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APPLICATION OF TRANSPORTATION MODEL TO SOLVE TANKERS' – ROUTING PROBLEM

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ABSTRACT

The problem of selecting minimum cost routes for tankers in distributing petroleum products and satisfying customers' requirement without scarcity in Nigeria remains a huge challenge to major marketers in the oil industries. The cost of transporting petroleum products from sources to destinations matters a lot to oil marketers because of the direct impact it has on their profits. The means of distributing petroleum products from refineries to depots or filling stations are tankers' routing and pipelines. In this research, we extended some existing tankers'-routing models in literature which use a discrete integer programming approach to determine efficient and effective distribution of petroleum products. Consequently, we developed a new transportation linear programming algorithm to determine minimum cost routes in the delivery of petroleum product from their supply centers (refinery) to demand centers (filling stations). The significance of the application we adopted in this research lies in the modified distribution approach to tackle the complexity involved when transportation problems are formulated as linear programming problem having several variables and constraints. In this research, we formulate a new version of transportation model of tankers' routing with the aim of reducing the cost of petroleum products delivery. The proposed transportation linear programming model was applied to a numerical example alongside other existing transportation algorithms. It is observed that, the new algorithm produced approximately the same total cost obtained by using other existing algorithms.

Keywords: Petroleum products, Transportation, Tankers'-routing, Linear programming**,** Refinery

INTRODUCTION

Transportation models are a type of linear programming problems (LPP). According to Ogumeyo and Panya (2024), transportation problems involve selection of minimum cost routes in which goods and services are shipped from a supply centre (origin) to demand centre (destination). Transportation problems formulated as LPPs are usually solved by using simplex algorithms. As reported in Gupta and Hira (2005), methods such as stepping stone and modified distribution have been developed to simplify the complexity involved when transportation problems are formulated as linear programming problems due to numerous variables and constraints. Gupta and Hira (2005) defines transportation problem as a linear programming problem which involves identifying minimum cost routes in delivering goods and services from their places of origin to destination. Transportation cost involves computation of entire cost of transporting people, products and materials from their sources to destinations or customers.

According to Salami (2014), the movement of goods from several sources to their destinations where they are needed belong to a category of linear programming known as transportation problem. Transportation problem was first presented and discussed by Hitchcock in1941 as stated in Salami (2014). Thereafter, Koopman (1947) developed a transportation system which involves using linear programming to determine optimum solution. Dantzi (1951) expanded the idea of linear programming to determine optimum solution of transportation problem involving complex variables and constraints. In recent times, transportation of petroleum products involving tankers' routing has been extensively applied to several models of oil industries. For examples, a tanker routing model which uses a discrete integer programming approach to determine efficient and effective distribution of petroleum products is developed in Agra *et al.* (2013) and was later expanded in Xu *et al.* (2021) model. Diz et al. (2017) applies tankers' routing model to a petroleum distribution pattern in Brazil while a model which describes downstream petroleum supply chain is reported in Kazemi and Szmerekovsky (2015). Similar models involving tanker routing are found in Rodrigues *et al.* (2016) and Stanzani et al. (2018).

The sum total cost on commodities delivery from sources to destinations matters a lot to production companies because of the direct impact it has on their profits as contained in Quan *et al.* (2018) and Shvetsov (2021). Hence, only minimum cost routes are selected during transportation process. Naqurney (2004) and Agureev and Akhromeshin (2020) state that a LPP technique can be used to solve transportation problems when the cost of taking a route only depends on the flow on that route. In this research, we aim at discussing transportation model in general and then apply it to tankers'–routing problem via linear programming techniques. The problem of tankers' routing was first investigated in Dantzig and Fulkerson (1954). Their model aims at finding the least number of tankers required to satisfy a fixed schedule in the delivery of petroleum products. Salami (2014) extended their tankers'-routing model by developing a transportation linear programming algorithm to determine minimum cost routes in the distribution of soft drinks in Nigeria. Major challenges faced by government in transporting both crude and refined petroleum products through pipelines are caused by the activities of illegal refiners and pipeline vandals. Nigerian Government budgets billions of naira annually to protect oil facilities and ensure un-interrupted supply of petroleum products as a part of her economic policy to boost the nation's revenue, Awariefe and Ogumeyo (2023).

This research aims at extending a tanker routing model which uses a discrete integer programming approach to determine efficient and effective distribution of petroleum products developed in Agra *et al.* (2013) and was later expanded in Xu *et al.* (2021) models by adding detail discussion to the concepts they used in order to formulate a new model of transportation linear programming which determines minimum cost routes in tankers'-routing problem. The significance of the application we adopted in this research lies in the modified distribution approach to tackle the complexity involved when transportation problem (especially tankers'-routing) are formulated as linear programming problem due to the presence of several variables and constraints.

MATERIALS AND METHODS

Description of the Model

Let $i=1,2,3\ldots, m$ be the number of loading depots of the petroleum products , at which tankers are loaded for deliveries to discharge centers, $j = 1,2,3 \ldots, n$. A time at which a tanker is to be loaded at *i depot* for making a delivery to *j* destination is fixed or known. The time duration it takes a tanker to move from any depot to another is also assumed to be known. In this model, we shall consider the case where all tankers are identical (so that they are interchangeable) and where deliveries are quantified in units of tanker capacity so that a tanker must make a full delivery. In other words, a tanker cannot make part delivery. Having made a delivery, a tanker can travel to any depot if it can get there in time to pick up another cargo for some selected destinations.

Model Assumptions

Assumptions associated with the model formulation and operations are as follow:

i. The product/ commodity being transported must be identical such that customers can accept them from any source.
ii. The number of tankers to be supplied and demanded is known otherwise a non- existing depot which takes excess supply of the product at no cost is added.
iii. The cost of shipment through each route is known.
iv. The tankers are available at the beginning of the period wherever they are needed irrespective of where they end up at the end of the period.

Methodology

The methodology applied includes

i. Russel's Approximation Method (RAM)
ii. Linear Programming Problem (LPP)
iii. Vogel's Approximation Method (VAM)
iv. Transportation Algorithm/ Northwest Corner Rule

The objective of the study is to formulate a transportation model which determines the minimum cost of shipping petroleum products from depots (Refinery centers) to their demand centers (filling stations).

Mathematical Notations/variables

The variables used in this research are defined as follows:

S_j = number of supply centre *for* $i = 1,2, \ldots m$

D_j = the number of demand centre for $j = 1,2,\ldots,n$

X_{ij} =number of tankers transported from supply centre i to destination j

C_{ij} = transportation cost per tanker.

Transportation Model Formulation

The mathematical expressions of the transportation model described above can be stated thus:

$$\text{Min } H = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij} \quad . \tag{1}$$

$$\sum_{j=1}^{n} X_{ij} \leq S_i \textit{ for } j=1,2,\ldots m \textit{ (supply constraint)} \tag{2}$$

$$\sum_{i=1}^{m} X_{ij} \geq D_j \textit{ for } j=1,2,\ldots n. \textit{ (demand constraint).} \tag{3}$$

$$X_{ij} \geq 0 \textit{ (Nonnegative constraints).} \tag{4}$$

In such cases, we employ at most *(m+n-1)* routes of feasible transportation schedule. Where the quantity supplied and the quantity demanded are the same then, we have a balanced transportation problem. This can mathematically be expressed as:

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j \,. \tag{5}$$

$$\text{If } \sum_{i=1}^{m} S_i \geq \sum_{j=1}^{n} D_j \,. \tag{6}$$

equation (6) implies that supply is either equal to or greater than demand.

If the supply and the demand are not the same in quantity then, the transportation problem TP is unbalanced. This can be expressed mathematically as:

$$\sum_{i=1}^{m} S_i \neq \sum_{j=1}^{n} D_j \,. \tag{7}$$

Theorem 1: (Existence of Feasible Solution). A necessary and sufficient condition required for a transportation problem to have a feasible solution, is that the quantity supplied must be equal to quantity demanded. That is

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j \,.$$

Proof: (a) Necessary Condition. Suppose, the solution that is feasible in a TP exits, then we shall have

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij} = \sum_{i=1}^{m} S_i \,. \tag{8}$$

$$\text{And } \sum_{j=1}^{n}\sum_{i=1}^{m} X_{ij} = \sum_{j=1}^{m} D_j \tag{9}$$

Equation (8) represents the quantity supplied while equation (9) is the demanded quantity. Since equation (8) and equation (9) are the same, it means $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$.

(b) Sufficient condition. Assuming the quantity supplied is equal to demanded quantity, then

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j = h(\text{say})\,. \tag{10}$$

Suppose k ($k \neq 0$) is a real number such that $X_{ij} = k_i D_j \neq 0$ for all *i and j*, then the value of k_i is given by

$\sum_{j=1}^{n} X_{ij} = \sum_{j=1}^{n} k_i D_j = k_i \sum_{j=1}^{n} D_j = hk_i$ or

$k_i = \frac{1}{h}\sum_{j=1}^{n} X_{ij} = \frac{S_i}{h}$.

Thus

$$X_{ij} = k_i D_j = \frac{S_i D_j}{h} \text{ for all } i \text{ and } j. \tag{11}$$

Since $S_i > 0$ and $D_j > 0$ for every i and j, then $S_j D_j / h \geq 0$ and hence a solution which is feasible exists, that is $x_{ij} > 0$.

Theorem 2: (Basic Feasible Solution). In any basic feasible solution, we have $m + n - 1$ basic variables and $m + n - 1$ independent constraint, where m rows is the supply constraint and n columns is the demand constraint equations.

Proof: In all mathematical formulations of transportation problems including the ones discussed in Sharma (2006) and Ekoko (2011), it is observed that, if m rows which represent supply constraints and n columns which represent demand constraints exist, then, we will have a total of $m + n$ constraints. But due to Theorem 1 which states that the quantity supplied and the quantity demanded must be equal out of $m + n$ constraint equations, one of the equations must be unused and consequently, removed. Hence, we have $m+n-1$ equation which are linearly independent. We can prove this if we add all the equations of the m rows and deducting from the sum of the first $n-1$ column equations, consequently obtaining the last column equation. That is,

$$\sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} - \sum_{j=1}^{n-1} \sum_{i=1}^{m} X_{ij} = \sum_{i=1}^{m} S_i -_i \sum_{j=1}^{n-1} D_j$$

$$\sum_{i=1}^{m} \left[\sum_{j=1}^{n} X_{ij} - \sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} - \sum_{i=1}^{m} X_{in}\right]_i = \sum_{i=1}^{m} S_j - \left[\sum_{i=1}^{n} D_j - D_n\right]$$

$$\sum_{i=1}^{m} X_{in} = D_n \,. \tag{12}$$

$$\text{since } \sum_{i=1}^{m} S_i = \sum_{j=1}^{m} D_j \,.$$

Unbalanced Transportation Problem

A necessary condition required for the existence of a feasible solution, is that the quantity supplied and quantity demanded must be equal as earlier stated in equation (6). That is,

$\sum_{i=1}^{m} S_i = \sum_{j=1}^{m} D_j$.

But that is not always true, since there could be cases in real life situations where supplied quantity does match the quantity required. Two cases can be derived from an unbalanced TP.

Case 1: If the supply is higher than demand, the constraints of the transportation problem can be mathematically stated as,

$\sum_{j=1}^{n} X_{ij} \leq S_i$ *for j=1,2,…m.* (13)

$\sum_{i=1}^{m} X_{ij} = D_j$ *for j=1,2,…n.* (14)

$X_{ij} \geq 0$ *for every i and j.* (15)

If we add $h_{i,n+1}$ as slack variable for ($i = 1,2, \ldots m$) in the first m equations, we obtain

$\sum_{j=1}^{m} X_{ij} + h_{i,m+1} = S_i$

$\sum_{i=1}^{m} \left[\sum_{i=1}^{m} X_{ij} + h_{i,n+1}\right] = \sum_{i=1}^{m} S_i$

$\sum_{i=1}^{m} X_{ij} + h_{i,n+1} = \sum_{i=1}^{m} S_i - \sum_{j=1}^{m} D_i$. available excess supply (16)

If we proceed further to denote the available excess supply by D_{n+1} then, the modified version of the TP can be written as

Min $H' = \sum_{i=1}^{m} \sum_{j=1}^{n} (C_{ij}X_{ij} + C_{i,n+1} h_{i,n+1})$. (17)

Subject to $\sum_{j=1}^{n} X_{ij} + h_{i,n+1} = S_i$; *i=1,2 ….,m* . (18)

$\sum_{j=1}^{n} X_{ij} = D_i$; j = *1, 2, …, n+1* . (19)

$X_{ij} \geq 0$ *for every i, j.* (20)

Note that $C_{i,\,n+1} = 0$ (i = 1,2…, m) and

$\sum_{i=1}^{m} S_i = D_j + D_{n+1}$ *or* $D_{n+1} = \sum_{i=1}^{m} S_i - \sum_{j=1}^{n} D_j$.

The mathematical analysis presented in equation (17)- (20) implies that, if the quantity supplied is higher than the quantity required (i.e $\sum_{i=1}^{m} S_i > \sum_{j=1}^{n} D_j$), then a dummy column (demand centre) which absorbs the surplus supply is added to the TP table. Then transportation cost per unit for the cells in this column is equated to zero. (See Ekoko (2011) and Ogumeyo and Panya (2024)).

Case 2: If demand is higher than the quantity supplied, the equation of constraints of the TP will appear as

$\sum_{j=1}^{n} X_{ij} = S_i$; *i =1,2,…m.* (21)

$\sum_{i=1}^{m} X_{ij} \leq D_j$; *j=1,2,….n.* (22).

$X_{ij} \geq 0$ *for every i, j.*

If we add slack variables h_{m+1} for *j(j=1,2,…n)* in the last n constraints, we obtain

$\sum_{j=1}^{n} X_{ij} = S_i$; *i =1,2,…m.* (23)

$\sum_{i=1}^{m} X_{ij} + h_{m+1,j} = D_j$; *j=1,2,…n.* (24)

$\sum_{j=1}^{n} h_{m+1,j} = \sum_{j=1}^{n} D_j - \sum_{i=1}^{m} S_i$. surplus demand (25)

If we denote the surplus demand by h_{m+1} then, we can rewrite the modified transportation problem as:

Minimize $H' = \sum_{i=1}^{m} \sum_{j=1}^{m} (C_{ij}X_{ij} + C_{m+1,j} h_{m+1,j})$. (26)

Subject to $\sum_{j=1}^{n} X_{ij} = S_i$; *i =1,2 ….,m+ 1* (27)

$\sum_{i=1}^{m} X_{ij} + h_{m+1,j} = D_j$; *j = 1, 2, …, n.* (28)

$X_{ij} \geq 0$ *for every i, j.*

Note that $C_{m+1,\,j} = 0$, for all *j* and

$\sum_{i=1}^{m} S_i + h_{m+1} = D_j$ or $\sum_{j=1}^{n} D_j$ or $h_{m+1} = \sum_{j=1}^{n} D_j - \sum_{i=1}^{m} S_i$.

Equations (26) – (28) imply that if total quantity demanded is higher than total quantity supplied (i.e $\sum_{j=1} D_j > \sum_{i=1} S_i$) then, we need to add a dummy row (supply centre) to the TP to absorb the surplus quantity demanded. In this case, the transportation cost for the dummy row is equated to zero.

Formulation of TP as LPP

If the size of transportation problem is very large such that applying the transportation simplex technique becomes computationally inefficient, then the TP can be formulated as a LPP. This is made possible by taking advantage of the duality relationships of its network. The general mathematical formulation of the transportation problem as earlier stated in Section 3.0 is:

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{m} C_{ij}X_{ij}$. (29)

Subject to

$\sum_{j=1}^{n} X_{ij} = S_i$ for I = 1, 2, …….. m (S constraints). (30)

$\sum_{i=1}^{m} X_{ij} = D_j$ for j = 1, 2, …….. n (D constraint) . (31)

Xij > 0 for every *i and j* (Nonnegative condition). (32)

Note that all S_i and D_j are positive number satisfying the equation

$\sum_{i=1}^{m} S_i = \sum_{i=1}^{m} D_j$ (Quantity supplied = Quantity Demanded) . (33)

The technology table for the TP is shown in Table 1 below for the case of a 3 rows x 4 columns (*m=3 and n=4) TP.*

Since the total quantity supplied and the total quantity demanded in equations (30) and (31) are equal, the TP has a dummy cell because if any *m+n-1* constraints in equations (30) and (31) are met, then the remaining constraint will also be met. The Table 1 is divided into an upper and a lower section.

Table 1 Technology table for transportation problem

U_{11}	U_{12}	U_{13}	U_{14}	U_{21}	U_{22}	U_{23}	U_{24}	U_{31}	U_{32}	U_{33}	U_{34}	
1	1	1	1									$= B_1$
				1	1	1	1					$= B_2$
								1	1	1	1	$= B_3$
1				1				1				$= M_1$
	1				1				1			$= M_2$
		1				1				1		$= M_3$
			1				1				1	$= M_4$
Q_{11}	Q_{12}	Q_{13}	Q_{14}	Q_{21}	Q_{22}	Q_{23}	Q_{24}	Q_{31}	Q_{32}	Q_{33}	Q_{34}	Minimize

The dual linear programming model corresponding to Table 1 is displayed in Table 2. Letters P and R are used to represent the dual variables corresponding to the supply and demand partition Table 1. The duality of the TP in Table 1 can be expressed as

Table 2: Duality of the TP

R_1	**R_2**	**R_3**	**P_1**	**P_2**	**P_3**	**P_4**	
1			1				$\le Q_{11}$
1				1			$\le Q_{12}$
1					1		$\le Q_{13}$
1						1	$\le Q_{14}$
	1		1				$\le Q_{21}$
	1			1			$\le Q_{22}$
	1				1		$\le Q_{23}$
	1					1	$\le Q_{24}$
		1	1				$\le Q_{31}$
		1		1			$\le Q_{32}$
		1			1		$\le Q_{33}$
		1				1	$\le Q_{34}$
B_1	B_2	B_3	M_1	M_2	M_3	M_4	Minimize

By applying the procedure described above, we have

Table 3: Supply and Demand Cost Structure

	1	2	3	SUPPLY
1	Q_{11} U_{11}	Q_{12} U_{12}	Q_{13}	B_1
2	Q_{21} U_{21}	Q_{22} U_{22}	Q_{23} U_{23}	B_2
3	Q_{31} U_{31}	Q_{32} U_{32}	Q_{33} U_{33}	B_3
Dd	M_1	M_2	M_3	

Using Table 3, the TP can be expressed as follows:

Minimize $Z = C_{11}X_{11} + C_{12}X_{12} + \ldots + C_{33}X_{33} = \sum \sum C_{ij}X_{ij}$ (34)

Subject to

$\sum_{j=1}^{n} X_{ij} = S_i$ *for i = 1 (1)m*, Supply. (35)

$\sum_{j=1}^{m} X_{ij} = D_j$ for j =1 (1)n, Demand. (36)

$X_{ij} \ge 0$ for *i =1(1)m, j = 1(1)n*, non-negativity condition. 37)

But the sum of the *m* equations in (35) is equal to the sum of the *n* equations in (36). This shows that one linear constraint is redundant i.e. equations (35) and (36) are not linearly independent. We can therefore, delete any one of the $m + n$ linear constraints in (35) and (36). If we delete say the constraint: $\sum_{j=1}^{n} X_{ij} = S_i$, then the reduced transportation problem becomes:

Minimize $C = C_{11}X_{11} + C_{12}X_{12} + \ldots + C_{33}X_{33}$. (38)

Subject to

$\sum_{j=1}^{n} X_{ij} = S_i$ for *i = 2 (1)m.* (39)

$\sum_{j=1}^{m} X_{ij} = D_j$ for *j=1(1)n.* (40)

$X_{ij} \ge 0$. (41)

In the next section, we present an algorithm known as Transportation Algorithm which we shall apply to a numerical solution in Section 4.

Transportation Algorithm

This method is sometimes referred to as transportation simplex method. It is a method of cross checking if the current solution to a transportation problem is optimal or can be improved. The algorithm is as follows:

Step 1: Associate with row *i* of the transportation table a multiplier U_i and with each column *j* a multiplier V_j. For each basic variable X_{ij} in the current solution, set $C_{ij} = U_i + V_j$. This gives $m + n - 1$ basic variable in $m + n$ unknown. The value of the multiplier U_i, and V_j are obtained from the simultaneous equations by assigning an arbitrary value to any one of the multipliers. (Often we set U_i with highest number of basic variables equal to zero. Break ties arbitrarily.

Step 2 For each non – basic X_{pq} *variable*, calculate $W_{pq} = C_{pq} - (U_p + V_q)$. W_{pq} gives the net increase or decrease in the quantity of the objection function (total transportation cost) as a result of increasing X_{pq} above zero level. The non – basic variable which will yield the largest per unit decrease in the cost is selected as the entry variable. If all $W_{pq} > 0$, stop, otherwise go to the next step

Step 3 Utilize the new route as fully as possible whilst still satisfying the supply and demand requirement. The quantities of X_{pq} are chosen so that the quantity of one basic variable in the present solution is reduced to zero.

Step 4 Determine the new solution and go to step 2.

Numerical Illustration

A refinery has three depots (supply centers) and three demand centers as shown in the table below. Transportation cost from any deport to any demand center is fixed in tens of thousands of naira (N10,000s) per tanker. Transport from supply center 1 to demand center 1 cost N7/tanker and N5/tanker from supply center 1 to demand center 2 and so on. The full data of the transport cost from each supply center to each demand center per tanker is summarized in Table 4 as follows:

Table 4. Supply and Demand Quantities

	CUSTOMERS			
	1	2	3	Supply source
1	7	5	6	5
2	3	4	3	6
3	2	5	1	1
Demand centers	4	3	5	

Use a transportation linear programming algorithm to determine the minimum cost routes tankers should take in delivering the products required at each depot.

Solution: By applying our model in section 3.0 to Table 4, the objective function and its constraints can be written as

Minimize $C = 7X_{11} + 5X_{12} + 6X_{13} + 3X_{21} + 4X_{22} + 3X_{23} + 2X_{31} + 5X_{32} + X_{33} + MS_1 + MS_2$

Subject to

Table 5: Transportation Problem in LP Form

S.V	X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	X_{31}	X_{32}	X_{33}	
S_1				X_{21}	X_{22}	X_{23}				$+ S_1 = 6$
S_2							X_{31}	X_{32}	X_{33}	$+ S_2 = 1$
X_{11}	X_{11}			X_{21}			X_{31}			$= 4$
X_{12}		X_{12}			X_{22}			X_{32}		$= 3$
X_{13}			X_{13}			X_{23}			X_{33}	$= 5$

$X_{ij}, S_i \geq 0$, *for* $i = 1(1)m, j=1(1)n$

Recall that S_i*'s* are artificial variables which are added to obtain an (m + n – 1) x (m + n – 1), that is (5 x 5) unit matrix which is needed in the initial computational form.

Thus, the tableau 1.1 is the reduced transportation problem in Tableau 1 .

Solution by Big-Method

The transportation problem (TP) in Table 4 formulated as LPP can be solved using the Big-M method. Considering the variable coefficients we have the following tableau:

Tableau 1. Initial Tableau

	-7	-5	-6	-3	-4	-3	-2	-5	-1	-M	-M	
C_1	X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	X_{31}	X_{32}	X_{33}	S_1	S_2	R.H.S
-M S_1	0	0	0	1	1	1	0	0	0	1	0	6
-M S_2	0	0	0	0	0	0	$\boxed{1}$	1	1	0	1	1
-7 X_{11}	1	0	0	1	0	0	1	0	0	0	0	4
-5 X_{12}	0	1	0	0	1	1	0	1	0	0	0	3
-6 X_{13}	0	0	1	0	0	1	0	0	1	0	0	5
$C_{ij} – Z_j$	0	0	0	M+4	M+1	M+3	M+5	M	M+5	0	0	

Since X_{31} and X_{33} has the largest simplex criterion, we can choose either X_{31} or X_{33} as pivot column. Choosing X_{31} arbitrarily the pivot fractions (r_i) are $r_2 = \frac{1}{1} \Rightarrow \quad r_2 = 1, \quad r_3 = \frac{4}{1} \quad r_3 = 4$, since r_2 is the smallest nonnegative fraction, row 2 becomes the pivot row.

Tableau 2: First iteration using the Big M Method

	-7	-5	-6	-3	-4	-3	-2	-5	-1	-M	-M	
C_1	X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	X_{31}	X_{32}	X_{33}	S_1	S_2	R.H.S
-M S_1	0	0	0	1	1	1	0	0	0	1	0	6
-2X_{31}	0	0	0	0	0	0	1	1	1	0	1	1
-7 X_{11}	1	0	0	1	0	0	0	-1	-1	0	-1	3
-5 X_{12}	0	1	0	0	0	0	0	1	0	0	0	3
-6 X_{13}	0	0	1	0	1	1	0	0	1	0	0	5
$C_{ij} – Z_j$	0	0	0	M+4	M+2	M+3	0	-5	0	0	-M-5	

In Tableau 3 above, X_{21} has largest value in the objective row function, hence the entering non-basic variable is X_{21}, the row fractions are $r_1 = \frac{6}{1} = 6$, $r_3 = \frac{3}{1} = 3$. Hence the departing basic variable is X_{11} since it corresponds to the smaller ratio.

Thus the pivot row is X_{21}. While X_{11} is the pivot column. Since there are still nonnegative values in the objective row function, the current solution is not optimal. Thus, by following the same procedure, we obtained Tableau 4 and 5 below.

Tableau 3: Second iteration using the Big M Method

C_1	X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	X_{31}	X_{32}	X_{33}	S_1	S_2	R.H.S
-M S_1	-1	0	0	0	1	1	0	1	-1	1	1	3
-2X_{31}	0	0	0	0	0	0	1	1	1	0	1	1
-7 X_{11}	1	0	0	1	0	0	0	-1	-1	0	-1	3 →
-5 X_{12}	0	1	0	0	0	0	0	1	0	0	0	3
-6 X_{13}	0	0	1	0	1	1	0	0	1	0	0	5
$C_j - Z_j$	0	0	0	0	M+2	M+3	0	-5	M-2	0	-1	

Tableau 4. Third iteration using the Big M Method

	-7	-5	-6	-3	-4	-3	-2	-5	-1	-M	-M	
C_1	X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	X_{31}	X_{32}	X_{33}	S_1	S_2	R.H.S
-3X_{23}	-1	0	0	0	1	1	0	1	1	1	1	3
-2X_{31}	0	0	0	0	0	0	1	1	1	0	1	1
-3X_{21}	1	0	0	1	0	0	0	-1	-1	0	-1	3 →
-5X_{12}	0	1	0	0	0	0	0	1	0	0	0	3
-6 X_{13}	1	0	1	0	0	0	-1	-1	0	-1	-1	2
$C_j - Z_j$	-1	0	0	0	-1	0	-8	-6	0	-M-3	-M-4	

Since all the terms in the objective function are non-positive, the optimal solution has been obtained. The optimal solution is ($X^*_{23} = 3,\ X^*_{31} = 1,\ X^*_{21} = 3,\ X^*_{12} = 3$ and $X^*_{13} = 2,$
$X_{11} = X_{22} = X_{32} = S_1 = S_1 = S_2 = 0$)
By substituting the above values into the objective function we have
$TotalTPCost = (N30000 \times 3) + (N20000 \times 1) + (N30000 \times 3) + (N50000 \times 3) + (N\ 60000 \times 2)$
$C = N470{,}000$

Solution by Russel's Approximation Method (RAM)
First iteration using the RAM, the initial basic feasible solution for Table 1

Tableau 7: First iteration using the RAM

	1	2	3	
1	7 W_{11}	5 3	6 2	$U_1 = 0$
2	3 3	4 W_{22}	3 3	$U_2 = -$
	2 1	5 W_{32}	1 W_{33}	$U_3 = -$

$V_1 = 5$ $V_2 = 5$ $V_3 = 6$, and $X_{12} = 3$, $X_{13} = 2$, $X_{21} = 3$, $X_{23} = 3$, $X_{31} = 1$.
We examine if the current solution is optimal
Set $C_{ij} = U_i + W_j$ for all basic variables

$5 = U_1 + V_2$
$6 = U_1 + V_3$
$3 = U_2 + V_1$
$3 = U_2 + V_3$
$2 = U_3 + V_1$
Set $U_1 = 0$, we have

$V_2 = 5$, $V_3 = 6$, $U_2 = -3$, $V_1 = 5$, $U_3 = -3$
$W_{pq} = C_{pq} - (Up + Vq)$
$W_{11} = C_{11} - (U_1 + V_1) = 7 - (0+5) = 2$
$W_{22} = 4 - (-3 + 5) = 2$
$W_{32} = 5 - (-3 + 5) = 2$
$W_{33} = 1 - (-3 + 6) = -2$

Since $W_{pq} < 0$ for some p and q, the solution is not optimal.
Therefore compute the second iteration. W_{33} is the entering variable since it yields largest per unit decrease.

Tableau 8: Second iteration using the RAM

1	2	3
7 W_{11}	5 3	6 2
3 3+y	4 W_{22}	3 3-y
2 1-y	5 W_{32}	1 y

We take y = 1.The new solution now is $X_{12} = 3$, $X_{13} = 2$, $X_{21} = 4$, $X_{23} = 2$, $X_{33} = 1$
We examine if the solution is optimal.

Tableau 9: Third iteration using the RAM

	1	2	3	
1	7 W_{11}	5 3	6 2	$U_1 = 0$
2	3 4	4 W_{22}	3 2	$U_2 =–3$
	2 W_{31}	5 W_{32}	1 1	$U_3 =$

$W_{11} = 7 – (0 + 5) = 2$
$W_{22} = 4 – (- 3 + 5) = 2$
$W_{31} = 2 – (- 5 + 5) = 2$
$W_{32} = 5 – (- 5 + 5) = 5$
Since Wpq $\geq$ 0 for all p and q, it means this is the optimal tableau. Hence the optimal solution of the total transportation cost is
(5 x 3) + (6 x 2) + (3 x 4) + (3 x 2) + 1 x 1) = N46.00

Analysis of Results

The transportation problem presented in Section 4.0, was solved by using the Big-M algorithm of the simplex method. The optimal solution is found in Tableau 4 which is the third iteration using the Big M Method. The basic variables which contributed to the optimal solution are
$\left(X^*_{23} = 3,\ X^*_{31} = 1,\ X^*_{21} = 3,\ X^*_{12} = 3\right.$ and $\left.X^*_{13} = 2,\right)$ "and the non-basic variables are"
$X_{11} = X_{22} = X_{32} = S_1 = S_1 = S_2 = 0)$
By substituting the above values into the objective function, we have: Total Transportation Cost = (N30,000 x 3) + (N20,000 x 1) + (N30,000 x 3) + (N50,000 x 3) + (N60,000 x2) = N470,000. This implies that the total minimum cost is N470,000, since the cost of the transportation is tens of thousands of naira per tanker. In other words, 3 tankers should be transported to depot X_{23} at the cost of N30,000 per tanker, 1 tanker the deport X_{31} at the cost N20,000, 3 tankers to depot X_{21} at the cost of N30,000 per tanker , 3 tankers to depot X_{12} at the cost N50,000 per tanker and 2 tankers to depot X_{13} at the cost N60,000 per tanker.
From the results obtained by using Russsel Approximation Method, the basic variables which contributed to the objective function are $X^*_{12} = 3$, $X^*_{131} = 2$, $X^*_{21} = 4$, $X^*_{23} = 2$ and $X^*_{33} = 1$,
) at the cost of N10,000 per tanker.
Hence, the optimal total cost of the transportation problem using Russel Approximation Method is:
(N50,000 X 3) + (N60, 000 2) + (N30 X 4) + (N30,000 X 2) + (N10,000X1) = N460,000. This optimal solution was obtained after the third iteration in Tableau 9.

CONCLUSION

The need for efficient distribution of petroleum products from refineries (supply centers) to demand centers cannot be over-emphasized considering their domestic and industrial importance in our daily activities. In this study, we have presented a transportation model which is being formulated as LPP, and consequently applied to solve a Tanker-Routing Problem of distribution of petroleum products from supply center (refinery depots) to demand center (filling stations). The aim of the study is to select minimum cost routes that will reduce the cost of shipping petroleum products from refinery depots to filling stations. Other methods of transportation model for determination of minimum cost routes such as NCR, VAM and RAM are being discussed. But the emphasis is on RAM and its algorithm. This is because we intend to use it to cross check the results we obtained from the Big-M method of LPP. It is observed that the new approach we presented, gives just one unit higher than the ones obtained by using NCR, VAM and the RAM. While the minimum total transportation cost obtained by using the Big-M approach is N470,000, that of the NCR, VAM and the RAM is N460, 000.

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