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CONSTRUCTION OF A THREE LEVEL SPLIT-LOT DESIGNS IN SUDOKU SQUARE DESIGN STRUCTURE

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ABSTRACT

In this research, a situation where blocking is required for a split lot design in order to tease out noise from the dependent variable (prevent factors other than that of interest influencing the outcome) was considered. Blocking for every stage of the design is administered, an elaborate construction procedure for the design was developed by infusing the ordinary split-lot design into a Sudoku Square design structure, the hybridisation gave it a convenient structure for the research. The linear model and the sum of squares of the design were derived, the ANOVA table was constructed and the table was used to analyse the whole system. The clear advantage of this design has been observed to be the additional source of variation, because the introduction of the Block Sum of Squares will reduce the Error Sum of Squares as a result makes it more efficient, but the fact that precision and cost are both functions of the number of sublots per step as well as the total number of items, comparison is complicated. The new design provides greater precision for main effects.

Keywords: Split-Plot, Split-lot, Sudoku Square design, Multistage process design, Blocking

INTRODUCTION

Split–plot originally developed by Fisher in 1925 for use in agricultural experiments is basically the modified form of randomized block designs. These designs are used in situations where complete randomization of runs within block is not possible. In 1998, the construction of split-lot factorial designs was pioneered by Mee and Bates (1998). They found designs in certain cases where there are many processing stages or many factors and at each stage one and only one factor is to be applied to setting of that stage. In fabrication of integrated circuits (IC's), the Authors noted it is accomplished through a vast sequence of processing steps. However, the silicon wafers on which the IC's are produced move through the process in lots of size 24 or more. In the experiment, some processing steps are applied to individual wafers, for other steps several wafers (or even several lots) are processed simultaneously as a group. They concluded that to facilitate experimentation with such a multistage batch process, "split-lot" experimental designs are attractive because they allow the experimental wafers to be split into sublots for processing. The designs were obtained by using different sets of factorial effects to define the composition of the sublots at each step. Then Butler (2004) described Split-lot designs, also as multiway split-unit designs, and as useful or essential in factorial experiments where there are multiple processing stages. He acknowledged that such experiments occur, for example, in the fabrication of integrated circuits in the semiconductor industry and in product assembly. The designs he stated, have a split-plot structure at each stage, so that settings of the factors are applied to sublots of experimental runs rather than to individual runs. He realized that it allowed experimental runs to be processed at much less expense than if the runs were completely randomized. In his work (construction of a two-level split lot design), split-lot designs are constructed for two, three, and four processing stages. The designs have minimum aberration under the split-lot structure and minimized the confounding of main effects and two-factor interactions with the sublots at each stage.

Split-lot designs for Multistage batch processes in the existing literature (Mee and Bates, 1998; Butler, 2004; Bingham, Sitter, Kelly, Moore, and Olivas, 2008) have all worked on split-lot designs that does not considered blocking. The neglect on designs that considers blocking will restrict experimenters carrying out experiments that requires it in experimentation, and it may lead to confounding blocking effects into other effects (i.e variability other than that of interest influencing the outcome, or overlapping effects as a result leading to inaccuracy - unexplained variability). The construction of a split-lot designs in Sudoku square designs structure will give it the required structure and visualize the randomization structure of the experiment for the purpose of this research. The Sudoku square designs had been extensively studied by many researchers (Hui–Dong and Ru–Gen, 2008, Subramani and Ponnuswamy, 2009 and so on). The construction and analysis of Sudoku square designs were extensively discussed by Subramani and Ponnuswamy (2009). Subramani, J. (2013) worked on the construction of graeco sudoku square designs of odd orders. Likewise, Danbaba (2016) studied the combined analysis of Sudoku square designs with same treatments, and also the construction and analysis of Samurai Sudoku. Danbaba and Shehu (2016) studied the combined analysis of Sudoku square designs with some common treatments. Shehu A. and Danbaba A. (2018) also studied the variance components of models of sudoku square design and Subramani, J. (2018) worked on the construction and analysis of sudoku square designs with rectangle, etc. However, the construction and analysis of split-lot designs in Sudoku square designs structure is not yet in the existing literature. Therefore, a key step of our approach in this research is to provide a procedure or design which will enable blocking (i.e control such variability) in split–lot design using the Sudoku square designs structure.

Construction of a Three-level Split-Lot Design in Sudoku Square Designs Structure

Many industrial experiments involve a sequence of processing stages, where at each stage the experimental units are partitioned into disjoint classes, with those in the same class assigned the same level of certain treatment factors. The standard Sudoku square has a 9 x 9 grids which we consider each grid to be a class of items to be administered the same level of treatment setting from the disjoint sets. This will enable the use of the three-level factorial design.

Consider the design in which nine sublots replicates are run together as one experiment. We process sublots of size 9 for each experimental factor. We will construct an 81-wafer design for which wafers processed together in sublots of previous stage will be reordered between sublots of the next stage and reordered again between levels of a factor. This design also considers that all three runs of each processing step must be completed before any processing can begin at the next step. Denote the four factors by X_i (i = 1, 2, 3, 4). When the wafers arrive at processing Step 1, they will be split into nine sublots of nine wafers each. These nine sublots will be processed separately and in random order - three at the 1st level of X_1 another three at the second level of X_1 and the last three at the third level of X_1. When the wafers arrive at Step 2, nine new sublots will be formed.

Each of these new sublots consists of three wafers from each of the sublots at Step 1. Three of the sublots will be processed separately at the three levels of X_2. When the wafers arrive at Step 3, nine new sublots will be formed by taking three wafers from each of the sublots at Step 2 to make a sublot. The three sublots will process separately, all at the three levels of X_3. The same goes for stage four. In this design for each batch process step other than the steps involving the experimental factors, the 81 wafers will be processed together.

In the split-lot design, suppose an experimenter design an experiment as a Sudoku square design of order m^2 $(m = 2,3,4,\ldots)$ and one possible arrangement is given in figure 3.1, with standard 9×9 - Sudoku square, row-blocks serve as stage one sublots which will receive the stage factor in it levels and rows within the row - blocks or column serves as order within the sublots, column blocks serves as stage two sublots to receive the stage factor in its levels, and sub-blocks or sub-squares are the stage 3 sublots to receive the stage factor in its levels and columns and rows within the sub-square are also order for treatments in each sublot, the stage 4 considers the items in each sublot of the 4^{th} stage which are the letters in rows within row-blocks. (A, B, C, D, E, F, G, H, I) are the split-lot or sublot items or Wafers in the case of Integrated Circuits in the case of Mee and Bates (1998) and Butler (2004).

Stage 2

		0			**1**			**2**		
		C1	C2	C3	C1	C2	C3	C1	C2	C3
0	R1	C	A	B	F	D	E	I	G	H
	R2	F	D	E	I	G	H	C	A	B
	R3	I	G	H	C	A	B	F	D	E
1	R1	A	B	C	D	E	F	G	H	I
	R2	D	E	F	G	H	I	A	B	C
	R3	G	H	I	A	B	C	D	E	F
2	R1	B	C	A	E	F	D	H	I	G
	R2	E	F	D	H	I	G	B	C	A
	R3	H	I	G	B	C	A	E	F	D

Stage 1 **Stage 4**

Stage 3

Figure 1: Split-lot arrangement with a standard 9×9 – Sudoku–square

Existing model

$Y_S = \mu + \sum_{i=1}^{m}(\alpha_{S_i(i)} + S_{i(i)}) + \sum_{i<j}\alpha_{S_iS_j(ij)} + \epsilon_S$ (1)

where m is the number of steps with sublots. S = (S_1,.., S_m) identifies the sublot number at each step. $\alpha_{S_i(i)}$ *is the effect of the i^{th} stage factor for the s_i^{th} sublot,* and $S_{i(i)}$ *is the error term associated with the s_i^{th} sublot of wafers processed together at the i^{th} stage.* $S_{i(i)}$~ *iid N(0,σ^2), and* $\alpha_{S_iS_j(ij)}$ *is the interaction effect between the i^{th} stage factor and j^{th} stage factor. Finally,* ϵ_S *is the remaining contribution to error* ϵ_S*- iid N(0, σ^2),*

The existing model by Mee & Bates (1998) does not take into consideration that blocking whichis necessary to eliminate nuisance effect. When blocking variable is administered to an experiment, it teases out a source of undesired variation in the dependent variable.

Proposed model

$Y_S = \mu + \sum_{i=1}^{m}(\alpha_{S_i(i)} + \gamma_{S_i(i)} + S_{i(i)}) + \sum_{i<j}\alpha_{S_iS_j(ij)} + \epsilon_S$ (2)

where:

m is the number of steps with sublots.

S = (S_1,.., S_m) identifies the sublot number at each step.

$\alpha_{S_i(i)}$ *is the effect of the i^{th} stage factor for the s_i^{th} sublot,* and

$\gamma_{S_i(i)}$ is the effect of the i^{th} stage block for the s_i^{th} sublot

$S_{i(i)}$ *is the error term associated with the s_i^{th} sublot of wafers processed together at the i^{th} stage,* $S_{i(i)}$ *~ iid N(0, σ^2),*

$\alpha_{S_iS_j(ij)}$ *is the interaction effect between the i^{th} stage factor and j^{th} stage factor. Finally,* ϵ_S *is the remaining contribution to error* ϵ_S*- iid N(0, σ^2).*

In this model it is assumed that the column blocks are stage 1 factor effects and the row blocks are stage 2 factor effects, the sub-blocks or sub-squares are the stage 3 factor effects and the letters considered row-wise are stage 4 effects. The split-lot design in Sudoku square design structure of order m^2 and its Analysis of Variance model together with the various assumptions are given in detail below:

The model in equation 3 is the linear model for split-lot designs using Sudoku square designs structure after being expanded from 2 to replace iterations. The distribution assumptions, derivation of sum-of-squares as well as the ANOVA table will be given in detail below:

$Y_{ijklp} = \mu + \alpha_i + \theta_j + \pi_{ij} + \beta_k + \delta_j + \varphi_{kj} + \gamma_l + \vartheta_j + r_{ij} + \tau_p + \rho_j + S_{pj} + \alpha\beta_{(ij)} + \alpha\gamma_{(il)} + \alpha\tau_{(ip)} + \beta\gamma_{(kl)} + \beta\tau_{(kp)} + \gamma\tau_{(lp)} + e_{ijklp}$ (3)

$where\ i = j = k = l = p = 1,2,\ldots,m$

Where $\mu = General\ mean\ effect$

μ: General mean of effect

α_i: first stage factor effect (stage treatment)

θ_j: first stage blocks of sublots

π_{ij}: error associated with first stage $\pi_{ij} \rightarrow N(0, \sigma^2)$

β_k: second stage factor effect (stage treatment)

δ_j: second stage block effects

φ_{kj}: error associated with second stage $\varphi_{kj} \rightarrow N(0, \sigma^2)$

γ_l: third stage factor effect (stage treatment)

ϑ_j: third stage block of sublots

r_{ij}: error associated with third stage $r_{ij} \rightarrow N(0, \sigma^2)$

τ_p: fourth stage factor effect (stage treatment)

ρ_j: fourth stage block of sublots

S_{pj}:error associated with fourth stage $S_{pj} \rightarrow N(0, \sigma^2)$

$\alpha\beta_{(ij)}, \alpha\gamma_{(il)}, \alpha\tau_{(ip)}, \beta\gamma_{(kl)}, \beta\tau_{(kp)}, \gamma\tau_{(lp)}$: two stage factor interactions, three and four stages factor interactions are neglected.

e_{ijklp}= residual error, $e_{ijklp} \overset{iid}{\Rightarrow} N(0, \sigma^2)$, independent of $\pi_{ij}, \varphi_{kj}, r_{ij}, S_{pj}$ $N = m^4$

Derivation of Sum-of-Squares for Split-lot Designs Using Sudoku Square Structure

From equation (3) the sum of squares of errors is

$$\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{p=1}^{m} e^2_{ijklp} = \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{p=1}^{m}(\mu + \alpha_i + \theta_j + \pi_{ij} + \beta_k + \delta_j + \varphi_{kj} + \gamma_l + \vartheta_j + r_{ij} + \tau_p + \rho_j + S_{pj} + \alpha\beta_{(ij)} + \alpha\gamma_{(il)} + \alpha\tau_{(ip)} + \beta\gamma_{(kl)} + \beta\tau_{(kp)} + \gamma\tau_{(lp)}) = 0 \quad (4)$$

Let assume that $L = \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{p=1}^{m} e^2_{ijklp}$

Differentiating equation (4) with respect to

$\mu, \alpha_i, \theta_j, \pi_{ij}, \beta_k, \delta_j, \varphi_{kj}, \gamma_l, \vartheta_j, r_{ij}, \tau_p, \rho_j, S_{pj}, \alpha\beta_{(ik)}, \alpha\gamma_{(il)}, \alpha\tau_{(ip)}, \beta\gamma_{(kl)}, \beta\tau_{(kp)}, \gamma\tau_{(lp)}$

respectively, and equating to zero, we obtain the following system of equations.

$$Y_{.....} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} = \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{p=1}^{m}Y_{ijklp} \quad (5)$$

$$Y_{i....} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \quad (6)$$

$$Y_{.j...} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \quad (7)$$

$$Y_{ij...} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \quad (8)$$

$$Y_{..k..} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \quad (9)$$

$$Y_{.jk..} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \quad (10)$$

$$Y_{...l.} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \quad (11)$$

$$Y_{.j.l.} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \tag{12}$$

$$Y_{....p} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \tag{13}$$

$$Y_{.j..p} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \tag{14}$$

$$Y_{i.k..} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \tag{15}$$

$$Y_{i..l.} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \tag{16}$$

$$Y_{i...p} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \tag{17}$$

$$Y_{..lk.} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \tag{18}$$

$$Y_{..k.p} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \tag{19}$$

$$Y_{...lp} = N\mu + m^3\sum_{i=1}^{m}\alpha_i + m^3\sum_{i=1}^{m}\theta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} + m^3\sum_{i=1}^{m}\beta_k + m^3\sum_{j=1}^{m}\delta_j + m^2\sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} + m^3\sum_{l=1}^{m}\gamma_l + m^3\sum_{j=1}^{m}\vartheta_j + m^2\sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} + m^3\sum_{p=1}^{m}\tau_p + m^3\sum_{j=1}^{m}\rho_j + m^2\sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} + m^2\sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} + m^2\sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} + m^2\sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} + m^2\sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} + m^2\sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \tag{20}$$

Assumptions

$$\sum_{i=1}^{m}\alpha_i = \sum_{i=1}^{m}\theta_j = \sum_{i=1}^{m}\sum_{j=1}^{m}\pi_{ij} = \sum_{i=1}^{m}\beta_k = \sum_{j=1}^{m}\delta_j = \sum_{k=1}^{m}\sum_{j=1}^{m}\varphi_{kj} = \sum_{l=1}^{m}\gamma_l = \sum_{j=1}^{m}\vartheta_j = \sum_{i=1}^{m}\sum_{j=1}^{m}r_{ij} = \sum_{p=1}^{m}\tau_p = \sum_{j=1}^{m}\rho_j = \sum_{p=1}^{m}\sum_{j=1}^{m}S_{pj} = \sum_{i=1}^{m}\sum_{k=1}^{m}\alpha\beta_{(ik)} = \sum_{i=1}^{m}\sum_{l=1}^{m}\alpha\gamma_{(il)} = \sum_{i=1}^{m}\sum_{p=1}^{m}\alpha\tau_{(ip)} = \sum_{i=1}^{m}\sum_{l=1}^{m}\beta\gamma_{(kl)} = \sum_{k=1}^{m}\sum_{p=1}^{m}\beta\tau_{(kp)} = \sum_{l=1}^{m}\sum_{p=1}^{m}\gamma\tau_{(lp)} \tag{21}$$

Equation (4) through (20) can be reduce to

$$\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{p=1}^{m}Y_{ijklp} - N\mu = 0 \tag{22}$$

$$m^3\sum_{i=1}^{m}Y_{i....} - N\mu = 0 \tag{23}$$

$$m^2\sum_{i}^{m}\sum_{j}^{m}Y_{ij...} - N\mu = 0 \tag{24}$$

$$m^3\sum_{i=1}^{m}Y_{..k..} - N\mu = 0 \tag{25}$$

$$m^2\sum_{j=1}^{m}\sum_{k=1}^{m}Y_{.jk..} - N\mu = 0 \tag{26}$$

$$m^3\sum_{l=1}^{m}Y_{...l.} - N\mu = 0 \tag{27}$$

$$m^3\sum_{j=1}^{m}Y_{.j,..} - N\mu = 0 \tag{28}$$

$$m^2\sum_{i=1}^{m}\sum_{j=1}^{m}Y_{ij...} - N\mu = 0 \tag{29}$$

$$m^3\sum_{p=1}^{m}Y_{....p} - N\mu = 0 \tag{30}$$

$$m^2\sum_{p=1}^{m}\sum_{j=1}^{m}Y_{.j.p.} - N\mu = 0 \tag{31}$$

$$m^2\sum_{i=1}^{m}\sum_{k=1}^{m}Y_{i.k..} - N\mu = 0 \tag{32}$$

$$m^2\sum_{i=1}^{m}\sum_{l=1}^{m}Y_{i..l.} - N\mu = 0 \tag{33}$$

$$m^2\sum_{i=1}^{m}\sum_{p=1}^{m}Y_{i..p.} - N\mu = 0 \tag{34}$$

$$m^2\sum_{k=1}^{m}\sum_{l=1}^{m}Y_{..kl.} - N\mu = 0 \tag{35}$$

$$m^2\sum_{k=1}^{m}\sum_{p=1}^{m}Y_{..kp.} - N\mu = 0 \tag{36}$$

$$m^2\sum_{l=1}^{m}\sum_{p=1}^{m}Y_{...lp} - N\mu = 0 \tag{37}$$

Solving equation (22) through (37) simultaneously, yields the following estimates of parameters

$\mu = \frac{G}{N}$, Where $N = m^4$ and $G =$

$$\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{p=1}^{m}Y_{ijklp} \tag{38}$$

$$SST = \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{p=1}^{m}Y_{ijklp} - \frac{G^2}{N} \tag{39}$$

$$SS_{X_1} = \sum_{i=1}^{m}\frac{\alpha_i}{m^3} - \frac{G^2}{N} \tag{40}$$

$$SS_{X_1}B = \sum_{i=1}^{m}\frac{\theta_j}{m^3} - \frac{G^2}{N} \tag{41}$$

$$SS_{X_1}E = \sum_{i=1}^{m}\sum_{j=1}^{m}\frac{\pi_{ij}}{m^3} - \frac{G^2}{N} \tag{42}$$

$$SS_{X_2} = \sum_{j=1}^{m}\frac{\beta_j}{m^3} - \frac{G^2}{N} \tag{43}$$

$$SS_{X_3}B = \sum_{j=1}^{m}\frac{\delta_j}{m^3} - \frac{G^2}{N} \tag{44}$$

$$SS_{X_2}E = \sum_{j=1}^{m}\sum_{k=1}^{m}\frac{\varphi_{kj}}{m^3} - \frac{G^2}{N} \tag{45}$$

$$SS_{X_3} = \sum_{i=1}^{m}\frac{\gamma_i}{m^3} - \frac{G^2}{N} \tag{46}$$

$$SS_{X_3}E = \sum_{j=1}^{m}\sum_{l=1}^{m}\frac{r_{jl}}{m^2} - \frac{G^2}{N} \tag{47}$$

$$SS_{X_4} = \sum_{p=1}^{m}\frac{\tau_p}{m^3} - \frac{G^2}{N} \tag{48}$$

$$SS_{X_4}B = \sum_{j=1}^{m} \frac{\rho_j}{m^3} - \frac{G^2}{N} \quad (49)$$

$$SS_{X_4}E = \sum_{j=1}^{m}\sum_{p=1}^{m} \frac{S_{jp}}{m^2} - \frac{G^2}{N} \quad (50)$$

$$SS_{X_1X_2} = \sum_{i=1}^{m}\sum_{k=1}^{m} \frac{\alpha\beta_{(ik)}}{m^2} - \sum_{i=1}^{m} \frac{\alpha_i}{m^3} - \sum_{k=1}^{m} \frac{\beta_k}{m^3} + \frac{G^2}{N} \quad (51)$$

$$SS_{X_1X_3} = \sum_{i=1}^{m}\sum_{k=1}^{m} \frac{\alpha\gamma_{(il)}}{m^2} - \sum_{i=1}^{m} \frac{\alpha_i}{m^3} - \sum_{l=1}^{m} \frac{\gamma_l}{m^3} + \frac{G^2}{N} \quad (52)$$

$$SS_{X_1X_3} = \sum_{i=1}^{m}\sum_{p=1}^{m} \frac{\alpha\tau_{(ip)}}{m^2} - \sum_{i=1}^{m} \frac{\alpha_i}{m^3} - \sum_{p=1}^{m} \frac{\tau_p}{m^3} + \frac{G^2}{N} \quad (53)$$

$$53SS_{X_2X_3} = \sum_{k=1}^{m}\sum_{l=1}^{m} \frac{\beta\gamma_{(kl)}}{m^2} - \sum_{k=1}^{m} \frac{\beta_k}{m^3} - \sum_{l=1}^{m} \frac{\gamma_l}{m^3} + \frac{G^2}{N} \quad (54)$$

$$SS_{X_2X_4} = \sum_{k=1}^{m}\sum_{p=1}^{m} \frac{\beta\tau_{(kp)}}{m^2} - \sum_{k=1}^{m} \frac{\beta_k}{m^3} - \sum_{p=1}^{m} \frac{\tau_p}{m^3} + \frac{G^2}{N} \quad (55)$$

$$SS_{X_2X_4} = \sum_{k=1}^{m}\sum_{p=1}^{m} \frac{\gamma\tau_{(kp)}}{m^2} - \sum_{l=1}^{m} \frac{\gamma_l}{m^3} - \sum_{p=1}^{m} \frac{\tau_p}{m^3} + \frac{G^2}{N} \quad (56)$$

$SSE =$ By subtraction

Table 1: ANOVA table for the split–lot design in Sudoku Square structure

Source	Sum of squares	Degrees of Freedom	Mean squares	F-Ratio (observed)
X_1	SSX_1	$m-1$	$MSX_1 = SSX_1/m-1$	MSX_1/MSX_1E
$Stage\ 1\ blocks$	SSX_1B	$m-1$	$MSX_1B = SSX_1B/m-1$	MSX_1B/MSX_1E
$Stage\ 1\ sublot\ variation$	SSX_1E	$(m-1)(m-1)$	$MSX_1E = SSX_1E/(m-1)(m-1)$	MSX_1E/MSE
X_2	SSX_2	$m-1$	$MSX_2 = SSX_2/m-1$	MSX_2/MX_2E
$Stage\ 2\ blocks$	SSX_2B	$m-1$	$MSX_2B = SSX_2B/m-1$	MSX_2B/MSE
$Stage\ 2\ sublot\ variation$	SSX_2E	$(m-1)(m-1)$	$MSX_2E = SSX_2E/(m-1)(m-1)$	MSX_2E/MX_2E
X_3	SSX_3	$m-1$	$MSX_3 = SSX_3/m-1$	MSX_3/MSX_3E
$Stage\ 3\ blocks$	SSX_3B	$m-1$	$MSX_3B = SSX_3B/m-1$	MSX_3B/MSX_3E
$Stage\ 3\ sublot\ variation$	SSX_3E	$(m-1)(m-1)$	$MSX_3E = SSX_3E/(m-1)(m-1)$	MSX_3E/MSE
X_4	SSX_4	$m-1$	$MSX_4 = SSX_4/m-1$	MSX_4/MSX_4E
$Stage\ 4\ blocks$	SSX_4B	$m-1$	$MSX_4B = SSX_4B/m-1$	MSX_4B/MSX_4E
$Stage\ 4\ sublot\ variation$	SSX_4E	$(m-1)(m-1)$	$MSX_4E = SSX_4E/(m-1)(m-1)$	MSX_4E/MSE
X_1X_2	SSX_1X_2	$(m-1)(m-1)$	$MSX_1X_2 = SSX_1X_2/(m-1)(m-1)$	MSX_1X_2/MSE
X_1X_3	SSX_1X_3	$(m-1)(m-1)$	$MSX_1X_3 = SSX_1X_3/(m-1)(m-1)$	MSX_1X_3/MSE
X_1X_4	SSX_1X_4	$(m-1)(m-1)$	$MSX_1X_4 = SSX_1X_4/(m-1)(m-1)$	MSX_1X_4/MSE
X_2X_3	SSX_2X_3	$(m-1)(m-1)$	$MSX_2X_3 = SSX_2X_3/(m-1)(m-1)$	MSX_2X_3/MSE
X_2X_4	SSX_2X_4	$(m-1)(m-1)$	$MSX_2X_4 = SSX_2X_4/(m-1)(m-1)$	MSX_2X_4/MSE
X_3X_4	SSX_3X_4	$(m-1)(m-1)$	$MSX_3X_4 = SSX_3X_4/(m-1)(m-1)$	MSX_3X_4/MSE
Error	SSE	By subtraction	$MSE = SSE/df$	
TOTAL	SST	m^4-1		

CONCLUSION

Comparing split-lot designs is complicated by the fact that precision and cost are both functions of the number of sublots per step as well as the total number of items. Introduction of the Block Sum of Squares by default will reduce the Error Sum of Squares which makes it more efficient. This design had captured some variability that has not been accounted for by ordinary split-lot designs.

When a design with 16 wafers experiment in a 4 x 4 Sudoku Square Design Structure is adopted, the procedure and structure becomes the same with the existing procedure and structure, but the Sudoku's additional source of variation still makes it more efficient. The proposed design is recommended in the field of industrial statistics, batch productions or even parts assembly firm. All these areas require stages before accomplishing any task and certain processing stages require splitting discrete units into sublots.

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