APPLICATION OF TRANSPORTATION MODEL TO SOLVE Tankers’ – ROUTING PROBLEM

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ABSTRACT
The problem of selecting minimum cost routes for tankers in distributing petroleum products and satisfying customers’ requirement without scarcity in Nigeria remains a huge challenge to major marketers in the oil industries. The cost of transporting petroleum products from sources to destinations matters a lot to oil marketers because of the direct impact it has on their profits. The means of distributing petroleum products from refineries to depots or filling stations are tankers’ routing and pipelines. In this research, we extended some existing tankers’-routing models in literature which use a discrete integer programming approach to determine efficient and effective distribution of petroleum products. Consequently, we developed a new transportation linear programming algorithm to determine minimum cost routes in the delivery of petroleum product from their supply centers (refinery) to demand centers (filling stations). The significance of the application we adopted in this research lies in the modified distribution approach to tackle the complexity involved when transportation problems are formulated as linear programming problem having several variables and constraints. In this research, we formulate a new version of transportation model of tankers’ routing with the aim of reducing the cost of petroleum products delivery. The proposed transportation linear programming model was applied to a numerical example alongside other existing transportation algorithms. It is observed that, the new algorithm produced approximately the same total cost obtained by using other existing algorithms.

Keywords: Petroleum products, Transportation, Tankers’-routing, Linear programming, Refinery

INTRODUCTION
Transportation models are a type of linear programming problems (LPP). According to Ogumeyo and Panya (2024), transportation problems involve selection of minimum cost routes in which goods and services are shipped from a supply centre (origin) to demand centre (destination). Transportation problems formulated as LPPs are usually solved by using simplex algorithms. As reported in Gupta and Hira (2005), methods such as stepping stone and modified distribution have been developed to simplify the complexity involved when transportation problems are formulated as linear programming problems due to numerous variables and constraints. Gupta and Hira (2005) defines transportation problem as a linear programming problem which involves identifying minimum cost routes in delivering goods and services from their places of origin to destination. Transportation cost involves computation of entire cost of transporting people, products and materials from their sources to destinations or customers.

According to Salami (2014), the movement of goods from several sources to their destinations where they are needed belong to a category of linear programming known as transportation problem. Transportation problem was first presented and discussed by Hitchcock in 1941 as stated in Salami (2014). Thereafter, Koopman (1947) developed a transportation system which involves using linear programming to determine optimum solution. Dantzig (1951) expanded the idea of linear programming to determine optimum solution of transportation problem involving complex variables and constraints. In recent times, transportation of petroleum products involving tankers’ routing has been extensively applied to several models of the oil industries. For examples, a tanker routing model which uses a discrete integer programming approach to determine efficient and effective distribution of petroleum products is developed in Agra et al. (2013) and was later expanded in Xu et al. (2021) model. Diz et al. (2017) applies tankers’ routing model to a petroleum distribution pattern in Brazil while a model which describes downstream petroleum supply chain is reported in Kazemi and Szmerekovsky (2015). Similar models involving tanker routing are found in Rodrigues et al. (2016) and Stanzani et al. (2018).

The sum total cost on commodities delivery from sources to destinations matters a lot to production companies because of the direct impact it has on their profits as contained in Quan et al. (2018) and Shvetsov (2021). Hence, only minimum cost routes are selected during transportation process. Naqurney (2004) and Agureev and Akhromeshin (2020) state that a LPP technique can be used to solve transportation problems when the cost of taking a route only depends on the flow on that route. In this research, we aim at discussing transportation model in general and then apply it to tankers’-routing problem via linear programming techniques. The problem of tankers’ routing was first investigated in Dantzig and Fulkerson (1954). Their model aims at finding the least number of tankers required to satisfy a fixed schedule in the delivery of petroleum products. Salami (2014) extended their tankers’-routing model by developing a transportation linear programming algorithm to determine minimum cost routes in the distribution of soft drinks in Nigeria. Major challenges faced by government in transporting both crude and refined petroleum products through pipelines are caused by the activities of illegal refiners and pipeline vandals. Nigerian Government budgets billions of naira annually to protect oil facilities and ensure uninterrupted supply of petroleum products as a part of her economic policy to boost the nation’s revenue, Awariefe and Ogumeyo (2023).

This research aims at extending a tanker routing model which uses a discrete integer programming approach to determine efficient and effective distribution of petroleum products developed in Agra et al. (2013) and was later expanded in Xu et al. (2021) models by adding detail discussion to the
concepts they used in order to formulate a new model of transportation linear programming which determines minimum cost routes in tankers'-routing problem. The significance of the application we adopted in this research lies in the modified distribution approach to tackle the complexity involved when transportation problem (especially tankers'-routing) are formulated as linear programming problem due to the presence of several variables and constraints.

**MATERIALS AND METHODS**

**Description of the Model**

Let \( i = 1, 2, 3, \ldots, m \) be the number of loading depots of the petroleum products, at which tankers are loaded for deliveries to discharge centers, \( j = 1, 2, 3, \ldots, n \). A time at which a tanker is to be loaded at \( i \) depot for making a delivery to \( j \) destination is fixed or known. The time duration it takes a tanker to move from any depot to another is also assumed to be known. In this model, we shall consider the case where all tankers are identical (so that they are interchangeable) and where deliveries are quantified in units of tanker capacity so that a tanker must make a full delivery. In other words, a tanker cannot make part delivery. Having made a delivery, a tanker can travel to any depot if it can get there in time to pick up another cargo for some selected destinations.

**Model Assumptions**

Assumptions associated with the model formulation and operations are as follow:

i. The product/commodity being transported must be identical such that customers can accept them from any source.

ii. The number of tankers to be supplied and demanded is known otherwise a non-existing depot which takes excess supply of the product at no cost is added.

iii. The cost of shipment through each route is known.

iv. The tankers are available at the beginning of the period wherever they are needed irrespective of where they end up at the end of the period.

**Methodology**

The methodology applied includes:

i. Russell’s Approximation Method (RAM)

ii. Linear Programming Problem (LPP)

iii. Vogel’s Approximation Method (VAM)

iv. Transportation Algorithm/ Northwest Corner Rule

The objective of the study is to formulate a transportation model which determines the minimum cost of shipping petroleum products from depots (Refinery centers) to their demand centers (filling stations).

**Mathematical Notations/variables**

The variables used in this research are defined as follows:

- \( S_i \) = number of supply centre for \( i = 1, 2, \ldots, m \)
- \( D_j \) = number of demand centre for \( j = 1, 2, \ldots, n \)
- \( X_{ij} \) = number of tankers transported from supply centre \( i \) to destination \( j \)
- \( C_{ij} \) = transportation cost per tanker.

**Transportation Model Formulation**

The mathematical expressions of the transportation model described above can be stated thus:

\[
\begin{align*}
\text{Min } H & = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}. \\
\sum_{j=1}^{n} X_{ij} & \leq S_i, \text{ for } i = 1, 2, \ldots, m \text{ (supply constraint)} \\
\sum_{i=1}^{m} X_{ij} & \geq D_j, \text{ for } j = 1, 2, \ldots, n \text{ (demand constraint).} \\
X_{ij} & \geq 0 \text{ (Nonnegative constraints).}
\end{align*}
\]

In such cases, we employ at most \((m+n-1)\) routes of feasible transportation schedule. Where the quantity supplied and the quantity demanded are the same then, we have a balanced transportation problem. This can mathematically be expressed as:

\[
\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j.
\]

If \( \sum_{i=1}^{m} S_i \geq \sum_{j=1}^{n} D_j \), equation (6) implies that supply is either equal to or greater than demand.

If the supply and the demand are not the same in quantity then, the transportation problem TP is unbalanced. This can be expressed mathematically as:

\[
\sum_{i=1}^{m} S_i \neq \sum_{j=1}^{n} D_j.
\]

**Theorem 1**: (Existence of Feasible Solution). A necessary and sufficient condition required for the transportation problem to have a feasible solution, is that the quantity supplied must be equal to quantity demanded. That is \( \sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j \).

**Proof**: (a) Necessary Condition. Suppose, the solution that is feasible in a TP exists, then we shall have

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} = \sum_{j=1}^{n} S_j \quad \text{(8)}
\]

\[
\sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} = \sum_{j=1}^{n} D_j \quad \text{(9)}
\]

Equation (8) represents the quantity supplied while equation (9) is the demanded quantity. Since equation (8) and equation (9) are the same, it means \( \sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j \).

(b) Sufficient condition. Assuming the quantity supplied is equal to demanded quantity, then

\[
\sum_{i=1}^{m} X_{ij} = \sum_{j=1}^{n} D_j = h \text{(say)}, \quad \text{(10)}
\]

Suppose \( k (k \neq 0) \) is a real number such that \( X_{ij} = k_i D_j \neq 0 \) for all \( i \) and \( j \), then the value of \( k_i \) is given by

\[
\sum_{j=1}^{n} X_{ij} = \sum_{j=1}^{n} S_j = k_i \sum_{j=1}^{n} D_j = k_i h \text{ or } k_i = \frac{h}{\sum_{i=1}^{n} D_j}.
\]

Thus

\[
X_{ij} = k_i D_j = \frac{S_j}{h}\text{ for all } i \text{ and } j. \quad \text{(11)}
\]

Since \( S_i > 0 \) and \( D_j > 0 \) for every \( i \) and \( j \), then \( S_i D_j / h \geq 0 \) and hence a solution which is feasible exists, that is \( x_{ij} > 0 \).

**Theorem 2**: (Basic Feasible Solution). In any feasible solution, we have \( m + n - l \) basic variables and \( m + n - l \) independent constraints, where \( m \) rows is the supply constraint and \( n \) columns is the demand constraint equations.

**Proof**: In all mathematical formulations of transportation problems including the ones discussed in Sharma (2006) and Ekoko (2011), it is observed that, if \( m \) rows which represent supply constraints and \( n \) columns which represent demand constraints exist, then, we will have a total of \( m + n \) constraints. But due to Theorem 1 which states that the quantity supplied and the quantity demanded must be equal out of \( m + n \) constraint equations, one of the equations must be unused and consequently, removed. Hence, we have \( m+n-1 \) equation which are linearly independent. We can prove this if we add all the equations of the \( m \) rows and deducting from the sum of the first \( n-l \) column equations, consequently obtaining the last column equation. That is,

\[
\begin{align*}
\sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} & = \sum_{i=1}^{m} S_i - \sum_{j=1}^{n} D_j \\
\sum_{i=1}^{m} S_i & = \sum_{j=1}^{n} D_j
\end{align*}
\]

\[
\sum_{i=1}^{m} X_{ij} = D_j, \quad \text{since } \sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j.
\]

Unbalanced Transportation Problem

A necessary condition required for the existence of a feasible solution, is that the quantity supplied and quantity demanded must be equal as earlier stated in equation (6). That is,
\[\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j.\]

But that is not always true, because there could be cases in real life situations where supplied quantity does match the quantity required. Two cases can be derived from an unbalanced TP.

Case 1: If the supply is higher than demand, the constraints of the transportation problem can be mathematically stated as:

\[\sum_{j=1}^{n} X_{ij} \leq S_i \quad \text{for } i = 1, 2, \ldots, m.\]  

\[\sum_{i=1}^{m} X_{ij} = D_j \quad \text{for } j = 1, 2, \ldots, n.\] 

\[X_{ij} \geq 0 \quad \text{for every } i, j.\] 

If we add \(h_{i+1}\) as slack variable for \((i = 1, 2, \ldots, m)\) in the first \(m\) equations, we obtain

\[\sum_{j=1}^{n} X_{ij} + h_{i+1} = S_i \quad \text{for any } i = 1, 2, \ldots, m.\] 

\[\sum_{i=1}^{m} \left[\sum_{j=1}^{n} X_{ij} + h_{i+1}\right] = \sum_{i=1}^{m} S_i - \sum_{j=1}^{n} D_j \quad \text{available excess supply}\] 

If we proceed further to denote the available excess supply by \(D_{a1}\) then, the modified version of the TP can be written as:

Minimize \(H' = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} + H \sum_{i=1}^{m} h_{i+1}\) .  

Subject to \(\sum_{j=1}^{n} X_{ij} = S_i \quad \text{for } i = 1, 2, \ldots, m\) .  

\(\sum_{i=1}^{m} X_{ij} = D_j \quad \text{for } j = 1, 2, \ldots, n\) .  

\(X_{ij} \geq 0 \quad \text{for every } i, j, i=1,2,3\ldots m+1\)  

\(X_{ij} = 0 \quad \text{for all } j, i \neq 1, j=1,2,3\ldots n\) 

\(C_{ij} = 0 \quad \text{for all } i, j \neq 1, 1\ldots n\) 

\(\sum_{j=1}^{n} X_{ij} = S_i \quad \text{for } i = 1, 2, \ldots, m\) 

\(\sum_{i=1}^{m} D_j = D_j \quad \text{for } j = 1, 2, \ldots, n\) 

Where \(S_i\) and \(D_j\) are positive numbers satisfying the supply and demand constraints respectively. The mathematical analysis presented in equation (17)-(20) implies that, if the quantity supplied is higher than the quantity required, then a dummy column (demand centre) which absorbs the surplus supply is added to the TP table. Then transportation cost per unit for the cells in this column is equated to zero. (See Ekokoh (2011) and Ogumeyo and Panya (2024)).

Case 2: If demand is higher than the quantity supplied, the equation of constraints of the TP will appear as

\[\sum_{i=1}^{m} X_{ij} = D_j \quad \text{for } j = 1, 2, \ldots, n.\] 

\[\sum_{j=1}^{n} X_{ij} \leq S_i \quad \text{for } i = 1, 2, \ldots, m.\] 

\[X_{ij} \geq 0 \quad \text{for every } i, j.\] 

If we add slack variables \(h_{m+1}\) for \(j = 1, 2, \ldots, n\) in the last \(n\) constraints, we obtain

\[\sum_{i=1}^{m} X_{ij} = S_i \quad \text{for } i = 1, 2, \ldots, m.\] 

\[\sum_{i=1}^{m} X_{ij} + h_{m+1} = D_j \quad \text{for } j = 1, 2, \ldots, n.\] 

\[\sum_{j=1}^{n} h_{m+1} = \sum_{i=1}^{m} D_j - \sum_{i=1}^{m} S_i \quad \text{surplus demand}\] 

If we denote the surplus demand by \(h_{m+1}\) then, we can rewrite the modified transportation problem as:

Minimize \(H' = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} + H \sum_{i=1}^{m} h_{m+1}\) .

Subject to \(\sum_{j=1}^{n} X_{ij} = S_i \quad \text{for } i = 1, 2, \ldots, m+1\) .

\(\sum_{i=1}^{m} X_{ij} + h_{m+1} = D_j \quad \text{for } j = 1, 2, \ldots, n\) .

\(X_{ij} \geq 0 \quad \text{for every } i, j, i=1,2,3\ldots m+1\)  

\(X_{ij} = 0 \quad \text{for all } j, i \neq 1, j=1,2,3\ldots n\) 

\(C_{ij} = 0 \quad \text{for all } i, j \neq 1, 1\ldots n\) 

\(\sum_{j=1}^{n} X_{ij} = S_i \quad \text{for } i = 1, 2, \ldots, m\) 

\(\sum_{i=1}^{m} D_j = D_j \quad \text{for } j = 1, 2, \ldots, n\) 

Where \(S_i\) and \(D_j\) are positive numbers satisfying the supply and demand constraints respectively. The mathematical analysis presented in equation (17)-(20) implies that, if the quantity supplied is higher than the quantity required, then a dummy column (demand centre) which absorbs the surplus supply is added to the TP table. Then transportation cost per unit for the cells in this column is equated to zero. (See Ekokoh (2011) and Ogumeyo and Panya (2024)).

### Table 1 Technology table for transportation problem

<table>
<thead>
<tr>
<th>U_{11}</th>
<th>U_{12}</th>
<th>U_{13}</th>
<th>U_{14}</th>
<th>U_{21}</th>
<th>U_{22}</th>
<th>U_{23}</th>
<th>U_{24}</th>
<th>U_{31}</th>
<th>U_{32}</th>
<th>U_{33}</th>
<th>U_{34}</th>
<th>= B_{1}</th>
<th>= B_{2}</th>
<th>= B_{3}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
<td>= M_{1}</td>
<td>= M_{2}</td>
<td>= M_{3}</td>
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<td>1</td>
<td>1</td>
<td>= M_{4}</td>
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</tr>
</tbody>
</table>

The dual linear programming model corresponding to Table 1 is displayed in Table 2. Letters P and R are used to represent the dual variables corresponding to the supply and demand partition Table 1. The duality of the TP in Table 1 can be expressed as...
U_1 S_1 = X U_2 Q, then the reduced transportation

center 1 cost N7/tanker and N5/tanker from

S_1 B_1 D_1 X_1 Q

SUPPLY = \sum U_1 X_1 M

C_B S_1 = Q_1 U_2

In the next section, we present an algorithm known as

Transportation Algorithm

which we shall apply to a numerical

solution in Section 4.

Transportation Algorithm

This method is sometimes referred to as transportation

simplex method. It is a method of cross checking if the current

solution to a transportation problem is optimal or can be

improved. The algorithm is as follows:

Step 1: Associate with row i of the transportation table a

multiplier U_i and with each column j a multiplier V_j,

Minimize

C = \sum C_{ij} X_{ij}

Subject to

\sum_{j=1}^{m} X_{ij} = S_i for i = 1 (1)m, Supply. (35)

\sum_{i=1}^{n} X_{ij} = D_j for j = 1 (1)n, Demand. (36)

X_{ij} \geq 0 for i = 1 (1)m, j = 1 (1)n, non-negativity condition. (37)

But the sum of the m equations in (35) is equal to the sum of

the n equations in (36). This shows that one linear constraint

is redundant i.e. equations (35) and (36) are not linearly

independent. We can therefore, delete any one of the m + n

linear constraints in (35) and (36). If we delete say the

constraint: \sum_{j=1}^{n} X_{ij} = S_i, then the reduced transportation

problem becomes:

Minimize C = C_{11} X_{11} + C_{12} X_{12} + \ldots + C_{33} X_{33}. (38)

Subject to

\sum_{j=1}^{n} X_{ij} = S_i for i = 2 (1)m. (39)

\sum_{i=1}^{n} X_{ij} = D_j for j = 1 (1)n. (40)

X_{ij} \geq 0. (41)

In the next section, we present an algorithm known as

Transportation Algorithm which we shall apply to a numerical

solution in Section 4.

Numerical Illustration

A refinery has three depots (supply centers) and three demand

centers as shown in the table below. Transportation cost from

any depot to any demand center is fixed in tens of thousands

of naira (N10,000s) per tanker. Transport from supply center

1 to demand center 1 cost N7/tanker and N5/tanker from

supply center 1 to demand center 2 and so on. The full data of

the transport cost from each supply center to each demand
center per tanker is summarized in Table 4 as follows:

Table 2: Duality of the TP

<table>
<thead>
<tr>
<th>R_1</th>
<th>R_2</th>
<th>R_3</th>
<th>P_1</th>
<th>P_2</th>
<th>P_3</th>
<th>P_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>1</td>
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<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>B_1</td>
<td>B_2</td>
<td>B_3</td>
<td>M_1</td>
<td>M_2</td>
<td>M_3</td>
<td>M_4</td>
</tr>
</tbody>
</table>

By applying the procedure described above, we have

Table 3: Supply and Demand Cost Structure

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>SUPPLY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Q_{11} U_{11}</td>
<td>Q_{12} U_{12}</td>
<td>Q_{13} U_{13}</td>
</tr>
<tr>
<td>2</td>
<td>Q_{21} U_{21}</td>
<td>Q_{22} U_{22}</td>
<td>Q_{23} U_{23}</td>
</tr>
<tr>
<td>3</td>
<td>Q_{31} U_{31}</td>
<td>Q_{32} U_{32}</td>
<td>Q_{33} U_{33}</td>
</tr>
</tbody>
</table>

Dd M_1 M_2 M_3

Using Table 3, the TP can be expressed as follows:

Minimize Z = C_{11} X_{11} + C_{12} X_{12} + \ldots + C_{33} X_{33} = \sum \sum C_{ij} X_{ij}

For each basic variable X_{ij} in the current solution, set
C_{ij} = U_i + V_j. This gives m + n – 1 basic variable in
m + n unknown. The value of the multiplier U_i and
V_j are obtained from the simultaneous equations by
assigning an arbitrary value to any one of the
multipliers. (Often we set U_i with highest number of
basic variables equal to zero. Break ties arbitrarily.

Step 2 For each non – basic X_{pq} variable, calculate W_{pq} = C_{pq} – (U_p + V_q). W_{pq} gives the net increase or decrease in
the quantity of the objection function (total
transportation cost) as a result of increasing X_{pq} above
zero level. The non – basic variable which will yield
the largest per unit decrease in the cost is selected as
the entry variable. If all W_{pq} > 0, stop, otherwise go
to the next step

Step 3 Utilize the new route as fully as possible whilst still
satisfying the supply and demand requirement. The quantities of X_{pq} are chosen so that the quantity of one
basic variable in the present solution is reduced to
zero.

Step 4 Determine the new solution and go to step 2.

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Table 4. Supply and Demand Quantities

<table>
<thead>
<tr>
<th>CUSTOMERS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Supply source</th>
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<tr>
<td>1</td>
<td>7</td>
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<td>5</td>
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<td>2</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Demand centers</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

Use a transportation linear programming algorithm to determine the minimum cost routes tankers should take in delivering the products required at each depot.

**Solution:** By applying our model in section 3.0 to Table 4, the objective function and its constraints can be written as

Table 5: Transportation Problem in LP Form

<table>
<thead>
<tr>
<th>S.V.</th>
<th>X_{11}</th>
<th>X_{12}</th>
<th>X_{13}</th>
<th>X_{21}</th>
<th>X_{22}</th>
<th>X_{23}</th>
<th>X_{31}</th>
<th>X_{32}</th>
<th>X_{33}</th>
<th>Supply source</th>
</tr>
</thead>
<tbody>
<tr>
<td>S_1</td>
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<td>6</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>S_2</td>
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<td>X_{11}</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>X_{12}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X_{13}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Recall that S_i’s are artificial variables which are added to obtain an (m + n - 1) x (m + n - 1) matrix which is needed in the initial computational form.

Thus, the tableau 1.1 is the reduced transportation problem in Table 1.

Table 1. Initial Tableau

<table>
<thead>
<tr>
<th>C_i</th>
<th>S_1</th>
<th>S_2</th>
<th>X_{11}</th>
<th>X_{12}</th>
<th>X_{13}</th>
<th>X_{21}</th>
<th>X_{22}</th>
<th>X_{23}</th>
<th>X_{31}</th>
<th>X_{32}</th>
<th>X_{33}</th>
<th>R.H.S</th>
</tr>
</thead>
<tbody>
<tr>
<td>S_1</td>
<td>-7</td>
<td>-5</td>
<td>-6</td>
<td>-3</td>
<td>-4</td>
<td>-3</td>
<td>-2</td>
<td>-5</td>
<td>-1</td>
<td>-M</td>
<td>-M</td>
<td></td>
</tr>
<tr>
<td>-M S_2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>-7 X_{11}</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>-5 X_{12}</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>-6 X_{13}</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>C_0 - Z_j</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>M+4</td>
<td>M+1</td>
<td>M+3</td>
<td>M+5</td>
<td>M</td>
<td>M+5</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Since X_{13} and X_{13} has the largest simplex criterion, we can choose either X_{13} or X_{33} as pivot column. Choosing X_{33} arbitrarily the pivot fractions (r_i) are r_{22} = \frac{1}{3}, r_{21} = \frac{1}{3}, r_{33} = \frac{1}{3}. Since r_{22} is the smallest nonnegative fraction, row 2 becomes the pivot row.

Table 2: First iteration using the Big M Method

<table>
<thead>
<tr>
<th>C_i</th>
<th>S_1</th>
<th>S_2</th>
<th>X_{11}</th>
<th>X_{12}</th>
<th>X_{13}</th>
<th>X_{21}</th>
<th>X_{22}</th>
<th>X_{23}</th>
<th>X_{31}</th>
<th>X_{32}</th>
<th>X_{33}</th>
<th>R.H.S</th>
</tr>
</thead>
<tbody>
<tr>
<td>-M S_1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>-2X_{11}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-7 X_{12}</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>-5 X_{13}</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>-6 X_{13}</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>C_0 - Z_j</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>M+4</td>
<td>M+2</td>
<td>M+3</td>
<td>0</td>
<td>-5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>M-5</td>
</tr>
</tbody>
</table>

In Tableau 3 above, X_{11} has largest value in the objective row function, hence the entering non-basic variable is X_{11}, the row fractions are r_{11} = \frac{6}{6} = 6, r_{13} = \frac{2}{6} = 3. Hence the departing basic variable is X_{11} since it corresponds to the smaller ratio.

Tableau 4: Tableau 5: Tableau 6

Thus the pivot row is X_{11}. While X_{12} is the pivot column. Since there are still nonnegative values in the objective row function, the current solution is not optimal. Thus, by following the same procedure, we obtained Tableau 4 and 5 below.
Table 3: Second iteration using the Big M Method

<table>
<thead>
<tr>
<th>C1</th>
<th>X11</th>
<th>X12</th>
<th>X13</th>
<th>X21</th>
<th>X22</th>
<th>X31</th>
<th>X32</th>
<th>X33</th>
<th>S1</th>
<th>S2</th>
<th>R.H.S</th>
</tr>
</thead>
<tbody>
<tr>
<td>-M</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-2X31</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-7X11</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>-5X12</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>-6X13</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cj-Zj</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>M+2</td>
<td>M+3</td>
<td>0</td>
<td>-5</td>
<td>M-2</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

Table 4. Third iteration using the Big M Method

<table>
<thead>
<tr>
<th>C1</th>
<th>X11</th>
<th>X12</th>
<th>X13</th>
<th>X21</th>
<th>X22</th>
<th>X31</th>
<th>X32</th>
<th>X33</th>
<th>S1</th>
<th>S2</th>
<th>R.H.S</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3X23</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-2X31</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-3X21</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>-5X12</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>-6X13</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>Cj-Zj</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-8</td>
<td>-6</td>
<td>0</td>
<td>-M-3</td>
<td>-M-4</td>
</tr>
</tbody>
</table>

Since all the terms in the objective function are non-positive, the optimal solution has been obtained. The optimal solution is

\[
X_{23} = 3, X'_{31} = 1, X'_{21} = 3, X'_{12} = 3, X'_{13} = 2,
\]

The optimal solution is

\[
X_{11} = X_{22} = X_{12} = S_1 = S_1 = S_2 = 0
\]

By substituting the above values into the objective function we have

\[
\text{Total Cost} = (N30000 \times 3) + (N20000 \times 1) + (N30000 \times 3) + (N50000 \times 3) + (N60000 \times 2)
\]

\[
C = N470,000
\]

Solution by Russell’s Approximation Method (RAM)

First iteration using the RAM, the initial basic feasible solution for Table 1

Table 7: First iteration using the RAM

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
V_1 = 5, \ V_2 = 5, \ V_3 = 6, \ \text{and} \ X_{11} = 3, X_{12} = 2, X_{21} = 3, X_{22} = 3, X_{31} = 1.
\]

We examine if the current solution is optimal

Set \( C_j = U_j + W_i \) for all basic variables

\[
\begin{align*}
5 &= U_1 + V_2 \\
6 &= U_1 + V_3 \\
3 &= U_2 + V_1 \\
3 &= U_2 + V_3 \\
2 &= U_3 + V_1 \\
0 &= U_1 = 0
\end{align*}
\]

\[
W_{pq} = C_{pq} - (Up + Vq)
\]

\[
\begin{align*}
W_{11} &= C_{11} - (U_1 + V_1) = 7 - (0 + 5) = 2 \\
W_{21} &= 4 - (-3 + 5) = 2 \\
W_{31} &= 5 - (-3 + 5) = 2 \\
W_{32} &= 1 - (-3 + 6) = -2
\end{align*}
\]

Since \( W_{pq} < 0 \) for some \( p \) and \( q \), the solution is not optimal.

Therefore compute the second iteration. \( W_{31} \) is the entering variable since it yields largest per unit decrease.
The transportation problem presented in Section 4.0, was solved by using the Big-M algorithm of the simplex method. The optimal solution is found in Table 4 which is the third iteration using the Big M Method. The basic variables which contributed to the optimal solution are

\[ (x'_{23} = 3, x'_{31} = 1, x'_{21} = 3, x'_{12} = 3 \text{ and} x'_{13} = 2) \]  

and the non-basic variables are

\[ X_{11} = X_{22} = X_{32} = S_1 = S_2 = S_3 = 0 \]

By substituting the above values into the objective function, we have: Total Transportation Cost = \((N30,000 \times 3) + (N20,000 \times 1) + (N30,000 \times 3) + (N50,000 \times 3) + (N60,000 \times 2)\) = N470,000. This implies that the total minimum cost is N470,000, since the cost of the transportation is tens of thousands of naira per tanker. In other words, 3 tankers should be transported to depot X_{21} at the cost of N30,000 per tanker, 1 tanker the depot X_{11} at the cost N20,000, 3 tankers to depot X_{23} at the cost of N30,000 per tanker, 3 tankers to depot X_{12} at the cost N50,000 per tanker and 2 tankers to depot X_{13} at the cost N60,000 per tanker.

From the results obtained by using Russel Approximation Method, the basic variables which contributed to the objective function are

\[ X'_{23} = 3, X'_{31} = 2, X'_{21} = 4, X'_{12} = 3, X'_{33} = 1, \]

at the cost \(N1,000 \text{ per tanker}.\)

Hence, the optimal total cost of the transportation problem using Russel Approximation Method is:

\[ (N50,000 \times 3) + (N60,000 \times 2) + (N30 \times 4) + (N30,000 \times 2) + (N10,000 \times 1) = N460,000. \]

This optimal solution was obtained after the third iteration in Table 9.

**CONCLUSION**

The need for efficient distribution of petroleum products from refineries (supply centers) to demand centers cannot be over-emphasized considering their domestic and industrial importance in our daily activities. In this study, we have presented a transportation model which is being formulated as LPP, and consequently applied to solve a Tanker-Routing Problem of distribution of petroleum products from supply center (refinery depots) to demand center (filming stations). The aim of the study is to select minimum cost routes that will reduce the cost of shipping petroleum products from refinery depots to filling stations. Other methods of transportation model for determination of minimum cost routes such as NCR, VAM and RAM are being discussed. But the emphasis is on RAM and its algorithm. This is because we intend to use it to cross check the results we obtained from the Big-M method of LPP. It is observed that the new approach we presented, gives just one unit higher than the ones obtained by using NCR, VAM and the RAM. While the minimum total transportation cost obtained by using the Big-M approach is N470,000, that of the NCR, VAM and the RAM is N460,000.

**REFERENCES**


