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PERT MATHEMATICAL MODEL DEVELOPMENT FOR THE SUPPLY OF DOMESTIC WELL-WATER TREATMENT PLANT

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ABSTRACT

In Nigeria most cities are water stressed, none of the cities are having regular water supply. Quality water for human consumption is a primary duty of the government but they have failed. Also, current cost of boreholes installation has increased beyond their capacity due to Naira devaluation in the world market. Water resources management requires constant monitoring in terms of its qualitative-quantitative values. This study presents potential impact of the Project Evaluation and Review Techniques (PERT) mathematical model software development for the supply of domestic well-water treatment plant. After testing software against the values then, the 37days calculated Expected Time (E_T) is unreliable because the risk involved in this project is (100 – 34.23) % = 65.77 %. However, a new completing date can be predicted by PERT mathematical model for supply of domestic well-water treatment plant by varying probability of completion to predict new date of completing the plant. Therefore, concerning this research the Initial probability was moved from 34.23% to 90%. By interpolation with 37 days; the new acceptable date was 97.28 days. The risk involved now (100 – 90) % reduced to 10%, a very good comfortable zone for the project. This information has really helped in two ways: for easy computation and decision making in water supply scheme project. This result is useful in industrialization, individuals, housing estate, small and medium scale industries where quality domestic well-water for crops production is a necessity. Hence, it is recommended that, the developed mathematical models performed satisfactorily upon validation with real life data while human and technical risk factors were established critical area to the safety management in manufacturing industry.

Keywords*:* Developed, Mathematical Model, Well-Water, Water Supply, Treatment Plant.

INTRODUCTION

Water is very important source of life. Drinking water supply is one of the prime services to be provided by government in any nation. Now days, A paramount issue is water i.e., its availability, quality and management. Because there is a supersonic growth in wide urban residential areas, as a result there is insufficient water supply. Thus, there is a need to provide better water management system (Wright, 2012). To satisfy the customer's requirement as well as to avoid faulty conditions there must be a better water supply management. Water supply management can be defined as "Dealing with water in the best possible way". Problem arises in the water supply management system due to improper water supply management and they will have to pay an equal amount irrespective of usage of water by individual houses (Amirtharajah and O' Melia, 2014).

Water is one of the most common substances known. It is a good solvent for many substances and rarely occurs in its pure form in nature. Natural water includes rain water, spring water, lake water, river water, well-water and sea water (Garg, 2019). Rain water is the purest form of natural water because it is formed as a result of the condensation of water vapour in the atmosphere. Spring water contains a considerable amount of mineral salts but very little suspended impurities such as dust, sand and dirt, so it is a good source of drinking water when treated. Some well-water contains a lot of clay and other mineral salts. One of the tools that are used to solve problems of surface water pollution is modeling of changes which take place in lake waters and associated water quality changes. Such a model can be used to predict water quality, taking into account changes that affect water quality factors or changes in their intensity. The accuracy of environmental assessment using models depends on the understanding of processes which occur in the environment and on appropriate choice of mathematical equations that describe them. On the other hand, it depends on available data sets, namely results of the measurement of water quality indicators, on which the estimation of parameters and coefficients of the model is based (Gromiec, 2018).

A well that is used as a source of drinking water should be sited away from sources of underground water pollution such as pit latrine and lined with bricks and covered. Water from deep wells tends to be less polluted than that from surface wells (Syed *et. al.,* 2015). Generally, it is safer to boil well-water before drinking. River water, lake water and sea water contain a lot of dissolved air, mineral salts, bacteria and organic remains. These waters have to be specifically purified before they can be used for drinking (Cleasby, 2013). Water source means either surface water includes streams, rivers, lakes or reservoirs and underground water such as springs, well or boreholes.

In recent years, computer simulation methods are becoming popular in scientific research, particularly as far as the research on the aquatic environment quality is concerned. Methods of computer analysis have become an independent branch of research, which substantially increases the research capabilities of modern eco-engineers (Holnicki et al., 2010).

Classification of Mathematical Models

The most general division of software modules used to model the processes occurringin the environment includes (Holnicki et al., 2010):

a) Physical models (laboratory); and

b) Mathematical models, including: Analytical models - based on exact solutions of the equations of mathematical physics, and Numerical models - using approximate solutions.

Depending on the complexity of computer simulation models of surface water quality, they can be divided into three groups:

One-Dimensional Models (1D), the simplest and most commonly used models in the analysis of river water quality, which assumes that significant changes in the parameters determining water quality occur only along the longitudinal profile of the water course. One-dimensional inflow-outflow models are based on measuring the concentration loads of water parameters flowing in and out. On this basis changes in the concentrations of all parameters are calculated. One-dimensional models are not designed to calculate the variation of concentration at the time, so one cannot get information on the specific hourly, daily and monthly water quality parameters. These programs do not describe the complex chemical, physical and biological reactions in water reservoirs which are an essential factor regulating changes in the water quality parameters. An advantage of these programs is that they can be applied quickly to any other water reservoir without pre-calibration and with small available database of measurements (Balcerzak, 2012).

Two-Dimensional Models (2D), which assume that significant changes in water quality

occur not only along but also in the longitudinal profile of the watercourse, and therefore it is necessary to analyse water quality at various depths. Two-dimensional models are used most often in the case of lakes, reservoirs or deep rivers and require more data and more analytical experience of the user than the one-dimensional models. They require careful calibration and are sensitive to changes of many parameters of water quality. The end result of these programs is a forecast of water quality parameters close to measurements of actual concentrations. Assessment of individual parameters can be performed for given time intervals, ie hour, day, week, month and year (Balcerzak, 2012).

Three-Dimensional Models (3D), which examine the spatial distribution of concentrations of simulated water-quality parameters. Three-dimensional models are used to simulate changes in water quality in the sea bays, lakes, dams and deep rivers; they require huge amounts of data and extensive analytical experience from the user. They are most rarely used because of high complexity of the analyzed issues. As with other areas of the environment, computer modeling of surface water quality issues requires a multidisciplinary approach in which mathematical methods and modern computer technologies are applied simultaneously with the basic techniques in the field of environmental engineering. This applies to the description of physical phenomena associated with the formation of pollutants, their distribution and the changes which take place in the aquatic environment, as well as the application of computational methods and models for hazard analysis or control of water quality in specific cases (Holnicki et al., 2010). From the standpoint of applications, the ultimate goal of the model is to use it as a supportive tool in monitoring and forecasting surface water quality, as well as while taking planning decisions.

Due to the extent and spatial scale, several categories of models are considered:

a) **Operational Models (OM)-** typically related to short-term forecasting and used to real-time control of water reservoirs or flow rate in order to maintain established parameters. Such models require automatic entering of current input data.

b) **Tactical Models(TM)-** associated with the use of operational decision-making in taking tactical actions in which relationships of the "input-output" type between the key parameters of the system are vital. The time horizon of this type of analysis covers a period of couple of days, weeks or even a season in the case of control of water quality in a river. In terms of tools used in the tactical model, steady-state analysis and quantitative economic and environmental instruments are used most frequently.

c) **Strategic Models (SM)-** refer to a longer time horizon, where one can analyze, predict, or plan the state of the environment as a projection of the current state, taking into account all the relevant trends. This type of modeling is based on an analysis of the results of computer simulation of various scenarios and analysis of its efficiency; and

d) **Directional Models (DM)**- concern long-term forecasts related to structural changes, testing the possibility of sustainable development and the evolution of the whole system. Properly chosen calculation programs are the tool that is used to model changes in the aquatic environment. They simulate the behaviour of the aquatic environment and the way its parameters change. These parameters include water temperature, wind strength and direction, the concentration of dissolved oxygen, salinity of water, the amount of biogenic compounds and other parameters depending on the complexity of a program (Bielak, 2011).The world's first surface water quality model was created in 1925 in the United States of America, by Streeter and Phelps in order to layout the location of drainage sewage outfalls on the Ohio River (Chapra, 2017). It was a one-dimensional model, based on the balance of oxygen and simple linear equations. Although since then more than 80 years have passed, basic approach to modeling of surface water quality has not changed, since the construction of all models is founded on three basic principles, ie the conservation of mass, momentum and energy. Ordinary and partial differential equations are used to describe the dynamics of phenomena occurring in the surface waters associated with the spread of various pollutants. For each physical quantity (in the considered control volume and time interval) the general law of conservation is as follows (Holnicki et al., 2010).:

Inflow outflow + source of change = accumulation

METHODOLOGY

Materials and Methods

The mathematical model was developed for domestic well-water treatment plant was evaluated using data obtain from a building site at Road Q Plot 17, Oba Ile Housing Estate, Akure North Local Government Area, Ondo State, Nigeria. This house is a storey building of 23 occupants of four flats of 3-bedrooms and one Boy's Quarters (BQ).

Development of Mathematical Model Equations for Water Requirements

(1) Bathing (Shower)

Assuming there are an adult and C children in the building. An adult is assumed to use 8 litres per minute for 5 minutes in bathing and 8 liters per minute for 4 minutes for a child and takes place twice in a day (Deolalikar, 2004).

Hence;

The quantity of water used in bathing
$= 8 \times 5 \times 2 \times A - 8 \times 4 \times 2 \times C$
$= 80A - 64C$
$= 16\ (5A-4C)$ (1)

(2) Flushing (Toilet)

Each occupant is assumed to use 10 litres per flushing (Deolalikar, 2004 and Innocence *et al.*, 2009) with a water closet (flush valve) and that each person in the residence goes to toilet three times in a day. Therefore, the quantity of water use in flushing the toilet is given as:

The quantity of water used in flushing $= 10 \times 3 \times A - 10 \times 3 \times C$

$= 30(A - C)$ (2)

(3) Washing of Clothes

Let X and Y be the least number of clothes that an adult and a child can wear in a day respectively, if 5 and 4 litres of water will wash a cloth for an adult and child respectively (Innocence *et al.*, 2009). The quantity of water for washing of clothes.

$= 5 \times A \times X - 4 \times C \times Y$

$= 5AX - 4CY$ (3)

(4) Cooking

If an adult eats twice a day with 4.5 litres of water per meal, and children thrice with 4litres meal (Innocence *et al.*, 2009).

Hence,

Quantity of Water for Cooking $= (9/2) \times 2 \times A - 4 \times 3 \times C$

$= 9A - 12C$

$= 3(3A-4C)$ (4)

(5) Drinking

If an adult drinks 4 litres in a day and children 3.5 litres per day. (Innocence *et al.*, 2009), then, Quantity of water fordrinking $= 4A + 3.5C$ (5)

(6) Washing of Plates and Utensils

Suppose it takes 3.75 (15/4) litres of water to wash a set of plate per meal.

Each cooking activities uses 5 litres thrice per day to wash utensils. (Deolalikar, 2004)

= (3 x 5/4) x 2 x A – (3 x 5/4) x 3 x C – 15

= (15/4) x 2A – (15/4) x 3 x C – 15

= 15/2A – 45/4C – 15

= 7.5A – 11.25C – 15 (6)

(7) Sanitation/Hygiene

Sanitation/Hygiene requirement differ by technology used for health benefits it is up to 20 l/c/d (Innocence et al., 2009) and it can be computed as:

Sanitation/Hygiene = 10(A + C) (7)

Total Demanded

Hence, the total quantity of water demanded is given as the addition of quantity of water for each of the activities. (That is, Bath + Toilet + Clothes + Cooking + Drinking + Washing Plates + Sanitation/Hygiene).

Calculation of Residential Water Requirements

The ultimate focused of this research study is mainly on basic residential water requirements, among the various types of buildings came across in the course of the study, a standard family home is considered and established as indices for developing model for other type of buildings. For this reason, let consider a storey building of 23 occupants in four flats of 3-bedrooms and one Boy's Quarters (BQ) to be used as a prototype for mathematical model equations with the following information. Some relevant information used in this report were gotten from a house that makes use of a small scale domestic well-water treatment plant which has a capacity of 5 m^3 located at Road Q, Plot 17, Oba Ile Housing Estate, Akure North LGA, Ondo State, Nigeria.

(1) number of adult in the building A = 4
(2) number of children in the building C = 4
(3) the least number of clothes an adult can wash in the residence per day X = 5
(4) the least number of clothes children can wash in the residence per day Y = 5
(5) adult and children goes to toilet three times per day;
(6) adult and children takes their bath twice per day;
(7) there is at least an open well installed with surface or well submersible water pump;
(8) a well stationed overhead tank with a minimum water capacity 4000 liters, and
(9) There is at least moderate power to the building to power pumping machine.

Table 1: Calculation of Quantity of Water Demand

S/N	Author(s) and Year	Types of Demand/ Activities	Mathematical Model Equations	Quantity Demanded for each Activities Litres/day	Gaps Identified
1.	Deolalikar, 2004	Bathing (showers)	80A + 64C 80(4) + 64(4)	576	The model developed depending on the technology associated with a house hold act.
2.	Deolalikar, 2004 and Innocence *et al.*, 2009	Toilet	30 (A + C) 30 (4 + 4)	240	The model was developed for only water used for toilet and domestic uses
3.	Innocence *et al.*, 2009	Washing (Clothes)	5.A x 4.C.Y 5.4.5 + 4.4.5	180	The model developed did not considered that user with pipe collection may consumed more per cloth than a household
4.	Innocence *et al.*, 2009	Cooking	9A + 13C 9.4 + 12.4	84	The model developed for food preparation
5.	Innocence *et al.*, 2009	Drinking	4A + 7/2C 4.4 + 7/2/4	30	The model developed depend on the environmental condition
6.	Deolalikar, 2004	Washing (Plate and Utensils)	15/2A + 45/4C + 15 15/2.4 + 45/4C+15	90	The model only considered the average daily requirement of water for bathing , cooking and washing
7.	Innocence *et al.*, 2009	Sanitation/Hygien e	10 (A + C) 10 (4 + 4)	80	The model is suitable for sanitation purposes
8.		Total Qty		1280	
		Dem. Unit / Person/day	Total / (A + C) = Total / (4 + 4)	1280/8 = 160	

It was observed from the above Table 2.2 that the quantity of water used for various activities were calculated through the developed mathematical model equation from Equations 2.2 to 2.8 to arrive at the total quantity of water needed. Therefore, dividing the total quantity of water needed by the total number of person in the building gives the daily water requirement per person from the above data supplied, the daily water requirement per person is 160 (160litres per person per day).

Equation Generation

A regression analysis can be used in order to: (i) characterize the relationship between the dependent and independent variables by determining the extent, direction, and strength of the association, (ii) seek a quantitative formula or equation to describe the dependent variable as a function of the independent variable(s), (iii) describe quantitatively or qualitatively the relationship between independent and dependent variables but control for the effects of other variables, (iv) determine which of several independent variables are important and which are not for predicting a dependent variable, (v) determine the best mathematical model for describing the relationship between a dependent variable and one or more independent variables, (vi) assess the interactive effects of two or more independent variables with regard to a dependent variable, (vii) compare several derived regression relationships, and (viii) obtain a valid and precise estimate of one or more regression coefficients from a larger set of regression coefficients in a given model (Rawling,2005). In this research, regression analysis has been used to seek a quantitative equation to describe the costs of treatment plant (dependent variable) as a function of treatment capacity and other parameters like area, feed capacity, etc. (independent variables). A number of set of observations (estimates in case of this research) can be plotted on a graph to get a scatter diagram. Basic questions to be dealt with in regression analysis are: (i) what is the most appropriate mathematical model to use – a straight line, a parabola, a log function, a power function, or what?, and (ii) how to determine the best-fitting model for the data? (Rawling, 2005). Common strategies to tackle first problem are: (i) forward method – begins with simply structured model and adds more complexity in successive steps, if necessary, (ii) backward method – begins with a complicated model and successively simplifies it, and (iii) model suggested from experience or theory. Two methods to solve second question are: (i) least-squares method, and (ii) minimum-variance method. Both these methods yield same solution (Rawling, 2005).

Order-of-magnitude estimates of projects can be done by making adjustments to cost of similar project, if available, with respect to variables like time, location and size. These adjustments require appropriate cost scale-up and location factors. Cost indexes are used to measure a given project to a basis and typically reference a base year, which is assigned an index value of 100 (Stein, 2002). The oldest cost index currently being used by the engineers is the Engineering News Record

(ENR) index, which started in 1909 (Grogan, 2004). Cost and location indexes available in the United States are listed in Appendix II. Inflation and location indexes use a base year or base location. These indexes can be used to estimate the cost of similar project at different time and/or location. Equations 2.9 and 2.10 can be used to estimate costs (adapted from Remer, (2008)).

Cost2 is Cost1 X (Cost Index2/Cost Index1) (8)

Where:

Cost 2 is estimated cost at time of construction

Cost1 is Actual/estimated historical cost

Inflation Index1 is Inflation index at construction/estimation of historical cost

Inflation Index 2 is Inflation index at time of construction

Cost2 is Cost1 X (Location Index 2/Location Index1) (9)

Where:

Cost2 is estimated cost at location of construction

Cost1 is Actual/estimated cost at location of project of available data

Location Index1 is Location index at location of project of available data

Location Index2 is Location index at location of construction

Gumerman, (2009) recommended two methods to update the construction, and operation and maintenance capital cost to current dollars. The first method was to use a single index (ENR/CCI was recommended). The second method recommended was to use different indexes for eight aggregated cost components. Cost data provided in USEPA reports (Gumerman, 2009) were divided into eight components for construction costs: excavation and site works (A), manufactured equipment (B), concrete (C), steel (D), labour (E), pipes and valves (F), electrical and instrumentation (G), and housing (H), and three components for operation and maintenance cost: energy (includes electricity (I), natural gas (J), and diesel (K)), labour (L), and maintenance material (M). Equation 2.9 gives these cost components and applicable indexes as suggested by Gumerman, (2009) and Qasim, (2002). Equation 2.10 - present worth and annual equivalent worth calculation. Present worth (PW) of annual operation and maintenance capital cost is a minimum sum that must be invested today at a given interest rate to pay for Operation and Maintenance (O & M) capital cost every year throughout the life of the water treatment plant. Equivalent annual cost is uniform series of expenditures at the end of each year that is equivalent to different non-uniform Operation and Maintenance (O & M) capital expenditures made during the life cycle of the treatment plant (Qasim, 2002). Equivalent annual cost is used to calculate cost per unit of water treated. Also, when different alternatives are considered, equivalent annual costs are used to compare and select the most cost effective alternative. The present worth of annual Operation and Maintenance(O & M) capital cost, and equivalent annual costs are obtained from Equations 2.11, and 2.12 respectively.

PW of annual O&M cost = (total annual O&M cost) x CRF–1 (10)

Equivalent annual cost, $/year = project PW x CRF (11)

Where,

O & M is Operation and Maintenance capital cost

PW is Present Worth

CRF is Capital Recovery Factor = i/(1 – (1 + i)–n)

IR is Interest Rate in design period or years; (Rawling, 2005).

Due-Date Prediction of Domestic Well-Water Treatment Plant for Household Community

The domestic well-water treatment plant exposes the feasibilities of using the dynamic method of controlling the terms of investment and construction projects during the underground construction, particularly the principles of ensuring the completion of construction projects to meet the due date, because in many cases project deadline are usually exceeded which leads to project failure. The aims and results of the projects plant installation deals with the relevant issues of project schedule control, methodology study based on application of schedule timeliness index and schedule progress index, indicating upon reaching the critical value of extreme deviations from the forecast project duration of targeted project implementation at any moment of time.

The suggestion herewith, is to calculate the minimum possible durations for each project activity and estimate the project duration safety margin coefficient of the project of underground construction. Based on the safety margin coefficient value it is suggested to define the border values for project shift from one status to another upon criteria of its timely completion possibility. The suggested methodology can be recommended for use by underground construction project managers in order to prevent a potential failure of project completion deadlines. The system indicating critical project time variance enables us to initiate the process of project schedule adjustment before point of no return and herewith prevent project failure. The studied methodology was implemented in modern project management software sphere.

The due-date prediction of domestic well water treatment plant for household community is a period right from the day of negotiation to the day of commission of the project. The following factors need to be put into consideration such as time allocation, capital materials need, proximity to the market, market value and numbers of workers and working forces. Quantity of delivery for example, cost services and reliability. For example, due-date prediction of plastic/bottle water. Besides that, the expiration date on bottled water has certain benefit for the manufacturer. Although water, in and of itself, does not go bad, the plastic bottles in which it is contained in does expire and will eventually start leaching chemicals into the water.

People also ask?

(i) Why do bottles of water have expiration date? – like science
(ii) Can water bottles expire?
(iii) How long is bottled water good if un-opened? and
(iv) Can water spoil?

Network Analysis and Its Terms

Network Analysis: is a generic term for a family of related techniques developed to aid management in the planning and control of project, (Ugwu, 2007).

PERT: The outcome of the team's efforts was the development of the network technique known as PERT (Project Evaluation and Review Technique). PERT was used to plan and control the development of the Polaris missile and was credited with saving two years in the missile's development. Since 1958 the technique has been developed and nowadays many variants exist which handle, in addition to basic time factors, costs, resources, probabilities and combinations of all these factors. A variety of names exist and some of the more commonly used are: Critical Path Planning (CPP), Critical Path Analysis (CPA), Critical Path Scheduling (CPS), Critical Path Method (CPM) and Project Evaluation and Review Technique (PERT) costs.

PERT Mathematical Model Developed for Due-Date Prediction

To determine the expected completion time for these procurement activities, all the time attached to each activity on the paths were added together and compared. The path with the longest time is the expected completion time for the procurement activities using Equations (1), (2), (3), (4) and (5) to determine: Expected Time (ET), Variance (V), Standard Deviation (SD) and Probability of Success (Z) of the date respectively.

The equations used are;

For Expected Time, "E_T" = $\frac{O_t + P_t + 4ML_t}{6}$ (12)

For the Variance, "V" = $[\frac{P_t - O_t}{6}]^2$ (13)

For the Standard Deviation, "δ" = $\sqrt{\sum_{i=1}^{n} V}$ (14)

To find the Total Number of Period, "T_P" = $E_T + V$ (15)

For the Normal Variant of Scheduled Time "Z"

$$\text{“Z”} = \frac{(T_P - E_T)}{V} \quad (16)$$

N.B: Find Z under Normal Distribution Table or by interpolation

Developed Model for Cost

The cost model considered: well-water unit, pressure filter, dosing, clean holding water tank, installation material and labour cost.

Nomenclature

Q is quantity of each item
P is price per unit item
Tp is total price of a set of similar components
EC is estimated cost of the whole project.
TE is total estimate to be charged by contractor.
TC is total cost of the major components.
IR is interest rate (percentage) = (n%)

$U_{1-6}C$ is total cost of the whole processing unit with labour and other miscellaneous expenses.
U_1C is cost of components in processing unit 1 (Well-Water Unit)
U_2C is cost of components in processing unit 2 (Pressure Filter Unit)
U_3C is cost of components in processing unit 3 (Dosing Unit)
U_4C is cost of components in processing unit 4 (Clean Water Holding Tank Unit)
U_5C is installation material cost
n is the number of items
U_6C is cost of labour

$$T_p = Q_i \times P_i = q_1p_1, q_2p_2, q_3p_3, \dots\dots q_np_n \quad (17)$$

$$TC = \left[\sum_{i=1}^{4} q_ip_i\right] = q_1p_1 + q_2p_2 + \dots. + q_4p_4 \quad (18)$$

$$U_{1-6}C = \left[\sum_{i=1}^{n} q_ip_i\right] = \left[\sum_{i=1}^{n} q_1p_1\right]u_1 + \left[\sum_{i=1}^{n} q_1p_1\right]u_2 + \dots \left[\sum_{i=1}^{n} q_1p_1\right]u_n \quad (19)$$

The Cost of Well-Water Unit Components

$$U_1C_1 = [pq]_{11} + [pq]_{12} + [pq]_{13} + [pq]_{14} + [pq]_{15} + [pq]_{16} + [pq]_{17} + [pq]_{18} + [pq]_{19} = \left[\sum_{i=1}^{9} q_1p_1\right]u_1 \quad (20)$$

Where $[qp]_{11}$ to $[qp]_{19}$are:

Pumping machine, PVC conduit pipe, valve fitting, socket fitting, elbow fitting, union fitting, well submersible pump, marine rope and T-fitting cost respectively.

The Cost of Pressure Filter Unit Components

$$U_2C = [pq]_{21} + [pq]_{22} + [pq]_{23} + [pq]_{24} + [pq]_{25} = \left[\sum_{i=1}^{5} p_iq_i\right]u_2 \quad (21)$$

Where $[pq]_{21}$ to $[pq]_{25}$are;
Filter tank, activated carbon, fine sand, coarse sand and gravel respectively.

The Cost of Dosing Unit Components

$$U_3C = [qp]_{31} + [qp]_{32} + [qp]_{33} = \left[\sum_{i=1}^{3} q_ip_i\right]u_3 \quad (22)$$

Where $[qp]_{31}$ to $[qp]_{33}$are;
Dosing pump, chemical solution tank and chlorine drum costs respectively.

The Cost of Retention Water Tanks Unit Components

$$U_4C = [qp]_{41} + [qp]_{42} = \left[\sum_{i=1}^{2} q_ip_i\right]u_4 \quad (23)$$

Where $[qp]_{41}$ and $[qp]_{42}$are;
Clean water holding tank, and raw water tank costs (Retention Tanks)

The Cost of Installation Material (IMC)

$$U_5C = [qp]_{51} + [qp]_{52} + [qp]_{53} + [qp]_{54} + [qp]_{55} = \left[\sum_{i=1}^{5} q_ip_i\right] u_5 \quad (24)$$

Where $[qp]_{51}$ to $[qp]_{52}$are;
Electric cable, electric switch, tipper of sharp sand, tipper of granite and a bag of cement costs respectively.

The Cost of Labour Cost (LC) Components

$$U_6C = [qp]_{61} + [qp]_{62} + [qp]_{63} + [qp]_{64} = \left[\sum_{i=1}^{4} q_ip_i\right] u_6 \quad (25)$$

Where $[qp]_{61}$ to $[qp]_{64}$are;

The Cost of plumbing labour, masonry labour, cost of electrical labour, and miscellaneous expenses respectively.

$$U_{1-6}C = \left[\sum_{i=1}^{6} q_ip_i\right]u_{1-6} = \left[\sum_{i=1}^{6} U_iC\right] = U_1C + U_2C + \dots\dots U_6C \quad (26)$$

IR= n/100 (27)

$$EC = \left[\sum_{i=1}^{6} q_ip_i\right] u_{1-6} + TE \quad (28)$$

$$TE = \left[\sum_{i=1}^{6} q_ip_i\right]u_{1-6} \times IR \quad (29)$$

RESULTS AND DISCUSSION

Table 2: Activities Expected Time and Variance Computation in Domestic Well-Water Treatment Plant.

S/N	Act.	Preced. Activity	Activity Description	A Optimistic Time (O_t)	B Most Likely Time (Ml_t)	C Pessimist Time (P_t)	Expected Time $E_T = O_t + P_t + 4ML_t / 6$	Variance = $[(P_t - O_t)/6]^2$ Or $V=[(C-A)/6]^2$
1	A	-	Feasibility Study-site identification -geophysical survey	2	4	6	24/6 = 4	0.4444
2	B	-	Design Plan	11	7	15	54/6 = 9	0.4444 *
3	C	A, B	Materials Selection/ Cost Estimation	2	5	8	30/6 = 5	1.0000 *
4	D	B,C	Construction Layout	3	6	9	36/6 = 6	1.0000 *
5	E	B, D	Well digging	4	8	12	48/6 =8	1.7777 *
6	F	C, D	Component Assembly	2	3	4	18/6 = 3	0.1111
7	G	E,F	Test-run of the System	5	7	9	42/6 = 7	0.4444 *
8	H	G	Commissioning	1	2	3	12/6 = 2	0.1111 *

The data collected in Table 2 was used for computing the Expected Time "E_T", Variance "V", Standard Deviation "SD" and Probability of Success "Z" of the date respectively. The PERT network of the project obtained from the Table 2 is given in the Figure 1.

Project Network

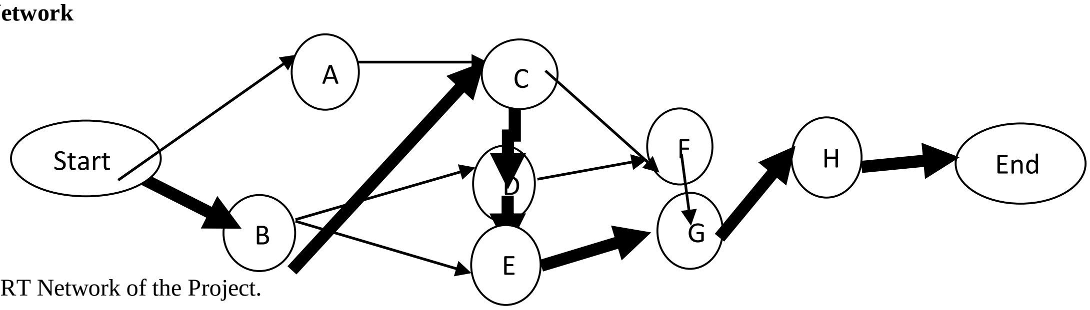


Fig. 1: PERT Network of the Project.

Determination of the critical path is done by computing each path duration from the Table 2 and Figure 1. The following underlisted paths were possible to be able to find the critical path for these activities. These paths are;

[A = 4, B = 9, C = 5, D = 6, E = 8, F = 3, G = 7 and H = 2]

Determine the critical paths and select the highest value for the project completion due-date

S/N	Activity's Paths	Procurement Activities	Total
1.	A → C → F → G → H	4 + 5 + 3 + 7 + 2	21
2.	A → C → D → F → G → H	4 + 5 + 6 + 3 + 7 +2	27
3.	A → C → D → E → G → H	4 + 5 + 6 + 8 + 7+ 2	32
4.	B → C → F → G → H	9 + 5 + 3 + 7 + 2	26
5.	B → C → D → F → G → H	9 + 5 + 6 + 3 + 7 + 2	32
6.	B → C → D → E → G → H	9 + 5 + 6 + 8 + 7 + 2	37
7.	B → D → E → G → H	9 + 6 + 8 + 7 + 2	32
8.	B → D → F → G → H	9 + 6 + 3 + 7 + 2	27
9.	B → E → G → H	9 + 8 + 7 + 2	26

Total Paths = 9

The critical path is

B → C → D → E → G → H = 9 + 5 + 6 + 8 + 7 + 2 = 37

Expected Completion Time, E_T = 37 days (sum of critical path), and

The Standard Deviation is given by:

B → C → D → E → G → H

0.4444 + 1.0000 + 1.0000 + 1.7777 + 0.4444 + 0.1111 = 4.7776

$V^2 = 4.7776$
$\therefore V = \sqrt{4.7776}$
$V = 2.1857 = 2.186$
$V = 2.2$ days
Total Number of Variance (V_T) = 2.186 days = 2.2 days
Total Expected Date = Expected Completion Time E_T + Variance

$$T_{ED} = E_T + V_T = 37 + 2.2 = 39.2 \text{ days} = 39 \text{ days}$$

Hence, the probability of domestic well-water completion not more than 39 days (1 month, 8 days) or later than the expected completion time is:

$$Z = \frac{\square\square\square - \square\square}{\square} = \frac{39-37}{2.2} = \frac{2}{2.2} = 0.909090$$

Under Normal Distribution Table, 0.9 under "0.9" = 0.3389 = 0.34 (Krajewski*et al.*, 2019)
If 0.900000 is 0.3389 (on the Normal Distribution Table)
Therefore, 0.909090 by interpolation will be

0.900000 → 0.3389
0.909090 → Z_n — By cross multiplication

$0.900000\, Z_n = 0.909090 \times 0.3389$

$$Z_n = \frac{0.909090 \times 0.3389}{0.900000} = \frac{0.308090601}{0.900000}$$

$Z_n = 0.34232289 = 0.3423$
$Z_n = 0.34232289 \times 100\%$
$Z_n = 34.23\%$

Since the Expected Time (E_T) is 37 days, this was used as value for completing household community well-water project due date. The variance was approximately 2.2 days, the probability of completing the plant installation and not more than 2.2 days later than 37 days was 34.23% then, the 37 days calculated expected time (E_T) is unreliable because the risk involved in this project is (100–34.23)% = 65.77%. However, a new completing due-date can be predicted by the developed mathematical modelfor cost prediction by varying probability of completion to predict new date of completing the plant installation. Therefore, concerning this research the initial probability was moved from 34.23% to 90%. The new acceptable due-date was predicted thus:

$$= Z_{nx} = \frac{\% \times E_T}{Z_n}$$

Therefore, $x = \frac{90 \times 37}{34.23} = \frac{3330}{34.23} = 97.28$ days, while the $Z_n = 90\%$.
The risk involved now (100 – 90) % reduced to 10%, this is in a very good comfortable zone for the project.

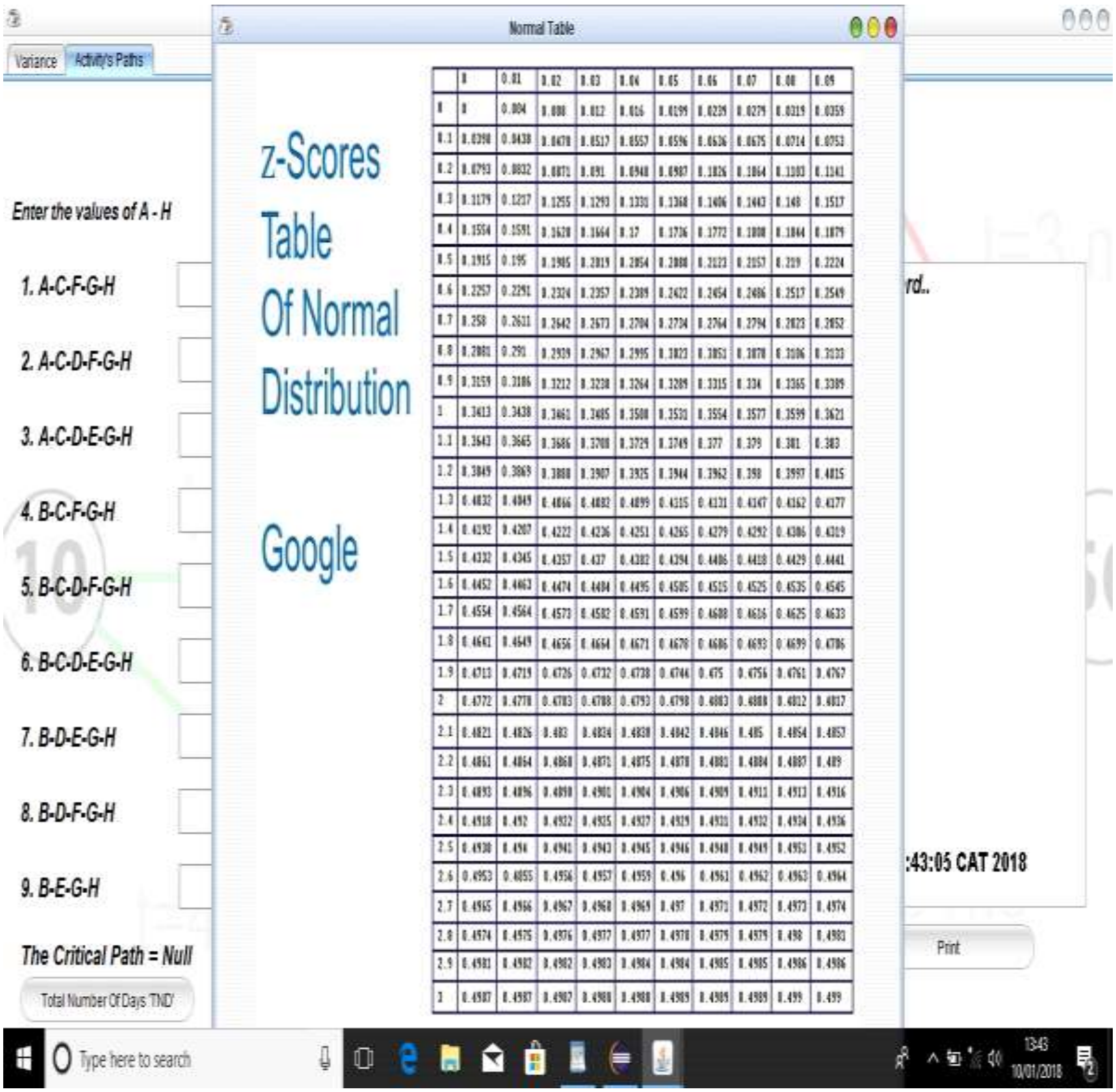
$$\begin{bmatrix} 90\% & 34.23\% \\ \downarrow & \downarrow \\ X & 37\text{days} \end{bmatrix}$$

Interpretation of Result using Interface

Using the above formula

$$= Z_{nx} = \frac{\% \times E_T}{Z_n}$$

The results obtained from PERT can be evaluated using Interface **for** Z-Scores Selection (Due Date).



	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0	0.004	0.008	0.012	0.016	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.091	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.148	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.17	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.195	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.219	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.258	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.291	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.334	0.3365	0.3389
1	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.377	0.379	0.381	0.383
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.398	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.437	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.475	0.4756	0.4761	0.4767
2	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.483	0.4834	0.4838	0.4842	0.4846	0.485	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.489
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.492	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.494	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.496	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.497	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.498	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.499	0.499

Fig 2: Interface for Z-Scores Selection (Due Date)

Operation: View the table carefully and enter the value of Z into the Zn field the click the Zn button to compute.
N.B: Incase Z-scores is not available directly, interpolate.
Pick the immediate low value of Z_l to the actual value Z, then pick the immediate high value of Z_h to the actual value Z.
Determine the new Z_n required. $Z_n = \frac{Z.Z_h}{Z_l}$
With reference to my case study and given the Table 2, the estimated cost for the project can be calculated using the mathematical model.

Table 3: Unit Price and Quantity of each Components

S/N	COMPONENTS	UNIT PRICE (P) (₦)	QUANTITY (Q)
1	Pumping Machine	30000	1
2	PVC Conduit Pipe	700	7
3	Socket Fitting	200	10
4	Elbow Fitting	200	10
5	Union Fitting	200	8
6	Valves Fittings	400	9
7	Well Submersible Pump	32000	1
8	T-Fitting	100	8
9	Marine Rope	3000	1
10	Activated Carbon	1000	2
11	Fine Sand (stone dust)	2000	1
12	Coarse Sand/Garnet	2000	2
13	Gravel	2000	1
14	Filter Tank	40000	1
15	Chemical Solution Tank	4000	1
16	Dosing Pump	45000	1
17	Chlorine Drum	35000	1
18	Clean Water Tank	30000	2
19	Plumbing Labour	30000	1
20	Masonry Labour	15000	1
21	Electrical Labour	10000	1
22	Electric Cable	500	20
23	Granite	30000	1
24	Sharp Sand	20000	1
25	Electric Switch	1000	3
26	Cement	2600	6
27	Miscellaneous Expenses	10000	

Source: Field Work, 2020.

Table 4: Computation of the Well-Water Unit [U_1] Cost

S/N	COMPONENTS	UNIT PRICE (p_i)	QUANTITY (q_i)	TOTAL COST (₦) TC= q_ip_i
1	Pumping Machine	30000	1	30000
2	PVC Conduit Pipe	700	7	4900
3	Socket Fitting	200	10	2000
4	Elbow Fitting	200	10	2000
5	Union Fitting	200	8	1600
6	Valves Fittings	400	9	3600
7	Well Submersible Pump	32000	1	32000
8	T-Fitting	100	8	800
9	Marine Rope	3000	1	3000
	U_1C	$[\sum_{i=1}^{9} q_ip_i]u_1$	=	79900

Source: Field Work, 2020.

Table 5: Computation of the Pressure Filter Unit [U_2] Cost

1	Activated Carbon	1000	2	2000
2	Fine Sand	2000	1	2000
3	Coarse sand/Garnet	2000	2	4000
4	Gravel	2000	1	2000
5	Filter Tank	40000	1	40000
	U_2C	$[\sum_{i=1}^{5} q_i p_i]u_2$	=	50000

Source: Field Work, 2020.

Table 6: Computation of the Dosing Unit [U_3] Cost

1	Chemical Solution Tank	4000	1	4000
2	Dosing Pump	45000	1	45000
3	Chlorine Drum	35000	1	35000
	U_3C	$[\sum_{i=1}^{3} q_i p_i]u_3$	=	84000

Source: Field Work, 2020.

Table 7: Computation of the Retention Tanks Unit [U_4] Cost

1	Clean Water Holding Tank	17500	1	17500
2	Raw Water Tank	17500	1	17500
	U_4C	$[\sum_{i=1}^{2} q_i p_i]u_4$	=	35000

Source: Field Work, 2020.

Table 8: Computation of the Installation Material (IM) [U_5] Cost

1	Electric Cable	500	20	10000
2	Granite	30000	1	30000
3	Sharp Sand	20000	1	20000
4	Electric Switch	1000	3	3000
5	Cement	2600	6	15600
	U_5C	$[\sum_{i=1}^{5} q_i p_i]u_5$	=	78600

Source: Field Work, 2020.

Table 9: Computation of the Labour Cost (LC)[U_6]

1	Plumbing Labour	30000	1	30000
2	Masonry Labour	15000	1	15000
3	Electrical Labour	10000	1	10000
4	Miscellaneous Expenses	5000	1	5000
	U_6C	$[\sum_{i=1}^{4} q_i p_i]u_6$	=	60000

Source: Field Work, 2020.

Table 10: Summary of the Computation of Percentage Contribution

S/No	Components	% Contribution
1	U_1 = Well Water Unit	17.18
2	U_2 = Pressure Filter Unit	10.75
3	U_3 = Dosing Unit	18.06
4	U_4 = Clean Water Tank Unit	7.53
5	U_5 = Material Installation Cost (MIC)	16.90
6	U_6 = Labour Cost (LC)	12.90
7	IR = Interest Rate of Total Estimate	16.67
	Total	100%

Source: Field Work, 2020.

In addition to these, the percentage contributions of each criterion to the total project estimate were found to be: 17.18; 10.75; 18.06; 7.53; 16.90; 12.90 and 16.67 percent respectively for the units as listed in Table 10.

DISCUSSION

The main criteria required for domestic well water treatment plant installation which are cost of mathematical model and due-date for the project has been identified in this research as: Well water, pressure filter, dosing, clean water tank, material installation, labour and rate of interest or total estimate required. The developed software model was evaluated using data obtained from a building site at Road Q Plot 17, Oba Ile Housing Estate, Akure North Local Government Area, Ondo State, Nigeria.

After testing software against the values in Table 2 then, the 37 days calculated Expected Time (E_T) is unreliable because the risk involved in this project is (100 – 34.23) % = 65.77 %. However, a new completing date can be predicted by the developed decision support system by varying probability of completion to predict new date of completing the plant. Therefore, concerning this research the Initial probability was moved from 34.23% to 90%. By interpolation with 37 days; the new acceptable date was 97.28 days. The risk involved now (100 – 90) % reduced to 10%, this is a very good comfortable zone for the project.

CONCLUSION AND RECOMMENDATION

This research has identified the parameters for budgeting and development of mathematical models. Mathematical models based on historical information collected were harmonized to generate an PERT network of the results. In recent years, computer simulation methods are becoming popular in scientific research, particularly as far as the research on the aquatic environment quality is concerned. This is done to avoid project failure, and maximize limited budget available and minimize cost. On the overall, the software that was developed can be used by construction consultants, project managers and site engineers to mathematical models of domestic well-water treatment plant.

Further work can be added tosolve problems of surface water pollution is modelling of changes which take place in lake waters and associated water quality changes. Such a model can be used to predict water quality, taking into account changes that affect water quality factors or changes in their intensity. The accuracy of environmental assessment using models depends on the understanding of processes which occur in the environment and on appropriate choice of mathematical equations that describe them. On the other hand, it depends on available data sets, namely results of the measurement of water quality indicators, on which the estimation of parameters and coefficients of the model is based. Methods of computer analysis have become an independent branch of research, which substantially increases the research capabilities of modern eco-engineers and this research to care for the manufacturing industries in other to ensure clean water consumption. This model can be improved to care for irrigation on farm land or animal husbandry. This project will be ideal economical for household, industries, institutions and for construction companies who have the mind of constructing to required standard. Hence, it is recommended that, the developed models performed satisfactorily upon validation with real life data while human and technical risk factors were established critical to the safety management in manufacturing industry.

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