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ASSESSMENT OF TRANSIENT VIBRATION EFFECTS OF A ROCKING STRUCTURE ON TWO-PARAMETER UPLIFTING FOUNDATION MODEL

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ABSTRACT

This study is on assessment of transient vibration of a rocking structure on two-parameter uplifting foundation model. This foundation model, Filonenko-Borodich (F-B) considers effect of continuity and interactions of the structure with the surrounding foundation surface. The foundation model aids uplift because of its elastic and flexible nature. During ground movement, some forces are generated which cause foundation to vibrate, rock and sometimes uplift due to increasing upward forces and then leading to reduction in the spring stiffness and increase in soil flexibility. This sometimes causes upward trend in the structural response which might affect structural integrity and stability. The use of damping and its effects on the structure response considering uplift are evaluated for a structure on two-parameter foundation. Equations describing the motion were developed by summing forces acting on the structure and foundation, considering equilibrium of moments for the conditions of before uplift and during uplift of the system in accordance using Newton's second law of motion, D'Alembert's principles. The resulting equations were solved by applying them in Duhamel Integral form and Simpson's method used for the numerical solution for the structure responses. The result showed an upward trend in the structural responses during uplifting hence uplift of building foundation is not always beneficial.

Keywords: Elastic foundation, Structural integrity, Transient vibration, Uplift, Resonance, Structure responses

INTRODUCTION

Soils bear low tensile force and they are not infinitely stiff (Li et al, 2014), hence during ground motion, soil can induce excessive load eccentricity on foundation causing temporary separation of structure-foundation from supporting soil (Chandra, 2014; Wang and Ishihara, 2020). This phenomenon result to occurrence of foundation uplift which means the detachment of the base of the structure from the supporting soil (Yim and Chopra, 1984; Givens et al, 2015). The uplift that will occur is expected to be small though it cannot be accessed for any observation (Yim and Chopra, 1984; and Xu and Skyrakos, 1996). This is because, the foundation is beneath ground. Foundation uplift is associated with some effects on structures like bridges, buildings, towers, rails, etc as increase in support flexibility results (Mergos and Kawashima, 2005). Here, the contact area between the soil and the foundation reduces leading to reduction in the soil stiffness. This reduction causes the natural period of oscillation to increase depending on the frequency content of the ground motion (Yim and Chopra, 1984; Celep and Guler, 1991). In consideration of short period structures, foundation uplift is necessary as the natural period of the soil is sensitive to the flexibility of the foundation (Celep and Guler, 1991).Determination of the load capacity and stiffness of the foundation is necessary to understand structural elements that are exposed to failure as uplifting of foundation can cause additional nonlinearity into structural system (Harden and Hutchinson, 2009). One can say that uplifting of foundation is a determinant on how the distribution and damage level of a structure depend on uplift response of its foundation from seismic ground motion (Anderson, 2003). During strong ground motions, the increasing upward forces from ground below due to earthquake forces makes the foundation to rise slowly (Hsiung, 1988) as in Figure 1. Careful considerations and analysis is required for buildings on elastic foundation that allows uplift. This is to make sure that the structure can safely withstand earthquake forces due to how flexible the foundation will be. As the structure vibrates and rock from one point to the other, that is from point '(o)' to point '(o')' as in Figure 1, there seems to be dissipation of energy every time the structure touches the base. But this energy loss can be dependent on the frequency of ground motion and as well structure-foundation parameters.

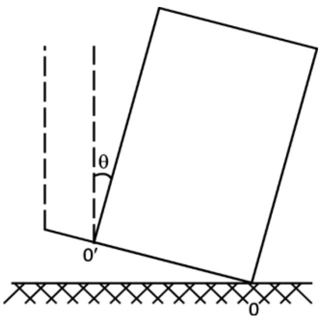

Figure 1: An uplifting structure

Serious damages to structures during earthquake ground motion are of serious concern to engineers as it leaves behind tens of thousands of human deaths and unquantifiable property as can be seen from the 2010 Haiti earthquake; 2011 Japan earthquake; 2017 Mexico earthquake; 2023 Turkey and Syrian earthquake to mention but a few, hence the need for more earthquake resistant structural systems which minimizes the negative impact of earthquakes. In recent times, many innovative earthquake resistant structural configurations like using base isolation devices, damping, applying post-tensioned tendons in beam-column connections have been developed and applied as a way to reducing the damaging earthquake effect. These have proven successful over the time as they elongate the system natural period (Moroni et al, 2012).

Damping occurrence makes a vibrating system to decay in amplitude of motion gradually by means of energy dissipation through several mechanisms. In practice, all engineering systems possess damping since energy is being dissipated by damping forces that retards motion (Smith, 1988; Clough and Penzien, 2003).Damping of the foundation incorporates the effects of energy loss due to waves from the vibrating foundation as well as hysteretic action in the soil (Givens et al, 2015).Damping helps to increase the period of natural frequency and make the resonant frequency somewhat lower than when there is no damping and in real structures values. Damping is not strictly due to viscosity but is mostly caused by friction at interfaces such as bolted connections, in joints of cladding and in cracks of reinforced concrete (Smith, 1988).In dynamic systems, damping tends to create a borderline between stability and instability thereby curbing resonance. Some of the known examples of damping in systems like friction at connections, micro cracks in concrete and friction in between the parts, etc that influence within the oscillatory system and also a form of base isolation used in earthquake resistant design to mitigate against motion of the system from seismic activities thereby reducing structural responses and protecting structural integrity.

Whenever ground shaking tends to increase unnecessarily, the absorbing damping system goes into action to reduce vibration. Psycharis and Jennings, 1983 worked on dissipation of energy and flexible foundations using two elastic foundation models viz; Two-Spring and Winkler foundations. They concluded that damping effects can be most efficiently represented by dashpots in parallel to the elastic soil springs as their result gave that there is increase in the rotation angle of the foundation mat from uplift. There was no clear effects of uplift on the structural deflections and resulting stresses where uplifting of foundation can cause reduction in the structural responses and as well increase in the structural response thereby altering structural demand (Psycharis and Jennings, 1983; Yongfeng et al, 2021; Amir and Mohammad, 2020). During structure rocking, it is characterized by their non-linearity that makes their behavior different from linear structures. There is tendency that rocking motion can significantly damage and affect the performance and functionality of the entire system (Thiers-Moggia and Malaga-Chuquitaype, 2020). Thiers-Moggia and Malaga-Chuquitaype, 2020 in their work proposed the fundamental dynamics of post-tensioned rocking structures and how using supplemental rotational inertia can be of benefit in reducing their seismic demand thereby improving their overall performance. Bonkowski et al, 2019 in their work showed the engineering analysis of strong ground rocking and how it affects tall structures. They also examined and investigated substantial structural response under combined translational and rotational excitations. A complicated pattern of translational-rocking seismic effects on slender structures was observed and it was noted that the presence of seismic rotations about horizontal axis sometimes increases the overall response (up to 30%), and also sometimes decreases the combined seismic response. Hence they concluded on the importance of rocking during ground motion that allowing tall structures to rock lowers the seismic response.

Yim and Chopra, 1983 also in their work on effects of transient foundation uplift on earthquake response of structures stated that damping of foundation brings about its tendency to uplift being reduced. Because of this, uplifting duration decreases slowly with each vibration cycle and the average contact area over a cycle increases too. Although uplift of the foundation increases the maximum downward edge displacement in the un-damped case, the effect of damping in the second mode during full contact, and the high frequency mode during uplift, which are excited when uplift is permitted, is so strong that downward edge displacement is actually slightly reduced. Towards the later phase of the earthquake occurrence, as the ground motion intensity decays the foundation uplift becomes negligible and full foundation contact is maintained for longer durations (Yim and Chopra, 1983).Uplift of structure can be reduced by installing dampers based on the soil-structure type (Mortezaie and Zamanian, 2021). Feng et al., 2021 showed in their works system that can provide higher damping so as to bring about reduction of amplitude of oscillation to minimum. The application of damping in systems to bring about stability of structure-foundation during vibration from earthquake ground motion is necessary especially for foundation on elastic medium. However, these studies were mostly performed on one-parameter foundation model (Winkler) which does not take into account the effect of continuity interactions among the spring elements. In this analysis, the elastic foundation is a two-parameter foundation model (Filonenko-Borodich) which accounts for continuity of the foundation surface. This study investigated the assessment of transient vibration of a rocking structure on two-parameter uplifting foundation model to maintain structural integrity and stability

MATERIALS AND METHODS

System Concept

The system is a rigid body on a two-parameter foundation model as in Figure 2. The top ends of the soil springs elements are connected to an elastic membrane that was stretched to a constant 'T'. This will control the continuity of the surrounding spring elements and the structure. Dashpots were introduced to the foundation models to dissipate the amount of energy going into the structure. The structure properties include total height 'H', width '2B', height from the supporting soil to the center of gravity 'h' and concentrated mass 'M' at center. The structure foundation is resting on the soil spring elements by gravity (vertically downwards) that is not bonded to supporting soil elements. From this, the supporting soil elements can provide upward force to the foundation and not a downward pull thereby allowing uplift to occur. In the cause of any ground motion, two foundation contacts conditions are established. Firstly, Full contact condition: when the base of the structure is in full contact with supporting soil. Here the equations of motion are linear for small displacements and the motion is governed by the standard classical theory of soil structure interaction and differential equation for single degree of freedom system. Secondly, when there is uplift condition where there is partial separation (uplift) of the base of structure from supporting soil elements. The equations of motion are highly non-linear here because of the different degree of contact between structure and foundation. The derivation of the governing equations of motion here is by considering lateral equilibrium of forces acting on the structure and the moment equilibrium of forces on the system. From previous researches, dynamic behavior of structures on elastic foundation shows that horizontal translation of the base effects is usually negligible hence horizontal translation is restricted in this analysis and there's no slippage between the base of structure and supporting soil. Applying Newton's second law of motion and D'Alembert's principle for the derivation of the equations of motion of this model, Figure 2 shows a two-dimensional structure on an elastic Filonenko-Borodich (F-B) foundation with its foundation parameters, 'K_f (soil spring parameter) and *T* (applied tension force)'. The elastic zone of the foundation is connected to a base which is assumed to be rigid. The structure properties includes: Spring stiffness 'P_{kf}' force; Weight of the structure acting at the centre 'W'; Angle of tilting of the structure 'α'; 'k_f' = *F* – *B* soil spring stiffness, with the vertical and horizontal components (x_G, y_G) of applied ground motion a_G.

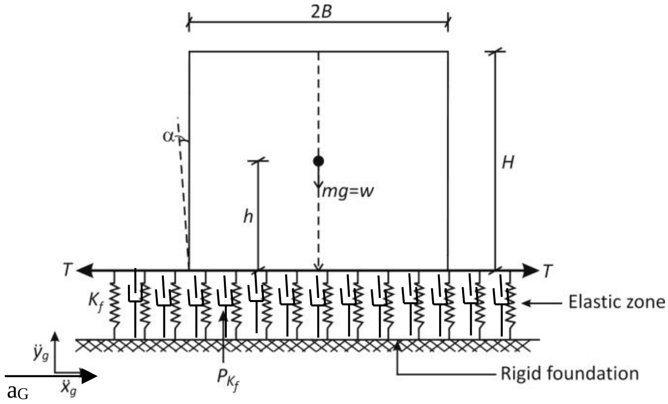

Figure 2: Sketch of structure on F-B foundation model with dashpots on full contact

As it was presumed, the structure cannot slip between foundation and supporting elements therefore the structure has two degrees of freedom which include the vertical motion measured from position of rest by vertical displacement and the rotation measured by the angle of tilting 'α' of the structure from the vertical. Also, it is assumed that the structure is not by any means bonded to soil elements and that it is resting on the foundation by gravity. This is from the fact that soil performs poorly in carrying tensile stresses for foundation uplift to occur. Figure 3 shows other system descriptions.

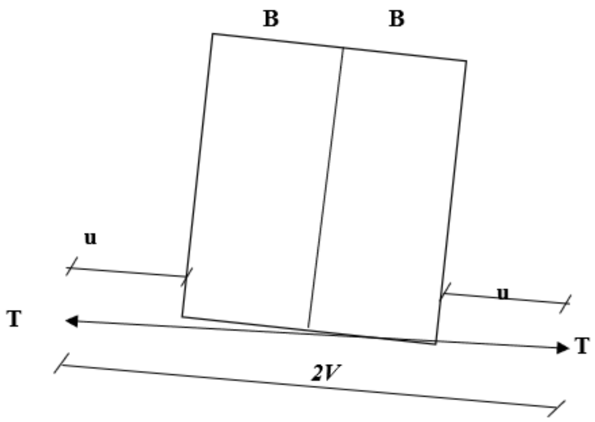

Figure 3: Structure on F-B foundation with descriptions

Equation of motion with damping effects

Considering Newton's second law of motion for equilibrium of forces acting on the system and by applying D'Alembert's principles, forces acting on the system include; Inertia force ($m\ddot{x}$), Resisting force (*kx*), Damping force $\left(c\dot{x}\right)$ and External force $-ma_G(t)$. Summing these forces and considering equilibrium of moments gives the resulting equations of motion for the vertical direction 'y' and rocking direction 'x' for before uplift and during uplift conditions;

Full contact

Vertical component;

$$m\ddot{y} + 2c_f V\dot{y} + 2k_f V y = -m\ddot{a}_{Gy} \quad (1)$$

Rocking component;

$$2I_c\ddot{\alpha} + \frac{2}{3}c_f V^3\alpha + \frac{2k_f V^3\alpha}{3} = -2mha_{G_x} \quad (2)$$

During uplift

Vertical component

$$m\ddot{y} - \frac{W}{2} - \frac{c_f}{2}V^2\dot{\alpha} + c_f B\dot{y} + k_f V y + \frac{k_f}{2}V^2\alpha = -ma_{Gy} \quad (3)$$

Rocking component

$$2I_c\ddot{\alpha} + \frac{2}{3}c_f V^3\dot{\alpha} + \frac{2k_f V^3\alpha}{3} + \frac{w^2 h}{8k_f V} - \frac{why}{2} = -2mh\ddot{a}_{G_x} \quad (4)$$

Where; C_f is F-B damping coefficient, k_f is F-B spring coefficient, $-ma_{G_y}$ and $-ma_{G_x}$ are the x and y component of the input ground motion, u covers the length of the elastic membrane from the edge of the building, V is taken from the center of the structure to the surrounding spring elements where the elastic membrane ended (this factor accounts for the continuity interactions of the foundation surface), I_c is the moment of inertia about the center of foundation base.

Solution of displacement equations

Equations (1) and (2) are for the vertical and rocking directions during full contact of structure-foundation and Equations (3) and (4) are for the vertical and rocking directions during structure-foundation uplifting. Applying Duhamel integral for solution of Equations (1), (2), (3) and (4) gave the displacement equations damped by an exponential decay factor for the vertical 'y' and rocking directions 'x' as;

Before uplift

Vertical direction;

$$y(t) = e^{-\omega\xi t}[y(0)\cos\omega_{d_7}t + \frac{\dot{y}(0)+\omega\xi y(0)}{\omega_{d_7}}\sin\omega_{d_7}t] - \frac{1}{\omega_{d_7}}\int_0^t \ddot{a}_{Gy}(\tau)e^{-\omega\xi t}\sin\omega_{d_7}(t-\tau)d \quad (5)$$

Rocking direction;

$$x(t) = e^{-\omega\xi t}[x(0)\cos\omega_{d_8}t + \frac{\dot{x}(0)+\omega\xi x(0)}{\omega_{d_8}}\sin\omega_{d_8}t] - \frac{1}{\omega_{d_8}}\int_0^t \frac{mh^2 a_{Gx}}{I_c}(\tau)e^{-\omega\xi t}\sin\omega_{d_8}(t-\tau)d\tau \quad (6)$$

During uplift

Vertical direction;

$$y(t) = e^{-\omega\xi t}[y(0)\cos\omega_{d_{7A}}t + \frac{\dot{y}(0)+\omega\xi y(0)}{\omega_{d_{7A}}}\sin\omega_{d_{7A}}t] - \frac{1}{\omega_{d_{7A}}}\int_0^t [a_{Gy} + \frac{W}{2m}](\tau)e^{-\omega\xi t}\sin\omega_{d_{7A}}(t-\tau)d\tau \quad (7)$$

Rocking direction;

$$x(t) = e^{-\omega\xi t}[x(0)\cos\omega_{d_9}t + \frac{\dot{x}(0)+\omega\xi x(0)}{\omega_{d_9}}\sin\omega_{d_9}t] - \frac{1}{I_c\omega_{d_9}}\int_0^t [mh^2 a_{Gx} + \frac{W^2h^2}{16k_fV}](\tau)e^{-\omega\xi t}\sin\omega_{d_9}(t-\tau)d\tau \quad (8)$$

Where; ω is normal frequency of the system

ω_{d7} is the damped vertical frequency before uplift

ω_{d8} is the damped rocking frequency before uplift

ω_{d7A} is the damped vertical frequency during uplift

ω_{d9} is the damped rocking frequency during uplift

Applying initial conditions

In the solution that follows, initial conditions of the system before uplift which is at rest conditions was applied and also the trigonometry identity applied as well.

Equations (5) and(6) become;

Before Uplift

Vertical direction

$$y(t) = -\frac{e^{-\omega\xi t}\sin\omega_{d_7}t}{\omega_{d_7}}\int_0^t \ddot{a}_{Gy}(\tau)e^{-\omega\xi t}\cos\omega_{d_7}td\tau + \frac{e^{-\omega\xi t}\cos\omega_{d_7}t}{\omega_{d_7}}\int_0^t \ddot{a}_{Gy}(\tau)e^{-\omega\xi t}\sin\omega_{d_7}td\tau \quad (9)$$

Rocking direction

$$x(t) = -\frac{e^{-\omega\xi t}\sin\omega_{d_8}t}{\omega_{d_8}}\int_0^t \frac{mh^2 a_{Gx}}{I_c}(\tau)e^{-\omega\xi t}\cos\omega_{d_8}td\tau + \frac{e^{-\omega\xi t}\cos\omega_{d_8}t}{\omega_{d_8}}\int_0^t \frac{mh^2 a_{Gx}}{I_c}(\tau)e^{-\omega\xi t}\sin\omega_{d_8}td\tau \quad (10)$$

The simple harmonic motion of Equations (9) and (10) before uplift is represented as;

For the Vertical Direction;

$$y(t) = e^{-\omega\xi t}(-A(t)\sin\omega_{d_7}t + B(t)\cos\omega_{d_7}t) \quad (11)$$

$$A(t) = \frac{1}{\omega_{d_7}}\int_0^t \ddot{a}_{Gy}(\tau)e^{-\omega\xi t}\cos\omega_{d_7}td\tau \quad (12)$$

$$B(t) = \frac{1}{\omega_{d_7}}\int_0^t \ddot{a}_{Gy}(\tau)e^{-\omega\xi t}\sin\omega_{d_7}td\tau \quad (13)$$

For The Rocking Direction;

$$x(t) = e^{-\omega\xi t}(-A(t)\sin\omega_{d_8}t + B(t)\cos\omega_{d_8}t) \quad (14)$$

$$A(t) = \frac{1}{\omega_{d_8}}\int_0^t \frac{mh^2 a_{Gx}}{I_c}(\tau)e^{-\omega\xi t}\cos\omega_{d_8}td\tau \quad (15)$$

$$B(t) = \frac{1}{\omega_{d_8}}\int_0^t \frac{mh^2 a_{Gx}}{I_c}(\tau)e^{-\omega\xi t}\sin\omega_{d_8}td\tau \quad (16)$$

Using Simpson's rule for the numerical integration of the vertical direction and rocking direction,

$$A(t) = -\frac{\Delta\tau}{3\omega}[A(t-2\Delta\tau) + a_G(t-2\Delta\tau)\cos\omega_d(t-2\Delta\tau)e^{-\omega\xi 2\Delta t} + 4a_G(t-\Delta\tau)\cos\omega_d(t-\Delta\tau)e^{-\omega\xi 2\Delta t} + a_G(t)\cos\omega_d t]\sin\omega t \quad (17)$$

$$B(t) = \frac{\Delta\tau}{3\omega}\left[B(t-2\Delta\tau) + a_G(t-2\Delta\tau)\sin\omega_d(t-2\Delta\tau)e^{-\omega\xi 2\Delta t} + 4a_G(t-\Delta\tau)\sin\omega_d(t-\Delta\tau)e^{-\omega\xi 2\Delta t} + a_G(t)\sin\omega_d t\right]\cos\omega t \quad (18)$$

Equations (17) and (18) gave the 'A' and 'B' values instrumental for evaluating the system displacement and velocity values from the earthquake ground motion.

During Uplift

In solving Equations (7) and (8) which are the displacement equations during the period of uplift, it follows the same process of before uplift above but the initial conditions of the system at this point is the time of start of uplift. During uplift, time of start of uplift takes place when static deflection of the system equals vertical displacement of the system.

Velocity equations

From the displacement Equations of (5) and (6) before uplift, velocity equations gave:

Vertical direction;

$$\dot{y}(t) = e^{-\omega\xi t}\cos\omega_{d7}t\int_0^t \ddot{a}_{Gy}(\tau)e^{-\omega\xi t}\cos\omega_{d_7}t d\tau + e^{-\omega\xi t}\sin\omega_{d7}t\int_0^t \ddot{a}_{Gy}(\tau)e^{-\omega\xi t}\sin\omega_{d_7}t d\tau \quad (19)$$

Rocking direction

$$\dot{x}(t) = e^{-\omega\xi t}\cos\omega_{d8}t\int_0^t \frac{mh^2 a_{Gx}}{I_c}(\tau)e^{-\omega\xi t}\cos\omega_{d_8}t d\tau + e^{-\omega\xi t}\sin\omega_{d8}t\int_0^t \frac{mh^2 a_{Gx}}{I_c}(\tau)e^{-\omega\xi t}\sin\omega_{d_8}t d\tau \quad (20)$$

Then the simple harmonic motion of the Equations (19) and (20) is represented as;

For the Vertical Direction,

$$y(t) = e^{-\omega\xi t}(A(t)\cos\omega_{d_7}t + B(t)\sin\omega_{d_7}t) \quad (21)$$

$$A(t) = \int_0^t \ddot{a}_{Gy}(\tau)e^{-\omega\xi t}\cos\omega_{d_7}t d\tau \quad (22)$$

$$B(t) = \int_0^t \ddot{a}_{Gy}(\tau)e^{-\omega\xi t}\sin\omega_{d_7}t d\tau \quad (23)$$

For the Rocking Direction**,**

$$x(t) = e^{-\omega\xi t}(A(t)\cos\omega_{d_8}t + B(t)\sin\omega_{d_8}t) \quad (24)$$

$$A(t) = \int_0^t \frac{mh^2 a_{Gx}}{I_c}(\tau)e^{-\omega\xi t}\cos\omega_{d_8}t d\tau \quad (25)$$

$$B(t) = \int_0^t \frac{mh^2 a_{Gx}}{I_c}(\tau)e^{-\omega\xi t}\sin\omega_{d_8}t d\tau \quad (26)$$

Displacement equations of motion with un-damped effects

The equations of motion for un-damped vibration for vertical and rocking directions of before uplift and during uplift of Equations (1) to (4) without the damping coefficient and its solution gives that;

Before Uplift

$$y(t) = -\frac{\sin\omega_7 t}{\omega_7}\int_0^t a_{Gy}(\tau)\cos\omega_7\tau d\tau + \frac{\cos\omega_7 t}{\omega_7}\int_0^\tau a_{Gy}(\tau)\sin\omega_7\tau d\tau \quad (27)$$

$$x(t) = -\frac{\sin\omega_8 t}{\omega_8}\int_0^t \frac{mh^2 a_{Gx}}{I_c}(\tau)\cos\omega_8\tau d\tau + \frac{\cos\omega_8 t}{\omega_8}\int_0^\tau \frac{mh^2 a_{Gx}}{I_c}(\tau)\sin\omega_8\tau d\tau \quad (28)$$

Equation (27) is represented as;

$$y(t) = -A(t)\sin\omega_7 t + B(t)\cos\omega_7 t \quad (29)$$

From which:

$$A(t) = \frac{1}{\omega_7}\int_0^t a_{Gy}(\tau)\cos\omega_7\tau d\tau \quad (30)$$

$$B(t) = \frac{1}{\omega_7}\int_0^t a_{Gy}(\tau)\sin\omega_7\tau d\tau \quad (31)$$

Equation (28) also represented as;

$$x(t) = -A(t)\sin\omega_8 t + B(t)\cos\omega_8 t \quad (32)$$

From which;

$$A(t) = \frac{1}{\omega_8}\int_0^t \frac{mh^2 a_{Gx}}{I_c}(\tau)\cos\omega_8\tau d\tau \quad (33)$$

$$B(t) = \frac{1}{\omega_8}\int_0^t \frac{mh^2 a_{Gx}}{I_c}(\tau)\sin\omega_8\tau d\tau \quad (34)$$

During Uplift

$$y(t) = y(0)\cos\omega_{7A}t + \frac{\dot{y}(0)}{\omega_{7A}}\sin\omega_{7A}t - \frac{1}{\omega_{7A}}\int_0^t \left[a_{Gy} + \frac{W}{2m}\right](\tau)\sin\omega_{7A}(t-\tau)d\tau \quad (35)$$

$$x(t) = x(0)\cos\omega_9 t + \frac{\dot{x}(0)}{\omega_9}\sin\omega_9 t - \frac{1}{I_c\omega_9}\int_0^t [\mathrm{mh^2a}_{Gx} + \frac{\mathrm{W^2h^2}}{16\mathrm{k_fV}}](\tau)\sin\omega_9(t-\tau)d\tau \quad (36)$$

The initial time of the system here is the time the static deflection of the system is equal to the vertical displacement of the system. Then the displacement equations finally becomes;

$$y(t) = y(0)\cos\omega_{7A} t + \frac{\dot{y}(0)}{\omega_{7A}}\sin\omega_{7A} t - (A(t)\sin\omega_{7A} t + B(t)\cos\omega_{7A} t) \quad (37)$$

$$x(t) = x(0)\cos\omega_9 t + \frac{\dot{x}(0)}{\omega_9}\sin\omega_9 t - [A(t)\sin\omega_9 t + B(t)\cos\omega_9 t] \quad (38)$$

Where for Equation (37);

$$A(t) = -\frac{1}{\omega_{7A}}\int_0^t [\mathrm{a}_{Gy} + \frac{\mathrm{W}}{2\mathrm{m}}](\tau)\cos\omega_{7A}\tau d\tau \quad (39)$$

$$B(t) = \frac{1}{\omega_{7A}}\int_0^t [\mathrm{a}_{Gy} + \frac{\mathrm{W}}{2\mathrm{m}}](\tau)\sin\omega_{7A}\tau d\tau \quad (40)$$

Similarly applying the procedure to Equation (38) where;

$$A(t) = -\frac{1}{I_c\omega_9}\int_0^t [\mathrm{mh^2a}_{Gx} + \frac{\mathrm{W^2h^2}}{16\mathrm{k_fV}}](\tau)\cos\omega_9\tau d\tau \quad (41)$$

$$B(t) = \frac{1}{I_c\omega_9}\int_0^t [\mathrm{mh^2a}_{Gx} + \frac{\mathrm{W^2h^2}}{16\mathrm{k_fV}}](\tau)\sin\omega_9\tau d\tau \quad (42)$$

The displacement equations with damping effect during uplift compared with one without damping is;
Vertical direction;

$$y(t) = e^{-\omega\xi t}[y(0)\cos\omega_{d_{7A}} t + \frac{\dot{y}(0)+\omega\xi y(0)}{\omega_{d_{7A}}}\sin\omega_{d_{7A}} t] - \frac{e^{-\omega\xi t}\sin\omega_{d_{7A}} t}{\omega_{d_{7A}}}\int_0^t [\mathrm{a}_{Gy} + \frac{\mathrm{W}}{2\mathrm{m}}](\tau)e^{-\omega\xi t}\cos\omega_{d_{7A}} t d\tau + \frac{e^{-\omega\xi t}\cos\omega_{d_{7A}} t}{\omega_{d_{7A}}}\int_0^t [\mathrm{a}_{Gy} + \frac{\mathrm{W}}{2\mathrm{m}}](\tau)e^{-\omega\xi t}\sin\omega_{d_{7A}} t d\tau \quad (43)$$

Rocking direction;

$$x(t) = e^{-\omega\xi t}[x(0)\cos\omega_{d_9} t + \frac{\dot{x}(0)+\omega\xi x(0)}{\omega_{d_9}}\sin\omega_{d_9} t] - \frac{e^{-\omega\xi t}\sin\omega_{d_9} t}{\omega_{d_9}}\int_0^t [\mathrm{mh^2a}_{Gx} + \frac{\mathrm{W^2h^2}}{16\mathrm{k_fV}}](\tau)e^{-\omega\xi t}\cos\omega_{d_9} t d\tau + \frac{e^{-\omega\xi t}\cos\omega_{d_9} t}{\omega_{d_9}}\int_0^t [\mathrm{mh^2a}_{Gx} + \frac{\mathrm{W^2h^2}}{16\mathrm{k_fV}}](\tau)e^{-\omega\xi t}\sin\omega_{d_9} t d\tau \quad (44)$$

The structural response with effects of damping for the uplifting system was then analyzed with a numerical example as seen below.

Method description

Damping was introduced to the system as a means of restraining or slowing down the vibratory motion of the structure. Figure 2 and Figure 3 showed the dimensional properties of the structure used in the formulation of displacement equations of motions with damping effects included since energy was dissipated by damping forces when the structure is in full contact and during uplifting applying the principle of Duhamel integral. Solution of the Duhamel Integral was done using Simpson's rule method of numerical integration of Equations. Damping effects on a system is to increase the period of natural frequency and bring down the resonant frequency. In most of engineering systems, the coefficient of damping is always less than ten percent and in so doing, makes the damping frequency and normal frequency equal (Smith, 1988; Clough and Penzien, 2003).This structural system being analyzed was assumed to be damped lightly, the damping frequency and normal frequency is the same in this work i.e.,, $\omega_D = \omega$. The coefficient of damping is 5% and the damping effects on the system's responses were evaluated using the formulated equations of motion. Numerical solution of the equations of motion for the conditions (full contact and during uplift condition) were done using Simpson's rule as Simpson's rule provides acceptable results and calculations for the structure response history and can also be applied to any arbitrary loading even where loads have been determined by experiment and can't be expressed analytically(Smith, 1988). In the case of un-damped system, the Simpson's ordinate multiplier used were '1' and '4' and for the damped system, Simpson's ordinate multiplier becomes damping ratio of 0.05 obtained as below;

Before Uplift

Vertical Direction

$[e^{-\omega\xi 2\Delta t}] = \mathrm{e}^{(-229.2\mathrm{x}0.05\times 2\times 0.0075)} = 0.84$

$[4e^{-\omega\xi\Delta t}] = 3.67$

Rocking Direction

$[e^{-\omega\xi 2\Delta t}] = 0.79 \quad [4e^{-\omega\xi\Delta t}] = 3.55$

During Uplift

Vertical Direction

$[e^{-\omega\xi 2\Delta t}] = 0.91[4e^{-\omega\xi\Delta t}] = 3.83$

Rocking Direction

$[e^{-\omega\xi 2\Delta t}] = 0.77 \quad [4e^{-\omega\xi\Delta t}] = 3.53$

These values act on the value of 'A' and 'B'

RESULTS AND DISCUSSIONS

Numerical Example

Using a system assumed with the following properties and subjected to impulsive horizontal 1940 El-Centro earthquake ground motion. The structure is of assumed mass of 4078kg acting at the center of the system. The mass is located at height 'h' = 3.2m from ground level. The stiffness of soil spring is $1.8\mathrm{x}10^6$ N/m^3 with the inertia moment about the center of base as $2.6\mathrm{x}10^4$ m^4. Evaluate the response history of the system with 5% damping applied with other properties as B = 7m, u = 1.5m, g = 9.81m/s^2. The El-Centro maximum ground acceleration is approximately 0.32G as seen in Figure 4. This was taken as the input acceleration that will be experienced by the system and 'G' is acceleration due to gravity in m/s^2.

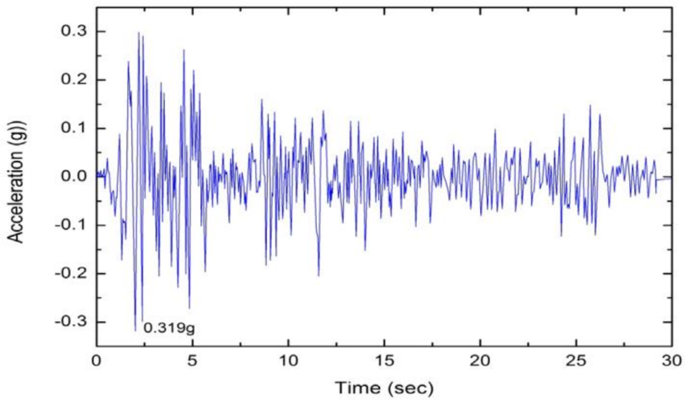

Figure 4: Time History of El-Centro, 1940 earthquake ground motion (the time history of acceleration of north-south component of the El-Centro, 1940 earthquake ground motion), Chopra, 2011

The damping frequency and normal frequency in this study are taken as equal. Damping as a form of base isolation is used in earthquake resistant design to bring down the system motion from seismic activity as it deflects and dissipates the seismic energy reducing the natural frequency of the system. Hence, thereby leading to reduction of the structural response as well as protecting the structural integrity. During vibration, there is reduction in the spring stiffness and increase in flexibility and whenever the ground shaking increases unnecessarily, the absorbing damping system goes into action to reduce vibration.

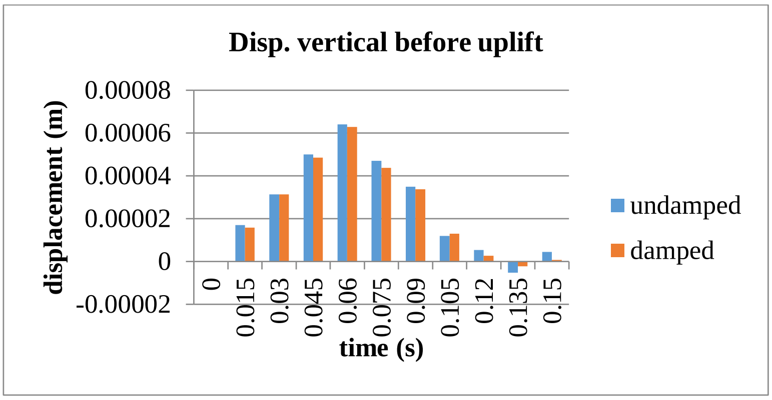

Figure 5: Damping effects on vertical displacement before uplift

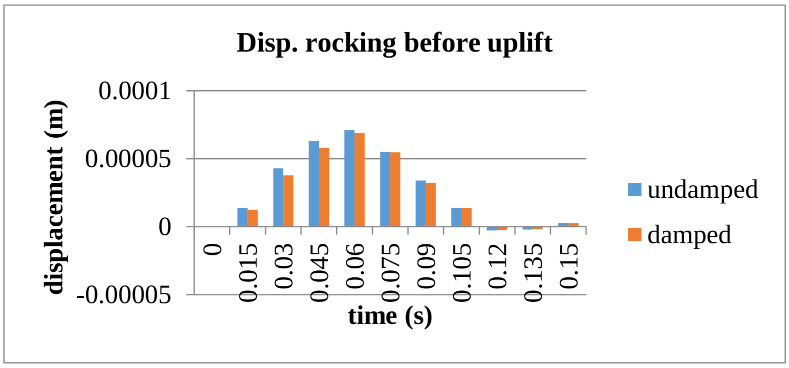

Figure 6: Damping effects on rocking displacement before uplift

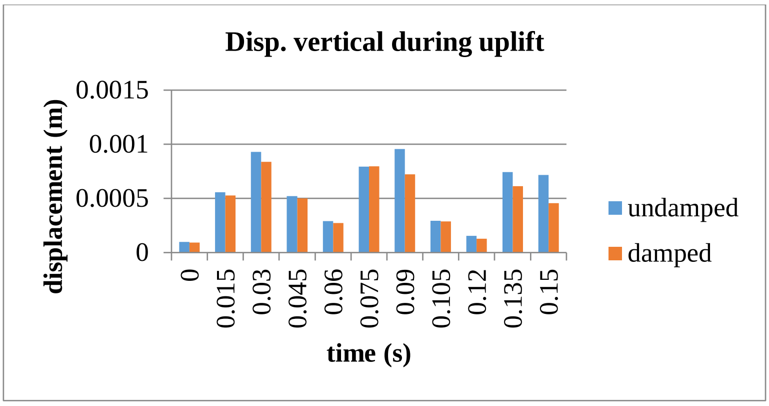

Figure 7: Damping effects on vertical displacement during uplift

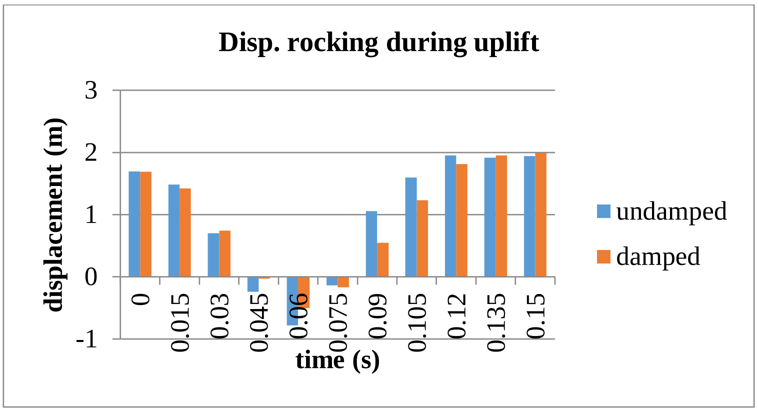

Figure 8: Damping effects on rocking displacement during uplift

Damping effects on displacement for vertical and rocking components are presented in Figure 5 to Figure 8 for the condition of full contact and during uplift. The vibration from earthquake ground motion increased the structural response during the time of structure uplifting as displacement during uplift was increased more than when the structure is on full contact(before uplift). This increase of structure displacement can be from the intensity of ground motion or from the system-foundation parameters. The use of dampers in the system reduced the displacement both in the vertical and rocking directions and also during uplifting. Figure 9 and Figure 10 showed effect of damping on velocity as it reduced on the vertical and rocking components for the different conditions (full contact and during uplift) as velocity of the system increased with the occurrence of uplift.

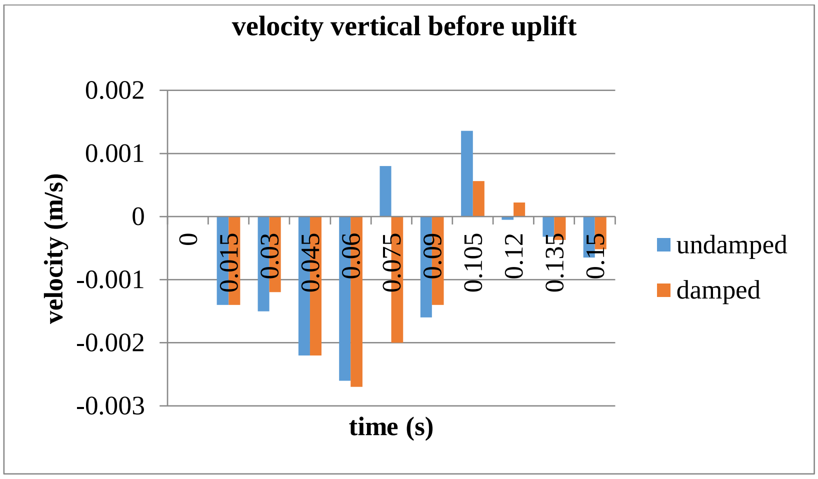

Figure 9: Damping effect on vertical velocity before uplift

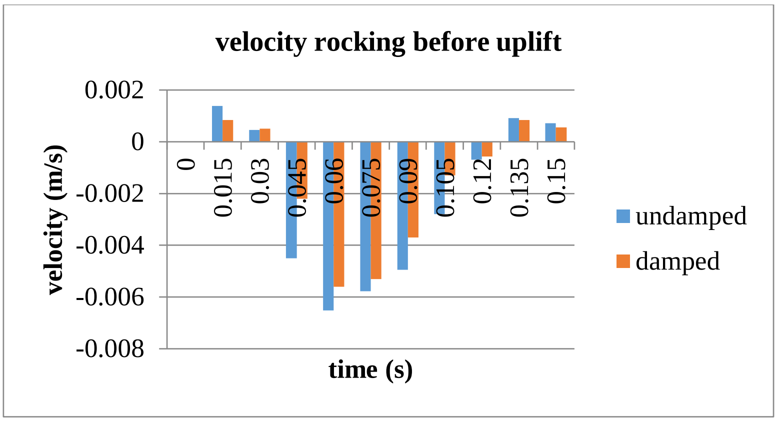

Figure. 10: Damping effect on rocking velocity before uplift

The base shear which is a representation of the expected lateral force from the ground motion at the base of the structure showed that at the occurrence of uplift there is upward trend in the forces generated for vertical and rocking directions. This can be inferred from the increased flexibility in the structure foundation as there is reduction in the soil stiffness. The damped and un-damped base shear effects can be seen that damping reduced the system response to uplift from Figure 11 to Figure 13.

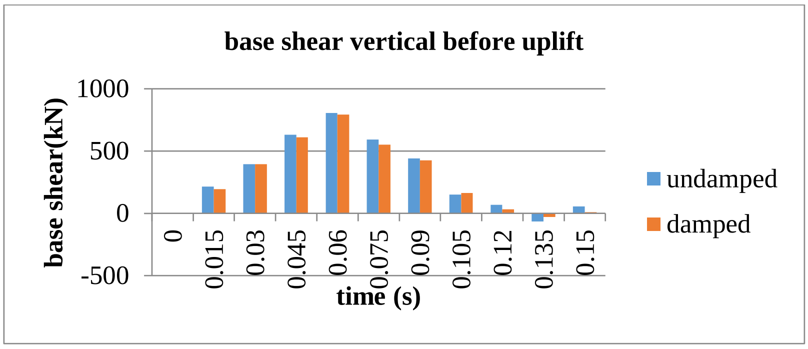

Figure 11: Damping effect on vertical base shear before uplift

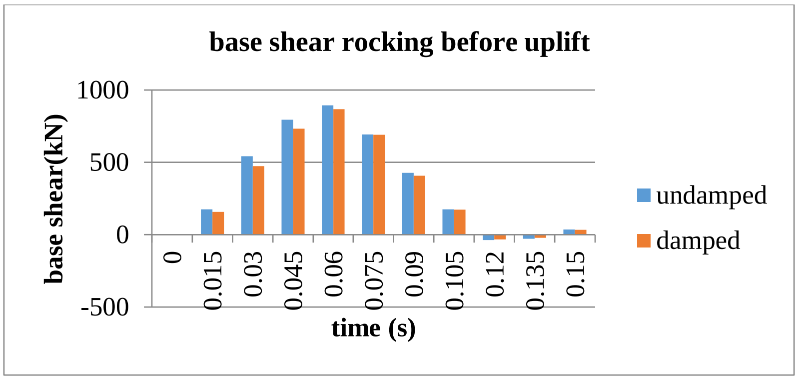

Figure 12: Damping effect on rocking base shear before uplift

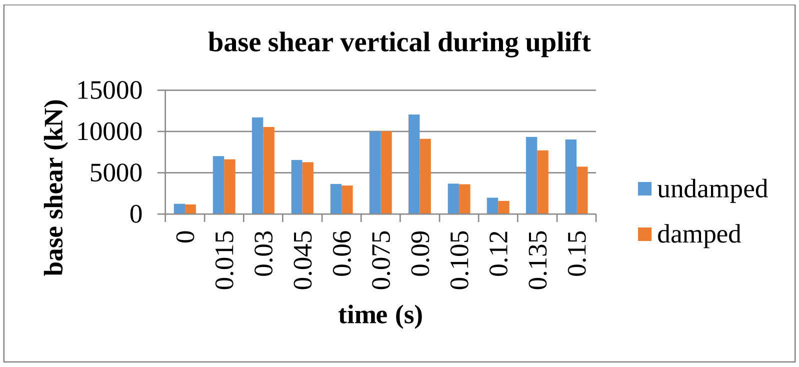

Figure 13: Damping effect on vertical base shear during uplift

i. The uplifting system showed to have natural frequency that increased during uplift in the rocking direction as seen from Table.1. This effect can be attributed to the continuity of the soil spring elements on the foundation surface (Bonkowski et al, 2019; Mortezaie and Zamanian, 2021; Okafor et al, 2023).

Table 1: Natural frequency and period of the uplifting structure

S/NO	Response	BEFORE UPLIFT		DURING UPLIFT	
		Vertical	**Rocking**	**Vertical**	**Rocking**
1	Natural frequency	36.47	50.13	18.24	53.27
2	Natural period	0.027	0.02	0.0548	0.0187

ii. There was slight decrease in the natural periods of oscillation of the structure as seen inTable.1 in the rocking component during structure uplifting. This is very important in foundation uplift because natural period of the soil is sensitive to how flexible the foundation is. (Ugwu et al, 2023; Bonkowski et al, 2019).
iii. The rocking response of the system was very sensitive to small changes of the ground motion details and system parameters. At the point of uplift, there is increased system response in Figure.5 to Figure.13.
iv. Structure uplifting leads to increase in the system response as in the case of displacement, velocity and base shear as in Figure5 to Figure13.
v. It is evident that rocking responses are more significant than the vertical responses during uplift occurrence from the analysis. This from the engineering point of view shows that one is to be normally more interested in the rocking response of the system than vertical response as in Table 1 and as in Figure5 to Figure13.
vi. Introduction of damping effects on the continuity interactions slowed down the frequency of the responses as in as in Figure5 to Figure13.

As the intensity of ground motion decays from damping effects, foundation uplifting reduces and tends to becomes negligible and full contact of structure to foundation base is maintained for longer durations. The application of damping to bring about stability of structure-foundation during vibration from earthquake ground motion is a contributory factor for an elastic medium.

CONCLUSION

In general, structure-foundation interaction during uplift sometimes is to bring about significant reduction of the fundamental frequency of the structure. When earthquake ground motion excitation is strong, the structure-foundation can become softer and more flexible which further reduces the fundamental frequency and increases the period of oscillation hence making the motion of the system non-linear. Since the system is now non-linear as a result of the strong excitation, this frequency is dependent on the type of excitation and reduces as amplitude of excitation increases. In this analysis, there was increase of the fundamental frequency during uplift and decrease in the period of oscillation which might be attributed to the effect of continuity interactions of the soil spring elements on the foundation surface. There was an upward trend in the response of the vibrating system which can be from the structure and the foundation surface. The uplifting system response to earthquake may be significantly different from response when there is no uplifting as uplifting depends on the nature and type of ground motion excitation, parameters of the structure and foundation. Foundation damping reduced the structure responses both when on full contact and during uplift. One can then say that not allowing uplift or bonding structures to the supporting soil and applying damping can slow down vibration effects from ground motion. The earthquake rocking component tends to have some form of irregular pattern which can be attributed to the complex non-linear state and swaying of the system during motion and it gave responses higher than the vertical component. But damping tried to maintain stability in the structural responses of the system. Hence bonding of foundations to the supporting soil can help to minimize uplift effects which sometimes are not beneficial as it can reduce or increase structure responses.

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