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SEMI-ANALYTIC SOLUTION OF MATHEMATICAL MODEL OF DISEASE DYNAMICS IN SCHOOL USING HOMOTOPY PERTURBATION METHOD (HPM)

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ABSTRACT

School is a place where the transmission of diseases is easy, resulting from the interaction between students and instructors. In this study, we consider the application of the Homotopy Perturbation Method (HPM) to solve a mathematical model of disease dynamics in schools. The model equation consists of five classes: the susceptible class S, the infected and careless class I_c, the infected and unhygienic class I_u, the infected and aware class I_a, and the recovered class R. The non-linear differential model equations are transformed into a series of linear differential equations, enabling faster derivation of approximate solutions.

Keywords: Mathematical Model, Disease Dynamics, Homotopy Perturbation Method

INTRODUCTION

Schools are the backbone of our society, responsible for shaping the minds of future generations through the transmission of knowledge and values. Nevertheless, their dense populations and interactive environments create an ideal setting for the rapid spread of infectious diseases. (Marili and Susana, 2023). Schools, by their very nature, bring together young people who often exhibit characteristics of a susceptible population, such as frequent contact and close proximity, creating an environment where outbreaks of infectious diseases can thrive. When disease outbreaks occur in schools, they can have devastating consequences on the physical and mental health of the student population, as well as the broader community. (Wang and Fang, 2020). Knowledge is power in the fight against infectious diseases. Strengthening students' and schools' understanding of disease prevention and treatment is essential to break the chain of transmission and create a safe and healthy environment for academic success.

Health literacy, comprising health knowledge and healthy behaviors, is a vital component in the prevention and control of diseases. Health education plays a crucial role in enhancing student knowledge and fostering appropriate behaviors, ultimately contributing to improved disease control and prevention. By promoting health knowledge, health education effectively slows the spread of infectious diseases. Therefore, it is essential to strengthen health education programs to enhance the health literacy of students, empowering them to make informed decisions and adopt healthy practices. (Wang and Fang, 2020, Jomell and Angelo, 2021).

Some authors have worked to solve model equations, Topman *et al.* (2023) examined the application of the Homotopy Pertubation method for solving the influenza virus model. Otoo *et al.* (2022) applied HPM to solve a system of non-linear differential model equations for the transmission dynamics of diarrhea. Ndidiamaka *et al.* (2021) solved a mathematical model of Ebola Virus disease transmission dynamics using HPM. The model is a coupled non-linear differential equation. Sinan (2020) applied HPM to solve a system of non-linear ordinary differential equations used to model rabies disease. The results obtained were compared with Runge-Kutta's fourth order and nonstandard finite difference methods, and it shows to be a reliable method of solution. Akogwu (2022) applied the Differential Transformation Method (DTM) to solve a mathematical model for Covid-19 transmission and vaccination in Nigeria. Azuaba *et al*., (2022) solved a substance abuse and domestic violence mathematical model using HPM. Buhe *et al.*, (2023) solved a non-linear mathematical model that studies the depletion of forest resources due to population growth and pressure.

MATERIALS AND METHODS

Model Formulation

The total population denoted by $N(t)$ is subdivided into five (5) classes (students) which are; the susceptible class S, the infected and careless class I_c, infected and unhygienic class I_u, infected and aware class I_a and lastly the recovered class R.

The model equations are given as:

$$\frac{dS}{dt} = (1-\rho_1-\rho_2-\rho_3)\Lambda - \frac{\beta}{N}(I_c+I_u+\varphi I_a)S + \omega R - \mu S \quad (1)$$

$$\frac{dI_c}{dt} = \rho_1\Lambda + \frac{\beta I_c S}{N} - (\psi_1+\kappa_1+\delta_1+\varepsilon_1+\mu)I_c \quad (2)$$

$$\frac{dI_u}{dt} = \rho_2\Lambda + \frac{\beta I_u S}{N} + \kappa_1 I_c - (\psi_2+\kappa_2+\delta_2+\mu)I_u \quad (3)$$

$$\frac{dI_a}{dt} = \rho_3\Lambda + \frac{\beta\varphi I_a S}{N} + \kappa_2 I_u + \varepsilon_1 I_c - (\psi_3+\delta_3+\mu)I_a \quad (4)$$

$$\frac{dR}{dt} = \psi_1 I_c + \psi_2 I_u + \psi_3 I_a - (\omega+\mu)R \quad (5)$$

For clarity, Tables 1 and 2 outline the definitions of the model's variables and parameters,
and Figure 1 presents a schematic overview of the model's structure.

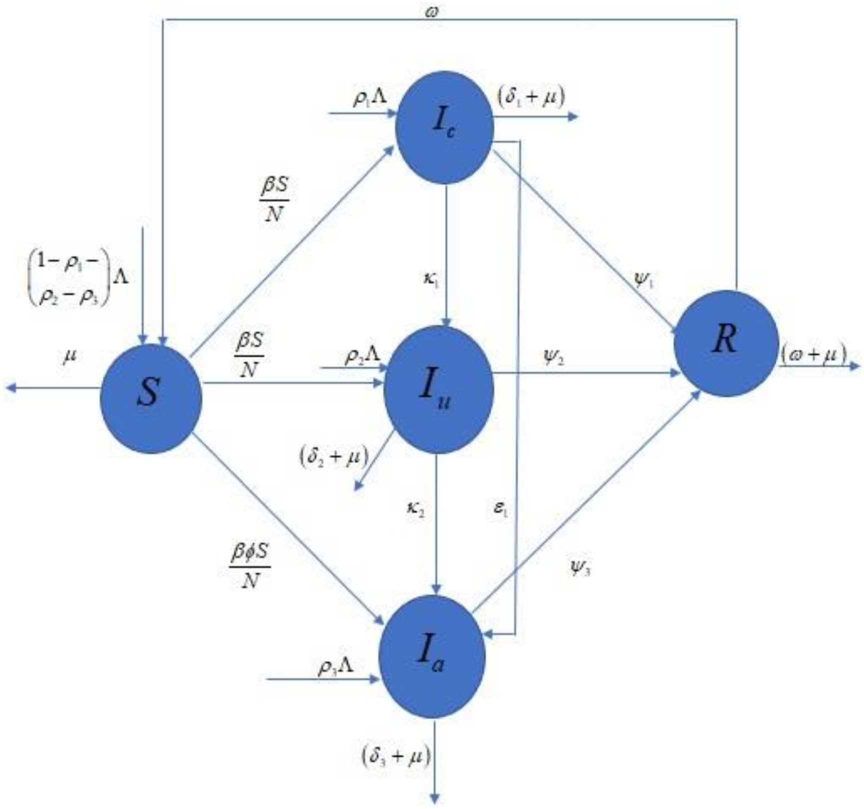

Figure 1: Schematic diagram of model

Table 1: Description of Variables of the Model

Variables	Description
S	Susceptible class
I_c	Infected and Careless class
I_u	Infected and Unhygienic class
I_a	Infected and Aware class
R	Recovered class

Table 2: Description of Parameters of the Model

Parameters	Interpretation	Value	Source
Λ	Recruitment Rate	15.26	Estimated
ρ_ii=1,2,3	Immigration rate	0.0,0.3,0.2	Assumed
β	Contact Rate	0.0025	Martcheva, 2015
φ	Effect of awareness in transmission rate	0.003	Assumed
ω	Rate of susceptibility after recovery	0.0014	Assumed
μ	Natural death rate	0.02	Oladejo and Oluyo 2022
κ_1	Movement from $I_c$to I_u	0.1	Assumed
κ_2	Movement from I_u to I_a	0.15	Assumed
ε_1	Movement from I_c to I_a	0.2	Assumed
δ_1	Disease induced death rate for I_c	0.003	Assumed
δ_2	Disease induced death rate for I_u	0.002	Assumed
ψ_ii=1,2,3	Treatment rate	0.11,0.105,0.25,	Martcheva, (2015)

Methodology

We solve the model equations using Homotopy Perturbation Method (HPM)

With initial conditions;

$S(0) = S_0; I_c(0) = I_{c0}; I_u(0) = I_{u0}; I_a(0) = I_{a0}; R(0) = R_0$ (6)

Let;

$S = a_0 + pa_1 + p^2a_2 + \ldots$ (7)

$I_c = b_0 + pb_1 + p^2b_2 + \ldots$ (8)

$I_u = x_0 + px_1 + p^2x_2 + \ldots$ (9)

$I_a = y_0 + py_1 + p^2y_2 + \ldots$ (10)

$R = z_0 + pz_1 + p^2z_2 + \ldots$ (11)

Applying HPM to (1) we have

$(1-p)\frac{dS}{dt} + p\left(\frac{dS}{dt} + \frac{\beta\alpha S}{N} + \mu S - \omega R - (1-\rho_1-\rho_2-\rho_3)\Lambda\right) = 0$ (12)

Where $\alpha = (I_c + I_u + \varphi I_a)$ (13)

Substituting (7) and (11) into (12) we have

$$(a_0^1 + pa_1^1 + p^2a_2^1 + \cdots) + p\left(\frac{\beta\alpha}{N} + \mu\right)(a_0 + pa_1 + p^2a_2 + \ldots) - p\omega(z_0 + pz_1 + p^2z_2 + \ldots) - p(1 - \rho_1 - \rho_2 - \rho_3)\Lambda = 0 \quad (14)$$

Comparing we have

$$p^0: a_0^1 = 0 \quad (15)$$

$$p^1: a_1^1 + \left(\frac{\beta\alpha}{N} + \mu\right)a_0 - \omega z_0 - (1 - \rho_1 - \rho_2 - \rho_3)\Lambda = 0 \quad (16)$$

$$p^2: a_2^1 + \left(\frac{\beta\alpha}{N} + \mu\right)a_1 - \omega z_1 = 0 \quad (17)$$

Applying HPM to (2) we have

$$(1-p)\frac{dI_c}{dt} + p\left(\frac{dI_c}{dt} + (\psi_1 + \kappa_1 + \delta_1 + \varepsilon_1 + \mu)I_c - \rho_1\Lambda - \frac{\beta I_c S}{N}\right) = 0 \quad (18)$$

Substituting (7) and (8) we have

$$(b_0^1 + pb_1^1 + p^2b_2^1 + \ldots) + p\tau_1(b_0 + pb_1 + p^2b_2 + \ldots) - p\rho_1\Lambda - p\frac{\beta}{N}(b_0 + pb_1 + p^2b_2 + \ldots)(a_0 + pa_1 + p^2a_2 + \ldots) = 0 \quad (19)$$

where $\tau_1 = \psi_1 + \kappa_1 + \delta_1 + \varepsilon_1 + \mu$ (20)

Comparing

$$p^0: b_0^1 = 0 \quad (21)$$

$$p^1: b_1^1 + \tau_1 b_0 - \rho_1\Lambda - \frac{\beta}{N}a_0 b_0 = 0 \quad (22)$$

$$p^2: b_2^1 + \tau_1 b_1 - \frac{\beta}{N}(a_0 b_1 + a_1 b_0) = 0 \quad (23)$$

Applying HPM to (3) we have

$$(1-p)\frac{dI_u}{dt} + p\left(\frac{dI_u}{dt} + \tau_2 I_u - \rho_2\Lambda - \frac{\beta I_u S}{N} - \kappa_1 I_c\right) = 0 \quad (24)$$

Where $\tau_2 = \psi_2 + \kappa_2 + \delta_2 + \mu$ (25)

Substituting (7),(8) and (9)

$$(x_0^1 + px_1^1 + p^2x_2^1 + \ldots) + p\tau_2(x_0 + px_1 + p^2x_2 + \ldots) - p\rho_2\Lambda - p\frac{\beta}{N}(x_0 + px_1 + p^2x_2 + \ldots)(a_0 + pa_1 + p^2a_2 + \ldots) - p\kappa_1(b_0 + pb_1 + p^2b_2 + \ldots) = 0 \quad (26)$$

Comparing

$$p^0: x_0^1 = 0 \quad (27)$$

$$p^1: x_1^1 + \tau_2 x_0 - \rho_2\Lambda - \frac{\beta}{N}a_0 x_0 - \kappa_1 b_0 = 0 \quad (28)$$

$$p^2: x_2^1 + \tau_2 x_1 - \kappa_1 b_1 - \frac{\beta}{N}(a_0 x_1 + a_1 x_0) = 0 \quad (29)$$

Applying HPM to (4) we have

$$(1-p)\frac{dI_a}{dt} + p\left(\frac{dI_a}{dt} + \tau_3 I_a - \rho_3\Lambda - \frac{\beta\varphi I_a S}{N} - \kappa_2 I_u - \varepsilon_1 I_c\right) = 0 \quad (30)$$

$$\tau_3 = \psi_3 + \delta_3 + \mu \quad (31)$$

Substituting (7),(8), (9) and (10) we have

$$(y_0^1 + py_1^1 + p^2y_2^1 + \ldots) + p\tau_3(y_0 + py_1 + p^2y_2 + \ldots) - p\rho_3\Lambda - p\beta\varphi(y_0 + py_1 + p^2y_2 + \ldots)(a_0 + pa_1 + p^2a_2 + \ldots) - p\kappa_2(x_0 + px_1 + p^2x_2 + \ldots) - p\varepsilon_1(b_0 + pb_1 + p^2b_2 + \ldots) = 0 \quad (32)$$

Comparing

$$p^0: y_0^1 = 0 \quad (33)$$

$$p^1: y_1^1 + \tau_3 y_0 - \rho_3\Lambda - \beta\varphi a_0 y_0 - \kappa_2 x_0 - \varepsilon_1 b_0 = 0 \quad (34)$$

$$p^2: y_2^1 + \tau_3 y_1 - \kappa_2 x_1 - \frac{\beta\varphi}{N}(a_0 y_1 + a_1 y_0) - \varepsilon_1 b_1 = 0 \quad (35)$$

Applying HPM to (5) we have

$$(1-p)\frac{dR}{dt} + p\left(\frac{dR}{dt} + (\omega + \mu)R - \psi_1 I_c - \psi_2 I_u - \psi_3 I_a\right) = 0 \quad (36)$$

Substituting (8),(9), (10) and (11) we have

$$(z_0^1 + pz_1^1 + p^2z_2^1 + \ldots) + p(\omega + \mu)(z_0 + pz_1 + p^2z_2 + \ldots) - p\psi_1(b_0 + pb_1 + p^2b_2 + \ldots) - p\psi_2(x_0 + px_1 + p^2x_2 + \ldots) - p\psi_3(y_0 + py_1 + p^2y_2 + \ldots) = 0 \quad (37)$$

Comparing

$$p^0: z_0^1 = 0 \quad (38)$$

$$p^1: z_1^1 + (\omega + \mu)z_0 - \psi_1 b_0 - \psi_2 x_0 - \psi_3 y_0 = 0 \quad (39)$$

$$p^2: z_2^1 + (\omega + \mu)z_1 - \psi_1 b_1 - \psi_2 x_1 - \psi_3 y_1 = 0 \quad (40)$$

Solving equations (15),(21),(27), (33) and (38) we have

$$a_0^1 = 0 \quad (41)$$

This implies

$$a_0 = S_0 \quad (42)$$

Similarly,

$$b_0 = I_{c0} \quad (43)$$

$$x_0 = I_{u0} \quad (44)$$

$$y_0 = I_{a0} \quad (45)$$

$$z_0 = R_0 \quad (46)$$

Substituting (42) – (46) into (16), (22), (28), (34) and (39) we have

$$a_1^1 + (\beta\alpha + \mu)S_0 - \omega R_0 - (1 - \rho_1 - \rho_2 - \rho_3)\Lambda = 0 \quad (47)$$

$$b_1^1 + \tau_1 I_{c0} - \rho_1\Lambda - \frac{\beta}{N}S_0 I_{c0} = 0 \quad (48)$$

$$x_1^1 + \tau_2 I_{u0} - \rho_2\Lambda - \frac{\beta}{N}S_0 I_{u0} - \kappa_1 I_{c0} = 0 \quad (49)$$

$$y_1^1 + \tau_3 I_{a0} - \rho_3\Lambda - \frac{\beta}{N}\varphi S_0 I_{a0} - \kappa_2 I_{u0} - \varepsilon_1 I_{c0} = 0 \quad (50)$$

$$z_1^1 + (\omega + \mu)R_0 - \psi_1 I_{c0} - \psi_2 I_{u0} - \psi_3 I_{a0} = 0 \quad (51)$$

Integrating (47) – (51) we have

$$a_1 = \left((1 - \rho_1 - \rho_2 - \rho_3)\Lambda - \left(\frac{\beta}{N}\alpha + \mu\right)S_0 + \omega R_0\right)t \quad (52)$$

$$b_1 = \left(\rho_1\Lambda + \frac{\beta}{N}S_0 I_{c0} - \tau_1 I_{c0}\right)t \quad (53)$$

$$x_1 = \left(\rho_2\Lambda + \frac{\beta}{N}S_0 I_{u0} + \kappa_1 I_{c0} - \tau_2 I_{u0}\right)t \quad (54)$$

$$y_1 = \left(\rho_3\Lambda + \frac{\beta}{N}\varphi S_0 I_{a0} + \kappa_2 I_{u0} + \varepsilon_1 I_{c0} - \tau_3 I_{a0}\right)t \quad (55)$$

$$z_1 = (\psi_1 I_{c0} + \psi_2 I_{u0} + \psi_3 I_{a0} - (\omega + \mu)R_0)t \quad (56)$$

Substituting (52) – (56) into (17), (23), (29), (35) and (40)

$$a_2^1 + \left(\frac{\beta}{N}\alpha + \mu\right)\left((1 - \rho_1 - \rho_2 - \rho_3)\Lambda - \left(\frac{\beta}{N}\alpha + \mu\right)S_0 + \omega R_0\right)t - \omega(\psi_1 I_{c0} + \psi_2 I_{u0} + \psi_3 I_{a0} - (\omega + \mu)R_0)t = 0 \quad (57)$$

$$b_2^1 + \tau_1\left(\rho_1\Lambda + \frac{\beta}{N}S_0 I_{c0} - \tau_1 I_{c0}\right)t - \frac{\beta}{N}\begin{pmatrix} S_0\left(\rho_1\Lambda + \frac{\beta}{N}S_0 I_{c0} - \tau_1 I_{c0}\right)t \\ +I_{c0}\left((1 - \rho_1 - \rho_2 - \rho_3)\Lambda - \left(\frac{\beta}{N}\alpha + \mu\right)S_0 + \omega R_0\right)t \end{pmatrix} = 0 \quad (58)$$

$$x_2^1 + \tau_2\left(\rho_2\Lambda + \frac{\beta}{N}S_0 I_{u0} + \kappa_1 I_{c0} - \tau_2 I_{u0}\right)t - \kappa_1\left(\rho_1\Lambda + \frac{\beta}{N}S_0 I_{c0} - \tau_1 I_{c0}\right)t - \frac{\beta}{N}\begin{pmatrix} S_0\left(\rho_2\Lambda + \frac{\beta}{N}S_0 I_{u0} + \kappa_1 I_{c0} - \tau_2 I_{u0}\right)t \\ +I_{u0}\left((1 - \rho_1 - \rho_2 - \rho_3)\Lambda - \left(\frac{\beta}{N}\alpha + \mu\right)S_0 + \omega R_0\right)t \end{pmatrix} = 0 \quad (59)$$

$$y_2^1 + \tau_3\left(\rho_3\Lambda + \frac{\beta}{N}\varphi S_0 I_{a0} + \kappa_2 I_{u0} + \varepsilon_1 I_{c0} - \tau_3 I_{a0}\right)t - \kappa_2\left(\rho_2\Lambda + \frac{\beta}{N}S_0 I_{u0} + \kappa_1 I_{c0} - \tau_2 I_{u0}\right)t - \frac{\beta}{N}\varphi\begin{pmatrix} S_0\left(\rho_3\Lambda + \frac{\beta}{N}\varphi S_0 I_{a0} + \kappa_2 I_{u0} + \varepsilon_1 I_{c0} - \tau_3 I_{a0}\right)t \\ +I_{u0}a_1 \end{pmatrix} - \varepsilon_1\left(\rho_1\Lambda + \frac{\beta}{N}S_0 I_{c0} - \tau_1 I_{c0}\right)t = 0 \quad (60)$$

$$z_2^1 + (\omega + \mu)(\psi_1 I_{c0} + \psi_2 I_{u0} + \psi_3 I_{a0} - (\omega + \mu)R_0)t - \psi_1\left(\rho_1\Lambda + \frac{\beta}{N}S_0 I_{c0} - \tau_1 I_{c0}\right)t - \psi_2\left(\rho_2\Lambda + \frac{\beta}{N}S_0 I_{u0} + \kappa_1 I_{c0} - \tau_2 I_{u0}\right)t - \psi_3\left(\rho_3\Lambda + \frac{\beta}{N}\varphi S_0 I_{a0} + \kappa_2 I_{u0} + \varepsilon_1 I_{c0} - \tau_3 I_{a0}\right)t = 0 \quad (61)$$

Solving (57) – (61)

$$a_2 = \left\{ \begin{matrix} w(\psi_1 I_{c0} + \psi_2 I_{u0} + \psi_3 I_{a0} - (\omega+\mu)R_0) - \\ \left(\frac{\beta\alpha}{N}+\mu\right)\left[(1-\rho_1-\rho_2-\rho_3)\Lambda - \left(\frac{\beta\alpha}{N}+\mu\right)S_0 + \omega R_0\right] \end{matrix} \right\} \frac{t^2}{2} \quad (62)$$

$$b_2 = \left\{ \begin{matrix} \frac{\beta}{N}\begin{pmatrix} S_0\left(\rho_1\Lambda + \frac{\beta S_0 I_{c0}}{N} - \tau_1 I_{c0}\right) \\ +I_{c0}\left((1-\rho_1-\rho_2-\rho_3)\Lambda - \left(\frac{\beta\alpha}{N}+\mu\right)S_0 + \omega R_0\right) \end{pmatrix} \\ -\tau_1\left(\rho_1\Lambda + \frac{\beta S_0 I_{c0}}{N} - \tau_1 I_{c0}\right) \end{matrix} \right\} \frac{t^2}{2} \quad (63)$$

$$x_2 = \left\{ \begin{matrix} \frac{\beta}{N}\begin{pmatrix} S_0\left(\rho_2\Lambda + \frac{\beta S_0 I_{u0}}{N} + \kappa_1 I_{c0} - \tau_2 I_{u0}\right) \\ +I_{u0}\left((1-\rho_1-\rho_2-\rho_3)\Lambda - \left(\frac{\beta\alpha}{N}+\mu\right)S_0 + \omega R_0\right) \end{pmatrix} \\ +\kappa_1\left(\rho_1\Lambda + \frac{\beta S_0 I_{c0}}{N} - \tau_1 I_{c0}\right) - \tau_2\left(\rho_2\Lambda + \frac{\beta S_0 I_{u0}}{N} + \kappa_1 I_{c0} - \tau_2 I_{u0}\right) \end{matrix} \right\} \frac{t^2}{2} \quad (64)$$

$$y_2 = \left\{ \begin{matrix} \kappa_2\left(\rho_2\Lambda + \frac{\beta S_0 I_{u0}}{N} + \kappa_1 I_{c0} - \tau_2 I_{u0}\right) \\ +\frac{\beta\varphi}{N}\left(S_0\begin{pmatrix} \rho_3\Lambda + \frac{\beta\varphi S_0 I_{a0}}{N} + \kappa_2 I_{u0} \\ +\varepsilon_1 I_{c0} - \tau_3 I_{a0} \end{pmatrix} + I_{u0}a_1\right) \\ +\varepsilon_1\left(\rho_1\Lambda + \frac{\beta S_0 I_{c0}}{N} - \tau_1 I_{c0}\right) \\ -\tau_3\left(\rho_3\Lambda + \frac{\beta\varphi S_0 I_{a0}}{N} + \kappa_2 I_{u0} + \varepsilon_1 I_{c0} - \tau_3 I_{a0}\right) \end{matrix} \right\} \frac{t^2}{2} \quad (65)$$

$$z_2 = \left\{ \begin{matrix} \psi_1\left(\rho_1\Lambda + \frac{\beta S_0 I_{c0}}{N} - \tau_1 I_{c0}\right) + \psi_2\left(\rho_2\Lambda + \frac{\beta S_0 I_{u0}}{N} + \kappa_1 I_{c0} - \tau_2 I_{u0}\right) \\ +\psi_3\left(\rho_3\Lambda + \frac{\beta\varphi S_0 I_{a0}}{N} + \kappa_2 I_{u0} + \varepsilon_1 I_{c0} - \tau_3 I_{a0}\right) \\ -(\omega+\mu)[\psi_1 I_{c0} + \psi_2 I_{u0} + \psi_3 I_{a0} - (\omega+\mu)R_0] \end{matrix} \right\} \frac{t^2}{2} \quad (66)$$

Setting$p = 1$ in equations (7) – (11) and substituting the appropriate values we have

$$S(t) = S_0 + \left((1-\rho_1-\rho_2-\rho_3)\Lambda - \left(\frac{\beta\alpha}{N}+\mu\right)S_0 + \omega R_0\right)t$$
$$+\left\{ \begin{matrix} \omega(\psi_1 I_{c0} + \psi_2 I_{u0} + \psi_3 I_{a0} - (\omega+\mu)R_0) - \\ \left(\frac{\beta\alpha}{N}+\mu\right)\left[(1-\rho_1-\rho_2-\rho_3)\Lambda - \left(\frac{\beta\alpha}{N}+\mu\right)S_0 + \omega R_0\right] \end{matrix} \right\} \frac{t^2}{2} + \ldots \quad (67)$$

$$I_c(t) = I_{c0} + \left(\rho_1\Lambda + \frac{\beta S_0 I_{c0}}{N} - \tau_1 I_{c0}\right)t$$
$$+\left\{ \begin{matrix} \frac{\beta}{N}\begin{pmatrix} S_0\left(\rho_1\Lambda + \frac{\beta S_0 I_{c0}}{N} - \tau_1 I_{c0}\right) \\ +I_{c0}\left((1-\rho_1-\rho_2-\rho_3)\Lambda - \left(\frac{\beta\alpha}{N}+\mu\right)S_0 + \omega R_0\right) \end{pmatrix} \\ -\tau_1\left(\rho_1\Lambda + \frac{\beta S_0 I_{c0}}{N} - \tau_1 I_{c0}\right) \end{matrix} \right\} \frac{t^2}{2} + \ldots \quad (68)$$

$$I_u(t) = I_{u0} + \left(\rho_2\Lambda + \frac{\beta S_0 I_{u0}}{N} + \kappa_1 I_{c0} - \tau_2 I_{u0}\right)t$$
$$+\left\{ \begin{matrix} \frac{\beta}{N}\begin{pmatrix} S_0\left(\rho_2\Lambda + \frac{\beta S_0 I_{u0}}{N} + \kappa_1 I_{c0} - \tau_2 I_{u0}\right) \\ +I_{u0}\left((1-\rho_1-\rho_2-\rho_3)\Lambda - \left(\frac{\beta\alpha}{N}+\mu\right)S_0 + \omega R_0\right) \end{pmatrix} \\ +\kappa_1\left(\rho_1\Lambda + \frac{\beta S_0 I_{c0}}{N} - \tau_1 I_{c0}\right) - \tau_2\left(\rho_2\Lambda + \frac{\beta S_0 I_{u0}}{N} + \kappa_1 I_{c0} - \tau_2 I_{u0}\right) \end{matrix} \right\} \frac{t^2}{2} + \ldots \quad (69)$$

$$I_a(t) = I_{a0} + \left(\rho_3\Lambda + \frac{\beta\varphi S_0 I_{a0}}{N} + \kappa_2 I_{u0} + \varepsilon_1 I_{c0} - \tau_3 I_{a0}\right)t$$
$$+\left\{ \begin{matrix} \kappa_2\begin{pmatrix} \rho_2\Lambda + \frac{\beta S_0 I_{u0}}{N} + \kappa_1 I_{c0} \\ -\tau_2 I_{u0} \end{pmatrix} \\ +\frac{\beta\varphi}{N}\left(S_0\begin{pmatrix} \rho_3\Lambda + \frac{\beta\varphi S_0 I_{a0}}{N} + \kappa_2 I_{u0} \\ +\varepsilon_1 I_{c0} - \tau_3 I_{a0} \end{pmatrix} + I_{u0}a_1\right) + \varepsilon_1\begin{pmatrix} \rho_1\Lambda + \frac{\beta S_0 I_{c0}}{N} \\ -\tau_1 I_{c0} \end{pmatrix} \\ -\tau_3\begin{pmatrix} \rho_3\Lambda + \frac{\beta\varphi S_0 I_{a0}}{N} + \kappa_2 I_{u0} \\ +\varepsilon_1 I_{c0} - \tau_3 I_{a0} \end{pmatrix} \end{matrix} \right\} \frac{t^2}{2} + \ldots \quad (70)$$

$$R(t) = R_0 + (\psi_1 I_{c0} + \psi_2 I_{u0} + \psi_3 I_{a0} - (\omega+\mu)R_0)t$$
$$+\left\{ \begin{matrix} \psi_1\left(\rho_1\Lambda + \frac{\beta S_0 I_{c0}}{N} - \tau_1 I_{c0}\right) + \psi_2\left(\rho_2\Lambda + \frac{\beta S_0 I_{u0}}{N} + \kappa_1 I_{c0} - \tau_2 I_{u0}\right) \\ +\psi_3\left(\rho_3\Lambda + \frac{\beta\varphi S_0 I_{a0}}{N} + \kappa_2 I_{u0} + \varepsilon_1 I_{c0} - \tau_3 I_{a0}\right) \\ -(\omega+\mu)[\psi_1 I_{c0} + \psi_2 I_{u0} + \psi_3 I_{a0} - (\omega+\mu)R_0] \end{matrix} \right\} \frac{t^2}{2} + \ldots \quad (71)$$

Equations (67) – (71) are the general solution to the model equation (1) to (5).

RESULTS AND DISCUSSION

In this section, numerical simulations are performed to examine the infection dynamics using the Homotopy Perturbation Method (HPM) scheme. Utilizing the parameter values listed in Table 2 and initial conditions $S(0) = 760, I_a(0) = 170, I_u(0) = 50, I_a(0) = 303, R(0) = 0$, the simulations are executed with MAPLE 18 software, yielding the results presented below.

Table 3: Solution of model with HPM

S(t)	$I_c(t)$	$I_u(t)$	$I_a(t)$	R(t)
760	170	50	303	0
790.0377550	123.8808341	81.61052758	291.5502092	96.3100951
819.6373815	96.69945886	99.39258699	279.7989917	186.2758326
848.7910957	80.67987217	109.3168171	268.9742507	270.63212717
877.4952012	71.23856260	114.8012858	259.5672960	350.11974595
905.7488574	65.67401425	117.79440862	251.6799474	425.41601752
933.5532476	62.39408216	119.40122661	245.2222146	497.10857347
960.9110042	60.46048278	120.24468419	240.0222584	565.69143286
987.8258012	59.32029391	120.67338903	235.8853527	631.57179868
1014.3020614	58.64767685	120.88068916	232.6236348	695.08119566
1040.3447438	58.25064051	120.97259992	230.0694472	756.48758758

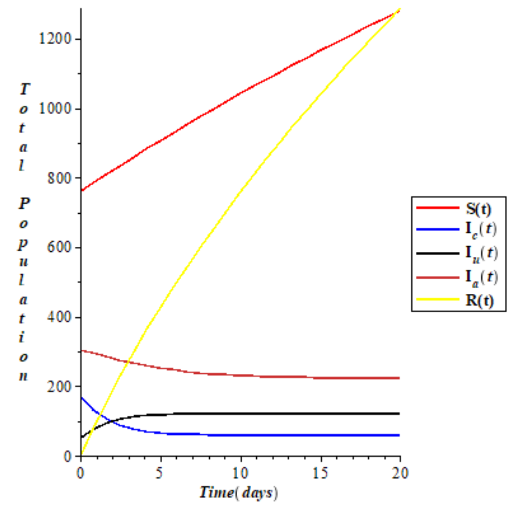

Figure 2: Solution of system of equation (1) using HPM

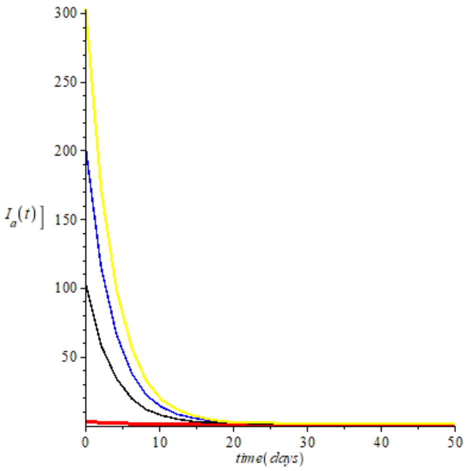

Figure 3: Global stability of disease-free point at varying initial values using HPM

Table 3 and Figure 2 present the solution for the total population using the proposed Homotopy Perturbation Method (HPM), illustrating the population dynamics at various time points with different initial values. Furthermore, Figure 3 reveals that a disease-free equilibrium (DFE) point exists, exhibiting local asymptotic stability within the population. Notably, the results indicate a persistent presence of susceptible students, while infected students decline to zero. Moreover, numerical simulations in Figure 2 demonstrate an increase in aware students over time. As shown in Figure 4, the endemic equilibrium exhibits global stability, ensuring that the system will eventually converge to a single steady-state solution, independent of initial values.

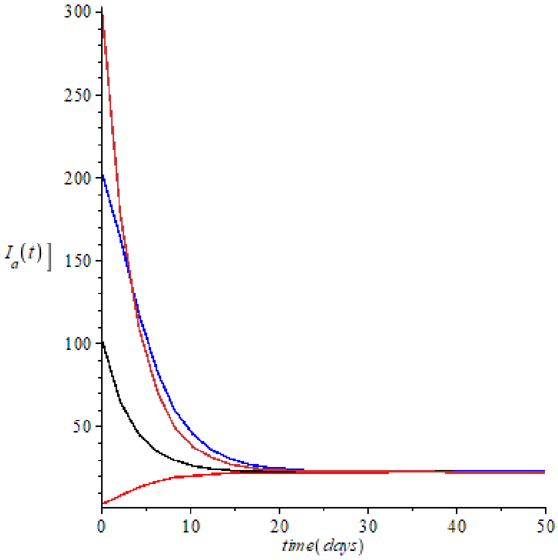

Figure 4: Global stability of endemic point at varying initial values using HPM

CONCLUSION

This research developed a mathematical model of disease dynamics in a school setting using non-linear ordinary differential equations. The Homotopy Perturbation Method (HPM) was successfully applied to solve the model equations, demonstrating its efficacy in tackling complex problems. The results show that HPM is a reliable and efficient technique with broad applications in scientific and engineering fields.

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