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FORMULATION OF BLOCK SCHEMES WITH LINEAR MULTISTEP METHOD FOR THE APPROXIMATION OF FIRST-ORDER IVPS

***Imoni S. O., Lanlege D. I., Atteh E. M. and Ogbondeminu J. O.**

Department of Mathematical Sciences, Federal University Lokoja, P.M.B. 1154, Lokoja, Kogi State

Corresponding Author's email: imoni4u@yahoo.com, Sunday.imoni@fulokoja.edu,ng

ABSTRACT

In this paper, formulation of an efficient numerical schemes for the approximation first-order initial value problems (IVPs) of ordinary differential equations (ODE) is presented. The method is a block scheme for some k-step linear multi-step methods ($k = 1, 2$ and 3) using the Hermite Polynomials a basis function. The continuous and discrete linear multi-step methods (LMM) are formulated through the technique of collocation and interpolation. Numerical examples of ODE have been examined and results obtained show that the proposed scheme can be efficient in solving initial value problems of first order ODE.

Keywords: linear multi-step method, ordinary differential equations, initial value problems, Hermite polynomials.

INTRODUCTION

Numerous problems in Sciences and Engineering are modelled using ordinary differential equations (ODEs). Most of these differential equations do not have analytical solutions which makes numerical methods an option for solving these problems. There are two major discrete variable methods for approximating the solutions of ODEs, namely, one step and linear multi-step methods.

In this paper, we proposed an efficient numerical scheme to solve numerically first order IVPs. The proposed method is a block scheme for some k-step linear multistep methods (for $k = 1, 2$ and 3) using Hermite polynomial as the basis functions. Also, we give the discrete methods used in block and implement it for solving some existing IVPs in the literature. In this paper, we consider the general form of the first order initial value problems.

$$y'(x) = f(x, y(x), \; y(x_0) = y_0 \tag{1}$$

Many researchers had developed interest on improving the numerical solution of initial value problems (IVPs) of ordinary differential equation. Consequently, the development of a class of methods called block methods is one of the outcomes. Okunuga and Ehigie (2009) derived two-step continuous and discrete linear multistep methods using power series as a basis function. Akinfenwa *et al.* (2011) developed a four step continuous block hybrid method with four non-step points for the direct solution of first order initial value problem. Odekunle *et al.* (2012) developed a continuous linear method using interpolation and collocation for the solution of first–order ODE with constant step size. James *et al.* (2013), proposed a continuous block method for the solution of second order IVPs with constant step size, the method was developed by interpolation and collocation of power series approximate solution to generate a continuous linear multistep method. A block procedure with linear multistep methods using Legendre polynomials was done by Abualnaja (2015). However, he did not include the block schemes. Okedayo *et al.* (2018) developed on modified Legendre collocation block method for solving initial value problems of first order ODEs. Also, Okedayo *et al*. (2018), developed a continuous Laguerre collocation block method for solving initial value problems of first order ordinary differential equations. However, in this paper, Hermite polynomial is used as a basis function to derive some block methods for the solution of first order initial value problem (1).

FORMULATION OF THE METHODS

We consider the approximate solution of the perturbed form of (1) in the power series below

$$y_k(x)=\sum_{i=0}^{k}c_i\psi_i(x),\quad x_n\le x\le x_{n+k} \tag{2}$$

where

$$\psi_i(x)=x^i,\ \ i=0,1,2,...,k \tag{3}$$

Substituting (2) into (1) and add $\lambda H_k(x)$ where λ is the perturbed term and $H_k(x)$ is the Hermite polynomial of degree k valid in $x_n\le x\le x_{n+k}$ we have

$$\sum_{i=0}^{k}c_i\psi_i'(x)=f(x,y)+\lambda H_k(x) \tag{4}$$

We shall consider cases where $k=1,2$ and 3 in (2) and (3)

The Hermite polynomial is given by $Hi(x),\ i=0,1,2,3$

$$H_0(x)=1,H_1(x)=2x,H_2(x)=4x^2-2,H_3(x)=8x^2-12x \tag{5}$$

These polynomials are gotten from the Hermite Rodrigue's formula

$$H_n(x)=e^{x^2}(-1)^n\frac{d^n}{dx^n}(e^{x^2}) \tag{6}$$

In this paper, we are going to use the set of polynomials in (5) to formulate the block schemes. Using these polynomials in (5) in the interval $[x_n,x_{n+k}]$, we introduce the change of variable to define the Hermite polynomial as

$$x=\frac{2\overline{x}-(x_{n+k}+x_n)}{(x_{n+k}-x_n)},\ \ k=1,2,3 \tag{7}$$

Abualnaja (2015)

For k =1,

In this case, take $H_1(x)=2x$, since k=1 and use equation (7). Collocate equation (7) at x_n and x_{n+1} and solve to obtain

$$x=\frac{2\overline{x}-(x_{n+k}+x_n)}{x_{n+k}-x_n}=\frac{2x_n-(x_{n+1}+x_n)}{x_{n+1}-x_n}=\frac{-x_{n+1}+x_n}{x_{n+1}-x_n}=-1 \tag{8}$$

Substitute the value of x into $H_1(x)=2x$ and obtain $H_1(x)=-2$

Also, following the same procedure for x_{n+1} we have

$$x=\frac{2\overline{x}-(x_{n+k}+x_n)}{x_{n+k}-x_n}=\frac{2x_{n+1}-(x_{n+1}+x_n)}{x_{n+1}-x_n}=\frac{x_{n+1}+x_n}{x_{n+1}-x_n}=1 \tag{9}$$

Hence $H_1(x)=2$

From equation (2.2), we deduce that $\omega_0'(x)=0$ and $\omega_1'(x)=1$. Then substituting into (2.3), we have

$$f(x,y)=c_1-\lambda H_1(x) \tag{10}$$

Thus collocate (10) at x_{n+1}, $i=0,1$ and interpolate (2) at $x=x_n$, we obtain a system of three equations with $c_i,(i=0,1)$

and parameter λ .The system of the equations are given as:

$$\begin{aligned} y_n &= c_0 + c_1 x \\ f_n &= c_1 + 2\lambda \\ f_{n+1} &= c_1 - 2\lambda \end{aligned} \qquad (11)$$

Solve (11) to obtain

$$\lambda = \frac{1}{4}(f_n - f_{n+1}),\ c_1 = \frac{1}{2}(f_n + f_{n+1}), c_0 = y_n - \frac{x_n}{2}(f_n + f_{n+1})$$

From (2) we obtain

$$\overline{y} = c_0 + c_1 x \qquad (12)$$

Now, the required numerical scheme of the proposed method for y_{n+1} will be obtain if we collocate (12) at $x = x_{n+1}$ and substitute the value of c_0, c_1 and λ . Hence, we obtain

$$y_{n+1} = y_n + \frac{h}{2}(f_n + f_{n+1}) \qquad (13)$$

Now for $k = 2$

In this case, $H_2(x) = 4x^2 - 2$, since $k = 2$ and use equation (7), then collocate the equation at x_n, x_{n+1} and x_{n+2} and solve to obtain

$$x = \frac{2\overline{x} - (x_{n+k} + x_n)}{x_{n+k} - x_n} = \frac{2x_{n+1} - (x_{n+2} + x_n)}{x_{n+2} - x_n} = \frac{-x_{n+2} + x_n}{x_{n+2} - x_n} = -1 \qquad (14)$$

Substitute the value of x into $H_2(x) = 4x^2 - 2$ and obtain Hence $H_2(x) = 2$

Using the same procedure for x_{n+1} to have

$$x = \frac{2\overline{x} - (x_{n+k} + x_n)}{x_{n+k} - x_n} = \frac{2x_{n+1} - (x_{n+2} + x_n)}{x_{n+2} - x_n} \qquad (15)$$

Now, put x_{n+1} to be $x_n + h$ and x_{n+2} to be equal to $x_n + 2h$ in equation (15) to obtain $x = 0$. Thus, substitute x into $H_2(x) = 4x^2 - 2$ to get $H_2(x_{n+1}) = -2$

Also for x_{n+2}, we have

$$x = \frac{2\overline{x} - (x_{n+k} + x_n)}{(x_{n+k} - x_n)} = \frac{2x_{n+2} - (x_{n+2} + x_n)}{(x_{n+2} - x_n)} = \frac{x_{n+2} - x_n}{x_{n+2} - x_n} = 1 \qquad (16)$$

Hence, we substitute the value of x into $H_2(x) = 4x^2 - 2$. Therefore $H_2(x_{n+2}) = 2$

From (3), we deduce that $\psi_0' = 0, \psi_1' = 1$ and $\psi_2' = 2x$. Then equation (4) reduces to the form

$$f(x, y) = c_1 + 2xc_2 - \lambda H_2(x) \qquad (17)$$

We now collocate (17) at $x_{n+i}, i = 0,1,2$ and interpolate (2) at $x = x_n$ to obtain a system of four equations with $c_i (i = 0,1,2)$ and parameter λ

$$\begin{aligned} y_n &= c_0 + c_1 x_n + c_2 x_n^2 \\ f_n &= c_1 + 2x_n c_2 - 2\lambda \\ f_{n+1} &= c_1 + c_2 x_{n+1} + 2\lambda \\ f_{n+2} &= c_1 + c_2 x_{n+2} - 2\lambda \end{aligned} \tag{18}$$

Solving the system (18) results to

$$\begin{aligned} \lambda &= \frac{1}{8}(2f_{n+1} - f_{n+2} - f_n) \\ c_2 &= \frac{1}{4h}(f_{n+2} - f_n) \\ c_1 &= \frac{1}{4h}(2hf_{n+1} - 2x_n f_{n+2} + 2x_n f_n - hf_{n+2} + 3hf_n) \\ c_o &= \frac{1}{4h}(4hy_n - 2hx_n f_{n+1} + hx_n f_{n+2} - 3hx_n f_n + x^2 f_{n+2} - x^2 f_n) \end{aligned} \tag{19}$$

From (2), we obtain

$$\bar{y} = c_0 + c_1 x + c_2 x^2 \tag{20}$$

The required numerical scheme is obtained if we collocate equation (19) at $x = x_{n+1}$ and substitute c_0, c_1, c_2 and λ as

$$y_{n+1} = y_n + \frac{h}{4}(f_{n+2} + 2f_{n+1} + f_n) \tag{21}$$

Now consider $k = 3$

In this case, take $H_2(x) = 8x^3 - 12x,$ since $k = 3$ and use equation (7). Collocate this equation at x_n, x_{n+1}, x_{n+2} and x_{n+3} and solve to have

$$x = \frac{2\bar{x} - (x_{n+k} + x_n)}{(x_{n+k} - x_n)} = \frac{2x_n - (x_{n+3} + x_n)}{x_{n+3} - x_n} = \frac{-x_{n+3} + x_n}{x_{n+3} - x_n} = -1 \tag{22}$$

Thus substitute the value of x into $H_3(x) = 8x^3 - 12x$ and obtain $H_3(x) = 4$

Following the same process for x_{n+1} we have

$$x = \frac{2x_{n+1} - (x_{n+3} + x_n)}{x_{n+3} - x_n}$$

Put $x_{n+1} = x_n + h$, $x_{n+3} = x_n + 3h$ and obtain

$$\frac{2(x_n + h) - (x_n + 3h + x_n)}{x_n + 3h - x_n} = -\frac{1}{3}.$$

By substituting the value of x into $H_3(x) = 8x^3 - 12x,$ then $H_3(x) = \frac{100}{27}$

Follow the same procedure for x_{n+2} and have

$x = \frac{2x_{n+2} - (x_{n+3} + x_n)}{x_{n+3} - x_n}$. Putting $x_{n+2} = x_n + 2h$, $x_{n+3} = x_n + 3h$ and substituting into the above equation to obtain

$\frac{2(x_n+2h)-(x_n+3h+x_n)}{x_n+3h-x_n}=\frac{1}{3}$. Substituting the value of x into $H_3(x)=8x^3-12x,$ then $H_3(x)=-\frac{100}{27}$

Following the same procedure for x_{n+3}, we have

$$x=\frac{2\overline{x}-(x_{n+k}+x_n)}{(x_{n+k}-x_n)}=\frac{2x_{n+3}-(x_{n+3}+x_n)}{x_{n+3}-x_n}=1$$

Thus, substitute the value of x into $H_3(x)=8x^3-12x,$ and have $H_3(x)=-4$

Thus, from equation (3), we deduce that $\psi_0'(x)=0,\psi_1'(x)=1,\psi_2'(x)=2x,\psi_3'(x)=3x^2$.

Equation (3) reduces to the form

$$f(x,y)=c_1+2xc_2+3x^2c_3-\lambda H_3(x) \tag{23}$$

We now collocate (23) at $x_{n+i}(i=0,1,2)$ and interpolate (2) at $x=x_n$ to obtain a system of five equations with $c_i(i=0,1,2,3)$ and parameter λ as

$$\begin{aligned}
y_n &= c_0+c_1x_n+c_2x_n^2+c_3x_n^3\\
f_n &= c_1+2c_2x_n+3c_3x_n^2-4\lambda\\
f_{n+1} &= c_1+2c_2x_{n+1}+3c_3x_{n+1}^2-\frac{100}{27}\lambda\\
f_{n+2} &= c_1+2c_2x_{n+2}+3c_3x_{n+2}^2+\frac{100}{27}\lambda\\
f_{n+3} &= c_1+2c_2x_{n+3}+3c_3x_{n+3}^2+4\lambda
\end{aligned} \tag{24}$$

Solving the system (24) resulted to

$$\lambda=\frac{9}{128}(f_n-3f_{n+1}+3f_{n+2}-f_{n+3})$$

$$c_3=\frac{1}{12h^2}(f_n-f_{n+1}-f_{n+2}+f_{n+3})$$

$$c_2=-\frac{1}{96h^2}(61hf_n-63hf_{n+1}-9hf_{n+2}+11hf_n+3+24f_nx_n-24f_{n+1}x_n-29f_n+2x_n+24f_nx_n)$$

$$\begin{aligned}c_1=-\frac{1}{96h^2}(&123h^2f_n-81h^2f_{n+1}+81h^2f_{n+2}-27h^2f_{n+3}+122hf_nx_n-126hf_{n+1}x_n-\\
&18hf_{n+2}x_n+22f_{n+3}x_n+24f_nx_n^2-24f_{n+1}x_n^2-24f_{n+2}x_n^2+24fn+3x_n^2)\end{aligned}$$

$$\begin{aligned}c_0=-\frac{1}{96h^2}(&123h^2f_nx_n-81h^2f_{n+1}x_n+81h^2f_{n+2}x_n-27h^2f_{n+3}x_n+61hf_nx_n^2-\\
&63hf_{n+1}x_n^2-9hf_{n+2}x_n^2+11hf_{n+3}x_n^2+8f_nx_n^3-8f_{n+1}x_n^3-8f_{n+2}x_n^3+8f_{n+3}x_n^3-\\
&96h^2y_n\end{aligned}$$

Again, from (2) we have

$$\overline{y}=c_0+c_1x+c_2x^2+c_3x^3 \tag{25}$$

The required numerical scheme is then obtained if we collocate (25) at x_{n+1} and substituting for c_0,c_1,c_2 and c_3 as

$$y_{n+1} = y_n + \frac{h}{48}(35 f_n - 13 f_{n+1} + 41 f_{n+2} - 15 f_{n+3}) \tag{26}$$

Now we formulate the block schemes for the polynomials of cases $k = 1, 2$ and 3

For $k = 1$, collocate equation (12) at $x = x_{n+1}, x_{n+2}, x_{n+3}$ to obtain

$$\begin{aligned} y_{n+1} &= y_n + \frac{h}{2}(f_n + f_{n+1}) \\ y_{n+2} &= y_n + h(f_n + f_{n+1}) \\ y_{n+3} &= y_n + \frac{3h}{2}(f_n + f_{n+1}) \end{aligned} \tag{27}$$

For $k = 2$, collocate equation (20) at $x = x_{n+1}, x_{n+2}, x_{n+3}$ to obtain

$$\begin{aligned} y_{n+1} &= y_n + \frac{h}{4}(f_n + 2 f_{n+1} + f_{n+2}) \\ y_{n+2} &= y_n + \frac{h}{2}(f_n + 2 f_{n+1} + f_{n+2}) \\ y_{n+3} &= y_n + \frac{3h}{4}(6 f_n + 2 f_{n+1} + f_{n+2}) \end{aligned} \tag{28}$$

For $k = 3$, collocate equation (25) at $x = x_{n+1}, x_{n+2}, x_{n+3}$ to obtain

$$\begin{aligned} y_{n+1} &= y_n + \frac{h}{48}(35 f_n - 13 f_{n+1} + 41 f_{n+2} - 15 f_{n+3}) \\ y_{n+2} &= y_n + \frac{h}{48}(33 f_n + 13 f_{n+1} + 67 f_{n+2} - 17 f_{n+3}) \\ y_{n+3} &= y_n + \frac{3h}{8}(f_n + 3 f_{n+1} + 3 f_{n+2} + f_{n+3}) \end{aligned} \tag{29}$$

ERORR ANALYSIS OF THE METHOD

In this section, we discuss the order, the error constant and convergence of the proposed block schemes. The proposed schemes in this paper belong to the class of linear multi-step method (LMM) which is of the form

$$\sum_{j=0}^{k} \alpha_j(x) y(x_{n+j}) = h \sum_{j=0}^{k} \beta_j(x) f(x_{n+j}) \tag{30}$$

According to Sastry (2008) and Lambert (1981), define the order and error constant associated with (30) to be the linear difference operator

$$\ell[y(x);h] = \sum_{j=0}^{k} \left\{ \alpha_j y(x+jh) - h\beta_j y'(x+jh) \right\} \tag{31}$$

Assuming that $y(x)$ is continuously differentiable on the interval $[a,b]$, we can expand the terms in (31) as a Taylor series about the point x to obtain the expansion

$$\ell[y(x);h] = c_0 y(x) + c_1 h y'(x) + c_2 h^2 y''(x) + \ldots + c_q h^q y^q(x) \tag{32}$$

where the constant coefficients $c_q, q = 0,1,\ldots$ are given as follows:

$$c_0 = \sum_{j=0}^{k} \alpha_j,\ c_1 = \sum_{j=0}^{k} (j\alpha_j - \beta_j),\ \ldots,\ c_q = \sum_{j=0}^{k} \left[\frac{1}{q!} j^q \alpha_j - \frac{1}{(q-1)!} j^{q-1} \beta_j \right], q = 2,3\ldots \tag{33}$$

We say that the methods are said to be order p if $c_0 = c_1 = c_2 = ... = c_p = 0, c_{p+1} \neq 0$

Therefore the c_{p+1} is the error constant.

Hence, we establish that (27), (28) and (29) are of the following orders and error constants respectively.

when $k = 1$, $p = 1$ and $c_{p+1} = \frac{1}{4}$

when $k = 2$, $p = 2$ and $c_{p+1} = -\frac{1}{6}$

when $k = 3$, $p = 4$ and $c_{p+1} = -\frac{864}{23040}$

NUMERICAL EXAMPLES

In this section, two initial value problems were solved using the proposed method for $k = 2$ and $k = 3$and compare the numerical results obtained with the exact solution and those obtained from using the method of Okedayo *et al* (2018) in order to test the efficiency of the schemes.

Example 1

Consider the following IVP

$$y'(x) = -y, y(0) = 1$$

with the exact solution $y(x) = e^{-x}$

Example 2

Consider the IVP

$$y'(x) = x - xy,\ y(0) = 0$$

With exact solution $y(x) = 1 - e^{-\frac{x^2}{2}}$

Notations: BLS = Block schemes derived in this paper, LM = Method of Okedayo *et al.* (2018), Exact = Exact Solution, |Exact-BLS| = the absolute value of the exact solution minus computed solution of the method derived in this paper and |Exact-LM| = the absolute value of the exact solution minus computed solution of Okedayo *et al.* (2018).

The numerical results of these examples are depicted in Tables 1, 2 3 and 4 with $k = 2$ and $k = 3$ with constant step size of $h = 0.1$respectively. In tables 1 and 3 we presented a comparison of the obtained numerical results using the proposed scheme with the exact solution and Table 2 and 4 presents the comparison of the results obtained from proposed scheme, the exact solution and those numerical results obtained from Okedayo *et al.* (2018).

Table 1: A comparison of numerical results of proposed Scheme at $k = 2$ with exact solution for Example 1

x-value	BLS $k = 2$	Exact	|Exact-BLS|
0.0	1.000000	1.000000	0.000000
0.1	0.905090	0.904837	4.253×10^{-4}
0.2	0.818181	0.818730	5.49×10^{-4}
0.3	0.749092	0.740818	8.274×10^{-3}
0.4	0.675227	0.670320	4.907×10^{-3}
i0.5	0.608761	0.606531	2.23×10^{-3}
0.6	0.548760	0.548811	5.1×10^{-3}
0.7	0.504028	0.496585	7.443×10^{-3}
0.8	0.448139	0.449328	1.189×10^{-3}
0.9	0.416232	0.406569	9.663×10^{-3}
1.0	0.373893	0.367879	6.014×10^{-3}

Table 2: A comparison of numerical results of proposed scheme at $k=3$ with exact solution and LM for Example 1

x-value	BLS $k=3$	LM	Exact	|Exact-BLS|	|Exact-LM|
0.0	1.000000	1.000000	1.000000	0.000000	0.000000
0.1	0.904808	0.905953	0.904837	2.9×10^{-5}	1.116×10^{-3}
0.2	0.818705	0.820856	0.818730	$5.0\times10^{-}5$	2.126×10^{-3}
0.3	0.740823	0.743857	0.740818	5.0×10^{-5}	3.039×10^{-3}
0.4	0.670304	0.674185	0.670320	1.6×10^{-5}	3.865×10^{-3}
0.5	0.606516	0.611143	0.606531	1.5×10^{-5}	4.612×10^{-3}
0.6	0.548820	0.554100	0.548811	9.0×10^{-6}	5.289×10^{-3}
0.7	0.496576	0.502486	0.496585	9.0×10^{-6}	5.901×10^{-3}
0.8	0.449321	0.455784	0.449328	7.0×10^{-6}	6.456×10^{-3}
0.9	0.406578	0.413527	0.406569	9.0×10^{-6}	6.958×10^{-3}
1.0	0.367876	0.375290	0.367879	3.0×10^{-6}	7.411×10^{-3}

Table 3: A comparison of numerical results of proposed scheme at $k=2$ with exact solution for Example 2

x-value	BLS $k=2$	Exact	|Exact-BLS|
0.0	0.000000	0.000000	0.000000
0.1	0.005452	0.004785	6.67×10^{-4}
0.2	0.018904	0.018901	1.97×10^{-4}
0.3	0.042261	0.033013	9.218×10^{-3}
0.4	0.058942	0.056862	2.078×10^{-3}
0.5	0.117427	0.115602	1.825×10^{-3}
0.6	0.121342	0.166729	4.5387×10^{-2}
0.7	0.178557	0.216929	1.1828×10^{-2}
0.8	0.235772	0.255531	1.9758×10^{-2}
0.9	0.274301	0.292022	1.37121×10^{-2}
1.0	0.327572	0.353368	2.5706×10^{-2}

Table 4: A comparison of numerical results of proposed scheme at $k=3$ with exact solution and LM for Example 2

x-value	BLS $k=3$	LM	Exact	|Exact-BLS|	|Exact-LM|
0.0	0.000000	0.000000	0.000000	0.000000	0.000000
0.1	0.005073	0.005892	0.004785	2.88×10^{-5}	1.027×10^{-4}
0.2	0.018214	0.020465	0.018901	6.87×10^{-4}	1.564×10^{-3}
0.3	0.039568	0.037475	0.033013	6.555×10^{-3}	4.402×10^{-3}
0.4	0.064584	0.078865	0.056862	7.222×10^{-3}	2.2003×10^{-3}
0.5	0.107592	0.120033	0.115602	8.01×10^{-3}	4.4431×10^{-3}
0.6	0.161899	0.159353	0.166729	4.83×10^{-3}	7.376×10^{-3}
0.7	0.212809	0.221055	0.216929	4.1214×10^{-3}	2.362×10^{-2}
0.8	0.250814	0.279245	0.255531	2.4716×10^{-2}	2.3715×10^{-2}
0.9	0.308725	0.328348	0.292022	1.6703×10^{-2}	3.6326×10^{-2}
1.0	0.384329	0.398478	0.353368	3.0961×10^{-2}	1.35202×10^{-1}

Tables 1, 2, 3 and 4 shows that the proposed schemes approximate the solutions of initial value problems given in Examples 1 and 2 as the absolute errors are convergent. Also, the absolute errors presented in Tables 2 and 4 show that the proposed schemes compared favourably with the method of Okedayo *et al.* (2015) applied to the given numerical examples.

CONCLUSION

In this research work, a class of three new block schemes for the approximation of initial value problems of first order ordinary differential equations using Hermite polynomial as a basis function has been obtained. The proposed method was used to solve numerically some initial value problems and the results compared with the exact solutions and the method of Okedayo *et al.* (2015). From the numerical results, it is observed that the new schemes were capable for solving first order IVPs as generated results compared favorably with the existing method and the exact solutions. The method is very simple to implement.

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