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A LINEAR PROGRAMMING APPROACH TO OPTIMIZING ENVIRONMENTAL RESOURCE MANAGEMENT IN URBAN AREAS

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ABSTRACT

This study investigates the optimization of environmental resource management in urban settings using a linear programming approach, focusing on ammonia production and wastewater management. The model addresses uncertainties in resource allocation by evaluating the cost implications of wastewater discharge during production. Key variables include water drawn from rivers, sludge discharge, and energy consumption. The study explores scenarios where zero-discharge policies are implemented, resulting in reduced water usage but increased energy consumption and higher production costs. The methodology employed linear programming to minimize the cost of ammonia production while ensuring water quality through regulated waste disposal. The findings indicate that minimizing river impurities through controlled waste discharge reduces water usage but escalates energy consumption, complicating cost management. Results from the numerical example show that under optimal conditions, 6,666.7 liters of water and 555.6 kg of sludge are needed to produce 41,666.7 kg of ammonia, with zero energy consumption in the process. The significance of these results lies in their potential to inform sustainable urban resource management policies that balance economic and environmental priorities. The study concludes that while achieving high water quality increases costs, it is crucial for sustainable ammonia production and environmental conservation, particularly in minimizing the ecological impact of industrial activities on water resources.

Keywords: Environmental Management, Linear Programming, Urban Resources, Sustainability, Optimization

INTRODUCTION

Urban environments face significant challenges in managing resources efficiently while minimizing environmental impact. Effective environmental management requires balancing resource supply and demand, reducing waste, and ensuring sustainability. The increasing urbanization and the necessity for sustainable development have sparked interest in understanding the relationship between urban growth and environmental conservation (Chen *et al.*, 2023). As cities expand, efficient resource management is crucial to mitigate environmental degradation and promote sustainability (Hassan *et al.*, 2020). Smart urbanization strategies, incorporating green technologies and efficient resource utilization, have emerged as key approaches to reduce environmental pollution and enhance resource management, (Xu, 2023) Urban areas are characterized by high population densities and significant resource consumption, leading to increased wastewater generation. For instance, Eludoyin et al. (2024) emphasize that the mismanagement of wastewater not only contributes to the deterioration of water quality but also exacerbates urban flooding, thereby increasing the vulnerability of urban populations to waterborne diseases (MacAfee, 2023). This assertion is supported by Ogwu and Idisi (2024), who argue that inadequate wastewater treatment facilities in urban settings lead to the discharge of untreated effluents into water bodies, creating a cycle of pollution that is difficult to break (Tuanaya, 2024). This research introduces a dynamic programming model in a linear programming form to optimize the management of urban resources.The application of linear programming models in wastewater management allows for the optimization of resource allocation, ensuring that limited resources are utilized efficiently to achieve maximum environmental benefits. Eludoyin, Olisa, and Idisi (2023) present a framework that integrates various factors such as treatment costs, environmental impacts, and regulatory compliance to develop sustainable wastewater management strategies (Avarand et al., 2023). This model not only aids in decision-making but also provides a quantitative basis for evaluating the trade-offs between different management options, thereby facilitating more informed policy-making (Varma, 2023). Furthermore, Ogumeyo and Idisi (2024) demonstrate that such models can help urban planners identify the most cost-effective solutions for wastewater treatment and resource recovery, ultimately contributing to a more sustainable urban environment (Varma, 2023). Traditional models often focus on specific resources or aspects of environmental management. These models have gained significant traction in recent years due to their ability to enhance sustainability, reduce costs, and minimize environmental risks. For instance, Fazli (2023) highlights the increasing popularity of mathematical models in planning water resource distribution, which not only reduces costs for urban stakeholders but also minimizes environmental risks associated with water management. Similarly, Xu (2023) emphasizes the importance of smart urbanization in reducing environmental pollution through technologies like smart grids and efficient waste management systems, showcasing the potential of mathematical models in promoting sustainable resource management in urban areas (Xu, 2023).

Moreover, the interwoven nature of urbanization and the ecological environment is a critical aspect that mathematical models can address to ensure coordinated development and sustainable outcomes. Chen *et al.* (2023) stress the significance of understanding and analyzing the relationship between urbanization and ecological efficiency to develop effective policies for environmentally sustainable urban development. By utilizing mathematical models, researchers can assess the impact of urbanization on environmental regulation efficiency, as demonstrated in the study by (Zhang *et al.*, 2021), which underscores the importance of integrating urban development with environmental resource governance to mitigate negative impacts on resources and the environment. Furthermore, the role of mathematical models extends to areas such as urban water management, where these models offer insights into optimizing resource allocation and managing water supply and demand effectively. Ogumeyo ,Omole (2014) developed a transportation linear programming algorithm to determine minimum cost routes in the delivery of petroleum product from their supply centers (refinery) to demand centers (filling stations). Shabani *et al.* (2020) present a multi-objective optimization model for water management that considers uncertainties in demand and supply, highlighting the value of mathematical modeling in addressing complex urban water challenges. Additionally, Ni *et al.* (2014) demonstrate the effectiveness of agent-based allocation models in urban water resource management, showcasing how mathematical approaches like the multi-agent Q-learning algorithm can optimize resource allocation and enhance efficiency in water management systems.

In the context of environmental sustainability, mathematical models can aid in assessing the impact of IoT technologies on resource optimization and environmental quality in urban environments. Singh (2024) explores the role of IoT in creating sustainable urban environments by focusing on areas like energy management, smart mobility solutions, and environmental monitoring, illustrating how mathematical modeling can support the optimization of resource usage and overall quality of life in cities. Additionally, Xu (2024) discusses innovative models of waste recycling in urban infrastructure, emphasizing the shift towards viewing waste as a renewable resource to achieve economic and environmental benefits, showcasing the transformative potential of mathematical modeling in promoting sustainability. Similarly, Shen & Liu (2022) highlight the positive impact of circular economy legislation on pollution reduction in urban settings, underscoring how mathematical models can support environmental quality improvements through effective waste management strategies.

Mathematical models play a pivotal role in optimizing environmental resource management in urban settings by providing a structured approach to decision-making, resource allocation, and sustainability initiatives. From water management to waste recycling and urban development, these models offer valuable insights into enhancing resource efficiency, reducing environmental risks, and promoting sustainable practices in urban environments. Models that deal with waste water treatment abound in literature in both scientific and engineering fields of research. In most cases, the goal of these models is to determine efficient techniques that could be used to enhance the quality of water for both domestic and industrial purposes, Mahlati *et al.* (2016). According to Li *et al.* (2015), water treatment involves processes such as physical elimination of suspended solids through sedimentation and filtration, bio-oxidization process which involves conversion of left-over suspended solids and dissolved organic impurities into settle-able solids which are then removed by sedimentation method. The final stage of water treatment involves the use of chemical substance to remove the impurities, Saremi *et al.* (2010).

Both domestic and industrial disposal of liquid and solid wastes in water bodies has been the major causes of water pollution and environmental degradation. Dabrowski *et al.* (2014), remark that urbanization and industrialization increased rate are major factors which contribute to water usage and pollution. This includes indiscriminate dumping of refuse, open defecations, etc into water bodies such as streams, rivers and seas. Finney *et al.* (1977), stated that Fan *et al.* (1971) were the first to apply a mathematical model to wastewater treatment in order to determine optimal policy which minimizes the cost of waste water treatment. The lack of adequate quality water supply in many regions in the world is traceable to water pollution caused by human activities. Basson *et al.* (1997) opined that if these activities continue, availability of quality water for human consumption will reach critical situations. Sasikumar *et al.* (1998), applied fuzzy optimization to enhance water quality management in streams and rivers. Their model was later followed by the work of Mujumdar and Vemula (2004).

Some of the mathematical models designed to address wastewater treatment include: Saremi *et al.* (2010) multi-optimal model which uses linear programming method to determine the level of water pollution in Haraz river in Iran. This model was closely followed by Liu *et al.* (2011) model which adopted integer programming techniques to ascertain the level of water pollution in Syros and Paros rivers in Greece. Optimal waste water management is also discussed in Gikas *et al.* (2015). Li *et al.* (2015), modified programming model and Jundiani (2024) integer programming model were applied to agricultural and urban water resource management respectively. The complexity of the above models suffer set back due to lack of availability of computational facilities. Hence, the need to develop precise models with less computation errors and complexity such as the one presented in this study becomes necessary.

The problem of water pollution can be solved by allowing wastewater undergo a treatment before its disposal. According to Mara (2004), biological activities of micro-organisms can be used to decompose organic compounds in the waste. Wastewater discharged into our water bodies is said to be of quality standards of domestic wastewater if it contains tolerated amount of pollutant elements void of harmful effects, Pratiwi *et al.* (2019). As contained in Inyim and Liengcharernsit (2012), wastewaters are bi-product of human activities with general waste that are dumped into water bodies. Hence, there is need to treat wastewaters in order to remove these wastes. Mathematical model which could assist policy makers to address the challenge of wastewaters management have been developed. These models have a common objective of describing the relationships between water pollutants and quality techniques for their removal by considering their physical, chemical and biological compositions. Inyims and Liengcharernsit (2012) pointed out that, linear programming, nonlinear programming, dynamic programming and integer programming techniques are used in most cases for wastewater treatment modeling.

In order to achieve quality wastewater treatment, Wuang and Huang (2014), opined that optimization approach is required to help relevant stakeholders assign and manage available resources. According Lan *et al.* (2015), optimization and integration approaches for modeling urban water use and treatment have proven to be the best methods in tackling water resources management. Stages involved in water system design include the sources of water supply, the type of treatment plants, type of equipment to convey the water to the consumption areas, Chung *et al.* (2009). Xu *et al.* (2024), remark that problem of uncertainty in data collections and analysis as a result of human error usually occur in managing water distribution network and treatment process. Consequently,a water supply and wastewater collection model which uses two-stage stochastic programming to address this uncertainty is developed in Naderi and Pishvaee, (2017). The model consists of three stages of water supply chain with the capacity to reduce system cost and uncertainty scenarios.A two-stage planning model for urban water supply resources management is also discussed in Qin and Xu, (2011). The model is made up of four components: sources of the water, plant treatment, water storage and water consumption region.

Wastewater management models aim at obtaining the optimal volume of water that enters and leaves a system by assessing the water network supply chain into the consumption region Dakht and Soleimapour, (2020). The water sources consist of rivers, streams, wells, dams, etc, while water treatment plants is made up of filters, disinfectants, drainage pools and air-condition devices. The distribution channels include: Pipelines, tubes and injector vessels or containers. Freshwater is needed for domestic, environmental and industrial development. Many regions of the world have inadequate water supply due to urbanization expansion, population growth and climate change, Zhou *et al.* (2019). Cosgrove and Loucks (2015) remarked that lack of adequate supply of fresh water has caused health hazard to our ecosystem as well as poor economic development in our nations. Hence, there is need to urgently address the problem by relevant stakeholders. In order to have long-term availability and sustainability of clean water, An *et al.* (2017), suggest that both scientific management and comprehensive planning approaches must be adopted by policy makers. Zhou (2019) stated that if standard quality data are available concerning the problem of acute shortage of clean water, mathematical models can be formulated to aid decision makers to translate developmental concepts into practical reality.

It is the duty of government to formulate policies which govern the exploration and use of natural resources. In this study, we focus on one of these natural resources – water. That is, water in rivers, streams, ponds, etc. Our approach integrates multiple resources, such as water, energy, and waste, considering their interdependence and dynamic interactions within urban systems. Urban resource management has been extensively studied, with models addressing various aspects such as water supply (Smith *et al.,* 2020), energy consumption (Johnson & Lee, 2019), and waste management (Davis, 2018). However, few models integrate these resources comprehensively. Our model builds on the work of Zhang *et al.* (2021), who developed a multi-resource management framework, and extends it by incorporating dynamic programming techniques to account for temporal variations in resource supply and demand.

This paper is an extension of the earlier work of Zhou *et al.* (2019) which applied a robust linear programming model to a regional water quality management under an unpredictable environment. The aim of this model is to deal with the problem of uncertainty encountered by previous models. In this paper, we proposed a linear programming model which aims at investigating the minimum cost of ammonia production when there is no limit imposed on the quantity of wastewater dumped into the river during production process.

Consequently, we established a control process which places limits on the variables as follows:

i. Amount of liquid waste allowed to return to the river during ammonia production process is fixed.
ii. Amount of sludge allowed to be dumped into the river is also known.
iii. The cost of drawing water from the river is fixed.

MATERIALS AND METHODS

Model Description

Fig. 1 describes ammonia production process which consists of a series of flows. These include flows of water from the river, used steaming, manufacturing, and cooling. The second type of flows is the flows of wastewater which is a bi-product of the production process. The third type is the flow of solid wastes extracted from the river water. The ammonia plant has four components. These are (a) The ammonia plant itself (b) Water cooling towers (c) Demineralizers which eliminate mineral from the water flows in the system (d) clarifier which extracts solid minerals from river water and produces sludge which is eliminated from the system. The ammonia plant produces feedstock and ammonia during the production process. Each of the activity in the four components described above is associated with a financial cost which is reflected in the objective function of the proposed model.

The followings are the variables associated with the proposed model.

x_1: Amount of water drawn from the river (thousands of liters)
x_2: Amount of sludge dumped into river (thousands of liters)
x_3: Amount of energy consumption (thousands of kilo watts)

Assumptions of the Model

The proposed model has the following assumptions:

i. Wastewater generated in ammonia plant must equal amount of wastewater sent to neutralizer or to river.
ii. Demineralizer wastewater generated must equal amount sent to injection well: evaporator, or to river.
iii. Sludge generated by clarifier must equal amount sent to landfill, river, or dryer.
iv. There is a legislated limit on sludge dumped into the river.

Methodology adopted in this study is the linear programming approach which is being used to model ammonia production process and its impact on the control and use of quality water. The objective function is to minimize the cost of ammonia production in the face of wastewater treatment measure adopted by government policy. The objective of the study is to: (a) control indiscriminate dumping of domestic and industrial solid wastes into water bodies. (b) Assess the economic implication in form of higher production cost as a result of ensuring high quality water supply. This study is extremely important in the sense that ammonia is used for producing fertilizers which farmers use to grow their crops. Hence, whatever policy that increases the cost of fertilizer will result to high cost of food prices. Consequently, this study aims at striking a balance between ensuring standard water quality at a minimum cost and at the same time maximizing profit from ammonia production by the firm.

Partial Flow Diagram of Ammonia Production

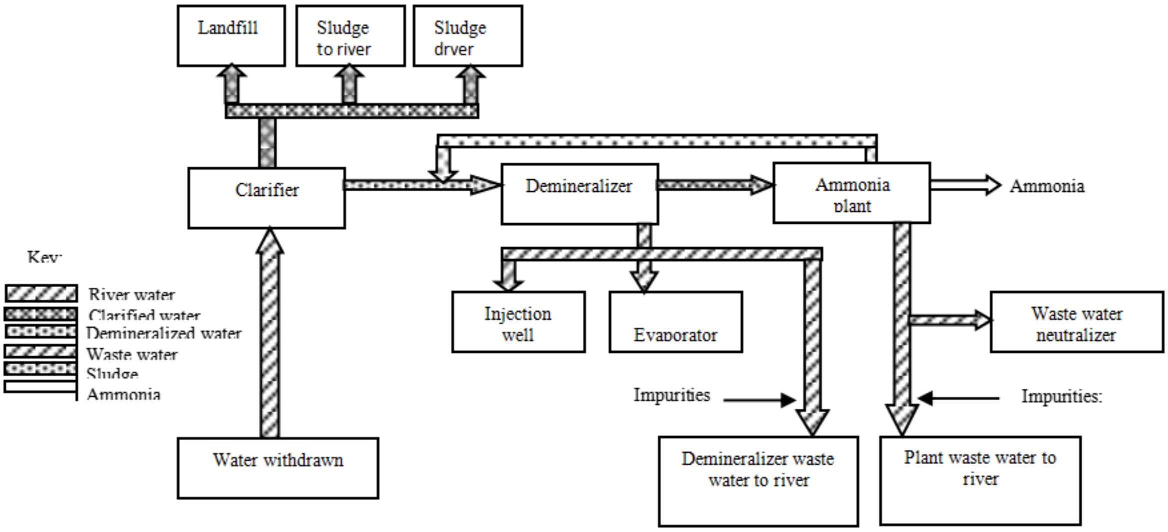

Figure 1: Coefficients flows involved in operating ammonia plants.

Model Formulation of the Linear Programming Problem

Step 1: Determine the number of units of each component (materials) of type 1, 2 and 3 that are to be optimized.

Step 2: Denote the units by variables such as $x_1, x_2, \dots, x_n$ respectively.

Step 3: State the non-negativity constraints for the variables, that is, $x_1, x_2, \dots, x_n \geq 0$ for $i = 1(1)n i = 1(1)n$

Step 4: State the objective function i.e. Minimize the cost of ammonia production.

$$Minimize\ Z = cx_1 + cx_2 + \dots + cx_n \quad (1)$$

$$Subject\ to$$

$$\left.\begin{array}{c} b_{11}x_1 + b_{12}x_2 + \dots + b_{1n}x_n = s_1 \\ b_{21}x_1 + b_{22}x_2 + \dots + b_{2n}x_n = s_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ b_{m1}x_1 + b_{m2}x_2 + \dots + b_{mn}x_n = s_n \end{array}\right\} \quad (2)$$

$$x_1, x_2, \dots, x_n \geq 0 \quad (3)$$

Equation (1) is the objective function,

Equation (2) is the constraint while Equation (3) is the Non-negativity.

Algorithm of the Linear Programming Model

The simplex method of the linear programming model is as follows:

Phase 1: State the problem in its initial computational form having m x n unit matrix and locate the most negative number in the bottom row of the simplex tableau, excluding the last column. This column in which this number appears is called work column. If more than one number appears to be most negative, choose one arbitrarily.

Phase 2: Form ratios by dividing each positive number in the work column, excluding the last row, into the element in the same row and last column. The element that produces the smallest ratio is called the pivot element. If more than one element produces the same ratio, select one arbitrarily. If none of the elements in the work column is positive, then the program can be concluded to have no solution.

Phase 3: Use elementary row operations to convert the pivot element to 1 and then reduce all other elements in the work column to zero.

Phase 4: Replace the *x*-variable in the pivot row and first column by the x-variable in the first row and pivot column. This new first column is the current set of basic variables.

Phase 5: Repeat step 1 through 4 until there are no negative numbers in the last row, excluding the last column.

Phase 6: Optimal solution is derived by assigning to each variable in the first column, that value in the corresponding row and last column. All other values are assigned the value zero. The associated Z, the optimal value of the objective function is the number in the row and last column for a maximization program but a negative number for a minimization problem.

RESULTS AND DISCUSSION

Numerical Illustration

The following table shows the volume of water (in 1000s liters), sludge (in 1000s kg) and the amount of energy required in the production of ammonia by XYZ company. Given the data in Table 1, use a linear programming algorithm to determine the minimum cost of ammonia production.

Table 1: Ammonia production constituents

Component type	**Constituents Per Unit**			
	River Water used (1000 ltrs)	**Sludge (1000 kg)**	**Energy consumption**	**Available resources**
1	1	6	1	10
2	1	6	1	10
3	2	3	1	15
4	2	3	1	15
Production cost per unit	6	3	4	

SOLUTION

The above problem can be written as

$Minimize\ P = 6x_1 + 3x_2 + 4x_3$

Subject to

$x_1 + 6x_2 + x_3 \leq 10$

$x_1 + 6x_2 + x_3 \leq 10$

$2x_1 + 3x_2 + x_3 \leq 15$

$2x_1 + 3x_2 + x_3 \leq 15$

$x_1, x_2, \ldots, x_7 \geq 0$

Expressing all the constraints in the $\leq$ form and adding the slack variables, the problem becomes:

$Minimize\ z = 6x_1 + 3x_2 + 4x_3 + 0x_4 + 0x_5 + 0x_6 + 0x_7$

Subject to

$x_1 + 6x_2 + x_3 + x_4 \qquad\qquad = 10$

$-x_1 - 6x_2 - x_3 \qquad + x_5 \qquad = 10$

$2x_1 + 3x_2 + x_3 \qquad\qquad + x_6 \quad = 10$

$x_1, x_2, \ldots, x_7 \geq 0$

$Minimize{:}\ z = 6x_1 + 3x_2 + 4x_3 + 0x_4 + 0x_4 + 0x_5 + 0x_6 + 0x_7$

Tableau 1

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_4	1	6	1	1	0	0	0	10
x_5	-1	-6	-1	0	1	0	0	-10
x_6	2	3	1	0	0	1	0	15
x_7	-2	**-3***	-1	0	0	0	1	-15
$(c_j - z_j)$	6	3	4	0	0	0	0	0

Since all the $(C_j - Z_j)$ values are non-negative, the above solution is optimal. However, it is infeasible because x_5 and x_7 have non-positive values. Since x_7 has the most non-positive value, it becomes the departing variable. c^r

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$(c_j - z_j)\ row$:	6	3	4	0	0	0	0
$x_7\ row$:	-2	-3	-1	0	0	0	1
$Absolute\ ratios$:	3	1	4	-	-	-	-

Since x_2 has the smallest ratio, it becomes the entering variable. Thus the element -3, marked by the asterisk, becomes the pivot element. Using elementary row operations, we obtain Tableau 2.

Tableau 2

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_4	**-3***	0	-1	1	0	0	2	-20
x_5	3	0	1	0	1	0	-2	20
x_6	0	0	0	0	0	1	1	0
x_2	2/3	1	1/3	0	0	0	-1/3	5
$(c_j - z_j)$	4	0	3	0	0	0	1	-15

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$(c_j - z_j)\ row$:	4	0	3	0	0	0	1
$x_4\ row$:	-3	0	-1	1	0	0	2
$Absolute\ ratios$:	3/4	-	3	-	-	-	-

Since x_1 has the smallest absolute ratio, it becomes the entering variable (E.V.). Thus the element -3, marked by the asterisk, becomes the pivot element. Using elementary row operations, we obtain Tableau 3.

Tableau 3

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_1	1	0	1/3	-1/3	0	0	-2/3	20/3
x_5	0	0	0	1	1	0	0	0
x_6	0	0	0	0	0	1	1	0
x_2	0	1	1/9	2/9	0	0	1/9	5/9
$(c_j - z_j)$	0	0	5/3	4/3	0	0	11/3	-125/3

Since all the variables have nonnegative values, the above optimal solution is feasible. The optimal and feasible solution is $x_1^* = 20/3$, $x_2^* = 5/9$, $x_3^* = 0$, and $z^* = 125/3$.

Discussion of results

From the numerical example presented in Section 4.0 we have three (3) computational tableaus. In Tableau 1, we observe that all the $(C_j - Z_j)$ values are non-negative values. Hence the above solution is optimal. However, it is infeasible since x_5 and x_7 coefficients have non-positive values. The departing variable in Tableau 1is x_7 since it has the most non-positive value. Since x_2 has the smallest ratio, it becomes the entering variable. Thus the element -3, marked by the asterisk, becomes the pivot element. Using elementary row operations, we obtain Tableau 2. In Tableau 2, x_1 has the smallest absolute ratio hence it becomes the entering variable (E.V.). Thus the element -3 marked by the asterisk in Tableau 2 becomes the pivot element. Using elementary row operations, we obtain Tableau 3. In Tableau 3, all the variables have nonnegative values in the objective function's row. Hence the above optimal solution is feasible. The optimal and feasible solution is $x_1^* = 20/3$, $x_2^* = 5/9$, $x_3^* = 0$ and $z^* = 125/3$. From Section 3.0, x_1 represents the volume of water (in 1000s of liters) drawn from the river, x_2 represents the amount of sludge discharged into the river in 1000s of kg while x_3 represents energy consumption in kilowatts. Hence from the optimal and feasible solution, the minimum cost of producing 41666.7kg of ammonia requires 6666.7 liters of water, 555.6 kg of sludge and zero kilowatts of energy consumption.

CONCLUSION

This paper is an extension of the earlier models which applied a robust linear programming model to a regional water quality management under an unpredictable environment. The aim of this model is to deal with the problem of uncertainty encountered by previous models. In this paper, we proposed a linear programming model which aims at investigating the minimum cost of ammonia production when there is no limit imposed on the quantity of wastewater dumped into the river during production process. Consequently, we established a control process which places limits on the variables during the model formulation as follows: (a) Amount of liquid waste allowed to return to the river during ammonia production process is fixed. (b) Amount of sludge allowed discharge into the river is also known. (c) The cost of drawing water from the river is fixed. From the result analysis of the numerical example, it is observed that when the amount of impurities dumped in the river is reduced, amount of energy consumption is increased while the quantity of water drawn from the river is decreased. Moreover, zero discharge of impurities into the river reduces the amount of water use, increases energy consumption and the cost of producing ammonia.

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