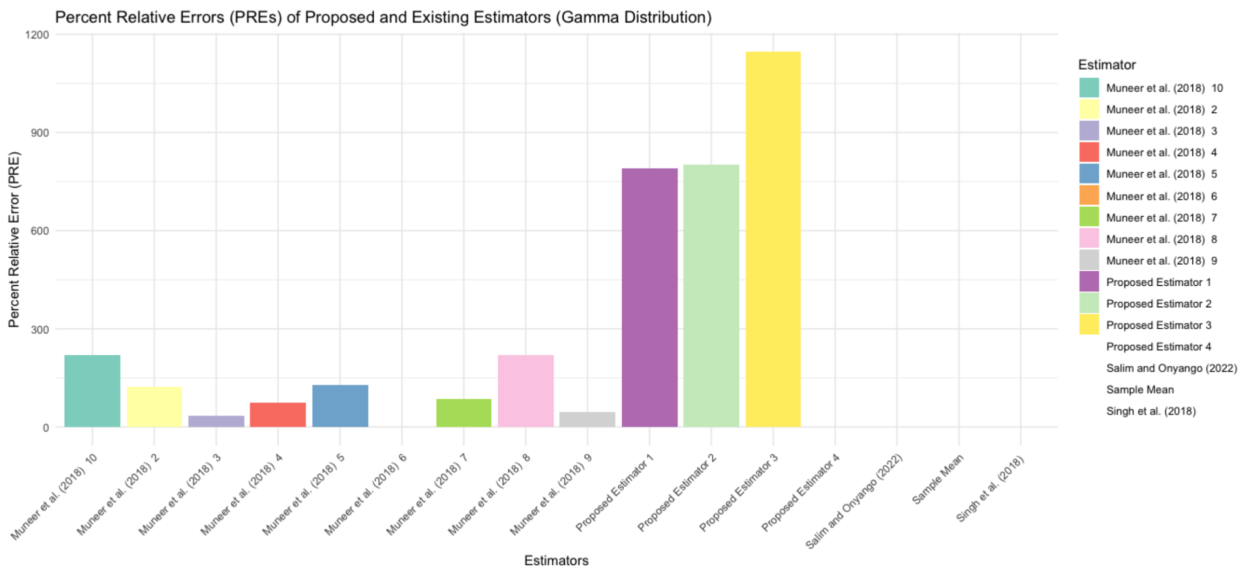
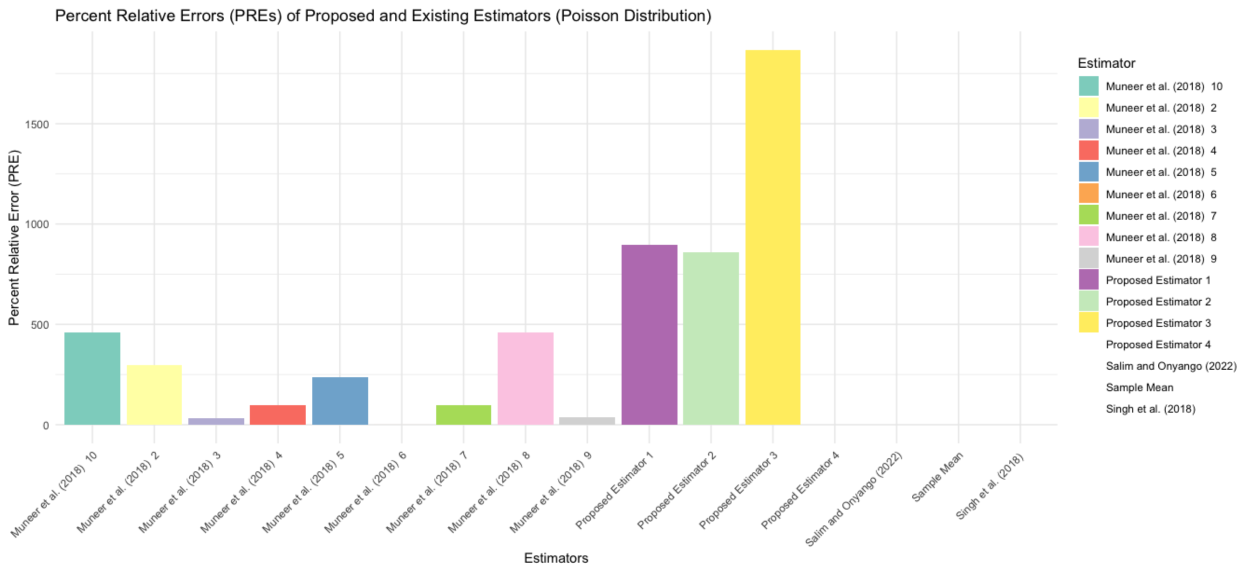


Table 5: Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from poison Distribution

Estimator	Bias	MSE	PRE
Sample mean	0.02252968	6.508619	100
θ_1	-1.840903e+113	5.938039e+227	1.096089e-225
SO (2022)λ	0.09002835	2.20565	460.7558
θ_2	0.4742451	19.57555	295.0884
θ_3	0.02252968	6.508619	33.24872
θ_4	0.1198424	6.75827	96.306
θ_5	-0.06961472	2.750671	236.6193
θ_6	121.6848	16301.8	0.03992577
θ_7	-0.05585562	6.508619	96.91564
θ_8	-0.05232447	6.715757	461.1598
θ_9	0.3083664	1.411359	38.16082
θ_{10}	-0.05232447	17.05576	461.1598
Proposed Estimators under imputation schemes			
G_1	0.271166	7.465962	894.3244
G_2	0.276815	0.7277694	861.1318
G_3	0.1049767	0.7558215	1867.092
G_4	-0.1308369	7.835589	1048.67



CONCLUSION

The study aimed to develop imputation-based estimators that reduce MSE, increase PRE, and improve efficiency compared to existing methods. The findings indicate that these estimators yield results closer to the true population mean in the presence of non-response and measurement errors, and their reliability was supported by consistency testing. These results suggest practical benefits for data analysis in fields where data accuracy is critical, such as medical and social sciences. Future research could further explore the performance of these estimators across diverse data contexts to validate their broad applicability.

ACKNOWLEDGMENT

The authors acknowledged Tertiary Education Trust Fund (TETUND) through the Federal Polytechnic Kaura Namoda, Zamfara State, for funding this research with a Grant Number TETF/DR&D/CE/POLY/KAURANAMODA/IBR/2023/VOL.1.

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