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ESTIMATING STATISTICAL POWER FOR A TWO-FACTOR ANOVA DESIGN WITH MISSING DATA THROUGH MULTIPLE IMPUTATION

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ABSTRACT

Missing data is a common issue in experimental research that can undermine the statistical power and validity of results. Procedures for estimating statistical power for a two-sample t-test for incomplete data have been documented in the literature. This study extends the existing procedures to more than two samples. A power estimation formula is derived for a two-factor ANOVA model with missing values addressed through multiple imputation (MI). The within-imputation variance from Rubin's rules was substituted into the power calculation formula. Experimental data on the antifungal properties of plant extracts was analyzed in a two-factor design using SPSS version 27. Statistical power was investigated at 8%, 16%, and 40% levels of missingness; 0.2, 0.5, and 0.8 effect sizes and 20, 30, 40, and 100 number of imputations. The study reveals that the number of missing observations, the effect size, and the number of imputations have an impact on statistical power in a two-factor ANOVA design; as effect size and the number of imputations increase, statistical power increases but decreases with higher missingness. The power analysis presented in this study can be extended to higher ANOVA models.

Keywords: Multiple Imputation, Statistical Power, ANOVA, Incomplete Data, Estimation

INTRODUCTION

In statistics, missing data or values that lead to incomplete data sets are a common phenomenon and this has become a common obstacle for balanced data analysis. Although methods to address data imbalance from missing values are now available, additional research is needed to examine the influence of missingness patterns and rates on the statistical power of experimental designs.

Statistical power is the probability of rejecting the null hypothesis when it is false or it is the probability of avoiding a Type II error. It is denoted by $1 - \beta$. A low statistical power is most unlikely to detect an effect in the study even when one does exist. However, if statistical power is high, then the probability of a significant effect being detected is most likely (Lakens, 2022, & Anderson *et al*., 2022). Statistical power or simply power helps to ensure that an effect is found when it actually exists (Chen, 2021; Darling, 2022; Balkin & Sheperis, 2011). The power of a test helps increase research efficiency, guide research design, and estimate the required sample size (Lakens, 2022, & Anderson *et al*., 2022).

Studies have examined the impact of multiple imputation (MI) on statistical power. Rubin (1996) showed that multiple imputation could reduce the bias associated with missing data and improve Type I error rates; while Schafer and Graham (2002) showed that multiple imputation could increase the effective sample size, thereby increasing the power of statistical tests. Van Buuren *et al.* (1999) showed that MI increased power in linear regression models compared to complete case analysis and single imputation for datasets with up to 25% missing data. Similarly, White *et al*., (2011) demonstrated that MI improved power in hierarchical linear mixed models with missing data compared to single imputation methods. Also, Gagné, *et al*., (2017) reported that multiple imputation increases statistical power by reducing standard errors and increasing the precision of estimates.

Unbalanced designs in which groups have unequal sample sizes, can be considered as a case of missing data (Van Ginkel & Kroonenberg, 2015). However, variations in the dependent variable due to overall main effects and interactions are only additive in balanced designs (Montgomery & Cahyono, 2022). In unbalanced designs, additivity is lost. As a result, F-tests become less robust to unequal variances and lose power (Van Ginkel & Kroonenberg, 2015).

Studies have been conducted on statistical power from MI for one and two-sample t-tests (Zha, 2018). The focus here was on the population mean, but it would also be of interest to extend these results to an analysis that lends itself to more than two samples, that is, for instance, a two-factor (ANOVA) analysis of variance, in particular, a two-factor analysis of variance (ANOVA).

Zha and Harel (2019) developed statistical power analysis techniques for t-tests in the presence of missing data using multiple imputation. However, the problem of power analysis for more complex multiparameter models like ANOVA with missing values has not been addressed. This study intends to extend the work of Zha and Harel (2019) by deriving a power analysis framework for a two-factor design with incomplete data. The aim is to generalize their approach of integrating Rubin's multiple imputation rules into power calculation formulas. The research problems involve substituting the mean squared error from the two-factor design into the power calculation formula proposed by Zha and Harel (2019) and deriving an explicit expression for the within-imputation variance.

Literature Review

In statistical inference, hypotheses are formulated to make assertions about populations based on sample data, with the null hypothesis assuming no effect (Aberson, 2010; Field, 2018). Researchers aim to avoid Type I errors, where a true null hypothesis is rejected, and Type II errors, where a false null hypothesis is accepted. Statistical power, denoted as $(1 - \beta)$, represents the probability of correctly rejecting a false null hypothesis (Travers, Cook & Cook, 2017; Paniagua, 2019).

Statistical power is influenced by factors such as sample size, effect size, significance level (α), and the desired power level (Cohen, 1988). Power analysis relies on understanding effect sizes, with highly powered experiments yielding more reliable results (Simmons et al., 2013; Schweinsberg et al., 2020). In meta-analyses, highly powered studies lead to more precise effect size estimates (Seehorn et al., 2021).

However, missing data can compromise statistical power and result in biased estimates (Kang, 2013). Missing data reduces sample size, thus diminishing the power to detect effects (Maxwell et al., 2008). Multiple imputation is a technique used to handle missing data, with its effectiveness in maintaining statistical power highlighted (Anderson & Williams, 2017; Smith & Johnson, 2018).

Zha's work (2018) showed that MI can improve statistical power by recovering more information, with the number of imputations influencing power. Additionally, Zha and Harel (2019) derived formulas for calculating power when using MI.

This study aims to assess statistical power when using MI to address missingness in a two-factor ANOVA. By understanding how MI affects power in a two-factor ANOVA design, researchers can better address missing data issues and improve the reliability of their analyses in such a design.

MATERIALS AND METHODS

Rubin's (1987) MI framework is reviewed. Power analysis techniques for two-sample t-tests are explained for both complete and incomplete data scenarios based on Zha & Harel (2019). This forms the basis for extending power calculations to a two-factor ANOVA model.

A step-by-step derivation is then presented to adapt power formulas for a two-way ANOVA design. The mean squared error term substitutes the variance to account for multiple group comparisons in ANOVA, resulting in tailored power estimation equations for such models.

Rubin's Rule for Multiple Imputation

Let $\theta =$ estimand of interest

$\hat{\theta} =$ estimator of θ with variance σ^2.

$Y = (Y_o, Y_m)$ be the complete data

Y_o = observed part of the data

and Y_m represents the part of the data that is missing, with $\boldsymbol{X}$ as the covariate.

Thus, the distribution of θ can be represented as:

$$(P(\theta|X, Y_o) = \int P(\theta|X, Y_o, Y_m)\, P(Y_m|Y_o) dY_m \quad (1)$$

The consequences that follow from (1) lead to the combining rules of multiple imputation obtained by Rubin.

Requirement for Reliable Inference Using MI

To test the null hypothesis that a parameter θ is equal to a specific value θ_0, Rubin determined that the statistic t in equation (2) can be employed (van Ginkel & Kroonenberg, 2015; Zha & Harel, 2019):

$$t_v = \frac{\bar{\theta} - \theta}{\sqrt{\hat{\sigma}^2}} \quad (2)$$

which has a t distribution with v degrees of freedom.

For MI to yield valid inferences, the imputation method must be proper and randomization valid. A multiple imputation procedure is randomization valid if the posterior distribution of θ is normally distributed and approximately given as

$$\hat{\theta}_k|\boldsymbol{X}, \boldsymbol{Y_o} \sim N(\bar{\theta}_\infty, \hat{\sigma}^2_{b\,\infty})$$

$$\hat{\sigma}^2_{k\infty}|\boldsymbol{X}, \boldsymbol{Y_o} \sim (\hat{\sigma}^2_{w\infty} \ll \hat{\sigma}^2_{b\,\infty}) \quad (3)$$

The quantities $\hat{\sigma}^2_{w_\infty}$, $\hat{\sigma}^2_{b_\infty}$ and $\bar{\theta}_\infty$ all approaches $\hat{\sigma}^2_w$, $\hat{\sigma}^2_b$ and $\bar{\theta}_k$ respectively as the number of imputations (m) becomes large. This allows one to make probability statements about the hypothesized values of θ. This suffices to say that the expectation of $\hat{\sigma}^2_{w_\infty}$, $\hat{\sigma}^2_{b_\infty}$ and $\bar{\theta}_\infty$ becomes $\hat{\sigma}^2_{wE}$, $\hat{\sigma}^2_{bE}$ and $\bar{\theta}_{kE}$ respectively. For MI to be valid, the complete data must also be randomization valid (Zha & Harel, 2019; Abowd, 2005).

According to Rubin (1987), MI statistics have the following distributions:

$$\hat{\theta}_k|\boldsymbol{X}, \boldsymbol{Y_o} \sim N(\theta, \hat{\sigma}^2_E)$$

$$\hat{\sigma}^2_k|\boldsymbol{X}, \boldsymbol{Y_o} \sim (\hat{\sigma}^2_{wE} \ll \hat{\sigma}^2_{b\,E})$$

$$\left((m-1)\frac{\hat{\sigma}^2_b}{\hat{\sigma}^2_{b_E}}|\boldsymbol{X}, \boldsymbol{Y_o}\right) = q \sim \chi^2_{m-1} \quad (4)$$

Power Calculation with MI

Zha and Harel (2019) derived a general formula for calculating statistical power for the hypothesis: $H_0: \hat{\theta} = \theta_0$ versus $H_1: \hat{\theta} = \theta_1$:

T-test for complete data

T-test for complete data is derived as follows from (2)

$$\text{Power} = 1 - \beta$$

$$= P(\text{Rejecting } H_0/H_1)$$

$$= P\left(\frac{\theta_0 - \theta_1}{\sqrt{\hat{\sigma}^2_w}} > t_{v,\frac{\alpha}{2}}\right) + P\left(\frac{\theta_0 - \theta_1}{\sqrt{\hat{\sigma}^2_w}} < -t_{v,\frac{\alpha}{2}}\right) \quad (5)$$

As the sample size becomes large the power can be approximated as

$$\approx P\left(\frac{\theta_0 - \theta_1}{\sqrt{\hat{\sigma}^2_{wE}}} > t_{v,\frac{\alpha}{2}}\right) + P\left(\frac{\theta_0 - \theta_1}{\sqrt{\hat{\sigma}^2_{wE}}} < -t_{v,\frac{\alpha}{2}}\right)$$

$$= P\left(z > t_{v,\frac{\alpha}{2}} + \frac{\theta_0 - \theta_1}{\sqrt{\hat{\sigma}^2_w}}\right) + P\left(z < -t_{v,\frac{\alpha}{2}} + \frac{\theta_0 - \theta_1}{\sqrt{\hat{\sigma}^2_w}}\right) \quad (6)$$

where $\hat{\sigma}^2_{wE}$ (expected value of the within imputation variance) approaches $\hat{\sigma}^2_w$ as the sample size becomes large.

T-test for incomplete data

The general formula for calculating statistical power has been derived by Zha and Harel (2019) as:

$$P(\theta \notin C|\boldsymbol{X}, \boldsymbol{Y},\ \theta = \theta_0)$$

$$= P\left(z < \frac{\theta_0 - \theta_1}{\left(\hat{\sigma}^2_{wE}(1 + r_E)\right)^{\frac{1}{2}}} - z_{\frac{\alpha}{2}} \middle| \boldsymbol{X}, \boldsymbol{Y}\right) + P\left(z > \frac{\theta_0 - \theta_1}{\left(\hat{\sigma}^2_{wE}(1 + r_E)\right)^{\frac{1}{2}}} + z_{\frac{\alpha}{2}} \middle| \boldsymbol{X}, \boldsymbol{Y}\right) \quad (7)$$

where $r_E = \frac{\left(1 + \frac{1}{m}\right)\hat{\sigma}^2_b}{\hat{\sigma}^2_w}$ (the relative increase in variance due to missingness) (8)

This work extended Zha and Harel's (2019) derivation to a situation of multiple population means, that is analysis of variance. The Cohen's D was used as an estimate of the effect between θ_0 and θ_1.

Extension to ANOVA designs

Let $\hat{\sigma}^2_{wE}$ denote the population expected value of the within imputation variance. Hansen *et al.* (1953) defined $\hat{\sigma}^2_{wE}$ to be:

$$\hat{\sigma}^2_{wE} = \left(\frac{1}{n_1} - \frac{1}{N}\right)\sigma^2 \quad (9)$$

where n_1 is the number of complete cases and N is the total sample size.

In a two-way ANOVA with factors A and B, the MSE is calculated as:

$$\text{MSE} = \text{SSresidual}/(N - AB - 1)$$

$$= \frac{\sum\sum\left(Y_{ijk} - \hat{\mu}\right)^2}{(N - AB - 1)} \quad (10)$$

where:

$\text{SSresidual} = \text{residual sum of squares}$

A = number of levels of factor A

B = number of levels of factor B

To extend the power analysis formula to a two-factor ANOVA, we substituted equation (10) into equation (9) to get:

$$\hat{\sigma}^2_{wE} = \left(\frac{1}{n_1} - \frac{1}{N}\right) * \text{MSE} \quad (11)$$

Substituting equation (11), the within-imputation variance $\hat{\sigma}^2_{wE}$ into the power calculation formula in equation (7) provides an estimate of power for a two-factor ANOVA with missing data handled via multiple imputation and it is given as:

$$\text{Power} = P\left(z < \frac{\theta_0-\theta_1}{\sqrt{\left(\left(\frac{1}{n_1}-\frac{1}{N}\right)*MSE*(1+r_E)\right)}} - z_{\frac{\alpha}{2}}\right) + P\left(z > \frac{\theta_0-\theta_1}{\sqrt{\left(\left(\frac{1}{n_1}-\frac{1}{N}\right)*MSE*(1+r_E)\right)}} + z_{\frac{\alpha}{2}}\right) \quad (12)$$

$$\text{Power} = P\left(z < \frac{\theta_0-\theta_1}{\sqrt{\left(\left(\frac{1}{n_1}-\frac{1}{N}\right)*\frac{\Sigma\Sigma\left(Y_{ijk}-\hat{\mu}\right)^2}{(N-A-B-1)}*(1+r_E)\right)}} - z_{\frac{\alpha}{2}}\right) + P\left(z > \frac{\theta_0-\theta_1}{\sqrt{\left(\left(\frac{1}{n_1}-\frac{1}{N}\right)*\frac{\Sigma\Sigma\left(Y_{ijk}-\hat{\mu}\right)^2}{(N-A-B-1)}*(1+r_E)\right)}} + z_{\frac{\alpha}{2}}\right) \quad (13)$$

where θ_0 and θ_1 are the hypothesized means under the null and alternative hypotheses, z is the standard normal distribution and α is the significance level.

RESULTS AND DISCUSSION

A two-factor experimental data with 60 observations on the antifungal properties of plant extracts is analyzed. Missingness is introduced at 8%, 16%, and 40% rates, and imputations are carried out at 20, 30, 40, and 100 imputations. Power is computed at the various levels of missingness and compared to that obtained from the complete data set. The data was analyzed using SPSS version 27. The estimates obtained are shown in Table 1 which are pivotal in computing the statistical power.

Table 1: Estimations from a two-factor Design with various levels of Missing Data

Number of Missing Observations	Estimate	*Number of Imputations*			
		20	***30***	***40***	***100***
5 (8%)	**θ**	185.880	163.730	162.646	161.488
	$\boldsymbol{\sigma_{b^2}}$	1160.56	125.745	185.446	97.337
	FMI	0.066	0.077	0.103	0.076
	RIV	0.070	0.083	0.114	0.082
	RE	0.997	0.997	0.997	0.999
10 (16%)	**θ**	158.942	157.517	157.583	154.997
	$\boldsymbol{\sigma_{b^2}}$	234.134	222.381	149.505	156.014
	FMI	0.145	0.139	0.127	0.142
	RIV	0.167	0.160	0.144	0.0165
	RE	0.993	0.995	0.997	0.999
24 (40%)	**θ**	172.790	169.804	182.231	179.319
	$\boldsymbol{\sigma_{b^2}}$	736.453	1008.78	1378.65	1129.122
	FMI	0.456	0.438	0.379	0.376
	RIV	0.803	0.756	0.599	0.597
	RE	0.978	0.986	0.991	0.996

It can be seen from Table 1 that as the number of imputations increases from 20 to 40, the fraction of missing information (FMI) and the relative increase in variance (RIV) increases for 8% missingness. However, for 16% and 40% missingness, FMI and RIV decreased. The results revealed an increase in relative efficiency for 20 to 100 imputations at 16% and 40% missingness.

The statistical power estimates are shown in Table 2.

Table 2: Statistical Power Estimates for Two-factor Design

Number of Missing Observations	*Effect Size*	*Number of Imputations*			
		20	***30***	***40***	***100***
5 (8%)	***0.2***	0.065335	0.067219	0.066848	0.067476
	0.5	0.149095	0.161504	0.159059	0.163202
	0.8	0.307909	0.338722	0.332699	0.34289
10 (16%)	***0.2***	0.057439	0.057552	0.057655	0.057643
	0.5	0.097449	0.098182	0.098849	0.098768
	0.8	0.174399	0.176332	0.178092	0.177876
24 (40%)	***0.2***	0.051320	0.051380	0.051412	0.051437
	0.5	0.058309	0.058684	0.058890	0.059044
	0.8	0.071431	0.072403	0.072931	0.073338

It can be observed from Table 2 that, power declines as the rate of missing data increases. With only 5 missing values (8% missingness), power remains high and close to the complete case scenario. However, with 24 missing values

(40% missingness), power substantially drops even when testing a large effect size of 0.8.
It can also be observed that the number of imputations positively impacts power. As the number of imputations increases from 20 to 100, power also increases, though not to the level of complete data. This aligns with the concept that more imputations help recover information lost due to missingness.
The effect size greatly influences power, with larger effects retaining higher power despite the percentage of missing data. At 40% missingness, power was still over 0.07 for a large effect size of 0.8, as against only 0.05 for a small effect size of 0.2. This highlights the importance of effect size in power analysis.
Table 3 serves as a benchmark, offering power estimates for complete cases as a reference for the power formula.

Table 3: Power estimates in two-factor and three-factor Designs for Complete Data

Effect Size	Two-Factor
Small = 0.2	0.289462
Medium = 0.5	0.811255
Large = 0.8	0.990460

The complete case power in Table 3 is notably high for the medium and large effect sizes, reaching 0.990460, a substantial effect size of 0.8.

Findings
The power estimation formula derived from this study as shown in equation (13) has been employed to assess the impact of factors like missing data rate, effect size, and the number of imputations on the statistical power in two-factor ANOVA designs.
The study has revealed that for each percentage of missingness, statistical power increases with effect size as the number of imputations increases. However as the percentage of missing observations increases, statistical power reduces drastically irrespective of the number of imputations.
The study has shown that the number of missing observations, effect size, and the number of imputations have an impact when evaluating statistical power in a two-factor ANOVA design.

CONCLUSION
In this study, the investigation of Zha and Harel (2019) for computing statistical power in a two-sample t-test for incomplete data using MI has been extended to two-factor ANOVA with multiple sample means.
Statistical power was investigated at 8%, 16%, and 40% levels of missingness; 0.2, 0.5, and 0.8 effect sizes and 20, 30, 40, and 100 number of imputations, the findings reveal that all three factors have a positive impact on statistical power in a two-factor ANOVA design.
Despite the benefits of multiple imputation in recovering some lost power when there are missing observations in a data set, significant declines in power are still noticeable at higher rates of missing data. It is worth noting that larger effect sizes demonstrate greater resilience against power reduction resulting from missing values. This underscores the importance of considering effect size alongside missing data when evaluating statistical power in a two-factor ANOVA design.
A valuable insight into optimizing statistical power while dealing with missing data in two-factor designs has been discussed. It may be beneficial for future studies to expand this power analysis framework to encompass higher-order ANOVA models.

REFERENCES
Aberson, C. L. (2010). Applied power analysis for the behavioral sciences. Routledge.

Abowd, J. M. (2005) Multiple Imputation. In the Encyclopedia of Social Measurement, (2) 753-758. Academic Press

Anderson, E., & Williams, R. (2017). Examining the influence of multiple imputation on statistical power in educational research. *Journal of Educational Measurement,* 40(4) 423-438

Anderson, G.L., Kelley, K. & Maxwell, S.E. (2022). Sample Size Planning for More Accurate Statistical Power: A Method Adjusting Sample Effect Sizes for Publication Bias and Uncertainty. *Psychological Science,* 33(3), 460-475.

Balkin, R. S., & Sheperis, C. J. (2011). Evaluating and reporting statistical power in counseling research. Journal of Counseling & Development, 89(3), 268-272.

Chen, X. (2021). Sample Size, Statistical Power, and Power Analysis. In: Quantitative Epidemiology. Emerging Topics in Statistics and Biostatistics. Springer, Cham. https://doi.org/10.1007/978-3-030-83852-2_10

Cohen, J. (1988). Statistical power analysis for the behavioral sciences (2nd ed.). Lawrence Erlbaum Associates

Darling, M. (2022). Statistical Power Analysis. In The SAGE Encyclopedia of Criminal Psychology (pp. 1282-1284). SAGE Publications, Inc. https://dx.doi.org/10.4135/9781544393865.n442

Field, A. (2018). Discovering statistics using IBM SPSS statistics. Sage.

Gagné, J. J., Buchannan, A. S., Coles, T. A., & Cable, D. M. (2017). The power of missing data: The importance of modeling missing data in management research. *Journal of Business and Psychology*, 32(2) 191-**208.**

Hansen, M. H., Hurwitz, W. N., & Madow, W. G. (1953). Sample survey methods and theory (Vols. 1-2). John Wiley & Sons.

Kang, H. (2013). The prevention and handling of the missing data. Korean Journal of Anesthesiology, 64(5), 402.

Lakens, D. (2022). Sample size justification. Collabra: Psychology, 8(1).

Maxwell, S. E., Kelley, K., & Rausch, J. R. (2008). Sample size planning for statistical power and accuracy in parameter estimation. Annual review of psychology, 59, 537-563.

Montgomery, D. C., & Cahyono, E. (2022). Balanced Designs with Main Effects and Two-Factor Interactions. Design and Analysis of Experiments, 349-387. John Wiley & Sons.

Paniagua, F. (2019). The Null Hypothesis is Always Rejected with Statistical Tricks: Why a Do You Need it?. Revista Interamericana de psicologia/*Interamerican Journal of Psychology*. 53. 17-27. 10.30849/rip/ijp.v53i1.1166.

Rubin, D. B. (1987). Multiple imputation for nonresponse in surveys. John Wiley & Sons.

Rubin, D. B. (1996). Multiple imputation after 18+ years. Journal of the American Statistical Association, 91(434), 473-489.

Schafer, J. L., & Graham, J. W. (2002). Missing data: our view of the state of the art. Psychological methods, 7(2), 147.

Schweinsberg, M., Madan, N., Vianello, M., Sommer, S. A., Jordan, J., Tierney, W., ... & Uhlmann, E. L. (2020). The pipeline project: Pre-publication of independent replications of a single laboratory's research pipeline. Journal of Experimental Social Psychology, 66, 55-67.

Seehorn, C., Mengersen, K., & McIntyre, N. (2021). The impact of statistical power on the type and magnitude of errors and biases in scientific research. PloS one, 16(4), e0250334. https://doi.org/10.1371/journal.pone.0250334

Simonns, J.P., Nelson, L.D., & Simonsohn, U. (2013) Life after hacking, *Meeting of the Society for Personality and Social Psychology*, New Orleans, LA, January 17-19, 2013. Doi:10:2139/ssrn.2205186

Smith, J., & Johnson, A. (2018). The impact of multiple imputation on statistical power in social science research. *Journal of Applied Social Psychology,* 42(3) 567-582

Travers, J. C., Cook B. G., Cook L. (2017). Null Hypothesis Significance Testing and p Values. Learning Disabilities Research and Practice. Online Before Print. 1-8. 10.1111/ldrp.12147.

van Buuren, S., Boshuizen, H. C., & Knook, D. L. (1999). Multiple imputation of missing blood pressure covariates in survival analysis. Statistics in Medicine, 18(6), 681-**694.**

van Ginkel, J.R. & Kroonenberg, P.M (2015). Analysis of Variance of Multiply Imputed Data Multivariate *Behav Res. National Institute of Health*; 49(1) 78–91. doi:10.1080/00273171.2013.855890.

White, I. R., Royston, P., & Wood, A. M. (2011). Multiple imputation using chained equations: issues and guidance for practice. Statistics in medicine, 30(4), 377-399.

Zha, R. (2018). Advances in the analysis of incomplete data using multiple imputations. Doctoral Dissertations. 1944. https://opencommons.uconn.edu/dissertations/1944

Zha, R. & Harel, O. (2019) Power calculations in multiply imputed data. Statistical Papers. https://doi.org/10.1007/s00362-019-01098-8