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IMPROVING THE LOCAL GEOMETRIC GEOID MODEL OF FCT ABUJA ACCURACY BY FITTING A HIGHER ORDER/DEGREE POLYNOMIAL SURFACE

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ABSTRACT

The improvement of the accuracy of a local geometric geoid model using the same data set (geoid heights) requires the fitting of a higher degree polynomial surface to the data set. Consequently, this paper presents improving the local geometric geoid model of FCT, Abuja accuracy by fitting a higher order polynomial surface. A fifth degree polynomial surface was fit to the existing geoid heights of 24 points used previously for the determination of the geometric geoid model of the study area to improve its accuracy. The least squares adjustment technique was applied to compute the model parameters, as well as the fit. The RMSE index was applied to compute the accuracy of the model. The computed accuracy (0.081m) of the model was compared with those of the previously determined geoid models (Multiquadratic, 0.110m and Bicubic, 0.136m models) of the study area to determine which of the models best fit the study area, as well as has the highest resolution. The comparison result shows that the fifth degree polynomial surface best fit the study area.

Keywords: Accuracy, Higher Degree, Polynomial, Geometric Geoid, model.

INTRODUCTION

The GNSS observation that uses the Global Positioning System (GPS)/Global Navigation Satellite System receivers gives the coordinates and elevations of stations at various points of measurements. The system elevations are relative to the ellipsoid. In other words, they are not to the Mean Sea Level (MSL), as well as the geoid. Therefore, they are not practical heights and not used for engineering construction purposes. The orthometric heights of points are obtained relative to benchmarks with spirit levelling whose procedure is tedious and time-consuming. Most times, the locations of benchmarks are far from project areas. Sometimes the closest ones are at 3km to 5km from project areas. To carry out spirit levelling for such distances to determine the orthometric height of temporal benchmarks (TBM) of project areas is not always easy. If all the turning points of a project area, are surveyed with the GNSS observation, then the positions and ellipsoidal heights of the observed points can be obtained. Since the ellipsoidal heights (h) are not useful for engineering purposes, there is a need to be converted to orthometric heights (H). The conversion of the ellipsoidal heights (h) to orthometric heights (H) requires geoid heights (N), as well as a local geoid model of the area. If the geoid heights (N) of points are known, then their orthometric heights are computed using the relation (Oluyori et al., 2018),

$$H = h - N \qquad (1)$$

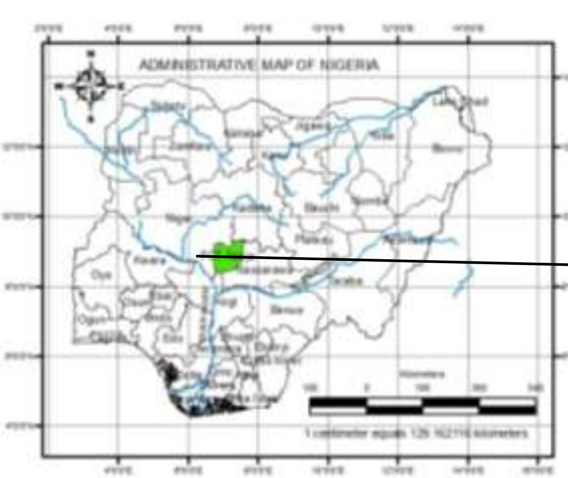
The Geoid is one of the Earth's shapes. Geoid surface is used to approximate the physical shape of the Earth. It is the equipotential surface of the Earth's gravity field which more or less coincides with the mean sea level (Borge, 2013). Ubajekwe (2011) also defined the geoid as the equipotential surface of the earth's attraction and rotation which coincides on average with the mean sea level in the open Ocean. It is the surface which coincides with the mean sea level assuming that the sea was free to flow under the land in small frictionless channels. Moritz and Hofmann (2005) stated that the geoid coincides with that surface to which the oceans would conform over the entire earth if free to adjust to the combined effect of the earth mass attraction (gravitation) and the centrifugal force of the earth's rotation. They also explained that the geoid is a surface along which the gravity potential is everywhere equal and to which the direction of gravity is always perpendicular when optical instruments containing gravity reference levelling devices are properly adjusted during observation coincides with the direction of gravity and are therefore perpendicular to the geoid. Civil engineers use the geoid as the reference surface for elevations while oceanographers use it for studies of ocean circulation, currents and tides. It is also valuable to geophysicists for displacement studies, geophysical interpretation of the Earth's crust, and prospecting (Borge, 2013).

The absence of the Nigeria local geoid model has resulted in the determination of the geoid models of local areas in the country. The determined geoid models have different accuracy. Most of them were determined using the geometric and the gravimetric methods while some others were the gravimetric-geometric method. The Local geoid model of the Federal Capital Territory (FCT), Abuja has been determined using the GPS/Levelling, as well as the geometric method. The comparison of two geometric geoid surfaces (Multiquadratic and Bicubic geoid surfaces), realized the local geoid model of the study area. The two surfaces were respectively 9 (fourth degree polynomial) and 10 (third degree polynomial) parameters. The accuracy obtained for the two geometric geoid models are respectively 0.110m and 0.136m and were recommended for application in the study area with more confidence in Multiquadratic geoid model as its accuracy was better than that of the Bicubic geoid surface. The obtained accuracy were a bit low. It was as a result of the number of points used in the study. Although the quadratic-based programs according to Schmidt *et al*. (2003) gives a more stable result, the high-order local surfaces can fit more complex landform features and reliable only for very accurate data from GNSS/GPS space methods. The accuracy of the local geometric geoid model increases as the number of observations, as well as points used in the study, approximates that of the model parameters/terms (Eteje *et al.,* 2019). According to Eteje *et al.,* (2019), the accuracy of the local geometric geoid model is highest at a unique solution that is, when the number of the observation is equal that of the model parameters, as well as terms. Here, the consideration is that the number of model terms increases as the degree increases. Considering Schmidt *et al*. (2003) and Eteje *et al.* (2019), the improvement of the local geometric geoid model of the Federal Capital Territory (FCT), Abuja requires a higher degree polynomial geoid surface whose number of model terms, as well as parameters approximates the number of observations/points used in the study. As a result, this study presents improving the local geometric geoid model of FCT, Abuja accuracy by fitting a higher order polynomial surface.

To realize this, a higher order (Quintic) polynomial geoid surface with 21 parameters and degree 5 is fit to the geoid heights of 24 points in the study area. These were the points and geoid heights used in the previous study. The fitting of the Quintic polynomial geoid surface to the geoid heights of the points requires the computation of the model parameters with the geoid heights and coordinates of the points using the least squares adjustment technique. The application of the coordinates of the points in the model parameters computation involves the computation of the study area centroid coordinates and finding the differences between the centroid coordinates and the coordinates of the points. Subsequently, the geoid heights and the differences between the centroid coordinates and the positions of the points are used for the computation of the model parameters. The accuracy of the model is computed using the Root Mean Square Error (RMSE) Index.

The Study Area

The Federal Republic of Nigeria consists of 36 states and Federal Capital, the FCT, Abuja. Nigeria is located between 4˚ and 14˚ latitude and 2˚ and 15˚ longitude occupying an area of 923768 km^2. The two major rivers in the country are Niger and Benue that meet at Lokoja. The FCT lies between 8˚ 15'N to 9˚ 12'N and 6˚ 27'E to 7˚ 23'E. Figures 1 and 2 are maps of Nigeria and Federal Capital Territory Area Councils respectively.



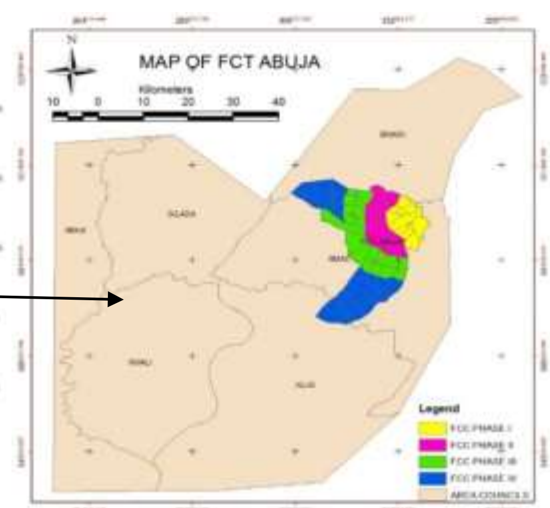


Figure1: Nigeria map with thirty-six states and FCT. Fig. 2: Map of FCT Area Councils
Source: Oluyori *et al.* (2018)

Polynomial Models

Polynomial geoid models, as well as surfaces, are fitted to the known geoid heights of points to enable the geoid heights of new points, be computed by interpolation using the coordinates of the points (Eteje *et al., 2019*). The Multiquadratic and Bicubic models had earlier been applied in the Federal Capital Territory, Abuja to determine its local geometric geoid model (Oluyori, 2019). In the attempts to improve the accuracy of the determined geoid model, having considered Schmidt *et al*. (2003) and Eteje *et al.* (2019), there is a need to apply a fifth-degree (Quintic) polynomial surface in the study area. The Multiquadratic and the Bicubic models as given by Oluyori (2019) and the fifth degree polynomial surface given by Awange *et al.* (2010) and Eteje *et al*. (2019) are respectively

$$N = a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy + a_6x^2y + a_7xy^2 + a_8x^2y^2 \quad (2)$$

$$N = a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2 + a_6x^3 + a_7x^2y + a_8xy^2 + a_9y^3 \quad (3)$$

$$N = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2 + a_6 x^3 + a_7 x^2 y + a_8 xy^2 + a_9 y^3 + \\ + a_{10} x^4 + a_{11} y^4 + a_{12} x^3 y + a_{13} x^2 y^2 + a_{14} xy^3 + a_{15} x^5 + a_{16} y^5 + a_{17} x^4 y + \\ + a_{18} x^3 y^2 + a_{19} x^2 y^3 + a_{20} xy^4 \quad (4)$$

Where,

N = Geoid height of point

$y = ABS(Y - y_o)$

$x = ABS(X - x_o)$

Y = Northing coordinate of observed station

X = Easting coordinate of observed station

y_o = Northing coordinate of the origin (average of the northing coordinates)

x_o = Easting coordinate of the origin (average of the easting coordinates)

$a_o, a_1, a_2, ..., a_n$ are the unknown model parameters.

Least Squares Adjustment Technique

The fitting of a polynomial surface to a set of geoid heights requires the computation of the polynomial, as well as the geometric geoid model parameters using the least squares technique. The least squares models for the computation of the geometric geoid model parameters are detailed in Eteje and Oduyebo (2018).

Root Mean Square Error (RMSE) Computation

The Root Mean Square Error (RMSE) is an indicator of accuracy. It is used for the computation of the accuracy of local geometric geoid models. Its application for accuracy computation in geoid modelling involves the comparison of the computed geoid heights obtained from the differences between the ellipsoidal and the orthometric heights and the model geoid heights of points. The RMSE index used for accuracy computation as given by Eteje and Oduyebo (2018) is

$$RMSE = \pm\sqrt{\frac{V^T V}{n}} \quad (5)$$

Where,

$V = (N)_{KNOWN} - (N)_{MODEL}$

$(N)_{KNOWN} =$ Point known geoid height

$(N)_{MODEL} =$ Point model geoid height

n = Number of points

METHODOLOGY

Data Acquisition

The data used in this study included the existing UTM zone 32 coordinates, existing orthometric heights and GNSS observation ellipsoidal heights of 24 control stations within the Federal Capital Territory (FCT), Abuja. The existing coordinates and orthometric heights of the points were obtained from the Surveying and Mapping Department of the Federal Capital Development Authority (FCDA), Abuja while their respective ellipsoidal heights were obtained from GNSS observation of the existing controls. These were the data used previously for the determination of the local geometric geoid model of FCT, Abuja and are detailed in Oluyori (2019) and Oluyori *et al.* (2019). Figure 2 shows the GNSS observation of the points while Table 1 shows the coordinates, orthometric heights and the ellipsoidal heights of the points.

FCT 260P Base Station FCT 2652S (Rover Station) FCT130P Base Station

Figure 2: GNSS Observation of the Control Stations

Table 1: Control Stations Coordinates, Orthometric Heights and Ellipsoidal Heights

CONTROL POINTS	COORDINATE REGISTER VALUE			GNSS ELLIPSOIDAL HEIGHT (m)
	EASTINGS (m) (e)X	NORTHINGS (m) (n)Y	EXISTING ORTHO. HEIGHTS, H (m)	
FCC11S	331888.114	998442.043	485.447	509.396
FCT260P	255881.175	993666.807	201.944	224.740
FCT103P	340639.766	998375.578	532.558	556.836
FCT12P	333743.992	1008308.730	735.707	760.192
FCT19P	337452.408	996344.691	635.644	659.824
FCT2168S	310554.927	1009739.930	431.087	455.274
FCT24P	322719.776	1001884.850	453.804	477.987
FCT276P	351983.716	1025998.314	625.572	649.848
FCT4154S	329953.882	1003831.280	476.981	501.232
FCT4159S	326124.422	1003742.860	452.230	476.553
FCT66P	299148.035	998114.283	297.111	321.115
FCT9P	329821.512	1007612.091	497.253	521.693
FCT35P	322183.380	992926.363	427.171	451.299
FCT57P	303234.270	992916.402	323.844	347.795
FCT4028S	330164.634	1001388.240	449.592	473.942
FCT53P	308943.361	993406.773	351.943	375.955
FCT4652S	329441.767	997474.808	462.711	487.113
FCT162P	270791.291	934625.533	189.696	215.091
FCT130P	330982.584	952889.869	695.608	719.383
FCT2327S	282526.612	973821.470	183.287	207.482
FCT2652S	271370.273	945385.429	138.952	163.741
FCT2656S	272644.591	941062.460	204.724	229.229
FCT83P	332954.205	987231.606	568.752	592.819
XP382	284074.729	983364.863	274.586	298.390

Data Processing

The positions and orthometric heights of the points were obtained from the Surveying and Mapping Department of the FCDA while the ellipsoidal heights were obtained from the GNSS processing results. The geoid heights of the points were computed by finding the differences between the GNSS ellipsoidal heights and the orthometric heights, as well as using equation (1). Table 2 shows the computed geoid heights of the points.

Table 2: Computed Geoid Heights of Stations

STATION	EASTINGS (m) (e)X	NORTHINGS (m) (n)Y	GEOID HEIGHT (m)
FCC11S	331888.114	998442.043	23.949
FCT260P	255881.175	993666.807	22.787
FCT103P	340639.766	998375.578	24.278
FCT12P	333743.992	1008308.730	24.485
FCT19P	337452.408	996344.691	24.180
FCT2168S	310554.927	1009739.930	24.187
FCT24P	322719.776	1001884.850	24.183
FCT276P	351983.716	1025998.314	24.276
FCT4154S	329953.882	1003831.280	24.251
FCT4159S	326124.422	1003742.860	24.323
FCT66P	299148.035	998114.283	24.004
FCT9P	329821.512	1007612.091	24.440
FCT35P	322183.380	992926.363	24.128
FCT57P	303234.270	992916.402	23.951
FCT4028S	330164.634	1001388.240	24.350
FCT53P	308943.361	993406.773	24.012
FCT4652S	329441.767	997474.808	24.402
FCT162P	270791.291	934625.533	25.395
FCT130P	330982.584	952889.869	23.775
FCT2327S	282526.612	973821.470	24.195
FCT2652S	271370.273	945385.429	24.789
FCT2656S	272644.591	941062.460	24.505
FCT83P	332954.205	987231.606	24.067
XP382	284074.729	983364.863	23.804

The computations of the model parameters and the accuracy of the Multiquadratic model, equation (2) and the Bicubic model, equation (3) were carried out with Microsoft Excel programs and are detailed in previous studies (Oluyori, 2019 and Oluyori *et al.,* 2019) that determined the local geometric geoid model of the study area. The model parameters of the fifth degree polynomial surface (equation (4)) were computed with the geoid heights of the points and the absolute differences between the control stations coordinates and the centroid point coordinates using the least squares technique. The accuracy of the higher degree polynomial model was computed using equation (5). The procedures for the computation of equation (4) parameters (the fit) and its accuracy are detailed in Eteje and Oduyebo (2018). The computations were also carried out with a Microsoft Excel program developed in this study for the application of the geometric geoid model in the study area.

RESULTS AND DISCUSSION

Figure 3 presents the plot of the existing/computed, Multiquadratic, Bicubic and the Quintic, as well as the fifth degree polynomial surface geoid heights. It was done to present graphically, the shapes of the plotted surfaces and determine which of the three surfaces best fit the computed geoid heights of the points, as well as has the highest resolution. In Figure 3, the green, red, orange and the blue lines respectively represent the shapes of the existing/computed, Multiquadratic, Bicubic and the Quintic geoid heights. It can be seen in Figure 3 that the blue line fits the green line most. This implies that the Quintic (fifth degree) polynomial surface has the highest resolution. It also confirms Schmidt *et al*. (2003) and Eteje *et al.* (2019) statements and conclusions.

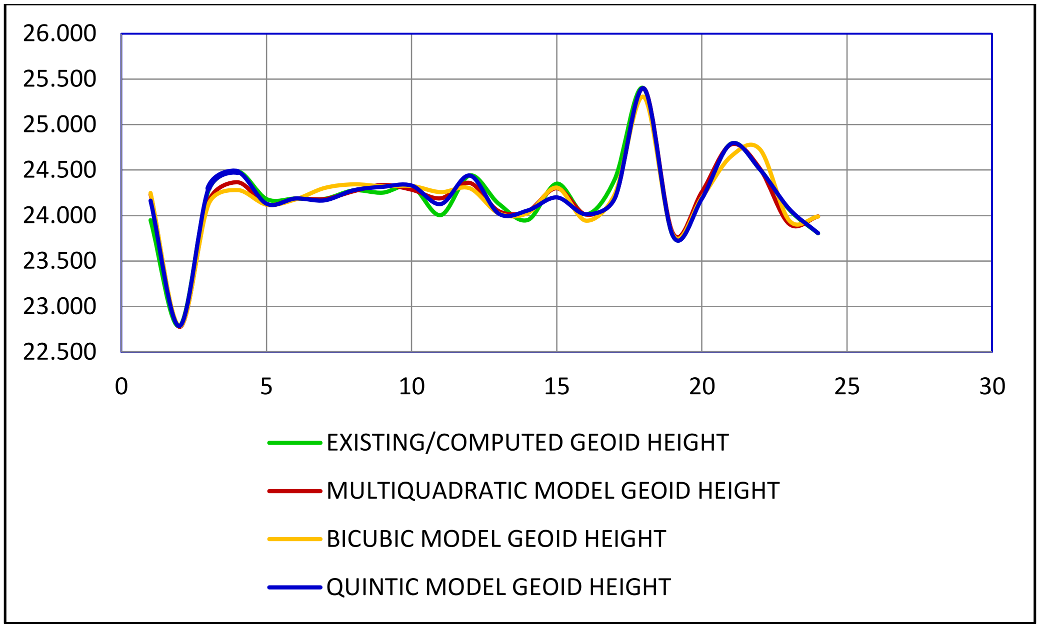


Figure 3: Plot of the Computed and the Three Models' Geoid Heights

Figure 4 presents the plot of the RMSE of the Multiquadratic, Bicubic and the Quintic, as well as the fifth degree polynomial surfaces. It was done to present the accuracy of the three surfaces and determine which of them has the highest accuracy, as well as resolution. The accuracy of the model varies inversely to the values of the computed RMSE, as well as the heights of the histogram bars. It can be seen in Figure 4 that the RMSEs of the Multiquadratic, Bicubic and the Quintic surfaces are respectively 0.110m, 0.136m and 0.081m which implies that the Quintic model has the highest accuracy, as well as resolution. It also confirms the improvement of the accuracy of the local geometric geoid model of the study area and agrees with Schmidt *et al.* (2003) and Eteje *et al.* (2019). The accuracy of the Multiquadratic model (9 parameters) that is higher than that of the Bicubic model (10 parameters) is as a result of its degree. It is a fourth-degree polynomial. Therefore, the accuracy of the models' increases in this order, Bicubic (third degree), Multiquadratic (fourth degree) and Quintic (fifth degree) surfaces. This also agrees with Eteje *et al.* (2019).

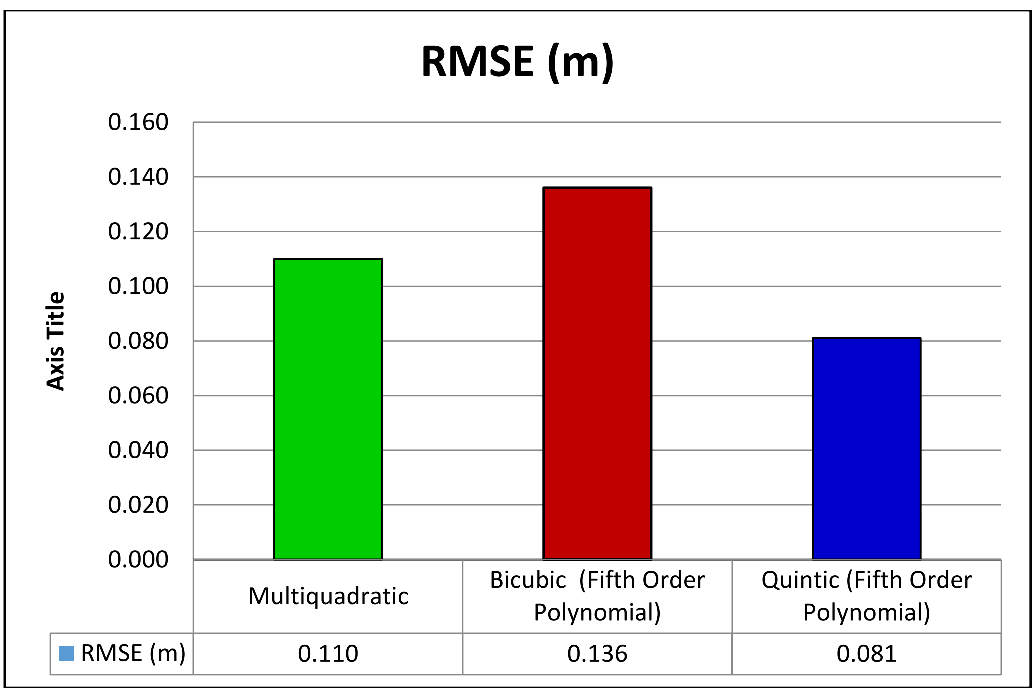


Figure 4: Plot of the Three Models' RMSE/Accuracy

CONCLUSION AND RECOMMENDATION

The study has improved the accuracy of the local geometric geoid model of the Federal Capital Territory (FCT), Abuja by fitting a higher degree (Quintic) polynomial surface to the existing geoid heights of the study area, computing the accuracy of the model and compared the computed accuracy with those of the existing geometric geoid models of the study area. The study shows that the Quintic polynomial geoid model has the highest accuracy, as well as resolution. Hence, improved the accuracy of the local geometric geoid model of the study area. The study has also confirmed the statement by Eteje *et al.,* (2019) that the accuracy of the local geometric geoid model increases as the degree of the model increases. It has as well developed a Microsoft Excel program for the application of the Quintic polynomial geoid model in the study area. The study recommends that whenever a highly reliable data set (geoid heights) is required in the study area, the Quintic polynomial geoid model, as well as program, should be applied.

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