



## MAGNETOHYDRODYNAMICS FLOW OF A FREE CONVECTIVE DUSTY FLUID BETWEEN TWO PARALLEL OSCILLATING VERTICAL POROUS PLATES WITH DISSIPATIVE EFFECTS

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#### ABSTRACT

This research investigates the unsteady Magnetohydrodynamics (MHD) flow of a free convective dusty fluid between two parallel oscillating vertical porous plates with dissipative effects. The dusty fluid is thought to be moving along the infinite plates as a result of an outside magnetic force. The temperature, velocity and pressure of the particles flowing along the channel are kept constant. The non-linear PDEs that govern the flow of the fluid are solved numerically using finite difference method and Maple 23 is used to implement and simulate the obtained scheme. The result are analyzed and presented inform of graphs. It is established that the effect of Reynolds number, viscoelastic parameter and Prandtl number on the velocity and temperature distribution are subdued by magnetic field.

Keywords: Magnetic field, Dusty fluid, Finite difference method, Convective flow, Maple 23

## INTRODUCTION

Magnetohydrodynamics (MHD) is a captivating and intricate field that combines principles of magnetism and fluid dynamics to explore how electrically conductive fluids behave under the influence of magnetic fields. This interdisciplinary area is crucial for understanding phenomena in astrophysics, plasma physics, and various industrial processes. At its core, MHD focuses on how magnetic fields affect the movement of electrically conductive fluids, and vice versa. The fundamental equations governing MHD include the Navier-Stokes equations (which describe fluid motion), Maxwell's equations (describing electromagnetism), and Ohm's law applied to moving mediums. These equations are interconnected and nonlinear, presenting challenges but also opportunities for insightful analysis and numerical exploration. MHD plays a pivot role in oil and gas production and many other fields like astrophysics.

Similarly, MHD helps explain the dynamics of the solar wind and its interaction with the planetary magnetosphere. The Earth's magnetic field, generated by the motion of molten iron in its outer core, is a result of MHD processes. This geodynamo effect is critical for maintaining the magnetic shield that protects Earth from harmful cosmic radiation. Additionally, MHD provides explanations for phenomena observed in the atmospheres and interiors of other celestial bodies in our solar system. In industry, MHD finds applications in metallurgy and materials processing, where precise control of molten metal flow is essential for producing high-quality materials.

The effects of radiation absorption on MHD Jeffrey fluid flow over a vertical plate through a porous material were investigated by Mopuri et al.(2021). The models for the temperature, concentration, and velocity distributions were obtained by deriving and solving the equations guiding the fluid flow using the perturbation approach. The effects of mass transfer on MHD oscillatory flow for Carreau fluid through an inclined porous channel under the impact of temperature and concentration at a slant angle on the flow center with the effect of gravity were examined by Dheia (2020) using a mathematical model. He examined the acquired graphs and explained the implications of various parameters that are useful for fluid movement.

Bang et al. (2022) examined the combined effects of radiation and chemical reaction on the MHD free convection flow across inclined plates encased in a porous medium of an electrically conductive incompressible viscous fluid. They consider the spontaneously initiated plate with changing mass diffusion and derived its equations, which the perturbation approach is used to solve analytically. The unstable heat transfer and incompressible MHD Poiseuille flow of a viscous liquid across a porous channel were investigated by Sreedhara et al.(2022). They believed that the fluid, whose viscosity varies with temperature, is influenced by an oscillating pressure gradient and confined by two horizontal plates.

Mustafa et al. (2021) studied the transient convection MHD flow between two infinite parallel plates with porous medium. They applied a periodic magnetic field that was perpendicular to the fluid while it was under a continuous pressurized gradient. After obtaining the dimensionless governing momentum and energy equations, a finite difference technique is used to solve them numerically. Unsteady MHD flow between two non-conducting infinite vertical plates in the presence of a uniformly inclined magnetic field was examined by Mrinmoy et al. in 2023. In their research, one of the plates is thought to be moving at a constant speed, while the other plate is thought to be adiabatic.

Under the influence of a uniform magnetic field applied normal to the surface, Prusty et al. (2020) investigated MHD free convective, dissipative boundary layer flow past a vertical surface embedded in a porous matrix with conjugate effect of thermophoresis and heat source in the presence of thermal radiation, chemical reaction, and constant suction. Together with their studies, they can demonstrate the solutal and thermal buoyancy effects. The free convective unstable fluctuating, MHD flow of an electrically conducting viscoelastic dusty fluid in a channel regulated by the upper plate's motion and an oscillating pressure gradient was examined by Farhad et al. (2020). Using the assumed periodic solutions, they reduced the coupled governing partial differential equations for the fluid and dust particle to an ordinary differential equation, which they eventually solved using the Poincare-Light Hill Perturbation Technique.

The effects of radiation on heat transfer and magnetohydrodynamic free convection flow through an extremely porous plate were examined by Nagaraju et al.

(2021). In their research, they verified and analyzed numerous parameter reactions on velocity, temperature, and concentration using graphs. Vidhya et al. (2020) investigated the effects of radiation and a heat source on unstable natural convective MHD flow passing through a vertical porous plate when a uniform magnetic field was supplied perpendicular to the flow direction. Sakthikala et al. (2020) examined the effects of suction and injection through a vertical channel with non-uniform wall temperature during MHD flow of secondgrade fluid through a vertical porous medium in a magnetic field. Using the regular perturbation approach, the governing equations under flow parameters on velocity, temperature, and concentration profiles are solved. The results are graphically displayed.

Linah et al. (2023) explored how mass and heats are transferred in (MHD) flow. Their focus is on understanding the flow over an inclined plate positioned near a semi-infinite porous plate. To tackle this problem, they combine the laws of electromagnetism with the Navier-Stokes equations to develop a comprehensive MHD flow model. Under the influence of a uniform transverse magnetic field, Ugwu et al. (2022) investigated the theoretical analysis of steady (MHD) free convective and mass transfer flow past an infinite vertical porous plate. They employ the Method of Lines (MOL) to solve non-dimensional coupled partial differential equations using numbers. The flow's velocity expression, temperature, and concentration profiles were primarily shown and discussed.

The MHD free convection and heat transfer fluid flow through a semi-infinite vertical porous plate with the impacts of chemical reaction were studied by Biswas et al. (2018). Following the presentation of the governing equations of the flow in terms of a system of partial differential equations (PDEs), the explicit finite difference method (EFDM) is used to solve the problems. Finally, using the graphics program tecplot-9, the acquired data are plotted and explained following the stability convergence test (SCT).

This work examines the unsteady MHD flow of a free convective dusty fluid between two parallel plates with viscous and magnetic dissipation, motivated by a few chosen research studies. The method of solving the associated system of partial differential equations has been selected to be the finite difference method. The generated systems of equations were implemented on MAPLE 23 in order to compute the necessary results for the impact of various parameters on the temperature and velocity profile.

# MATERIALS AND METHODS

### Formulation of the problem

It is assumed that the dusty fluid is flowing along the infinite plates in an upright orientation along the x-axis with the y-axis taken normal to the plate due to the external magnetic force. When t<0, the fluid's thermal state (T) and that of the plates are equal. The temperature outright increased or reduced to  $T_w$  while the plates started from rest advances in its own plane with uniform velocity  $U_0$  at t > 0. With the exception of the influence of density change with temperature, which has only been taken into account in the external force component, the fluid properties in this case are assumed to be adiabatic. The buoyancy force acting on the dust particles is ignored. Following Saffman's (1962) work on a dusty fluid, Marble (1963) provided the following governing equations for two-dimensional incompressible flow:

Continuity Equation  $\nabla \cdot \bar{u} = 0,$  (1) Momentum Equation

$$\left[\frac{\partial \bar{u}}{\partial t} + (\nabla \cdot \bar{u})\bar{u}\right] = -\nabla P_{\rho} + \nabla(\mu \nabla \cdot \bar{u}) + \bar{F}_{p} + J \times \beta_{0} - \frac{\mu}{\kappa_{1}}\bar{u}$$
(2)

$$\rho c_p \left[ \frac{\partial \bar{T}}{\partial t} + (\nabla \cdot \bar{u}) \bar{T} \right] = K_2 \nabla^2 \bar{T} + Q_p + \varphi_{VD} + \varphi_{MD} \quad (3)$$
$$\nabla \cdot \bar{u}_p = 0. \quad (4)$$

$$\rho_n \left[ \frac{\overline{u}_p}{\overline{u}_p} + \left( \nabla \cdot \overline{u}_n \right) \overline{u}_n \right] = -\nabla P_n - \overline{F}_n \tag{5}$$

$$\rho_p c_p \left[ \frac{\partial \bar{T}_p}{\partial t} + \left( \nabla \cdot \bar{u}_p \right) \bar{T}_p \right] = \bar{u}_s - Q_p \tag{6}$$

The viscosity and volume fraction of the solid particle pseudo-fluid are ignored.

- *ū*, *T̄*, *P* and ρ denote the velocity, temperature, pressure, and density of the fluid, respectively, while subscript "p" indicates corresponding properties of the particle phase.
- μ and c<sub>f</sub> denotes viscosity and specific heat of fluid while c<sub>p</sub> is the specific heat of particles.
- J represent current density,
- $\beta_0$  is a uniform magnetic field,
- *K*<sub>1</sub> and *K*<sub>2</sub> are the permeability and thermal conductivity of the fluid respectively,
- $\overline{F}_p$  is an absolute fluid particles interaction force per unit volume,
- $\varphi_{VD}$  and  $\varphi_{MD}$  are viscosity and magnetic dissipation respectively,
- Q<sub>p</sub> is total thermal interaction between the fluid and the particle phase per unit volume,
- *u<sub>p</sub>* is velocity of dust particles and *P<sub>p</sub>* is the mass of the dusty particles per unit volume of the fluid.

When the particle's relative velocity is used to calculate the Reynolds number and it is less than one, Stokes' law describes the force that accelerates the particle to the fluid speed, where  $K = 6\pi\mu a$  is a stokes' constant and a is radius of the dust particle. Presuming that N represents the particle number density  $\bar{F}_p = 6\pi N\mu a (\bar{u}_p - \bar{u}) = \rho_p \frac{(\bar{u}_p - \bar{u})}{\tau_m}$  is the expression for the total contact force per unit volume. The relaxation time, denoted by  $\tau_m = \frac{m}{6\pi\mu a}$ , is the amount of time that the particle phase's velocity decreases to  $(\frac{1}{e})$  times its original value in relation to the fluid phase and the mass of each particle is denoted by m. Similarly,  $Q_p = \frac{pc_s(T_p - T)}{\tau_\tau}$  gives the overall thermal interaction between the fluid and the particle phase's thermal relaxation period is represented by  $\tau_{\tau} = \frac{mc_s}{4\pi Ka}$  (the particle phase's temperature in relation to the fluid is  $(\frac{1}{e})$  times its original value). Typically, simplifying assumptions are made for diluted suspensions in most studies on dusty fluids. We make following assumptions in this study;

- i. The particle's number density N, or the quantity of dust particles per unit volume of the
- ii. mixture is constant.
- iii. The approximation by Boussinesq is accurate.
- iv. It is assumed that the dust particles are all undeformable, spherical in shape, and have the same mass and radius.
- v. The pressure has the same velocity vector and temperature locally since the solid particles spread sparsely and do not interact.

As a result, the pressure related to the particle cloud is minimal as this assumption eliminates the randomness in local particle motion. Consequently, the mixture's total pressure will equal the fluid pressure, *p*. Since the plate is infinite in the x-direction, the only functions that determine the flow amounts are y and t. Therefore,  $\bar{u} \equiv u(y, t)$ ,  $\bar{u}_p \equiv u_p(y, t)$ ,  $\bar{T} \equiv T(y, t)$ ,  $\bar{T}_p \equiv T_p(y, t)$  and  $\gamma = \gamma_p = 0$ . As a result, the fluid and particle phases' continuity equations are both satisfied. Clearly, the governing equations of motion for the viscous, unsteady, incompressible, dusty, and particle phases hold the same values. So the governing equations (1-6) become;

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \rho g + k N (u_p - u) - \sigma \beta_0^2 u - \frac{\mu}{\kappa_1} u$$
(7)

$$\frac{\partial u_p}{\partial t} = -\frac{K}{m} \left( u_p - u \right) \tag{8}$$

$$\rho c_p \frac{\partial T}{\partial t} = K_2 \frac{\partial^2 T}{\partial y^2} + \frac{\rho c_s (I_p - T)}{\tau_T} + \mu (\frac{\partial u}{\partial y})^2 + \beta_0^2 u^2 \qquad (9)$$
$$\frac{\partial T_p}{\partial t} = \frac{T - T_p}{\tau_T} \qquad (10)$$

u(y,t),  $u_p(y,t)$ , are respectively for the velocities of the fluid and particle, while *P* represent pressure. From equation (7), we get;

$$-\left(\frac{\partial P}{\partial x} + \rho_{\infty} g\right) = 0 \tag{11}$$

Taking away  $-\frac{\partial P}{\partial x}$  from equation (7) and (11) we deduced,  $\rho \frac{\partial u}{\partial x} = u \frac{\partial^2 u}{\partial x} + \rho (\rho_{\infty} - \rho) + k N_0 (u_{\infty} - u) - \sigma \beta_0^2 u - \frac{\mu}{2} u$ 

$$\rho \frac{\partial}{\partial t} = \mu \frac{\partial}{\partial y^2} + g(\rho_{\infty} - \rho) + k N_0 (u_p - u) - \sigma \beta_0^2 u - \frac{1}{\kappa_1}$$
(12)

NOTE:  $\rho_{\infty} = \rho$ , and they are both constant in all terms exceptet the bouyancy term  $g(\rho_{\infty} - \rho)$ . From equation above we get;

$$g\beta\rho_{\infty}(T - T_{\infty}) = g(\rho_{\infty} - \rho)$$
(13)  
By substituting equation (13) into (12), we obtain,  
$$\rho\frac{\partial u}{\partial t} = \mu\frac{\partial^2 u}{\partial y^2} + g\beta\rho_{\infty}(T - T_{\infty}) + kN(u_p - u) - \sigma\beta_0^2 u - \frac{\mu}{K_1}u,$$
(14)

where  $\beta$  is the coefficient of volume expansion and  $\mu$  is the fluid's kinematic viscosity.

The initial and boundary conditions of the problem are given by;

For 
$$t \leq 0$$
;  $u = u_p = 0$ ;  $\theta = \theta_p = 0$  (15)  
 $y = 0$ ;  
 $t > 0$ ;  $\begin{cases} u = u_0 + \varepsilon u_0 e^{i\omega t}; u_p = u_0 + \varepsilon u_0 e^{i\omega t} \\ T = T_w + \varepsilon (T_w + T_\infty) e^{i\omega t}; T_p = T_w + \varepsilon (T_w + T_\infty) e^{i\omega t} \end{cases}$   
 $y = h$ ;  $\{u \to 0; u_p \to 0; T \to T_\infty; T_p \to T_\infty, \}$  (16)  
Considering non-dimensional variables;  
 $\begin{cases} y = \frac{u_0 \bar{y}}{\gamma}; u = \frac{\bar{u}}{u_0}; u_p = \frac{\bar{u}_p}{u_0}; \theta = \frac{T - T_\infty}{T_w - T_\infty}; y = \frac{c_s}{c_p}; t = \frac{\bar{t}}{\gamma} u_0^2; \\ \rho = \frac{\mu c_p}{\kappa}; \alpha = \frac{N_0 m}{\rho}; \lambda = \frac{\tau_p u_0}{\gamma}; \theta_p = \frac{T_p - T_\infty}{T_w - T_\infty}; p = \frac{\mu c_p}{k}; E = \frac{u_0^2}{c_p (T_w - T_\infty)}; \end{cases}$ 

(17) Where *G* is Grashof value, *Ec* is the Eckert value, Pr is the Prandtl value,  $\propto$  is our concentration variable and  $\lambda$  is the nondimentional relaxation time, then, equation (14), (8), (9) and (10) give us;

$$\frac{\partial u}{\partial t} = \frac{1}{\text{Re}} \frac{\partial^2 u}{\partial y^2} + G\theta + \frac{\alpha}{\lambda} (u_p - u) - \frac{\text{Ha}^2 u}{\text{Re}} - \frac{\text{Ku}}{\text{R}}; \quad (18)$$

$$\frac{\partial u_p}{\partial u_p} = -\frac{1}{2} (u_n - u); \quad (19)$$

$$\frac{\partial t}{\partial t} = -\frac{1}{\lambda} (u_p - u); \qquad (19)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} + \frac{2\alpha}{3\lambda\Pr} (\theta_p - \theta) + Ec(\frac{\partial u}{\partial y})^2 + HaEcu^2; (20)$$

$$\frac{\partial \theta_p}{\partial t} = -\frac{2}{3\lambda\gamma} (\theta_p - \theta); \qquad (21)$$
$$t < 0; \ \mu = \mu_p = 0; \ \theta = \theta_p = 1;$$

$$t \le 0, \ u = u_p = 0, \ v = v_p = 1,$$
  
 $y = 0; \ t > 0;$ 

$$\begin{split} u &= 1 + \varepsilon e^{i\omega t}; \quad u_p = 1 + \varepsilon e^{i\omega t}; \quad \theta = 1 + \varepsilon e^{i\omega t}; \\ \theta_p &= 1 + \varepsilon e^{i\omega t}; \end{split}$$

$$y \to \infty; \ u \to 0; \ u_p \to 0; \ \theta \to 0; \ \theta_p \to 0$$
 (22)

By adopting finite different methods FDMs, we solve equation (18), (19), (20) and (21). By replacing the available coefficients in the equations with their finite difference quotients, the following finite difference equations are produced;

$$\binom{(u_{i,j+1}-u_{i,j})}{\Delta t} = \frac{1}{\text{Re}} \left( \frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{(\Delta y)^2} \right) + G\theta_{i,j} + \frac{\alpha}{\lambda} \left( u_{p(i,j)} - u_{p(i,j)} \right) - \frac{\text{Ha}^2 u_{(i,j)}}{\text{Re}} - \frac{\text{Ku}_{(i,j)}}{\text{Re}}$$
(23)  
$$\binom{(u_{p(i,j+1)}-u_{p(i,j)})}{(u_{p(i,j+1)}-u_{p(i,j)})} = \frac{1}{1} \left( u_{p(i,j)} - u_{p(i,j)} \right)$$
(24)

$$\frac{1}{\Delta t} = -\frac{1}{\lambda} \left( u_{p(i,j)} - u_{(i,j)} \right)$$
(24)  
$$n \left( \frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} \right) = \frac{1}{\Pr} \left( \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta y)^2} \right) + \frac{2\alpha}{3\lambda Pr} \left( \theta_{p(i,j)} - \theta_{(i,j)} \right) + Ec \left( \frac{u_{i,j+1} - u_{i,j}}{\Delta} \right)^2 + HaEc u_{(i,j)}^2$$
(25)

$$\left(\frac{\theta_{p(i,j+1)}-\theta_{p(i,j)}}{\Delta t}\right) = -\frac{2}{3\lambda\gamma} \left(\theta_{p(i,j)}-\theta_{(i,j)}\right)$$
(26)

In the equation (23), (24), (25) and (26) the subscript *i* denotes *y* and *j* denotes *t*. Taken  $\Delta y$  to be 0.1.

From initial conditions;

$$\begin{cases} u(0,0) = 0, \ u_p(0,0) = 0; \\ \theta(0,0) = 0, \ \theta_p(0,0) = 0 \end{cases},$$
(27)  
For all *i* except *i* = 0,  

$$(u(i,0) = 1, \ u_p(i,0) = 1; )$$

Based on the boundary circumstances,  $(u(0, i) = 1 + se^{i\omega j}, u_i(0, i) = 1 + se^{i\omega j})$ 

$$\begin{cases} u(0,j) = 1 + \varepsilon e^{i\omega j}, \ u_p(0,j) = 1 + \varepsilon e^{i\omega j}; \\ \theta(0,j) = 1 + \varepsilon e^{i\omega j}, \ \theta_p(0,j) = 1 + \varepsilon e^{i\omega j} \end{cases}, \text{ for all } j.$$

$$(29)$$

By analytical solutions of (18) for  $\propto = 0$  and taking y = 5 as the point of  $y = \infty$  as all of  $u, u_p, \theta, \theta_p$  approach zero (0) at  $y \sim 5$  for all values of P and G. So, we take

$$u(5,j) = 0, u_p(5,j) = 0, \theta(5,j) = 0, \theta_p(5,j) = 0, (30)$$

This is correct for every j and align with the boundary condition. In terms of velocities and temperatures, the velocities at the end of the time step, ,  $u_{i,j+1}$  and  $u_{p(i,j+1)}$  , i= 1 to 5, are iterated from (22) and (24). Likewise,  $\theta_{i,j+1}$  and  $\theta_{p(i,i+1)}$  are iterated from (25) and (26). This is continuously done until t = 1, (j = 100). These iterations were carried out and the graphs of u and  $u_p$  against y and  $\theta$  and  $\theta_p$  against ywere generated using MAPLE-23 package. The numerical results acquired for  $\propto = 0$  is compared with the results acquired by the previous researchers and there is no contradiction. Also, for the accuracy of these results, there is comparison between results acquired and the analytic solution at t = 0.2, 0.3, 0.5. For every range of y values the absolute agreement was established with greatest error not up to one percent. The convergence was also established for minute values of  $\Delta t;$ namely,  $\Delta t =$ 0.001, 0.002, 0.003, 0.004, 0.005. The difference in the results is trivial. It has been established by previous researchers that G < 0 corresponds to plate gaining heat and G > 0 corresponds to plate losing heat.

# **RESULTS AND DISCUSSION**

The effect of the relevant non-dimensional parameter on the velocity and temperature profile at extremely high and low constant values of magnetic field (Ha) were given and analyzed with the help of a graph as a result of iterating the preceding numerical approach using Maple 23.



Figure 1: Effects of the concentration parameter ( $\alpha$ ) on the graphs of velocity and temperature against time for Pr = 0.7,  $\gamma = 0.6$ , K = 0.2, Ha = 2, Gr = 5,  $\lambda = 2$ , RO = 2, Ec = 0.001, R = 0.1,  $\varepsilon = 0.01$ ,  $\omega = 1$ 

Figure 1, confirmed that as the concentration parameter ( $\alpha$ ) increases, the velocity and the temperature of the flow decreases this is because the concentration of particles of the fluid has direct impact on viscosity of the fluid which can be determined by multiple factors that are not limited to particle

size, chemical reaction and interactions. The reduction in temperature of the plate increases the chemical bond of the flow and this directly reduces the velocity of the flow.



Figure 2: Effects of the Hartmann number (*Ha*) on the graphs of velocity and temperature against time for Pr = 0.7,  $\gamma = 0.6$ , K = 0.2, Gr = 2,  $\lambda = 2$ ,  $\alpha = 0.1$ , RO = 2, Ec = 0.001, R = 0.1,  $\varepsilon = 0.01$ ,  $\omega = 1$ 

Figure 2, shows that the increase in Hartmann number (Ha) brings about a decrease in the velocity of the flow due to its magnetic effect. For channel flow between two parallel plates with an external magnetic field perpendicular to the channel walls, the velocity of the (MHD) flow in the presence of an external magnetic field reduces.



Figure 3: Effects of the Grashof number (*Gr*) on the graphs of velocity and temperature against time for Pr = 0.7,  $\gamma = 0.6$ , K = 0.2, Ha = 2,  $\lambda = 2$ ,  $\alpha = 0.1$ , RO = 2, Ec = 0.001, R = 0.1,  $\varepsilon = 0.01$ ,  $\omega = 1$ 

Figure 3, confirms that the increase in the value of Grashof number (Gr) implies the increase in the wall temperature and this makes the bond(s) between the fluid to become weaker which make the strength of the internal friction to decrease, then the gravity to becomes stronger enough to speed-up the velocity of the flows.



Figure 4: Effects of the nondimentional relaxation time ( $\lambda$ ) on the graphs of velocity and temperature against time for Pr = 0.7,  $\gamma = 0.6$ , K = 0.2, Ha = 2, Gr = 5,  $\alpha = 0.1$ , R = 2, Ec = 0.001, R = 0.1,  $\varepsilon = 0.01$ ,  $\omega = 1$ 

Figure 4, verifies that as nondimentional relaxation time ( $\lambda$ ) increases the temperature and velocity of the particles decreases because the thermal effect of the plate drastically decreases with time and contributes to the reduction in velocity of the flow.



Figure 5: Effects of the Stoke's constant value (*K*) on the graphs of velocity and temperature against time for Pr = 0.7,  $\gamma = 0.6$ , Ha = 2, Gr = 5,  $\lambda = 2$ , a = 2, RO = 2, Ec = 0.001, R = 0.1,  $\varepsilon = 0.01$ ,  $\omega = 1$ 

Figure 5, shows that as the Stoke's constant value (K) is increasing the velocity of the flow decreases, this validate the physical property Stoke's constant as drag force.



(c)

(d) Figure 6: Effects of the Prandtl (Pr) on the graphs of velocity and temperature against time for  $\gamma = 0.6, K = 0.2, Ha =$ 2, Gr = 5,  $\lambda = 2$ ,  $\alpha = 2$ , RO = 2, Ec = 0.001, R = 0.1,  $\varepsilon = 0.01$ ,  $\omega = 1$ 

Figure 6, show that the velocity and temperature of the flow decreases as the Prandtl (Pr) value increases, as a dimensionless number that provides a measure of the efficiency of transport by momentum diffusivity to thermal diffusion. Heat diffuses very quickly in liquid metals ( $Pr \leq$ 1) and very slowly in oils ( $Pr \ge 1$ ) relative to momentum.

## CONCLUSION

The unsteady MHD flow of a free convective dusty flow between two parallel plates with viscous and magnetic dissipation has effect on the flow of the fluid.

- On the basis of obtained results; the following are observed. i. The velocity and the temperature of the flow decreases
  - as concentration parameter increases. The increase in Hartmann number brings about a ii. decrease in the velocity of the flow due to its magnetic effect.

The increase in the value of Grashoff number implies iii. the increase in the velocity of the flow.

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