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MAXIMIZING PROFIT POTENTIAL IN SUPPLY CHAIN FROM SUPPLIER-RETAILER INVENTORY REPLENISHMENT IN RURAL AREAS

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ABSTRACT
Successful collaborative relationship enhances profitability in the delivery of products or services to customers in a supply. This study focuses on the impact of successful relationships in supply chain management on customer exploration and profitability. By implementing a strong strategy, more customers can be brought into the supply chain, leading to increased productivity and ultimately, higher profitability. The main objective is to determine the optimal quantity levels in supplier relationships that minimize long-term costs in the supply chain. A mathematical model incorporating shipment consolidation was utilized to derive the profit function. Numerical examples were then employed to illustrate the developed models. The results demonstrate variations in total quantity and relevant costs based on factors such as retailer input, supplier replenishment quantities, optimal order quantities, and total relevant costs within the supply chain, ultimately contributing to enhanced profitability. Our result shows that. there is an increase in the supply chain profitability when the retailer orders greater number of items from the supplier.

Keywords: Delivery, Inventory, Replenishment, Shipment consolidation, Supply chain

INTRODUCTION

Supply chain management (SCM) is the management of operations that are involved in the procurement of raw materials, its processing into finished goods, and distribution to the end consumer. That is, supply chain management is a process of transforming raw material into intermediate products and thereafter into finished items, which are then distributed to consumers (Subham et al., 2023). According to Md. Rasidul et al. (2023), supply chain management relies heavily on logistic operations that encompass everything from sourcing raw materials to delivering the final product. That is, strategically managing the procurement, movement and storage of materials, parts, finished inventory through the manufacturing industry and its marketing channels in such a way that profitability are maximized through the cost-effective fulfillment of orders. This process involves product development, distribution, finance and customer services (O'Byrne, 2016); in which different actors play different roles for the distribution of the final products to the clients (Subham et al., 2023).

Supply chain management is the heart of any business, Nurul et al, (2020). It can be achieved not only by improving processes internally but also by working with suppliers, customers and most notably partners like intermediaries and distributors (Thu-Trang and Toan, 2020). According to Anisha et al., (2023), supply chain management (SCM) is an important element of competitive strategy to increase the efficiency of organization and profits no matter the services they render or product produced. Its practices attracted much attention from both practitioners and researchers (Bag et al., 2020); it has a significant effect on firm performance in the Fast-moving consumer goods (Ayorinde, 2024); and there is profitability when the operational efficiency is boosted (Abigail, 2024).

The profit maximization is when appropriate logistics are used to convey the demand to the supply point (supplier) and delivers the supply to the demand point (retailer) and by exploring more customers from the rural areas.

Nikos (2007) as cited in Simon (2012), maximized profit in supply chain processes should involve a combination of people, systems and technology in collaboration with external partners. Adyang (2012) argued that proper supply chain practices led to higher profitability. Muthoni (2010) suggested that enhancement of operational excellence in the retail service increased service quality, customer satisfaction and service performance.

Zohreh and Amir (2018) explained that the most important features that can be mentioned in order to manage supply chain orders for profitability are long-term orders earnings, increased customer loyalty and long-term cooperation with the company. Minimizing the total costs, involves forward flows in order to reduce fixed and variable costs and increase customer responsiveness. Applied dispatch volume limit increases both the ordering cycle and the total annual costs. It keeps inventory value at the lowest possible rate, operates several processes such as Vendor Managed Inventory [VMI], to manage inventory of the customer and store goods at the customer location (Judit et al., 2017). They should employ a faster means of delivery of the end products to customers with greater accuracy by using track shipments than you could with a manual system to ensure the products reached their customers destinations safely and on time (*O'Byrne, 2016)*.

Also to maximize profits in the supply chain, more customers should be explored from the rural areas. The problem is critical due to the fact that transportation infrastructure may be limited; by using the company heavy vehicles

This study seeks to investigate the possibility of an increase in profitability in the supply chain when management employs smaller vehicles and shipment consolidation for the delivering of finished products to customers located in rural areas.

Shipment consolidation allows small loading vehicles to enhance the service level since many of the customers have small scale businesses and are unable to buy a huge quantity and combination of two or more orders or multiple small batches into a single larger quantity to be dispatched for delivery (Qishu, 2011). Since an individual in the rural area is unable to get a full truckload, the supplier will hold order arriving from customers in that destination (outbound quantities) for a period of time, and then dispatch them on the same vehicle (Bookbinder *et al., 2012*). It is believed that this strategy will ensure continuous performance improvement of the supply chain from huge losses to profitability and a higher level of production. This study shall answer the following questions: what is the optimal values of the supplier quantity ($\boldsymbol{S_Q}$) and retailer ($\boldsymbol{R_Q}$) that minimized the expected long-run average cost in a supply chain management? What is the Variation of the optimality of replenishment quantity of the supplier and retailer and total relevant cost?

To answer these questions we considered that shipment is made only when a certain quantity of outstanding demand is accumulated. A consolidation cycle begins immediately after the previous dispatch, and ends upon arrival of the order which causes the accumulative weight to reach the supplier or the retailer quantity. The cycle length is random, depending on the interarrival times between orders. The load dispatched to replenish the supplier's inventory from the manufacturer is often greater than that delivered to the retailers from the supplier.

MATERIALS AND METHODS

In this study, a group of retailers who initiate the ordered replenishment quantity from the supplier who is also a dealer of the same products was considered. The supplier replenished her quantity from the manufacturer. The quantity ordered from the manufacturer by the supplier is not the same as the quantity supplied to the retailers. The ordered quantity by the retailers from the supplier is consolidated to the capacity of the delivery vehicle and may vary due to stochastic demand from customers. Demands from supplier to the retailer and from the retailer to consumers follow a compound Poisson distribution.

We also considered the delivery cycle to be the time interval between two consecutive deliveries. The delivering cost of the supplier and retailers is composed of a fixed cost which is incurred when there is a positive replenishment quantity. The delivering cost is a linear variable cost and linearly proportional to the quantity delivered. This cost includes; cost for loading products on vehicles from the supplier consolidated center, transporting them and unloading at the retailer center as shown in Figure 1. The reorder points of the supplier and the retailers are determined to be zero.

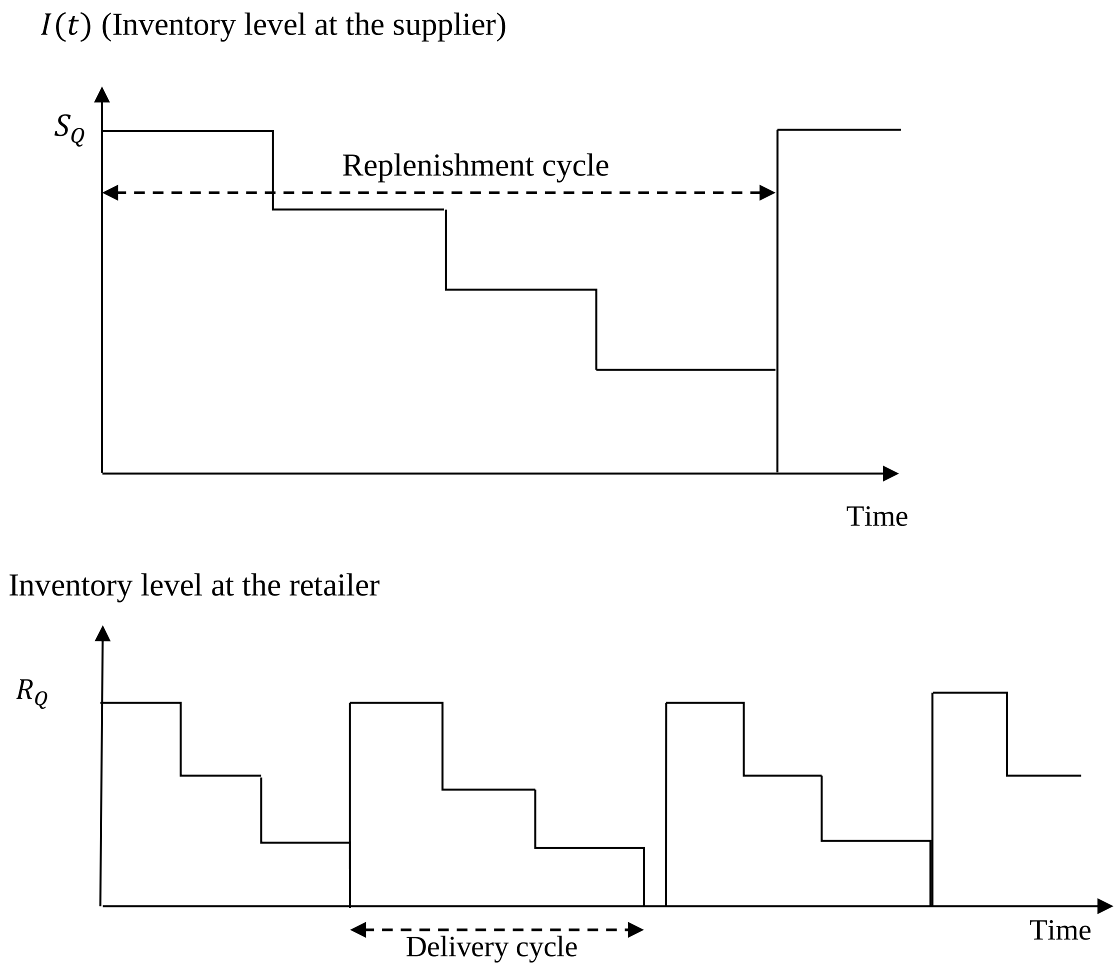

Figure 1: Inventory levels of the supplier and retailer

Mathematical Model

The mathematical model developed in Jac-Hun (2010) was considered for the replenished quantity delivered to the retailer center from the supplier.

Let Q be the size of a replenishment quantity and r the number of deliveries in a cycle, S_Q the replenishment quantity of the supplier and R_Q the delivery quantity of the retailers

T_i: Inter-arrival time between the arrivals of the $(n-1)^{st}$ and the n^{th} retailers

A_t: Arrival time of the n^{th} retailer ($A_t = \sum_{i=1}^{n} T_i$)
λ: Arrival rates of the customers
$\frac{1}{\lambda}$: The mean of the inter-arrival time of customers
$N(t)$: Number of orders that have arrived by time t, ($N(t) = max\{n/A \leq t_t\}$);
it is assumed that this follows the Poisson distribution with mean λt
d_n: Demand quantity (or weight) of the n^{th} retailer
μ: Mean of demand quantities
σ^2: Variance of the demand quantities
D_n : Cumulative demand quantity of the first n retailers ($D_n = \sum_{i=1}^{n} d_i$)
$N_2(x)$: Minimum number of retailers whose cumulative demand quantity exceeds, i.e., $N_2(x) = min\{n/D_n > x\}$
$L^j(x)$: Minimum number of retailers whose cumulative demand quantity exceeds x in the j^{th} deliver cycle.
h_S: The inventory holding cost per unit per unit time at the supplier
h_R: The inventory holding cost per unit per unit time at the retailer.
$I_S(t)$: Inventory level of the supplier at time t
$I_R(t)$: Inventory level of the retailer at time t
C_R: The cost replenishing one unit at supplier
A_R:The fixed cost of replenishing the inventory at the retailer from the supplier
C_D: The cost of delivering one unit from the supplier to the retailer;
A_D: The fixed cost of delivering of a shipment from the supplier to the retailer;
K: Number of delivery cycles within replenishment cycles (a random variable)
$F(x)$: Distribution of $D_{N_2(S_R)}$, the sum of demand quantities of the customers that arrive during a delivery cycle, i.e., $F(x) = P\{ D_{N_2(S_R)} \leq x \}$
$F^{(k)}(x)$: k-fold convolution of $F(x)$
$C(S_Q, R_Q)$: The expected long-run average cost incurred when the order-up-to-levels of the supplier and the retailer are S_Q and R_Q respectively.

Assumptions of the Model
To achieve the quantity-based dispatching model for the coordinated supply chain, the followings are the assumptions of the model:
a) The inventory level is under continuous review
b) The load is dispatched whenever the size of demands is accumulated
(c) The mean and variance of the quantities is known to each supplier
(d) Inter-arrival times of the order quantities are mutually independent.
(e) Shortages are not allowed.
(f) There are an integer number of delivery cycles in each replenishment cycle.
(g) The distances between the supplier and retailers are not very large.
h) Lead times for inventory replenishments are fixed and negligibly short, often measured in hours rather than days or weeks. That is,
i) The supplier and the retailers are located within a short distance from the manufacturing plant, often within the same region (Proximity to Suppliers). This geographical closeness allows for quick transportation of parts.
ii) The suppliers operate on a schedule that aligns with the manufacturer's production needs, delivering components multiple times a day. This ensures that the manufacturer has just enough inventory to meet production demands without overstocking. (Frequent Deliveries)

An optimal solution for our problem was obtained by applying the renewal theory (Çetinkaya and Lee, 2000). It is assumed that the delivery decision is made on a recurrent basis.
Let T_i $(i = 1, 2, \cdots, K)$ be instants the demands have been accumulated to a level of S_Q and R_Q and a dispatch takes place. T_K, the time instant an inventory replenishment took place and the replenishment arrives (lead time is assume to be zero). The main objective is to obtain the optimal values (S_Q) and (R_Q) that will minimize the average long-run cost of the system, to using the renewal reward theorem, given by

$$TC(\mathrm{S_Q}, \mathrm{R_Q}) = \frac{\mathrm{E[Replenishment\ cycle\ cost]}}{\mathrm{E[Replenishment\ cycle\ length]}} \qquad (1)$$

We considered:
inter-arrival times of demands $\{T_n: n \geq 1\}$ are exponentially distributed.
The parameter, λ. $d_n, n = 1, 2, 3, \cdots$ are random variables representing the demand quantity of the n^{th} customer, and d_n's are assumed to be identically and independently distributed and are independent of $N_1(t)$ as well.
The variables involved in the replenishment/delivery cost per cycle are obtained as follows:
1) For expectation delivery cycle length, let $N_2(R_Q)$ equal the delivery cycle for the arrival number of customers at the retailer center. Then by applying Wald's equation (Ross, 1996), we obtain;
The expected delivery cycle length $= E[T_i]E[N_2(R_Q)]$.
But the expected inter-arrival time of the customer $E[T_i] = \frac{1}{\lambda}$ implies $[N_2(R_Q)] = \frac{R_Q}{E[d_n]} + 1$
Since $\lim_{S_R \to \infty} \frac{N_2(R_Q)-1}{R_Q} = \frac{1}{E[d_n]}$ as in Ross (1996)

$$\text{Thus,} \quad E[Length\ of\ Delivery\ ] = E\left[A_{N_2(R_Q)}\right] = E\left[\sum_{i=1}^{N_2(R_Q)} T_i\right] = E[T_i]E[N_2(R_Q)] = E[T_n]\left(\frac{R_Q}{E[d_n]} + 1\right) = \frac{R_Q+\mu}{\lambda\mu} \qquad (2)$$

2) Retailer expected delivery quantity depends on
the number of customers arriving at the retailer denoted by $N_2(R_Q)$
the delivery quantity given as $D_{N_2(R_Q)}$
That is
$E\left[D_{N_2(R_Q)} - R_Q\right] = \frac{E[d_n^2]}{2E[d_n]}$ by the inspection paradox (Ross, 1996),
Therefore the retailer expected delivery quantity

$$E[N_2(R_Q)] = R_Q + E\left[D_{N_2(R_Q)} - R_Q\right] = R_Q + \frac{E[d_n^2]}{2E[d_n]} = R_Q + \frac{Var[d_n]+E[d_n]^2}{2e[d_n]} = \frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu} \qquad (3)$$

3) The expected number of delivery within a replenishment cycle is obtained by first of all;
Obtaining the expected value and variance of the number of delivery within a replenishment cycle.
Considering $D_{N_2(R_Q)}$in equation (2) to follow Poisson distribution with parameter $\frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}$ i.e., $E\left[D_{N_2(R_Q)}\right] = \frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}$, where the value of $D_{N_2(R_Q)} \leq R_Q$.

Taking $F^{(k)}(S_Q)$ as the distribution function of the Poisson distribution with parameter $k\left(\frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}\right)$, then $F^{(k)}(S_Q)$ is the $k-fold$ convolution of the Poisson with $\frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}$.

K is expressed as $K=min\left\{k/\sum_{j=1}^{k}D_{L^j(R_Q)}>S_Q\right\}$, The event $\{K\geq k\}$ is equivalent to $\left\{\sum_{j=1}^{k-1}D_{L^j(R_Q)}\leq S_Q\right\}$ and hence $P\{K\geq k\}=P\left\{\sum_{j=1}^{k-1}D_{L^j(R_Q)}\leq S_Q\right\}=F^{(k-1)}(S_Q)$.

Therefore, the distribution function of K is expressed as

$$P\{K\leq k\}=1-F^{(k)}(S_Q)=1-\sum_{i=0}^{S_Q}\frac{k\left(\frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}\right)^i esp\left(-k\left(\frac{2\mu R_Q+\mu^2+\sigma^2}{2\mu}\right)\right)}{i!} \tag{4}$$

This equation represents the distribution function of the (S_Q+1)-stage Erlang (Gamma) distribution with $mean=\frac{2\mu(S_Q+1)}{2\mu R_Q+\mu^2+\sigma^2}$ and $Variance=\frac{4\mu^2(S_Q+1)}{(2\mu R_Q+\mu^2+\sigma^2)^2}$.

Therefore, the approximate expected value of the number of delivery is given as

$$E[K]=\frac{2\mu(S_Q+1)}{2\mu R_Q+\mu^2+\sigma^2} \tag{5}$$

and the variance of the number of deliveries

$$Var[K]=\frac{4\mu^2(S_Q+1)}{(2\mu R_Q+\mu^2+\sigma^2)^2} \tag{6}$$

4) Expected length of Replenishment cycle is obtained from Wald's equation (Ross,1996), by multiplying expected length of delivery cycle with expected number of delivery cycles within a replenishment. That is,

$$E\left[\sum_{j=1}^{K}X_i\right]=E[K]E[X]$$

$where\ X_1,X_2,\ldots$ are independent and identically distributed random variables with finite expectations and K is a stopping time for $X_1,X_2,\ldots$ such that $E[K]<\infty$. The stopping time for $X_1,X_2,\ldots$ if the event $\{K=k\}$ is independent of $X_{k+1},X_{k+2,}\ldots,k\geq 1$. Thus, the approximate expected replenishment cycles is obtain from equations (2) and (4) given as,

$$E[Replenishment\ cycle\ length]=E[Delivery\ cycle\ length].E[K]$$
$$=\frac{2(S_Q+1)(R_Q+\mu)}{\lambda(2\mu R_Q+\mu^2+\sigma^2)} \tag{7}$$

5) Expected replenishment quantity to the supplier is obtained by multiplying the expected delivery quantity in a delivery cycle by expected number of delivery cycles within a replenishment cycle since there are K delivery cycles. Then From (3) and (5) we get

$$E[Replenishment\ quantity]=E[K]E\left[D_{2(R_Q)}\right]=S_Q+1 \tag{8}$$

6) To obtain the expected inventory holding cost at the retailer in a delivery cycle we first express the inventory level of the retailer in a delivery cycle as;

$$I_R(t)=\begin{cases}R_Q & if\ 0\leq t<F_1\\ R_Q-D_1 & if\ F_1\leq t<F_2\\ R_Q-D_2 & if F_2\leq t<F_3\\ \vdots & \\ R_Q-D_{N_2(R_Q)-1} & if\ F_{N_2(R_Q)-1}\leq t\leq F_{N_2(R_Q)}\end{cases}$$

Thus, the expected inventory holding cost is obtained by,

$$h_RE\left[R_QT_1+(R_Q-D_1)T_2+(R_Q-D_2)T_3+\cdots+\left(R_Q-D_{N_2(R_Q)-1}\right)T_{N_2(R_Q)}\right]$$
$$=h_RE\left[R_Q\sum_{i=1}^{N_2(R_Q)}T_i-\sum_{i=1}^{N_2(R_Q)-1}D_iT_{i+1}\right]$$
$$=h_RR_QE[T_i]E[N_2(R_Q)]-h_RE\left[E\left[\sum_{i=1}^{N_2(SR_Q)-1}D_iT_{i+1}\right]\right]$$
$$=h_RR_QE[T_i]E[N_2(R_Q)]-h_RE\left[E\left[\sum_{i=1}^{N_2(R_Q)-1}D_iT_{i+1}/N_2(R_Q)=m+1\right]\right]$$
$$=h_RR_QE[T_i]E[N_2(R_Q)]-h_RE\left[E\left[\sum_{i=1}^{m}D_iT_{i+1}/N_2(R_Q)=m+1\right]\right]$$
$$=h_RR_QE[T_i]E[N_2(R_Q)]-h_RE[T_i]E\left[E\left[\sum_{i=1}^{m}D_i/N_2(R_Q)=m+1\right]\right]$$

Here, we assume $d_i,i=1,\cdots,N_2(R_Q)-1$, follows an exponential distribution and $D_1,\cdots,D_m$, for $m=N_2(R_Q)-1$, the cumulative demand quantities, to be mutually independent random variables following uniform distribution with range $(0,R_Q)$ from the relationship between the arrival times of the Poisson arrival process and the uniform distribution (Ross, 1996).

Thus, $E[D_i]$ as $\frac{R_Q}{2}$, and

$$E\left[\sum_{i=1}^{m}D_iT_{i+1}/N_2(R_Q)=m+1\right]=mE[D_iT_{i+1}]=mE[D_i]E[T_{i+1}]=m\frac{1}{\lambda}\frac{R_Q}{2}.$$

Thus,

$$E\left[E\left[\sum_{i=1}^{m}D_iT_{i+1}/N_2(R_Q)=m+1\right]=E\left[m\frac{1}{\lambda}\frac{R_Q}{2}\right]=\frac{1}{\lambda}\frac{R_Q}{2}E[m]\right]=\frac{1}{\lambda}\frac{R_Q}{2}E[N_2(R_Q)-1]$$

Since $E[N_2(R_Q)]=\frac{R_Q}{\mu+1}$, the expected inventory holding cost at the retailer in a delivery cycle is estimated as

$$h_RR_QE[T_i]E[N_2(R_Q)-1]-h_R\frac{1}{\lambda}\frac{R_q}{2}E[N_2(R_Q)-1]\ =h_R\left(R_Q\frac{1}{\lambda}\left(\frac{R_Q}{\mu}+1\right)-\frac{1}{\lambda}\frac{R_Q}{2}\frac{R_Q}{\mu}\right)$$
$$=\frac{h_RR_Q(R_Q+2\mu)}{2\lambda\mu} \tag{9}$$

Also, first expression of the inventory level of the supplier is given as

$$I_S(t) = \begin{cases} S_Q & if\ 0 \le t < F_{L^j(R_Q)} \\ S_Q - D_{L^j(R_Q)} & if\ F_{L^j(R_Q)} \le t < \sum_{j=1}^{2} F_{L^j(R_Q)} \\ S_Q - \sum_{j=1}^{2} D_{L^j(R_Q)} & if\ \sum_{j=1}^{2} F_{L^j(R_Q)} \le t < \sum_{j=1}^{3} F_{L^j(R_Q)} \\ \vdots & \\ S_Q - \sum_{j=1}^{K-1} D_{L^j(R_Q)} & if\ \sum_{j=1}^{K-1} F_{L^j(R_Q)} \le t \le \sum_{j=1}^{K} F_{L^j(R_Q)} \end{cases}$$

Thus, the expected inventory holding cost at the supplier in a replenishment cycle is given as

$$h_S E\left[S_S F_{L^j(R_Q)}\right] + \left(S_Q - D_{L^j(R_Q)}\right) F_{L^2(R_Q)} + \cdots + \left(S_Q - \sum_{j=1}^{K-1} D_{L^j(R_Q)}\right) F_{L^K(R_Q)}$$

$$= h_S E\left[S_Q \sum_{j=1}^{K} F_{L^j(R_Q)} - \sum_{i=2}^{K}\left\{F_{L^j(R_Q)} \sum_{j=1}^{2} D_{L^j(R_Q)}\right\}\right]$$

$$= h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E\left[\sum_{i=2}^{K}\left\{F_{L^j(R_Q)} \sum_{j=1}^{i-1} D_{L^j(R_Q)}\right\}\right]$$

$$= h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E\left[\sum_{i=2}^{K}\left\{F_{N_2(R_Q)} \sum_{j=1}^{i-1} D_{L^j(R_Q)}\right\}\right]$$

$$= h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E\left[E\left[\sum_{i=2}^{K}\left\{F_{N_2(R_Q)} \sum_{j=1}^{i-1} D_{L^j(R_Q)}\right\}/K = k\right]\right]$$

$$= h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E\left[E\left[F_{N_2(R_Q)}\right] E\left[\sum_{i=2}^{K} \sum_{j=1}^{i-1} D_{L^j(R_Q)}/K = k\right]\right]$$

$$= h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E[F_{N_2(R_Q)}] E\left[E[\sum_{i=2}^{K} \sum_{j=1}^{i-1} D_{L^j(R_Q)}/K = k]\right].$$

But

$$E\left[E[\sum_{i=2}^{K} \sum_{j=1}^{i-1} D_{L^j(R_Q)}/K = k]\right] = E\left[E[\sum_{j=1}^{k-1}(k-j)\, D_{L^j(R_Q)}/K = k\right]$$

$$= E\left[D_{N_{2(R_Q)}}\right] E\left[E[\sum_{j=1}^{k-1}(k-j)/K = k]\right] = E\left[D_{N_{2(R_Q)}}\right] E\left[\frac{K^2 - K}{2}\right]$$

$$= E\left[D_{N_{2(R_Q)}}\right] \frac{Var[K] + E[K]^2 - E[K]}{2}$$

Thus, the expected inventory holding cost of the supplier is given as

$$h_S S_Q E[K] E\left[F_{N_{2(R_Q)}}\right] - h_S E\left[F_{N_2(R_Q)}\right] E\left[D_{L^j(R_Q)}\right] \frac{Var[K] + E[K]^2 - E[K]}{2}$$

$$= \frac{h_S(S_Q+1)(S_Q+\mu)(4\mu S_Q + 4\mu R_Q + 2\mu^2 + 2\sigma^2 - 8\mu)}{4\lambda\mu(2\mu R_Q + \mu^2 + \sigma^2)} \quad (10)$$

By the renewal reward theorem the optimal values (S_Q) and (R_Q) that minimizes the average long-run cost of the system is given by

$$TC(S_Q, R_Q) = \frac{\text{E[Replenishment cycle cost]}}{\text{E[Replenishment cycle length]}}$$

Adding equations (3), (5), (8), (9), and (10) and substituting in expected replenishment cost, we obtain:

$$TC(S_Q, R_Q) = \frac{\lambda\mu A_R}{(S_Q+1)} + \frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R + C_D)}{2(R_Q+\mu)} + \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)(R_Q+\mu)} + \frac{h_S(S_Q+1)}{2} + \frac{(h_S+h_R)(R_Q+\mu)}{2} + \frac{h_S(\sigma^2-\mu^2-6\mu)}{4\mu} + \lambda\mu(C_R + C_D) \quad (11)$$

Since all demands at the planned period will be eventually satisfied through the replenishment and delivery processes, the cost terms related to unit replenishment cost (C_R) and unit delivery cost (C_D) are not affected by the decision variables, (i.e., the order-up- to-levels). That is, the same quantity is replenished regardless of the order-up-to levels.

Therefore the minimization problem that will minimize the average long-run cost is given by

$$\min_{Q,T} TC(S_Q, R_Q)$$

$$Subject\ to\ S_Q, R_Q \ge 0$$

The optimal solution on $S_Q{}^*$ $and, R_Q{}^*$ are obtained from (11) by differentiating the average long-run cost $TC(S_Q{}^*, R_Q{}^*)$with respect to R_Q and S_Q, and equating to zero.

$$\frac{\partial C(S_Q,R_Q)}{\partial S_Q} = \frac{-\lambda\mu A_R}{(S_Q+1)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)^2(R_Q+\mu)} + \frac{h_S}{2} \quad (12)$$

and

$$\frac{\partial C(S_Q,R_Q)}{\partial R_Q} = -\frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)}{2(R_Q+\mu)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)(R_Q+\mu)^2} + \frac{(h_S+h_R)R_Q}{2} \quad (13)$$

We note that the cost function $TC(S_Q, R_Q)$ is strictly convex for any positive S_Q and R_Q. Thus, the unique global minimum for any positive S_Q and R_Q can be obtained by setting equations (12) and (13) to zero.

$$\frac{\partial C(S_Q,R_Q)}{\partial S_Q} = \frac{-\lambda\mu A_R}{(S_Q+1)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)^2(R_Q+\mu)} + \frac{h_S}{2} = 0 \text{ and}$$

$$\frac{\partial C(S_Q,R_Q)}{\partial R_Q} = -\frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)}{2(R_Q+\mu)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)(R_Q+\mu)^2} + \frac{(h_S+h_R)R_Q}{2} = 0$$

That is for $\frac{\partial C(S_Q,R_Q)}{\partial S_S} = 0$, we get

$$0 = -\frac{\lambda\mu A_R}{(S_Q+1)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)^2(R_Q+\mu)} + \frac{h_S}{2}$$

$$\frac{h_S}{2} = \frac{\lambda\mu A_R}{(S_Q+1)^2} + \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)^2(R_Q+\mu)}$$

$$(S_Q+1)^2 = \frac{2\lambda\mu A_R(R_Q+\mu) + \lambda(\sigma^2-\mu^2)A_R}{2h_S(R_Q+\mu)} = \frac{2\lambda\mu A_R R_Q + \lambda A_R(\sigma^2+\mu^2)}{2h_S(R_Q+\mu)}$$

$$S_Q = \sqrt{\frac{2\lambda\mu A_R R_Q + \lambda A_R(\sigma^2+\mu^2)}{2h_S(R_Q+\mu)}} - 1 \quad (14)$$

For value of R_Q, $\frac{\partial C(S_Q,R_Q)}{\partial R_Q} = 0$

$$0 = -\frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)}{2(R_Q+\mu)^2} - \frac{\lambda(\sigma^2-\mu^2)A_R}{2(S_Q+1)(R_Q+\mu)^2} + \frac{(h_S+h_R)R_Q}{2}$$

$$0 = \frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)(S_Q+1) - \lambda(\sigma^2-\mu^2)A_R + (h_S+h_R)(S_Q+1)(R_Q+\mu)^2 R_Q}{2(S_S+1)(S_R+\mu)^2}$$

$$(h_S+h_R)(S_Q+1)(R_Q+\mu)^2 R_Q = 2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)(S_Q+1) + \lambda(\sigma^2-\mu^2)A_R$$

$$(R_Q+\mu)^2 = \frac{\{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)\}(S_Q+1) + \lambda(\sigma^2-\mu^2)A_R}{(h_S+h_R)(S_Q+1)}$$

$$R_Q = \sqrt{\frac{\{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)\}(S_Q+1) + \lambda(\sigma^2-\mu^2)A_R}{(h_S+h_R)(S_Q+1)}} - \mu \tag{15}$$

The optimal pair is then given by

$$(R_Q^*, S_Q^*) = \left(\sqrt{\frac{\{2\lambda\mu A_D - h_R\mu^2 + \lambda(\sigma^2-\mu^2)(C_R+C_D)\}(S_Q+1) + \lambda(\sigma^2-\mu^2)A_R}{(h_S+h_R)(S_Q+1)}} - \mu, \sqrt{\frac{2\lambda\mu A_R R_Q + \lambda A_R(\sigma^2+\mu^2)}{2h_S(R_Q+\mu)}} - 1\right) \tag{16}$$

The approximated cost function is derived from equation (11) by letting $\mu^2 = \sigma^2$ since demand is compound Poisson and demand quantities followed an exponential distribution.

$$TC(S_Q, R_Q) = \frac{\lambda\mu A_R}{(S_Q+1)} + \frac{2\lambda\mu A_D - h_R\mu^2 + \lambda(\mu^2-\mu^2)(C_R+C_D)}{2(R_Q+\mu)} + \frac{\lambda(\mu^2-\mu^2)A_R}{2(S_Q+1)(R_Q+\mu)} + \frac{h_S(S_Q+1)}{2} + \frac{(h_S+h_R)(R_Q+\mu)}{2} + \frac{h_S(\mu^2-\mu^2-6\mu)}{4\mu} + \lambda\mu(C_R+C_D)$$

$$TC(R_Q, S_Q) = \frac{\lambda\mu A_R}{(S_Q+1)} + \frac{2\lambda\mu A_D - h_R\mu^2}{2(R_Q+\mu)} + \frac{h_S(S_Q+1)}{2} + \frac{(h_S+h_R)(R_Q+\mu)}{2} - \frac{3h_S}{2} + \lambda\mu(C_R+C_D)$$

$$TC(R_Q, S_Q) = \frac{\lambda\mu A_R}{(S_Q+1)} + \frac{\lambda\mu A_D}{(R_Q+\mu)} - \frac{h_R\mu^2}{2(R_Q+\mu)} + \frac{h_S(S_Q+1)}{2} + \frac{h_S(R_Q+\mu)}{2} + \frac{h_R(R_Q+\mu)}{2} - \frac{3h_S}{2} + \lambda\mu(C_R+C_D)$$

$$TC(R_Q, S_Q) = \frac{\lambda\mu A_R}{(S_Q+1)} + \frac{\lambda\mu A_D}{(R_Q+\mu)} + h_S\left\{\frac{S_Q+R_Q+\mu}{2} - 1\right\} + \frac{h_R}{2}\left\{R_Q + \mu - \frac{\mu^2}{R_Q+\mu}\right\} + \lambda\mu(C_R+C_D) \tag{17}$$

The optimal pair is then given by

$\frac{\partial C(S_Q,R_Q)}{\partial S_Q} = 0$, we get $-\frac{\lambda\mu A_R}{(S_Q+1)^2} + \frac{h_S}{2} = 0$

$$\frac{\lambda\mu A_R}{(S_Q+1)^2} = \frac{h_S}{2} \Rightarrow (S_Q+1)^2 = \frac{2\lambda\mu A_R}{h_S}$$

$$S_Q^* = \sqrt{\frac{2\lambda\mu A_R}{h_S}} - 1 \tag{18}$$

For value of R_Q,

$$\frac{\partial C(S_Q,R_Q)}{\partial R_Q} = -\frac{2\lambda\mu A_D - h_R\mu^2}{2(R_Q+\mu)^2} + \frac{(h_S+h_R)}{2} = 0$$

$$\frac{2\lambda\mu A_D - h_R\mu^2}{2(R_Q+\mu)^2} = \frac{(h_S+h_R)}{2} \Rightarrow (R_Q+\mu)^2 = \frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}$$

$$R_Q^* = \sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}} - \mu \tag{19}$$

$$(R_Q^*, S_Q^*) = \left(\sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}} - \mu, \sqrt{\frac{2\lambda\mu A_R}{h_S}} - 1\right) \tag{20}$$

The corresponding optimal costs is

$$TC(R_Q, S_Q) = \frac{\lambda\mu A_R}{\left(\sqrt{\frac{2\lambda\mu A_R}{h_S}} - 1 + 1\right)} + \frac{\lambda\mu A_D}{\left(\sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}} - \mu + \mu\right)} + h_S\left\{\frac{\sqrt{\frac{2\lambda\mu A_R}{h_S}} - 1 + \sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}} - \mu + \mu}{2} - 1\right\} + \frac{h_R}{2}\left\{\sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}} - \mu + \mu - \frac{\mu^2}{\sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}} - \mu + \mu}\right\} + \lambda\mu(C_R+C_D)$$

$$TC(R_Q, S_Q) = \frac{\lambda\mu A_R}{\left(\sqrt{\frac{2\lambda\mu A_R}{h_S}}\right)} + \frac{\lambda\mu A_D}{\left(\sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}}\right)} + h_S\left\{\frac{\sqrt{\frac{2\lambda\mu A_R}{h_S}} - 1 + \sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}}}{2} - 1\right\} + \frac{h_R}{2}\left\{\sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}} - \frac{\mu^2}{\sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}}}\right\} + \lambda\mu(C_R+C_D)$$

$$TC(R_Q^*, S_Q^*) = \frac{\sqrt{2\lambda\mu A_R h_S}}{2} + \lambda\mu A_R\frac{\sqrt{(2\lambda\mu A_D - h_R\mu^2)(h_S+h_R)}}{2\lambda\mu A_D - h_R\mu^2} + h_S\left\{\frac{\sqrt{\frac{2\lambda\mu A_R}{h_S}} + \sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}} - 3}{2}\right\} + \frac{h_R}{2}\left\{\sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}} - \frac{\mu^2\sqrt{2\lambda\mu A_D - h_R\mu^2(h_S+h_R)}}{2\lambda\mu A_D - h_R\mu^2}\right\} + \lambda\mu(C_R+C_D) \tag{21}$$

This is a lower bound of (R_Q^*, S_Q^*) for any positive values of R_Q and S_Q,

$$\text{i.e}\, TC(R_Q^*, S_Q^*) \geq \frac{\sqrt{2\lambda\mu A_R h_S}}{2} + \lambda\mu A_R\frac{\sqrt{(2\lambda\mu A_D - h_R\mu^2)(h_S+h_R)}}{2\lambda\mu A_D - h_R\mu^2} + h_S\left\{\frac{\sqrt{\frac{2\lambda\mu A_R}{h_S}} + \sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}} - 3}{2}\right\} + \frac{h_R}{2}\left\{\sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S+h_R)}} - \frac{\mu^2\sqrt{2\lambda\mu A_D - h_R\mu^2(h_S+h_R)}}{2\lambda\mu A_D - h_R\mu^2}\right\} + \lambda\mu(C_R+C_D) \tag{22}$$

for any $R_Q, S_Q \geq 0$.

RESULTS AND DISCUSSION

The optimal values of S_Q and R_Q that minimized the expected long-run average cost from the models developed corresponded with Jae-Hun (2010), That is,

(a) If $2\lambda\mu A_R < h_S$ and $2\lambda\mu A_D < \mu(h_S + 2h_R)$, $S_Q{}^* = 0$ and $R_Q{}^* = 0$.

Since the cost for replenishing products is less than the cost of holding inventories, both the supplier and the retailer use a policy which satisfies the requirement from downstream members of the supply chain without carrying inventory but with immediate replenishments from upstream members.

b) If $2\lambda\mu A_R < h_S$ and $2\lambda\mu A_D \geq \mu(h_S + 2h_R)$, $S_Q{}^* = 0$ and $R_Q{}^* = \sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S + h_R)}} - \mu$

Since the cost of holding inventories at the supplier is greater than the cost of replenishing products from the outside supplier, the supplier does not hold inventory. There is a single delivery cycle within a replenishment cycle.

c) If $2\lambda\mu A_R \geq h_S$ and $2\lambda\mu A_D \geq \mu(h_S + 2h_R)$,

$S_Q{}^* = \sqrt{\frac{2\lambda\mu A_R}{h_S}} - 1$ and $R_Q{}^* = 0$.

Since the cost of holding inventories at the retailer is greater than the cost of delivering products from the supplier to the retailer, when needed the retailer does not hold inventory. In this case, there may be multiple delivery cycles within a replenishment cycle, that is, replenishment occurs when the cumulative demands exceeds the order-up-to level of the supplier while delivery occurs when there is demand at retailer.

d) If $2\lambda\mu A_R \geq h_S$ and $2\lambda\mu A_D < \mu(h_S + 2h_R)$,

$S_Q{}^* = \sqrt{\frac{2\lambda\mu A_R}{h_S}} - 1$ and

$R_Q{}^* = \sqrt{\frac{2\lambda\mu A_D - h_R\mu^2}{(h_S + h_R)}} - \mu$.

Both members hold inventories, since the cost of holding inventories is less than the cost of replenishment or delivery. If the order-up-to level of the supplier is smaller than that of the retailer, there is a single delivery cycle within a replenishment cycle. Otherwise, there may be multiple delivery cycles within a replenishment cycle.

Numerical Example

In this section, we include a numerical example as in Lan, et al, (2014).to illustrate the proposed model and their associated optimal policies. Let's simulate some hypothetical data for a company's inventory replenishment costs. Consider the following parameters:

Fixed replenishment cost per unit $(A_R) = 200$,

Fixed delivery cost per unit $(A_D) = 10$,

Retailer holding cost per unit per month $(h_R) =$ Supplier holding cost per unit per month$(h_S) = 1$. The arrival rate, $(\lambda) = 1$. The mean of the demand size, $(\mu) = 1$,

Retailer carrying cost per unit $(C_R) =$ delivery cost per unit $((C_D) = 1$.

The optimal solution for the retailer quantity $R_Q{}^* = 2.08$ from equation (19) and supplier quantity $S_Q{}^* = 19.00$ from equation (18). Based on the data, the total minimum cost $TC(R_Q{}^*, S_Q{}^*) = 1257.53$ from equation (22).

Using the algorithm, the computational results are shown in Table 1 and the Fixed delivery cost per unit verses Retailer's quantity is shown graphically in Figure 1. From this Table 1, the Fixed delivery cost per unit verses Supplier's quantity is shown in Figure 2. As in the literature of (Sundara and Uthayakumar ,2017).

Sensitivity analysis

To illustrate the behaviour of the parameters we vary one parameter at a time while keeping other at based values. The values of $R_Q{}^*$, $S_Q{}^*$ and $TC(R_Q{}^*, S_Q{}^*)$ are rounded to the nearest two decimal places. The results of the varying parameters are presented in Table 1.

Table 1: Variation of the optimality of replenishment quantity of the retailer and supplier and total relevant cost

No	A_R	A_D	h_R	h_S	λ	μ	$R_Q{}^*$	$S_Q{}^*$	$TC(R_Q{}^*, S_Q{}^*)$
1	200	10	1	1	1	1	2.08	19.00	1257.53
2	200	20	1	1	1	1	3.42	19.00	1790.99
3	200	30	1	1	1	1	4.43	19.00	2197.20
4	200	40	1	1	1	1	5.28	19.00	2538.60
5	200	10	1	2	1	3	1.12	23.50	7470.92
6	200	20	1	2	1	4	2.93	27.28	16684.87
7	200	30	1	2	1	5	4.57	30.62	28787.06
8	200	40	1	2	1	6	6.17	33.64	43866.62
9	200	10	2	1	3	2	4.11	47.99	22056.83
10	200	20	2	2	3	4	6.58	47.99	101686.7
11	200	30	2	2	3	5	9.58	53.77	175025.5
12	200	40	2	1	3	6	15.35	83.85	230729.8
13	200	10	3	1	1	3	4.30	68.28	70130.64
14	200	20	3	2	4	5	7.04	62.25	240925.2
15	400	30	3	3	6	4	11.23	79.00	877486.6
16	400	40	3	4	4	2	7.47	39.00	212234.6
17	400	10	4	3	5	4	2.93	72.03	388129.6
18	400	20	3	2	3	6	5.06	83.85	398420.4
19	400	30	6	3	4	2	5.12	45.189	204968.2
20	400	40	4	5	6	5	10.99	68.28	1726590

From the computed values of the retailer and supplier optimal replenishment quantity and minimum total cost of the supply chain, we observed that the optimality replenishment quantity of retailer, supplier and minimum total cost of the supply chain increases with increase in the parameters.

Table 1 also shows that as the individual parameters such as the fixed replenishment cost (A_R), increases from 200 to 400, fixed delivery cost (A_D), from 10 to 40 the inventory cost per unit of the retailer (h_R), from 1 to 6, the inventory cost per unit at the supplier (h_S), from 1 to 5, arrival rate, (λ), from 1 to 6 and the mean of the demand size, (μ) from 1to 6, the retailer's and supplier's optimal replenishment quantity and minimum relevant total cost of the supply chain increases due to the carrying cost . At fixed replenishment cost (A_R) of 200, fixed delivery cost (A_D) of 10, replenishment cost per unit (C_R), and delivery cost per unit (C_D) to be 1 and $\lambda = \mu = h_R = h_S = 1$, the value of the minimum relevant cost of the supply chain is 257.53 which agree with minimum total cost of the supply chain as given in Optimization-simulation jointly correcting approach (OSJCA) by Shao-Fu *et al.*, 2006. This minimum total average long-run cost 1257.53 is at fixed replenishment cost$(A_R) = 200$, fixed delivery cost $(A_D) = 10$, the unit inventory holding cost for the retailer $(h_R) = 1$, arrival rate, $(\lambda) = 1$, mean of the demand size, $(\mu) = 5$, optimal replenishment quantity of retailer $\left(R_Q^*\right) = 2.08$ and optimal replenishment quantity of retailer $\left(S_Q^*\right) = 19.00$.

Furthermore, Table 1 shows that the lowest retailer quantity is at serial number 5, the supplier quantity is at serial number 1, 2, 3, and 4 while the minimum cost of the supply chain is in serial number 1. The highest quantity of the retailer is at serial number 12 that of the supplier quantity is at serial number 12 and 18, while the total supply chain cost is in serial number 20.

In general, the Table 1 shows the analysis of the variation of total relevant cost of the supply chain with respect to the simultaneous variation of retailer's and supplier's replenishment quantity. This result reveals that there is a general increase in the optimal order quantity of the retailer, supplier and total relevant cost of the supply chain if there is an increase in the arrival rate, (λ), the mean of the demand size, (μ), fixed replenishment cost (A_R) and fixed delivery cost (A_D) as reported in Shao-Fu *et al.* (2006).

The fixed delivery cost per unit and the retailer's quantity is also represented in Figure 1.

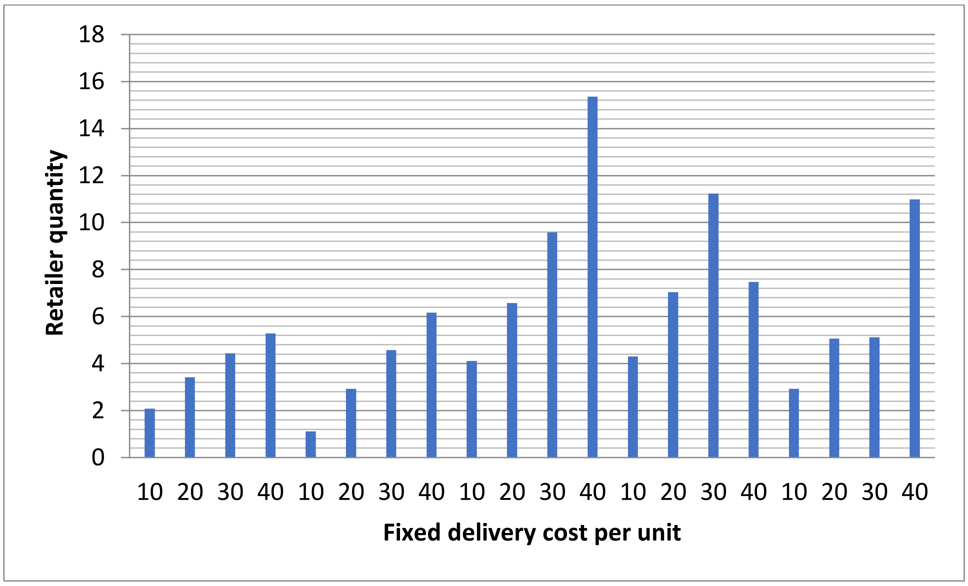

Figure 1: Fixed delivery cost per unit verses Retailer's quantity

Figure 1 shows that the retailer quantity increases with increase in fixed delivery cost. Table 1 and Figure 1 shows that the highest retailer quantity is 15.35 when the fixed delivery cost is 40, the retailer holding cost at 2, supplier holding cost at 1, arrival rate 3 and inter-arrival time 6. Closely followed is at 11.23 when the delivery cost is 30, the retailer and supplier holding costs are 3 with the arrival rate 6 and inter-arrival time 4.

The relationship between an increasing fixed delivery cost and the supplier quantity supply from the manufacturer is shown in Figure 2.

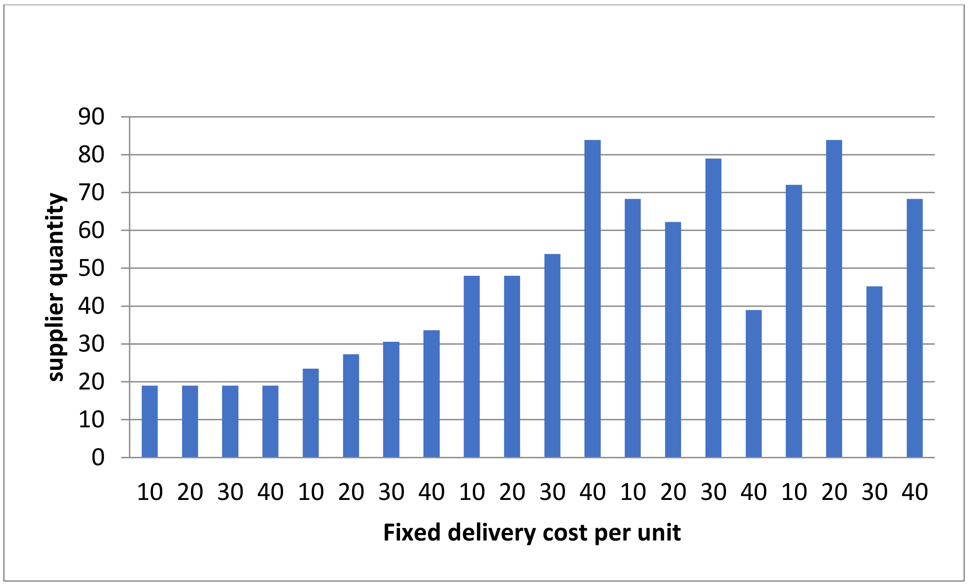

Figure 2: Fixed delivery cost per unit verses Supplier's quantity

Figure 2 shows that at $h_R = h_S = \lambda = \mu = 1$ and varying A_D from 10, 20, 30, 40 the supplier quantity is 19.00. It started increasing when h_S and μ increases. The highest increase in the quantity of the supplier is at the fixed delivery cost of 40 when the quantity is at 83.85 with $h_R = 2, h_S = 1, \lambda = 3$ and $\mu = 6$. And at serial number18 when fixed replenishment cost per unit (A_R) is 400, fixed delivery cost is 20 at $h_R = 3, h_S = 2, \lambda = 3$ and $\mu = 6$.

CONCLUSION

This paper considered a supply chain consisting of a group of retailers who initiate the ordered replenishment quantity from the supplier who is also a dealer of the same products. The ordered is consolidated to the capacity of the delivery vehicle and may vary due to stochastic demand from customers. Demands to consumers follow a compound Poisson distribution and the delivery cycle is the time interval between two consecutive deliveries. The delivering cost is composed of a fixed cost which is incurred when there is a positive replenishment quantity. The delivering cost is a linear variable cost and linearly proportional to the quantity delivered. This cost includes; cost for loading products on vehicles from the supplier consolidated center, transporting them and unloading at the retailer center. The reorder points of the supplier and the retailers are determined to be zero.

A mathematical model developed by Jac-Hun (2010) was used to investigate the optimal quantity of the retailer and the profitability in the supply chain for the replenished single product to the retailers center from the supplier.

Several properties for obtaining a closed-form expression for approximated long-run average cost, were developed, a mathematical model developed by Jac-Hun (2010) was applied and the renewal reward theory was used to determine the order-up-to level of each member of the supply chain that minimizes the long-run average cost.

The result shows that there is a general increase in the optimal order quantity of the retailer and total relevant cost of the supply chain if there is an increase in the arrival rate (λ) and the mean of the demand size (μ). When the value of k increases with fixed replenishment cost (A_R), fixed delivery cost (A_D), the retailer inventory holding cost and supplier inventory holding cost and the total relevant cost of the supply chain increases due to the fact that demand increases linearly at the retailer point as the value of the arrival rate and the mean of the demand increases. Consequently, there is an increase in the supply chain profitability if the retailer orders greater number of items from the supplier.

The managerial benefits of the study are as follows: companies can reduce stockouts and overstock situations, leading to improved operational efficiency, improved efficiency by optimizing inventory replenishment between suppliers in rural areas. Companies can achieve cost savings by minimizing excess inventory and reducing transportation costs through more efficient supply chain management and enhanced collaboration by implementing supplier-supplier inventory replenishment strategies. This can foster better collaboration and communication between suppliers, leading to stronger relationships and more effective partnerships.

Suggestions for further research

i. Optimization algorithms: Further research can focus on developing advanced optimization algorithms to maximize profit potential in supply chain management, considering various factors such as demand variability, lead times, and transportation costs.
ii. Technology integration: Exploring the integration of emerging technologies such as block chain, IoT, and AI in supplier-supplier inventory replenishment may enhance visibility, transparency, and efficiency in rural supply chains.
iii. Sustainability considerations: Future research can investigate the impact of supplier-supplier inventory replenishment on sustainability metrics such as carbon footprint, waste reduction, and social responsibility in rural areas.

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