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A NEW DUAL HEURISTIC ALGORITHM FOR FINDING THE INITIAL BASIC FEASIBLE SOLUTION FOR A TRANSPORTATION PROBLEM

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ABSTRACT

Because determining the best initial basic feasible solution (IBFS) for a transportation problem is so crucial, numerous authors have expended a great deal of energy developing effective algorithms that will result in the lowest possible cost of moving products from a given source to a destination. The goal of this work was to develop an efficient dual algorithm for finding an initial basic feasible solution to a transportation problem (TP). Two distinct algorithms that produce the same IBFS make up our suggested approach. Compared to some popular methods in the literature, Using four numerical examples, the Row Minimum Method (RMM), Column Minimum Method (CMM), Least Cost Method (LCM), Extremum Difference Method (EDM), Northwest Corner Method (NWCM), Vogel's Approximation Method (VAM), etc. In comparison to the other heuristic techniques compared with the optimal dictate solution modified distribution (MODI), the proposed heuristic approach (PS-DESPAN) approximation was shown to provide a better starting solution (a solution that is extremely close to the optimal solution).

Keywords: Feasible solution, Initial basic feasible solution, Optimum solution, Heuristic Transportation models, Source, Destination, Transportation unit cost, Allocation

INTRODUCTION

One of the most recent issues that has impacted businesses, enterprises, and organizations is the issue of high transportation costs resulting from the increase in the price of premium motor spirit (PMS) in Nigeria. This has, in effect, influenced the cost of manufacturing and transportation. It is necessary to find ways to supply products at comparatively low costs in order for businesses and organizations to lower their overall transportation costs while meeting supply and demand constraints without sacrificing product quality or cost (materials cost, transportation cost, fuel cost, packing cost, maintenance cost, advertisement cost and transportation cost). The only way to address these anomalies is to identify reliable and effective models that will lower the cost of transportation. Because of the rising cost of living brought on by rising food and fuel prices, among other factors, the cost of moving people, products, and services has increased alarmingly in recent years. It is crucial to note that transportation plays a significant role in human activity since it facilitates and promotes social and economic interactions. One field of operations research is transportation problems, which have many applications in resource allocation, inventory control, production planning, scheduling, logistics, and supply chain management to lower costs and improve services. Organizations are constantly concerned with determining the appropriate and most cost-effective mode of transportation, the quantity to be given, and the area to be delivered.

The goal is to keep the cost of transporting goods between different places to a minimum while meeting the needs of each arrival area and making sure every supply location is working to capacity. Organizations are under tremendous pressure to find more efficient and cost-effective ways to develop, plan, purchase, and provide goods and services to clients in the highly competitive market of today, when the cost of labor, fuel, and raw materials is also rising. It gets harder to decide how and when to deliver goods to clients in the amounts they desire while staying inside a tight budget. The goal of transportation models is to solve this problem.

Because transportation issues are among the most urgent, serious, and strategic difficulties facing many businesses and organizations, and because they can only be resolved by a transportation algorithm that is robust, consistent, reliable, and all-encompassing. Organizations occasionally choose to make these choices based only on common sense or intuitive reasoning, eschewing the use of heuristic or quantitative models but it is undisputable that, reliability and efficiency in the initial basic feasible solution (IBFS) can only be achieved by using a scientific methodology.

A transportation challenge is usually solved by starting with the identification of a basic, workable solution and working methodically through better revisions until the ideal solution is reached. Essentially, there are two steps involved in identifying the best solution for transportation issues. One, the application of available heuristic techniques, such as Extremum difference approach (Kassana and Kumar,2005), Vogel's Approximation (Hamdy, 2007), Row Minima, Column Minima, Highest Cost Difference (Khan, 2012), and Column Pointer approach (Khan, 2012) the first stage of the feasible solution is determined while using the Modified Distribution technique or Stepping Stone, the feasible solution is examined for optimality (Charnes and Cooper, 1954).

Numerous researches have shown that there is no one optimal approach for determining the first fundamental solution of a transportation problem, despite the Vogel approximation being often considered to be superior its weaknesses were reported to be computational complexity and time wastage. While some of these heuristic models are capable of finding a basic, workable solution quite fast, the solutions they produce are frequently not very effective at minimizing the overall cost. On the other hand, some heuristics might take longer to locate a fundamental workable solution but good at decreasing overall costs. Therefore, a balance between finding a workable solution as soon as possible and obtaining an ideal workable solution must be struck.

This study has jumped on the bandwagon of discovering efficient methods using some of the heuristics that are now accessible to generate the lowest cost of transferring a consignment from a source to a destination in less time and with less computational effort. By creating a better heuristic method that produces a better solution than the widely used methodologies in the literature, this work closed this gap. The Vogel approximation method (VAM) and the least cost method have seen the most recent adjustments to the transportation problem; nevertheless, as the VAM is one of the computationally hard methods, the majority of the modifications are likewise computationally laborious. The concept of least cost was utilized by the novel algorithm provided in this study. Numerical comparisons were made between the suggested approach and other heuristic models.

Numerous writers have put up solutions to the transportation dilemma. Hitcock created the least squares method in 1941, which is a systematic approach to solving transportation problems. It involves assigning as much of the problem's total cost to the cell with the lowest cost.

Another model, known as the north-west corner method, was created by Charnes and Cooper in 1953. It begins allocation from the transportation problem's upper top corner cell. The Vogel approximation approach was later introduced by Reinfield and Vogel in 1958, provides a more satisfactory first basic viable solution than both the least cost and the north-west corner methods. Few reported literatures about the heuristic transportation models include:

Khan (2012) presented another approach called the highest cost difference method (HCDM) defined by use of a "pointer cost," or the difference between the highest cost and the next highest cost in each of the rows and column of the transportation table of the problem and make allocations to the cell with the smallest cost corresponding to the highest three pointer cost.

In a similar vein, Hussain and Ahmad (2020) introduced a novel technique known as the least cost mean method, which uses the mean of the lowest or next lowest row or column in the cost matrix to extract a row and column penalty and produce a better initial basic feasible solution.

Another transportation model known as the total opportunity cost matrix was created by Kirca and Satir (1990). It is calculated by adding the opportunity cost for each row and each column.

Karagul-Sahin approximation method was also devised to determining the initial fundamental feasible solution of a transportation problem. This method's first basic workable answer was contrasted with six more widely used techniques with 24 examples from the literature. In 17 of the 24 situations, the suggested strategy provided the most optimal preliminary possible answer with the least amount of computation. In the end, they stated that their approach is both simpler like the Vogel approximation method (Karagul and Sahin ,2020)

MATERIALS AND METHODS

Method of Data Analysis

The analysis of finding the IBFS was done manually and some with the aid of software's like: LINGO (version 19.0), AZMATH and Tora. Software. The data used for this study is a secondary data collected from the literatures. See appendix.

The general mathematical model of transportation problem

Let there be M sources of supply $S_1, \ S_2, \ . \ . \ ., \ S_m$. having a_i (I = 1,2,3,…..m) units of supply (or capacity), respectively to be transported to *n*-destinations $D_1, \ D_2, \ . \ . \ . \ D_n$ with b_i (*i*= 1,2,3,….n) units of demand respectively. Let C_{ij}be the cost of shipping one unit of the commodity from the source *i* to the destination *j*. If X_{ij}represents the number of units shipped from source *i* to destination *j*. Mathematically, the transportation problem can be stated as follows:

Objective Function

Minimize (Total cost) Z = $\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\, X_{ij}$ (1)

Subject to the constraints

$\sum_{j=1}^{n} X_{ij} = a_{ij}, i = 1, \ 2, \ 3, \ldots\ldots, \ , \ m$

(Supply Constraints). (2)

$\sum_{i=1}^{m} X_{IJ} = b_{ij}, \ j = \ 1, \ 2, \ 3, \ldots\ldots, \ n$

(Demand Constraints). (3)

$\sum_{j=1}^{n} X_{ij} \geq D_j$ for *i* = 1,2, 3...*n*

$Wih\ all\ X_{ij} = 0$

First, the total amount of products to be transported from the source could not be greater than its supply; second, the total amount of items to be delivered to a destination could not be greater than its demand. When the overall supply is less than the total demand, there is an imbalanced transportation case. i.e.

$\sum_{I=1}^{m} S_I \neq \sum_{J=1}^{n} D_i$ (4)

This can be fixed by setting up a phantom source or destination for each instance in which supply outpaces demand. For any fictitious source or destination, the unit transit cost is $0. In order to streamline the process of identifying the best option, a transportation tableau is created. The sources, destinations, supply, demand, unit cost of transportation, and allocations for transportation are all included in this table.

Table 1: Transportation Table of the Transportation problem

	D_1	D_2	…………………	D_n	Supply a_i
S_1	C_{11}	C_{12}		C_{1n}	a_1
	X_{11}	X_{12}	…………………	X_{1n}	
S_2	C_{21}	C_{22}		C_{2n}	a_2
	X_{21}	X_{22}	……………………	X_{2n}	
.	.	.	.	.	.
.	.	.		.	.
.	.	.		.	.
S_m	C_{m1}	C_{m2}	………………….	C_{mn}	a_m
	X_{m1}	X_{m2}		X_{mn}	
Demand			…………………		
b_i	b_1	b_2		b_n	$\sum_{I=1}^{m} S_I = \sum_{J=1}^{n} D_i$

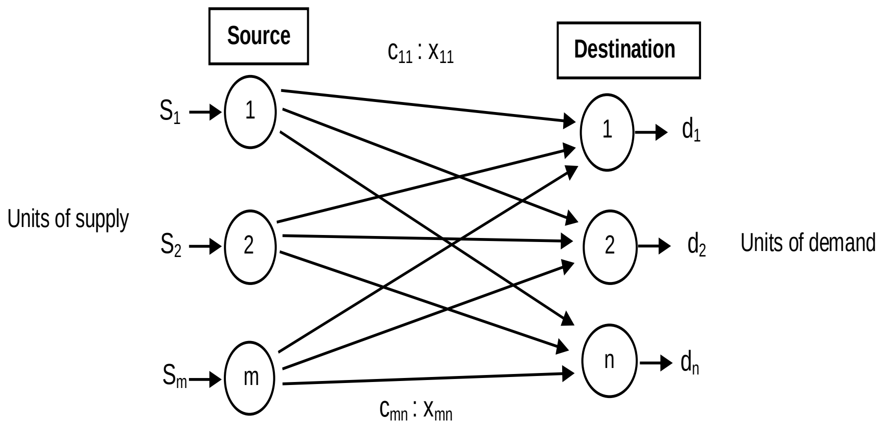

Figure 2: Network Representation of Transportation Problem

In the generalized model above there are (*m+n*) constraints, one for each source of supply. Since all (*m+n*) constraints are equations, therefore one of this equation is extra redundant. The extra equation (constraints) can be derived from other constraint without affecting the feasible solution. It follows that any feasible solution for a transportation problem must have exactly (*m+n-1*) non negative basic variables (or allocations) X_{ij}satisfying the rim-conditions.

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Also, the initial basic feasible solutions obtained should be closer to the optimum solution because the closer they are the fewer the number of iterations leading to optimality (Munapo,2021). The essence of computing the optimality condition is to ascertain how much the initial basic feasible solution (IBFS) is from the optimal solution. This can obtain by computing the percentage Deviance (PD), given as:

$$PD = \frac{Basic \quad Solution \quad - \quad Optimum \quad Solution}{Optimum \quad Solution} \times 100 \quad (5)$$

The PD is used to access how closer or nearer the Basic feasible solution is to the Optimum Solution (Hussain and Ahmed, 2020).

Variations in Transportation Problem

The variation encountered when solving transportation problem include:

(a) The Unbalanced Transportation Problem

Thi**s** is practically a case of unbalanced demand and supply. For a feasible solution to exist, it is necessary that the total demand be equal to the total supply. That is:

$$\sum_{I=1}^{m} S_I = \sum_{J=1}^{n} D_i \quad (6)$$

But situation can arise where the total demand is not equal to the total supply. The two following cases can arise in that situation: The transportation table may need to have a dummy column (row) inserted in order to absorb surplus supply (demand) in the event that it surpasses demand(supply) overall. Since the column and row cells reflect product items that are neither created nor shipped, the unit transportation cost of those cells is set to zero.

(b) Maximization Transportation Problem

Cost minimization challenges are typically approached via the lens of transportation issues. But it can also be applied to tackle various other situations where the goal is to maximize the overall profit for each route (i,j). The objective function in terms of total profit is then stated as follows:

Maximize (Total profit) $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$. (7)

The procedure for solving such problems is the same as that of the minimization with little adjustments made in the Vogel approximation method for finding initial solution. By multiplying the profit matrix by a unit negative number, the maximization problem can be transformed into a minimization problem. Secondly, the problem can alternatively be handled directly by maximizing it, which involves deducting all profit from the highest profit in the matrix and solving it using the standard approach. Lastly, by making allocations to the maximum profit cells in order to find the initial feasible solution.

(c) Prohibited Transportation Routes

Shipments of commodities from particular sources to certain destinations may not be feasible in the event of road hazards such as floods, conflict, traffic jams, snowfall, etc. These circumstances can be resolved by allocating a very high cost, say M (or ∞), to a particular route or cell and then using any available transportation problem-solving technique.

Assumption of the Transportation Model

i. Total quantity of the items available at different warehouses is equal to the total requirement of demand at different destinations.
ii. Items can be transported conveniently from all sources to various destinations.
iii. The unit transportation cost of the items from all sources to their destinations is known.
iv. The transportation cost on a given route is directly proportional to the number of units shipped on that route.
v. The objective is to minimize the total transportation cost for savannah as a whole and not for individual supply and distribution centers.

RESULTS AND DISCUSSION

Procedure for finding an initial basic feasible solution

The procedure for all algorithm adopted in this study can be found in the literature. Furthermore, for identification, this new approach will be called PS- Despan approximation and the procedure for the proposed method is represented below:

The Novelty of Our Algorithm

We worked on the modification of the least cost method by allocating directly to the cells with the lowest cost and crossing out the highest cost cell. Movement is done row - wise starting from the last row. We deliberately avoided computing penalty cost which is the most common means of modifying transportation algorithm by many authors due to its tediousness.

Illustrative example for finding initial basic feasible solution.

The secondary data collected from literatures was used to compare eight transportations models.

The initial basic feasible solutions for all the methods can be seen in the appendix I

Comparative study

The Table below shows a comparison of the proposed method and other existing methods using numerical examples.

Table 2: Comparative study of different Methods of finding the initial basic feasible solution

Methods	**Total Transportation Cost**			
	Example 1	**Example 2**	**Example 3**	**Example 4**
Least Cost	475(2)	814(5)	37(1)	112(4)
North West Method	520(8)	1015(7)	41(7)	116(7)
Vogel Approximation	475(2)	779(1)	37(1)	102(2)
Row Minimum	505(7)	1110(8)	37(1)	112(4)
Column Minimum	475(2)	779(1)	37(1)	116(7)
Extremum Difference Method	475(2)	779(1)	62(8)	100(1)
Pointer Cost Method	475(2)	814(5)	37(1)	112(4)
Proposed Method	**460(1)**	**779(1)**	**37(1)**	**109(3)**
Optimum Solution by MODI	**435**	**743**	**33**	**100**

In the Table 2 above, the initial basic solution is closely the same indicating that the methods are similar in nature but our proposed method (PM), Vogel approximation method (VAM), least cost (LCM) and the pointer method (PCM) gives better result. We can also see that the North West Corner Method (NWCM) allocate to the cell without considering the unit transportation cost and therefore yields a worst initial solution that is far wide from optimal solution. Row Minimum Method (RMM) and Column Minimum Method (CMM) considers unit transportation cost row and column wise respectively and so sometimes they yield best starting solution and sometimes not. The Vogel approximation method is producing a good initial basic feasible solution which is in line with the submission made by numerous studies. Overall, the proposed method outperforms all the other seven methods.

Table 3: A percentage deviance from the optimal solution

Methods	**Percentage Deviance**			
	Example 1	**Example 2**	**Example 3**	**Example 4**
Least Cost	9.195%	9.556%	12.121%	12%
North West Method	19.540%	36.608%	24.242%	16%
Vogel Approximation	9.195%	4.845%	12.121%	2%
Row Minimum	16.092%	49.394%	12.121%	12%
Column Minimum	9.195%	4.845%	12.121%	16%
Extremum Difference Method	9.195%	4.845%	87.879%	0%
Pointer Method	9.195%	9.556%	12.121%	12%
Proposed Method	**5.747%**	**4.845%**	**12.121%**	**9%**
Optimum Solution (MODI)	**435**	**743**	**33**	**100**

Table 3 demonstrates how closer the IBFS is to the optimum solution. It is worthy of note that IBFS can be below or above the optimum solution. In essence, the IBFS been greater or less than the optimal solution is not a concern but rather how nearer is it to the optimal solution from both ends. The table above shows how closer (Percentage wise) is the IBFS of each of the heuristic to the optimum solution.

Table 4: Approaches yielding the same initial basic feasible solutions

Method/ SN	**Example 1**	**Example 2**	**Example 3**	**Example 4**
1	VAM	VAM	VAM	PCM
2	CMM	CMM	CMM	-
3	LCM	PM	LCM	LCM
4	PCM	EDM	RMM	RMM

Table 4 above, shows the approaches that produced the same allocations in the same cell with equal units of allocations and the same initial basic feasible solution for each of the examples. We can easily deduce that there is a sort of relationship between VAM, CMM, LCM and PM. Another generalization that can be made from this result is the fact that: From numerical example 1 in Table 5 (See appendix), we noticed that, there are the same allocations with equal unit cost allocations made to the following cells;

$x_{12} = 15, x_{22} = 0, x_{23} = 15, x_{24} = 10, x_{31} = 5, x_{34} = 5$

For Vogel approximation method (VAM), least cost method (LCM), Column Minimum method (CMM), and the Pointer Cost method (PCM) yielding an IBFS of 475-unit cost.

Making reference to numerical example 2 in Table 6 (See appendix), we can see that the Vogel approximation method, Proposed Method, column minimum and the extremum difference method have the same cost allocations with equal unit allocations at the same cells

$x_{11} = 5, x_{14} = 2, x_{23} = 7, x_{24} = 2, x_{32} = 8, x_{34} = 10$
yielding the same initial basic feasible solution of 779-unit cost which is the least from all the comparisons.

In example 3 Table 7 (See appendix), also the same cost allocations with equal unit allocations are made to the following cell
$x_{11} = 0, x_{12} = 1, x_{22} = 5, x_{23} = 2, x_{33} = 7$
yielding an initial basic feasible solution of 37-unit cost for least cost method, Vogel approximation method, Row minimum method, column minimum method and the pointer cost method, the lowest of all the comparisons made. Lastly, in example 4, there is still the same allocation with equal unit cost allocation made to the following cells $x_{11} = 6, x_{22} = 1, x_{31} = 1, x_{32} = 4, x_{33} = 3, x_{34} = 2$
in the least cost method, Row Minimum method, and the Pointer Cost approach yielding an initial basic feasible solution of 112-unit cost. From these findings we conclude that there might be some iterative relationships between the Vogel approximation method, column minimum and the least cost method.

CONCLUSION

Undoubtedly, the logistics costs incurred by corporations and commercial organizations will be significantly impacted by the appropriateness, effectiveness, and adaptability of the transportation process. In this work, we created a novel dual approach for locating a transportation problem's first fundamentally workable solution. We evaluated our new algorithm's efficiency with four numerical examples. The proposed heuristic was discovered to be mathematically simpler and have a significantly higher IBFS when compared to some well-known traditional heuristic algorithm. We conclude from the results that our suggested approach is appropriate for determining the feasible solution of a transportation problem.

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APPENDIX I

Table 5: Initial Basic Feasible Solutions for the First Cost Marix

Table 35: Initial Basic Feasible Solution using least Cost Method

	1	2	3	4	Supply
1	10	15 2	20	11	**15**
2	12	0 7	15 9	10 20	**25**
3	5 4	14	16	5 18	**10**
Demand	**5**	**15**	**15**	**15**	

$Solution: x_{34} = 5, x_{12} = 5, x_{22} = 0, x_{23} = 15, x_{24} = 10, x_{31} = 5$
$Min(z) = 30 + 0 + 135 + 200 + 20 + 90 = 475$

Table 36: Initial Basic Feasible Solution using North West Corner Method

	1	2	3	4	Supply
1	5 10	10 2	20	11	**15**
2	12	5 7	15 9	5 20	**25**
3	4	14	16	10 18	**10**
Demand	**5**	**15**	**15**	**15**	

$Solution: x_{11} = 5, x_{12} = 10, x_{22} = 5, x_{23} = 15, x_{24} = 5, x_{34} = 10$
$Min(Z) = 50 + 20 + 35 + 135 + 100 + 180 = 520$

Table 37: Initial Basic Feasible Solution using Vogel App. Method

	1	2	3	4	Supply
1	10	[15] 2	20	11	**15**
2	12	[0] 7	[15] 9	[10] 20	**25**
3	[5] 4	14	16	[5] 18	**10**
Demand	**5**	**15**	**15**	**15**	

$Solution: x_{12} = 15, x_{22} = 0, x_{23} = 15, x_{24} = 10, x_{31} = 5, and x_{34} = 5$

$Min(Z) = 30 + 0 + 280 + 135 + 200 + 20 = 475$

Table 38: Initial Basic Feasible Solution using Row Minimum Method

	1	2	3	4	Supply
1	[0] 10	[15] 2	20	11	**15**
2	[5] 12	7	[15] 9	[5] 20	**25**
3	4	14	16	[10] 18	**10**
Demand	**5**	**15**	**15**	**15**	

$Solution: x_{11} = 0, x_{12} = 15, x_{21} = 5, x_{23} = 15, x_{24} = 5, x_{34} = 10$

$Min(Z) = 0 + 30 + 60 + 135 + 100 + 180 = 505$

Table 39: Initial Basic Feasible Solution using Column Minimun Method

	1	2	3	4	Supply
1	10	[15] 2	20	11	**15**
2	12	[0] 7	[15] 9	[10] 20	**25**
3	[5]	14	16	[5] 18	**10**
Demand	**5**	**15**	**15**	**15**	

$Solution: x_{12} = 15, x_{22} = 0, x_{23} = 15, x_{24} = 10, x_{31} = 5, x_{34} = 5$

$Min(Z) = 30 + 0 + 135 + 200 + 20 + 90 = 475$

Table 40: Initial Basic Feasible Solution using Exremum Difference Method

	1	2	3	4	Supply
1	10	[15] 2	20	[0] 11	**15**
2	12	7	[15] 9	[10] 20	**25**
3	[5] 4	14	16	[5] 18	**10**
Demand	**5**	**15**	**15**	**15**	

$Solution: x_{12} = 15, x_{14} = 0, x_{23} = 15, x_{24} = 10, x_{31} = 5, x_{34} = 5$

$Min(Z) = 30 + 0 + 135 + 200 + 20 + 90 = 475$

Table 41: Initial Basic Feasible Solution using Pointer Cost Method

	1	2	3	4	Supply
1	10	[15] 2	20	11	**15**
2	12	[0] 7	[15] 9	[10] 20	**25**
3	[5] 4	14	16	[5] 18	**10**
Demand	**5**	**15**	**15**	**15**	

$Solution: x_{12} = 15, x_{22} = 0, x_{23} = 15, x_{24} = 10, x_{31} = 5, x_{34} = 5$

$Min(Z) = 30 + 0 + 135 + 200 + 20 + 90 = 475$

Table 42: Initial Basic Feasible Solution using proposed Method

	1	2	3	4	Supply
1	10	2	20	[7] 11	**15**
2	12	[8] 7	[1] 9	20	**25**
[5]	4	14	[6] 16	[7] 18	**10**
Demand	**5**	**15**	**15**	**15**	

$Solution: x_{12} = 0, x_{14} = 15, x_{22} = 10, x_{23} = 15, x_{31} = 5, x_{32} = 5$

$Min(Z) = 0 + 165 + 70 + 135 + 20 + 70 = 460$

Table 6: Initial Basic Feasible Solutions for the Second Cost Marix

Table 11: Initial Basic Feasible Solution using least Cost Method

	1	2	3	4	Supply
1	19	30	50	[7] 10	**7**
2	[2] 70	30	[7] 40	60	**9**
3	[3] 40	[8] 8	70	[7] 20	**18**
Demand	**5**	**8**	**7**	**14**	

$Solution: x_{14} = 7, x_{21} = 2, x_{23} = 7, x_{31} = 3, x_{32} = 8, x_{34} = 7$
$Min(Z) = 70 + 140 + 280 + 120 + 64 + 140 = 814$

Table 12: Initial Basic Feasible Solution using North West Corner Method

	1	2	3	4	Supply
1	[5] 19	[2] 30	50	10	**7**
2	70	[6] 30	[3] 40	60	**9**
3	40	8	[4] 70	[14] 20	**18**
Demand	**5**	**8**	**7**	**14**	

$Solution: x_{11} = 5, x_{12} = 2, x_{22} = 6, x_{23} = 3, x_{33} = 4, x_{34} = 14$
$Min(Z) = 95 + 60 + 180 + 120 + 280 + 280 = 1015$

Table 13: Initial Basic Feasible Solution using Vogel App. Method

	1	2	3	4	Supply
1	[5] 19	30	50	[2] 10	**7**
2	70	30	[7] 40	[2] 60	**9**
3	40	[8] 8	70	[10] 20	**18**
Demand	**5**	**8**	**7**	**14**	

$Solution: x_{11} = 5, x_{14} = 2, x_{23} = 7, x_{24} = 2, x_{32} = 8, x_{34} = 10$
$Min(Z) = 95 + 20 + 280 + 120 + 64 + 200 = 779$

Table 14: Initial Basic Feasible Solution using Row Minimum Method

	1	2	3	4	Supply
1	19	30	50	[7] 10	**7**
2	70	[8] 30	[1] 40	60	**9**
3	[5] 40	8	[6] 70	[7] 20	**18**
Demand	**5**	**8**	**7**	**14**	

$Solution: x_{14} = 7, x_{22} = 8, x_{23} = 1, x_{31} = 5, x_{31} = 5, x_{33} = 6, x_{34} = 7$
$Min(Z) = 70 + 240 + 40 + 200 + 420 + 140 = 1110$

Table 15: Initial Basic Feasible Solution using Column Minimum Method

	1	2	3	4	Supply
1	[5] 19	30	50	[2] 10	**7**
2	70	30	[7] 40	[2] 60	**9**
3	40	[8] 8	70	[10] 20	**18**
Demand	**5**	**8**	**7**	**14**	

$Solution: x_{11} = 5, x_{14} = 2, x_{23} = 7, x_{24} = 2, x_{32} = 8, x_{34} = 10$
$Min(Z) = 95 + 20 + 280 + 120 + 64 + 200 = 779$

Table 16: Initial Basic Feasible Solution using Extremum Difference Meth

	1	2	3	4	Supply
1	[5] 19	30	50	[2] 10	**7**
2	70	30	[7] 40	[2] 60	**9**
3	40	[8] 8	70	[10] 20	**18**
Demand	**5**	**8**	**7**	**14**	

$Solution: x_{11} = 5, x_{14} = 2, x_{23} = 7, x_{24} = 2, x_{32} = 8, x_{34} = 10$
$Min(Z) = 95 + 20 + 280 + 120 + 64 + 200 = 779$

Table 17: Initial Basic Feasible Solution using pointer cost Method

	1	2	3	4	Supply
1	19	30	50	[7] 10	**7**
2	[2] 70	30	[7] 40	60	**9**
3	[3] 40	[8] 8	70	[7] 20	**18**
Demand	**5**	**8**	**7**	**14**	

$Solution: x_{14} 7, x_{21} = 2, x_{23} = 7, x_{31} = 3, x_{32} = 8, x_{34} = 7$
$Min(Z) = 70 + 140 + 280 + 120 + 64 + 200 = 814$

Table 18: Initial Basic Feasible Solution using Proposed Method

	1	2	3	4	Supply
1	[5] 19	30	50	[2] 10	**7**
2	70	30	[7] 40	[2] 60	**9**
3	40	[8] 8	70	[10] 20	**18**
Demand	**5**	**8**	**7**	**14**	

$Solution: x_{11} = 5, x_{14} = 2, x_{23} = 7, x_{24} = 2, x_{32} = 8, x_{34} = 10$
$Min(Z) = 95 + 280 + 120 + 64 + 200 = 779$

Table 7: Initial Basic Feasible Solutions for the Third Cost Marix

Table 19: Initial Basic Feasible Solution using Least Cost Method

	1	2	3	Supply
1	[5] 0	2	[1] 1	6
2	2	[5] 1	[2] 5	7
3	2	4	[7] 3	7
Demand	5	5	10	

$Solution: x_{11} = 5, x_{13} = 1, x_{22} = 5, x_{23} = 2, x_{33} = 7$

$Min(Z) = 0 + 1 + 5 + 10 + 21 = 37$

Table 20: Initial Basic Feasible Solution using North West Corner Metho

	1	2	3	Supply
1	[5] 0	[1] 2	1	6
2	2	[4] 1	[3] 5	7
3	2	4	[7] 3	7
Demand	5	5	10	

$Soluion: x_{11} = 5, x_{12} = 1, x_{22} = 4, x_{23} = 3, x_{33} = 7$

$Min(Z) = 0 + 2 + 4 + 15 + 21 = 41$

Table 21: Initial Basic Feasible Solution using Vogel App. Method

	1	2	3	Supply
1	[5] 0	2	[1] 1	6
2	2	[5] 1	[2] 5	7
3	2	4	[7] 3	7
Demand	5	5	10	

$Solution: x_{11} = 5, x_{13} = 1, x_{22} = 5, x_{23} = 2, x_{33} = 7$

$Min(Z) = 0 + 1 + 5 + 10 + 21 = 37$

Table 22: Initial Basic Feasible Solution using Row Minimum Method

	1	2	3	Supply
1	[5] 0	2	[1] 1	6
2	2	[5] 1	[2] 5	7
3	2	4	[7] 3	7
Demand	5	5	10	

$Solution: x_{11} = 5, x_{13} = 1, x_{22} = 5, x_{23} = 2, x_{33} = 7$

$Min(Z) = 0 + 1 + 5 + 10 + 21 = 37$

Table 23: Initial Basic Feasible Solution using Column Minimum Method

	1	2	3	Supply
1	[5] 0	2	[1] 1	6
2	2	[5] 1	[2] 5	7
3	2	4	[7] 3	7
Demand	5	5	10	

$Solution: x_{11} = 5, x_{13} = 1, x_{22} = 5, x_{23} = 2, x_{33} = 7$

$Min(Z) = 0 + 1 + 5 + 10 + 21 = 37$

Table 24: Initial Basic Feasible Solution using Extremum Difference Method

	1	2	3	Supply
1	[5] 0	2	[1] 1	6
2	2	1	[7] 5	7
3	2	[5] 4	[2] 3	7
Demand	5	5	10	

$Solutionx_{11} = 5, x_{13} = 1, x_{23} = 7, x_{32} = 5, x_{33} = 2$

$Min(Z) = 0 + 1 + 35 + 20 + 6 = 62$

Table 25: Initial Basic Feasible Solution using pointer cost Method

	1	2	3	Supply
1	[5] 0	2	[1] 1	6
2	2	[5] 1	[2] 5	7
3	2	4	[7] 3	7
Demand	5	5	10	

$Solution: x_{11} = 5, x_{13} = 1, x_{22} = 5, x_{23} = 2, x_{33} = 7$

$Min(Z) = 0 + 1 + 5 + 10 + 21 = 37$

Table 26: Initial Basic Feasible Solution using Proposed Method

	1	2	3	Supply
1	0	2	[6] 1	6
2	2	[5] 1	[2] 5	7
3	[5] 2	4	[2] 3	7
Demand	5	5	10	

$Solution: x_{13} = 6, x_{22} = 5, x_{23} = 2, x_{31} = 5, x_{33} = 2$

$Min(Z) = 6 + 5 + 10 + 10 + 6 = 37$

Table 8: Initial Basic Feasible Solutions for the Fourth Cost Marix

Table 27: Initial Basic Feasible Solution using Least Cost Method

	1	2	3	4	Supply
1	[6] 2	3	11	7	6
2	1	[1] 0	6	1	1
3	[1] 5	[4] 8	[3] 15	[2] 9	10
Demand	7	5	3	2	

$Solution: x_{11}=6, x_{22}=1, x_{31}=1, x_{32}=4, x_{33}=3, x_{34}=2$

$Min(Z)=12+0+5+32+45+18=112$

Table 28: Initial Basic Feasible Solution using North West Corner Method

	1	2	3	4	Supply
1	[6] 2	3	11	7	6
2	[1] 1	0	6	1	1
3	[0] 5	[5] 8	[3] 15	[2] 9	10
Demand	7	5	3	2	

$Solution: x_{11}=6, x_{21}=1, x_{31}=0, x_{32}=5, x_{33}=3, x_{34}=2$

$Min(Z)=12+1+0+40+45+18=116$

Table 29: Initial Basic Feasible Solution using Vogel App. Method

	1	2	3	4	Supply
1	[1] 2	[5] 3	11	7	6
2	1	0	6	[1] 1	1
3	[6] 5	8	[3] 15	[1] 9	10
Demand	7	5	3	2	

$Solution: x_{11}=1, x_{12}=5, x_{24}=1, x_{31}=6, x_{33}=3, x_{34}=1,$

$Min(Z)=2+15+1+30+45+9=102$

Table 30: Initial Basic Feasible Solution using Row Minimum Method

	1	2	3	4	Supply
1	[6] 2	3	11	7	6
2	1	[1] 0	6	1	1
3	[1] 5	[4] 8	[3] 15	[2] 9	10
Demand	7	5	3	2	

$Solution: x_{11}=6, x_{22}=0, x_{31}=1, x_{32}=4, x_{33}=3, x_{34}=2$

$Min(Z)=12+0+32+45+18=112$

Table 31: Initial Basic Feasible Solution using Column Minimum Method

	1	2	3	4	Supply
1	[6] 2	3	11	7	6
2	[1] 1	0	6	1	1
3	[0] 5	[5] 8	[3] 15	[2] 9	10
Demand	7	5	3	2	

$Solution: x_{11}=6, x_{21}=1, x_{31}=0, x_{32}=5, x_{33}=3, x_{34}=2$

$Min(z)=12+1+0+40+45+18=116$

Table 32: Initial Basic Feasible Solution using Extremum Difference Method

	1	2	3	4	Supply
1	2	[5] 3	[1] 11	7	6
2	1	0	[1] 6	1	1
3	[7] 5	8	[1] 15	[2] 9	10
Demand	7	5	3	2	

$Solution: x_{12}=5, x_{13}=1, x_{23}=1, x_{31}=7, x_{33}=1, x_{34}=2$

$Min(z)=15+11+6+35+15+18=100$

Table 33: Initial Basic Feasible Solution using pointer cost Method

	1	2	3	4	Supply
1	[6] 2	3	11	7	6
2	1	[1] 0	6	1	1
3	[1] 5	[4] 8	[3] 15	[2] 9	10
Demand	7	5	3	2	

$Solution: x_{11}=6, x_{22}=0, x_{31}=1, x_{32}=4, x_{33}=3, x_{34}=2$

$Min(Z)=12+0+5+32+45+18=112$

Table 34: Initial Basic Feasible Solution using Proposed Method

	1	2	3	4	Supply
1	2	[1] 3	[3] 11	[2] 7	6
2	1	[1] 0	6	1	1
3	[7] 5	[3] 8	15	9	10
Demand	7	5	3	2	

$Solution: x_{12}=1, x_{13}=3, x_{14}=2, x_{14}=2, x_{22}=0, x_{31}7, x_{32}=3$

$Min(Z)=3+33+14+0+35+24=109$