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ECONOMIC ORDER QUANTITY (EOQ) MODEL: WEIBULL AMELIORATING ITEMS WITH CONSTANT DEMAND RATE

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ABSTRACT

In this study, we investigate the behavior of inventory items having the potential of incurring significant increase in their utilities or values while in stock in accordance to the Weibull distribution law. The amelioration of an inventory occurs when it remains in stock but depletes as a result of constant demand rate. The Weibull amelioration function plays the role of depicting the amelioration state of an inventory whereas the constant demand function describes the different possible stages of market demand. We describe the application of simple, yet analytical, approach for successful derivation and analyses of the underlying inventory system. We furthermore, analyze the model while providing, for the model, the necessary but sufficient and optimal replenishment policy. We use numerical results and examples to better illustrate the solution and successfully carryout the sensitivity analysis and the evaluation of the model.

Keywords: Weibull amelioration, Holding cost, Purchasing cost, The cycle length, The rate of amelioration

INTRODUCTION

Ameliorating items are generally known to be those items that are responsible for incurring the gradually increase in the quantity and or quality of inventory items/commodities. Examples of such items include agricultural products such as fishes, fruits, broilers, husbandry, estate properties, stock shares, and so on. Perishable items like fruits and other food stuffs are examples of inventories that undergo depletion by direct spoilage. Physical depletion over time may occur in volatile liquids such as turpentine, gasoline and so on, where a considerable amount of stocked items are lost through evaporation. The presence of depletion due to deterioration is noticed in some items in store such as blood in banks, radioactive substances, pharmaceutical products and the likes. The depletion happens due to spoilage process the items undergo. A lot of literature has been written concerning the control of the inventory of ameliorating and deteriorating items. Such literature was pioneered by Whitin (1957), where goods stored in the fashion industry for a specific period of time were studied. Thereafter, Ghare and Schrade (1963) initiated the research in inventory by developing a simple EOQ model while the rate of decay is assumed constant, which was later extended to a varying deterioration rate by Covert and Philip (1973). There, it was assumed that the deteriorating function takes the form of a two parameter Weibull distribution. Misra (1975), extended the work by including a finite a replenishment rate. In Sarker *et.al* (1997), a variant form of EOQ model is studied while considering level dependence in the presence of deterioration. Additional work by Sh-Tyan & Hui-Ming (2006), was on integrated production type model developed for both the deteriorating and ameliorating items, following Weibull distribution, while including time discount.

Mishra & Singh (2011) developed the EOQ model that combines the ameliorating and deteriorating items, however, by taking into consideration the effect of inflation and the value of time and money. Items like Chickens (preferably broilers), Ducks, Pigs and the likes were studied as when kept in the farm industry their market value increases they grow, however, decreases as a result of expenses of feeding and costs due to natural disasters such as disaster. Gwanda and Sani (2012) considered an EOQ for items that ameliorate according to the linear demand rate function. Optimal analysis is carried-out in determination of the optimal time needed for inclusion and exclusion of inventories in order that the average cost is minimized. The remarkable results were obtained allowed a careful investigation of sensitive parameters on the decision variables.

Samanta (2017) had a quite different work on an EOQ model; fuzzy inventory model having two parameters for deteriorating items following Weibull law was introduced. Critical examination of the development of a fuzzy-type inventory model is given while the associated demand rate function is linear and follows a two-parameter Weibull deterioration law that is fully backlogged.

The resulting costs for deterioration, holding and shortage were assumed to be hexagonal fuzzy members. In Khatri and Gothi (2018), an EOQ model was developed to study the behavior of the model when the ameliorating rate follows the Weibull distribution law at the onset of the production activity. In another study, the demand rate function is presumably an exponential time dependent function and exponentially ameliorating, Devyani (2018). These studies, focused on developing new EOQ models using Weibull distribution law and at the same time provided full detail analyses for finding optimal solutions of the generated model. Chandra et. al. (2013) developed an EOQ having Weibull ameliorating for items with smaller ameliorating rate following linear demand rate. Gwanda in (2019), generated an EOQ model taking into consideration both the ameliorating and deteriorating items where the demand rate and holding costs are respectively and exponentially increasing and linear in time. Ahmad & Hudu (2019) considered an ordering policy suitable for ameliorating items in retail having unconstrained capital with constant demand rate. Optimal control study carried out was meant to obtain an optimal replenishment cycle time in a way that total variable cost T_{vc} is minimized. For that to be achieved, throughout the cycle, both the ameliorating rate and holding cost are assumed constant, however, linear time dependent demand rate while no shortage is recognized. In a recent study by Gwanda (2018), a particular fluctuation nature of the demand function rate is studied for ameliorating items whereas the demand rate is taken as a function of holding cost over a period of time. Hwang (1997) carried out the pioneered work on an EOQ and Partial Selling Quantity (PSQ) models in relating the ameliorating items based on the hypothesis that the time taken during the amelioration follows the law of Weibull distribution. An extended version of EOQ was deeply studied where the demand function is constant, Gwanda & Sani (2011). In a further study by Tripathi and Sang (2012), similar to that of Gwanda & Sani (2011), an EOQ model was studied for constant demand rate except a completely backlogged and shortages for deteriorating items is taken. Their model considered the shortages as completely backlogged whose production rate is proposed to be a demand rate function. Han-Wen *et. al.*, (2017) studied an amelioration inventory model with Weibull distribution of two parameters where the demand rate is a constant function as well. There, a new amelioration model was developed and the detailed analyses of the procedure for finding optimal solution were provided. Furthermore, In their work, improved results for the existing works on inventory system were established. See the Han-Wen *et. al.* (2017) and the references therein for further details.

In Gwanda et. al. (2023), the extended EOQ model was studied. The study combined both ameliorating and deteriorating and took the demand function to be linear dependent. Our work considers ameliorating case only, however, an ameliorating Weibull function and constant demand rate. We intended to explore what the introduced ameliorating Weibull function could suggests depending on the several parameter choices.

Preliminary assumptions

Here, we make the following assumptions to capture specific case of study:

i. We assume the underlying inventory system has only one item and a single stock point. That's is to say a particular item is studied at a time while in stock at a particular instance.
ii. We do not allow shortages in the underlying inventory system of the study.
iii. The amelioration is expected to take place only at the time while the inventory items are unfailingly in the stock.
iv. Finally, the deterioration is either assumed insignificant or relatively zero.

Notation

Some of the parameters involved in the model are defined as follows.

i. T: the complete cycle length of the inventory system;
ii. The Weibull amelioration law is defined by $A(t) = \alpha\beta(t-\mu)^{\beta-1}$ with α>0 as a scale
iii. parameter, β > 0 the shape parameter, and μ> 0 as the location parameter.
iv. C_0: the ordering cost
v. R: the constant demand rate
vi. C_h: the holding cost (constant)
vii. C: the purchasing
viii. α: the rate of amelioration
ix. $I(t)$: an instantaneous inventory level for $0 \leq t \leq T$
x. I_0: The ordering quantity per cycle at *time* $t = 0$
xi. I_T: the amount of on hand inventory in the interval $[0, T]$
xii. T^*: Optimal time for a complete inventory cycle;
xiii. : Total variable cost, and the total variable cost at optimal T^*:= T^*_{vc}

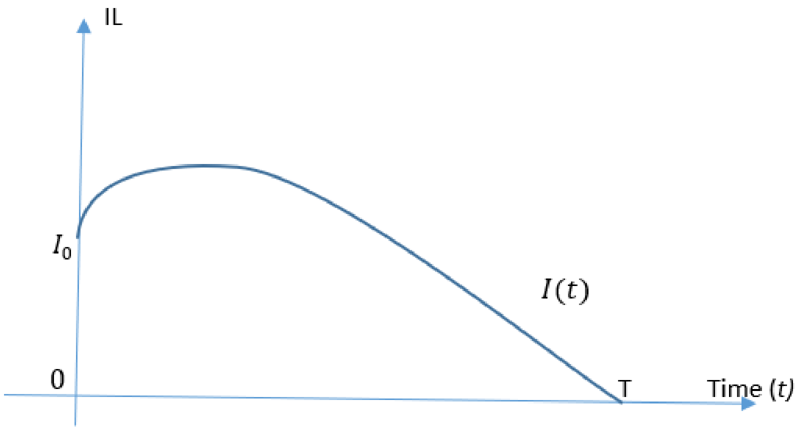

Figure 1: An illustration of the Inventory system with varying inventory level, with I_0 as the initial inventory level and $I(t)$ being the inventory level at particular instance t.

MATERIALS AND METHODS

Let $A(t)$ a function representing the ameliorating that follows the Weibull distribution law having three parameters α, β and μ:

$A(t) = \alpha\beta(t-\mu)^{\beta-1}, \qquad \beta > 0$

Suppose further that, D (t) represents the demand rate and $I(t)$ denotes the instantaneous *on-hand inventory* at time $t \geq 0$.

The equation governing the instantaneous inventory level within and on the interval $0 \leq t \leq T$ reads:

$$\frac{dI(t)}{dt} - A(t)I(t) = -R \qquad (1)$$

where

$$\frac{d}{dt}I(t) - \alpha\beta(t-\mu)^{\beta-1}I(t) = -R \qquad (2)$$

The solution to the equation (2) is easily obtained by using integrating factor as follows: multiplying through by the factor $e^{-(t-\mu)^\beta}$ we have:

$$\frac{d}{dt}I(t)e^{-(t-\mu)^\beta} - \alpha\beta(t-\mu)^{\beta-1}\,I(t)e^{-\alpha(t-\mu)^\beta} = Re^{-\alpha(t-\mu)^\beta}$$

$$I(t)e^{-\alpha(t-\mu)^\beta} = -R\int e^{-\alpha(t-\mu)^\beta}\,dt \qquad (3)$$

Upon approximation of the exponential function via Taylor's series, the solution involving the first two terms of the Taylor's series yields:

$$I(t)e^{-\alpha(t-\mu)^{\beta}} = -R\int(1-\alpha(t-\mu)^{\beta})\ dt \quad (4)$$

$$I(t)e^{-\alpha(t-\mu)^{\beta}} = -R\left(t-\frac{\alpha(t-\mu)^{\beta}}{(\beta+1)}\right)+C \quad (5)$$

$$I(t) = -Re^{-\alpha(t-\mu)^{\beta}}\left(t-\frac{\alpha(t-\mu)^{\beta+1}}{(\beta+1)}\right)+Ce^{-\alpha(t-\mu)^{\beta}} \quad (6)$$

Applying the boundary condition that $t=0,\ I(t)=I_0$ we found that:

$$I_0 = Re^{\alpha(t-\mu)^{\beta}}\left(-\frac{\alpha(t-\mu)^{\beta}}{(\beta+1)}\right)+Ce^{-\alpha(t-\mu)^{\beta}} \quad (7)$$

Hence,

$$C = I_0e^{-\alpha(-\mu)^{\beta}}\left(-\frac{\alpha(t-\mu)^{\beta}}{(\beta+1)}\right) \quad (8)$$

Substituting the value for C into equation (6), one gets

$$I(t) = Re^{\alpha(t-\mu)^{\beta}}\left[-\left(t-\frac{\alpha(t-\mu)^{\beta+1}}{(\beta+1)}\right)+\left[I_0e^{-\alpha(-\mu)^{\beta}}+\left(t-\frac{\alpha(t-\mu)^{\beta+1}}{(\beta+1)}\right)\right]\right] = Re^{\alpha(t-\mu)^{\beta}}\left(t-\frac{\alpha(t-\mu)^{\beta+1}}{(\beta+1)}\right)+ I_0e^{-\alpha(-\mu)^{\beta}}e^{\alpha(t-\mu)^{\beta}}+R\left(t-\frac{\alpha(t-\mu)^{\beta+1}}{(\beta+1)}\right)e^{\alpha(t-\mu)^{\beta}}$$

Similarly, using $I(T)=0$, we obtain

$$I_0 = Re^{\alpha(T-\mu)^{\beta}}\left[-\left(T-\frac{\alpha(T-\mu)^{\beta+1}}{(\beta+1)}\right)+I_0e^{-\alpha(-\mu)^{\beta}}+\left(-\frac{\alpha(-\mu)^{\beta+1}}{(\beta+1)}\right)\right]$$

$$I_0 = Re^{\alpha(T-\mu)^{\beta}}\left[\left(T-\frac{\alpha(T-\mu)^{\beta+1}}{(\beta+1)}\right)-\left(-\frac{\alpha(-\mu)^{\beta+1}}{(\beta+1)}\right)\right]$$

Substituting equation (11) into equation (9), we get;

$$I(t) = -Re^{\alpha(t-\mu)^{\beta}}\left(t-\frac{\alpha(t-\mu)^{\beta+1}}{(\beta+1)}\right)+R\left(-\frac{\alpha(-\mu)^{\beta+1}}{(\beta+1)}\right)e^{\alpha(t-\mu)^{\beta}}e^{\alpha(t-\mu)^{\beta}} \cdot\left[e^{\alpha(t-\mu)^{\beta}}\left(R\left(T-\frac{\alpha(T-\mu)^{\beta+1}}{(\beta+1)}\right)-R\left(-\frac{\alpha(-\mu)^{\beta+1}}{(\beta+1)}\right)\right)\right]$$

$$= -Re^{\alpha(t-\mu)^{\beta}}\left(t-\frac{\alpha(t-\mu)^{\beta+1}}{(\beta+1)}\right)+R\left(t-\frac{\alpha(-\mu)^{\beta+1}}{(\beta+1)}\right)e^{\alpha(t-\mu)^{\beta}}+R\left(t-\frac{\alpha(t-\mu)^{\beta+1}}{(\beta+1)}\right)e^{\alpha(t-\mu)^{\beta}} - R\left(t-\frac{\alpha(t-\mu)^{\beta+1}}{(\beta+1)}\right)e^{\alpha(t-\mu)^{\beta}}$$

$$= R\left(T-\frac{\alpha(T-\mu)^{\beta+1}}{(\beta+1)}\right)e^{\alpha(t-\mu)^{\beta}}-Re^{\alpha(t-\mu)^{\beta}}\left(-\frac{\alpha(t-\mu)^{\beta+1}}{(\beta+1)}\right)$$

Again, by applying Taylor's series expansion on the exponential terms while taking the first two terms, we have

$$I(t) = R\big(1+\alpha(t-\mu)^{\beta}\big)\left(T-\frac{\alpha(T-\mu)^{\beta+1}}{(\beta+1)}\right)-R\left[t-\frac{\alpha(t-\mu)^{\beta+1}}{(\beta+1)}+(t-\mu)^{\beta}-\frac{\alpha^2(t-\mu)^{\beta}}{(\beta+1)}\right]$$

The resulting amount of the *on-hand inventory* for complete cycle T is generally defined by

$$I_T = \int_0^T I(t)dt$$

This yields

$$I_T = R\left[\frac{T^2}{2}-\frac{\alpha^2}{(\beta+1)^2}\left((t-\mu)^{\beta}-(-\mu)^{\beta+1}-\frac{(T-\mu)^{2\beta+2}}{2}\right)+\frac{2\alpha}{(\beta+1)(\beta+2)}\left((t-\mu)^{\beta+2}-(-\mu)^{\beta+2}-\frac{\alpha(-\mu)^{2\beta+2}}{2}\right) -\frac{T}{(\beta+1)}\big(\alpha(T-\mu)^{\beta+1}-\mu^{\beta+1}\big)\right]$$

Therefore, the inventory holding over a complete cycle length T is

$C_h = i \cdot C \cdot I_T$

where i is the change of inventory.

The corresponding amelioration amount (A_m) defined over the interval $[0,T]$ is

$A_m = \alpha \cdot I_T$,

while the total variable cost T_{vc} reads

Ordering cost (C_0) + Inventory holding cost (C_h) – cost of ameliorated amount (A_m):

$T_{vc} = C_0 + C_h - A_m$.

Then, the total variable cost per unit cycle period is given by

$$T_{vc}(T) = \frac{C_0}{T}+\frac{1}{T}(i\cdot C-\alpha)I_T$$

This is equivalently written as

$$T_{vc}(T) = \frac{C_0}{T} + \frac{(iC-\alpha)}{T}R\left[\frac{T^2}{2} - \frac{\alpha^2}{(\beta+1)^2}\left((T-\mu)^{\beta} - (-\mu)^{\beta+1} - \frac{(T-\mu)^{2\beta+2}}{2}\right) + \frac{2\alpha}{(\beta+1)(\beta+2)}\left((T-\mu)^{\beta+2} - (-\mu)^{\beta+2} - \frac{\alpha(-\mu)^{2\beta+2}}{2}\right) - \frac{T}{(\beta+1)}\left(\alpha(T-\mu)^{\beta+1} - \mu^{\beta+1}\right)\right]$$

When equation (15) is differentiated with respect to T one obtains

$$\begin{aligned}\frac{dT_{vc}(T)}{dT} = {} & -\frac{C_0}{T^2} - \frac{1}{T^2}\Bigg((Ci-\alpha)R\Bigg(\frac{1}{2}T^2 - \frac{\alpha^2\left((T-\mu)^{\beta} - (-\mu)^{\beta+1} - \frac{1}{2}(T-\mu)^{2\beta+2}\right)}{(\beta+1)^2} \\ & + \frac{2\alpha\left((T-\mu)^{\beta+2} - (-\mu)^{\beta+2} - \frac{1}{2}\alpha(-\mu)^{2\beta+2}\right)}{(\beta+1)(\beta+2)} \\ & - \frac{T\left(\alpha(T-\mu)^{\beta+1} - (-\mu)^{\beta+1}\right)}{\beta+1}\Bigg)\Bigg) + \frac{1}{T}\Bigg((Ci-\alpha)R\Bigg(T \\ & - \frac{\alpha^2\left(\frac{(T-\mu)^{\beta}\beta}{T-\mu} - \frac{1}{2}\frac{(T-\mu)^{2\beta+2}(2\beta+2)}{T-\mu}\right)}{(\beta+1)^2} + \frac{2\alpha(T-\mu)^{\beta+2}}{(\beta+1)(T-\mu)} \\ & - \frac{\alpha(T-\mu)^{\beta+1} - (-\mu)^{\beta+1}}{\beta+1} - \frac{T\alpha(T-\mu)^{\beta+1}}{T-\mu}\Bigg)\Bigg)\end{aligned} \tag{17}$$

For optimal value of T minimizing T_{vc} per unit time, we take $\frac{d[T_{vc}(T)]}{dT}$ =0, which simplifies further to

$$\begin{aligned} & 2(-\mu)^{2\beta+2}R\alpha^3\beta + 2(T-\mu)^{\beta}R\alpha^3\beta - 2(-\mu)^{\beta+1}R\alpha^3\beta - (T-\mu)^{2\beta+2}R\alpha^3\beta \\ & - 4(T-\mu)^{\beta+2}R\alpha^2\beta + 4(-\mu)^{\beta+2}R\alpha^2\beta + 8(T-\mu)^{\beta+1}RT\alpha^2 + 4(T-\mu)^{2\beta+1}RT\alpha^3 - 4(T-\mu)^{\beta}RT^2\alpha^2 - 4(T-\mu)^{\beta-1}RT\alpha^3\beta + 6(T-\mu)^{2\beta+1}RT\alpha^3\beta \\ & - 10(T-\mu)^{\beta}RT^2\alpha^2\beta + 4(T-\mu)^{\beta+1}RT\alpha^2\beta^2 - 2C(-\mu)^{2\beta+2}R\alpha^2 i \\ & - 4C(T-\mu)^{\beta}R\alpha^2 i + 4C(-\mu)^{\beta+1}R\alpha^2 i + 2C(T-\mu)^{2\beta+2}R\alpha^2 i + 12(T-\mu)^{\beta+1}RT\alpha^2\beta + 4C(T-\mu)^{\beta+2}R\alpha i - 4C(-\mu)^{\beta+2}R\alpha i \\ & - 2(T-\mu)^{\beta}RT^2\alpha^2\beta^3 - 2(T-\mu)^{\beta-1}RT\alpha^3\beta^2 + 2(T-\mu)^{2\beta+1}RT\alpha^3\beta^2 - 8(T-\mu)^{\beta}RT^2\alpha^2\beta^2 - CRT^2\beta^3 i - 4CRT^2\beta^2 i - 5CRT^2\beta i + 2RT^2\alpha \\ & + 2(-\mu)^{2\beta+2}R\alpha^3 + 4(T-\mu)^{\beta}R\alpha^3 - 4(-\mu)^{\beta+1}R\alpha^3 - 2(T-\mu)^{2\beta+2}R\alpha^3 - 4(T-\mu)^{\beta+2}R\alpha^2 + 4(-\mu)^{\beta+2}R\alpha^2 + 2C_0\beta^3 + 8C_0\beta^2 + 10C_0\beta \\ & - 4C(T-\mu)^{\beta+1}RT\alpha\beta^2 i - 12C(T-\mu)^{\beta+1}RT\alpha\beta i + 2C(T-\mu)^{\beta}RT^2\alpha\beta^3 i + 2C(T-\mu)^{\beta-1}RT\alpha^2\beta^2 i - 2C(T-\mu)^{2\beta+1}RT\alpha^2\beta^2 i + 8C(T-\mu)^{\beta}RT^2\alpha\beta^2 i \\ & + 4C(T-\mu)^{\beta-1}RT\alpha^2\beta i - 6C(T-\mu)^{2\beta+1}RT\alpha^2\beta i + 10C(T-\mu)^{\beta}RT^2\alpha\beta i \\ & + 4C_0 - 2C(-\mu)^{2\beta+2}R\alpha^2\beta i - 2C(T-\mu)^{\beta}R\alpha^2\beta i + 2C(-\mu)^{\beta+1}R\alpha^2\beta i \\ & + C(T-\mu)^{2\beta+2}R\alpha^2\beta i + 4C(T-\mu)^{\beta+2}R\alpha\beta i - 4C(-\mu)^{\beta+2}R\alpha\beta i \\ & - 8C(T-\mu)^{\beta+1}RT\alpha i - 4C(T-\mu)^{2\beta+1}RT\alpha^2 i + 4C(T-\mu)^{\beta}RT^2\alpha i \\ & + RT^2\alpha\beta^3 + 4RT^2\alpha\beta^2 - 2CRT^2 i + 5RT^2\alpha\beta = 0 \end{aligned} \tag{18}$$

The associated economic order quantity EOQ is evaluated as follows

$$EOQ = I_0 = \frac{Re^{\alpha(-\mu)\beta}}{(\beta+1)}\left(\alpha(T-\mu)^{\beta+1} - (\beta+1)T + \alpha(-\mu)^{\beta+1}\right) \tag{19}$$

RESULTS AND DISCUSSION

Using the equations (18) and (19) we carry-out the numerical experiments which seeks for the optimal value of T. Several values of the mentioned parameters are investigated in five (5) experiments whose corresponding outputs are described in the table below:

Table 1: Five numerical results showing the sensitivity of the parameters.

SN	C	C_0	i	R	α	β	μ
1	3500	15000	0.45	160	15	80	0.24
2	2500	15000	0.48	140	20	60	0.28
3	1800	25000	0.33	400	30	60	0.25
4	5000	30000	0.49	300	15	50	0.22
5	3000	57000	0.67	130	45	120	0.34

Table 2: Outputs in searching of the optimal values of T and I

SN	T^*	EOQ^*	$T_{vc}(T^*)$
1	127 days	1 $unit$	86534
2	156 days	1 $unit$	70399
3	176 days	3 $unit$	106207
4	105 days	2 $unit$	209358
5	244 days	1 $unit$	170650

Table 3: Results of the sensitivity analysis for Table 1 and Table 2 for the determination of the sensitive parameter on the decision variables

Parameters involved	Percentage change in the parameters	Change in the result		
		T^*	$T_{vc}(T^*)$	EOQ^*
C	−50%	42	−30	92
	−25%	15	−14	56
	−5%	2	−3	39
	5%	−3	2	31
	25%	−11	12	21
	50%	−19	23	10
C_0	−50%	−30	−29	−5
	−25%	−13	−13	18
	−5%	−3	−3	32
	5%	2	2	39
	25%	11	11	51
	50%	22	22	66
i	−50%	42	−30	92
	−25%	15	−13	56
	−5%	2	−3	39
	5%	−3	2	31
	25%	−11	12	20
	50%	−19	23	10
α	−50%	0	0	35
	−25%	0	0	35
	−5%	0	0	35
	5%	0	0	35
	25%	0	0	35
	50%	0	0	−35
β	−50%	0	0	165
	−25%	0	0	80
	−5%	0	0	42
	5%	0	0	−29
	25%	0	0	−9
	50%	0	0	−8
μ	−50%	0	0	36
	−25%	0	0	36
	−5%	0	0	36
	5%	0	0	36
	25%	0	0	36
	50%	0	0	36
R	−50%	36	−28	99
	−25%	11	−12	98
	−5%	−1	−1	98
	5%	−6	−1	−98
	25%	−15	13	−98
	50%	−21	24	−97

Sensitivity Analysis

Table 3 highlights the results of the stability analysis carried out on the involved parameters. It shows the effect in the change on parameters affects the inventory variables EOQ, T^* and T_{vc}^*. It is apparent from the table that variation of these parameters result in changes in some of those variables, where change in signs of the values of those inventory variables indicate how sensitive the parameters are on them. On the other hand, where changes are not detected, it is an indication of how insensitive the corresponding parameter is.

In Table 2, the sensitivity analysis shows that the model is highly sensitive to change in the parameters R C, C_0 and i and moderately sensitive to changes in the parameters β, α and μ.

Discussion

Here we discuss the effect of the changes in the values of the involved parameters on decision variables as contained in Table 3.

The Table 3 indicates that almost all decision variables are sensitive with respect to the changes on the parameter values except. From table 3, we have seen and observe the following results:

i. Sensitivity of the optimal cycle length T^*: there is a significant increases in T^* when C_0 increases, however, decreasing behavior is observed with the increase in the parameters C, R and i. It can be noticed that T^* remains in sensitive on α, β and μ.
ii. The optimal total variable cost $T_{vc}(T)^*$ increases with increase in the parameters C_0 C, R and i but slowly decreasing in α, β and μ. Hence, T_{vc}^* is sensitive to these parameters.
iii. The sEOQ^* undergoes increments due to increment in the parameters C_0 and R but decrement as a result of increment the parameters C, β and . It however, remains insensitive on the variation of the Weibull parameters α, β and μ.

CONCLUSION

In this article, an EOQ model having inventory items that follow Weibull ameliorating function has been proposed. Sensitivi
ty analyses have been established where the Weibull parameters have shown significant influence on inventory variables like EOQ and T^*. The results of the analyses have been presented in tabular form where sensitive parameters are detected through the sudden change in the values of the inventory variables.

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