FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645 - 2944
Vol. 8 No. 2, April, 2024, pp 286 - 300
DOI: https://doi.org/10.33003/fjs-2024-0802-2291

POWER MEDIAN-BASED ESTIMATORS OF FINITE POPULATION MEAN

***[1]Yusuf Ajibola Yahya, [2]Ahmed Audu and [2]Yunusa, Mojeed Abiodun**

[1]Department of Statistics, Federal Polytechnic, Kaura Namoda, Zamfara, Nigeria
[2]Department of Statistics, Usmanu Danfodiyo University, Sokoto, Nigeria.

*Corresponding authors' email: yusaji710@gmail.com

ABSTRACT

In this paper, median based mean estimators for estimating finite population mean are proposed. The proposed estimators were obtained by transforming estimators in literature utilizing mean of auxiliary variable into median based estimators with the aim of obtaining estimators with higher efficiency. The mean square error of the proposed estimators was obtained up to the first order of approximation using Taylor series approach and the optimum values of the unknown of the estimators were obtained by means of partial derivative of the mean square error and equating to zero. A Numerical study was carried out to support the fact that the proposed estimators are more efficient as compared to the existing ones, as the proposed estimators have the least mean squared error at optimum values of the unknown constants and have higher percentage relative efficiency (PRE). This implies that the proposed estimators are more efficient than the traditional ones considered in the study.

Keywords: Bias, Efficiency, Simple random sampling, Mean square error, Ratio estimator, Auxiliary information

INTRODUCTION

In the field of sample survey, the use of estimators such as ratio, product, exponential ratio type, exponential product and regression estimation methods that use auxiliary information for estimation purpose is a common practice. These estimators perform well in the presence of auxiliary variable (auxiliary information)

Many studies in the field of sample survey have established the fact behind the use of auxiliary information at both planning and estimation stages aids in enhancing the efficiency of estimators for estimation of parameters like population mean, population variance, population coefficient of variation etc. Authors such as Cochran (1940), Murthy (1964), Sisodia and Dwivedi (1981), Bahl and Tuteja (1991), Perri (2007), Upadhyaya et al. (2011), Singh and Kumar (2011), Hassan et al. (2019), Muili and Audu (2019), Muili et al. (2020), Hassan et al. (2020), Audu and Singh (2021), Yunusa et al. (2021), Audu et al. (2021) have made use of auxiliary information for the development of estimators and estimation. Hafeez et al. (2020) transformed the estimator of Bahl and Tuteja (1991) to median based estimator that uses median of the study variable y as auxiliary information and in their work, they demonstrated the efficiency of the transformed estimator over existing ones.

In the current study, we aimed at proposing some median based estimators in the presence of median of the study variable that is highly efficient and can produce estimate closer to the population mean of the study variable by modification of Muili and Audu (2019) and Yunusa et al. (2021) estimators.

Consider a finite population of N distinct and identifiable units $W = \{W_{1,}W_{2,}W_3, \ldots. W_N\}$. Let a sample of size n be drawn from the population by simple random sampling without replacement (SRSWOR). Assuming that interest is is to obtain regression estimate of the mean of a random variable Y from the sample using a relative variable X as supplementary information and assuming that the total of X is known from source outside the survey.

The following notations are used in this paper

N: Population size

$\bar{M}$: Average of sample medians

n : Sample size

m : Sample median of the study variable

$f = N^{-1}n$Sampling fraction

$\rho = Cov(y,x)\left(S_x S_y\right)^{-1}$ Correlation coefficient between y and x

$\rho_{ym} = Cov(y,m)\left(S_m S_y\right)^{-1}$ Correlation coefficient between y and m

$S_y^2 = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2$ Population variance of the study variable

$S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \bar{X})^2$ Population variance of the auxiliary variable

$Cov(y,x) = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \bar{Y})(x_i - \bar{X})$ Population covariance between y and x

$Cov(y,m) = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \bar{Y})(m_i - \bar{M})$ Population covariance between y and m

$\bar{X} = N^{-1}\sum_{i=1}^{N} X_i$: Population mean of the auxiliary variable X

$\bar{Y} = N^{-1}\sum_{i=1}^{N} Y_i$: Population mean of the study variable Y

$\bar{x} = n^{-1}\sum_{i=1}^{n} x_i$: Sample mean of the auxiliary variable X

$\bar{y} = n^{-1}\sum_{i=1}^{n} y_i$: Sample mean of the study variable Y

$C_x = \bar{X}^{-1}S_x$: Population coefficient of variation of the auxiliary variable X

$C_y = \bar{Y}^{-1}S_y$: Population coefficient of variation of the study variable Y

$\gamma = n^{-1}(1-f)$: Correction factor

Also, we define:

$$V(m) = \left[{}^N C_n\right]^{-1}\sum_{i=1}^{{}^N C_n}(m_i - \bar{M})^2,$$

$$Cov(\bar{y},m) = \left[{}^N C_n\right]^{-1}\sum_{i=1}^{{}^N C_n}(m_i - \bar{M})(y_i - \bar{Y}),$$

$$Cov(\bar{y},\bar{x}) = (1-f)\left(n(N-1)\right)^{-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2,$$

$$V(\bar{x}) = \gamma S_x^2.$$

Some existing estimators in literature

To estimate the finite population, some authors in literature have given various estimation methods and they include:

The usual sample mean estimator is defined as

$\bar{y} = n^{-1}\sum_{i=1}^{n} y_i$ (1)
The variance of the estimator is given by
$V(\bar{y}) = \gamma S_y^2$ (2)
Cochran (1940) introduced the ratio method of estimation as
$\bar{y}_R = \frac{\bar{y}}{\bar{x}}\bar{X}$ (3)
The mean square error of the ratio estimator is given by
$MSE(\bar{y}_R) = V(\bar{y}) + \bar{Y}^2\bar{X}^{-2}V(\bar{x}) - 2\bar{Y}\bar{X}^{-1}Cov(\bar{y},\bar{x})$ (4)
Murthy (1964) introduced the product method of estimation as
$\bar{y}_P = \frac{\bar{y}}{\bar{X}}\bar{x}$ (5)
The mean square error of the ratio estimator is given by
$MSE(\bar{y}_P) = V(\bar{y}) + \bar{Y}^2\bar{X}^{-2}V(\bar{x}) + 2\bar{Y}\bar{X}^{-1}Cov(\bar{y},\bar{x})$ (6)
Bahl and Tuteja (1991) introduced the exponential ratio method of estimation as
$\bar{y}_{BT} = \bar{y}\,exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$ (7)
The mean square error of the ratio estimator is given by
$MSE(\bar{y}_{BT}) = V(\bar{y}) + 0.25\bar{Y}^2\bar{X}^{-2}V(\bar{x}) - \bar{Y}\bar{X}^{-1}Cov(\bar{y},\bar{x})$ (8)
Muili and Audu (2019) introduced a modified ratio mean method of estimation as
$\bar{y}_J = \bar{y}\left(\frac{\bar{X}+n}{\bar{x}+n}\right)^{\alpha_0}$ (9)
Where, $\alpha_0 = \frac{(\bar{X}+n)\rho C_y}{\bar{X}C_x}$.
The minimum mean square error of the ratio estimator is given by
$MSE(\bar{y}_j) = V(\bar{y})(1-\rho^2)$ (10)
Hassan et al. (2020) introduce a new regression estimator as
$\bar{y}_Z = \bar{y} + r(\bar{X}-\bar{x})$ (11)
Where, r is an estimate of the population correlation coefficient ρ
The mean square error of their estimator is given by
$MSE(\bar{y}_Z) = V(\bar{y}) + \rho^2V(\bar{x}) - 2\rho Cov(\bar{y},\bar{x})$ (12)
Hafeez et al. (2020) introduced median based exponential ratio method of estimation as
$\bar{y}_H = \bar{y}\,exp\left(\frac{\bar{M}-m}{\bar{M}+m}\right)$ (13)
The mean square error of the ratio estimator is given b
$MSE(\bar{y}_H) = V(\bar{y}) + 0.25\bar{Y}^2\bar{M}^{-2}V(\bar{x}) - \bar{Y}\bar{M}^{-1}Cov(\bar{y},m)$ (14)
Yunusa et al. (2021) introduced an efficient exponential type method of estimation as
$\bar{y}_M = 2^{-1}\bar{y}\left(\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} + exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right)$ (15)
where $\alpha = \frac{1}{2} - \frac{2\rho C_y}{C_x}$.
The minimum mean square error of the ratio estimator is given by
$MSE(\bar{y}_M) = V(\bar{y})(1-\rho^2)$ (16)

MATERIALS AND METHODS

Proposed estimator

Following the estimation strategy of Hafeez et al. (2020), we proposed Muili and Audu (2019) and Yunusa et al. (2021) estimators and other new estimators in terms of median based estimators as
$\bar{y}_{M1} = \bar{y}\left(\frac{\bar{M}+n}{m+n}\right)^{\alpha_1}$ (17)
$\bar{y}_{M2} = 2^{-1}\bar{y}\left\{\left(\frac{m}{\bar{M}}\right)^{\alpha_2} + exp\left(\frac{\bar{M}-m}{\bar{M}+m}\right)\right\}$ (18)
$\bar{y}_{M3} = 2^{-1}\bar{y}\left\{\left(\frac{\bar{M}+n}{m+n}\right)^{\alpha_3} + exp\left(\frac{\bar{M}-m}{\bar{M}+m}\right)\right\}$ (19)
$\bar{y}_{M4} = 2^{-1}\bar{y}\left\{\left(\frac{m}{\bar{M}}\right)^{\alpha_4} + exp\left(2\frac{\bar{M}-m}{\bar{M}+m}\right)\right\}$ (20)
$\bar{y}_{M5} = 2^{-1}\bar{y}\left\{\left(\frac{\bar{M}+n}{m+n}\right)^{\alpha 5} + exp\left(2\frac{\bar{M}-m}{\bar{M}+m}\right)\right\}$ (21)
$\bar{y}_{M6} = 2^{-1}\bar{y}\left\{\left(\frac{\bar{M}}{m}\right)^{\alpha_6} + exp\left(2\frac{\bar{M}-m}{\bar{M}+m}\right)\right\}$ (22)
$\bar{y}_{M7} = 2^{-1}\bar{y}\left\{\left(\frac{m+n}{\bar{M}+n}\right)^{\alpha 7} + exp\left(2\frac{\bar{M}-m}{\bar{M}+m}\right)\right\}$ (23)
Where, $\alpha_i(i = 1,2,3,4,5,6,7)$ are unknown functions to be estimated through minimization of $\bar{y}_{Mi}(i = 1,2,3,4,5,6,7)$.
In order to obtain the MSE of the proposed estimator, let us define
$e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}, e_1 = \frac{m-\bar{M}}{\bar{M}}$, such that $\lim_{N\to n}|e_h| \approx 0, h = 0,1.$
This implies that,
$\bar{y} = \bar{Y}(1+e_0), m = \bar{M}(1+e_1)$.
$$\left.\begin{array}{l} E(e_0) = E(e_1) = 0, E(e_0^2) = \frac{V(\bar{y})}{\bar{Y}^2}, \\ E(e_1^2) = \frac{V(m)}{\bar{M}^2}, E(e_0e_1) = \frac{cov(\bar{y},m)}{\bar{Y}\bar{M}} \end{array}\right\} \quad (24)$$

By expressing equations (17)-(24) in term of sampling error terms, we have
$$\bar{y}_{M1} = \bar{Y}(1+e_0)\left(\frac{\bar{M}+n}{\bar{M}(1+e_1)+n}\right)^{\alpha_1} \quad (25)$$
$$\bar{y}_{M2} = 2^{-1}\bar{Y}(1+e_0)\left[\left(\frac{\bar{M}(1+e_1)}{\bar{M}}\right)^{\alpha_1} + exp\left[\frac{\bar{M}-\bar{M}(1+e_1)}{\bar{M}+\bar{M}(1+e_1)}\right]\right] \quad (26)$$
$$\bar{y}_{M3} = 2^{-1}\bar{Y}(1+e_0)\left[\left(\frac{\bar{M}+n}{\bar{M}(1+e_1)+n}\right)^{\alpha_3} + exp\left[\frac{\bar{M}-\bar{M}(1+e_1)}{\bar{M}+\bar{M}(1+e_1)}\right]\right] \quad (27)$$
$$\bar{y}_{M4} = 2^{-1}\bar{Y}(1+e_0)\left[\left(\frac{\bar{M}(1+e_1)}{\bar{M}}\right)^{\alpha 4} + exp\left[2\frac{\bar{M}-\bar{M}(1+e_1)}{\bar{M}+\bar{M}(1+e_1)}\right]\right] \quad (28)$$
$$\bar{y}_{M5} = 2^{-1}\bar{Y}(1+e_0)\left[\left(\frac{\bar{M}+n}{\bar{M}(1+e_1)+n}\right)^{\alpha_5} + exp\left[2\frac{\bar{M}-\bar{M}(1+e_1)}{\bar{M}+\bar{M}(1+e_1)}\right]\right] \quad (29)$$
$$\bar{y}_{M6} = 2^{-1}\bar{Y}(1+e_0)\left[\left(\frac{\bar{M}}{\bar{M}(1+e_1)}\right)^{\alpha 6} + exp\left[2\frac{\bar{M}-\bar{M}(1+e_1)}{\bar{M}+\bar{M}(1+e_1)}\right]\right] \quad (30)$$
$$\bar{y}_{M7} = 2^{-1}\bar{Y}(1+e_0)\left[\left(\frac{\bar{M}(1+e_1)}{\bar{M}}\right)^{\alpha 7} + exp\left[2\frac{\bar{M}-\bar{M}(1+e_1)}{\bar{M}+\bar{M}(1+e_1)}\right]\right] \quad (31)$$

By simplifying equations (25) to (31) to first order of approximation, we have,
$$\bar{y}_{M1} = \bar{Y}\left[1 + e_0 + \alpha_1\theta e_1 + \frac{(\alpha_1^2-\alpha_1)\theta^2}{2}e_1^2 + \alpha_1\theta e_1 e_0 e_1\right] \quad (32)$$
$$\bar{y}_{M2} = \bar{Y}\left[1 + e_0 + \left(\frac{\alpha_2}{2} - \frac{1}{4}\right)e_1 + \left(\frac{\alpha_2^2-\alpha_2}{4} + \frac{3}{16}\right)e_1^2 + \left(\frac{\alpha_2}{2} - \frac{1}{4}\right)e_0e_1\right] \quad (33)$$
$$\bar{y}_{M3} = \bar{Y}\left[1 + e_0 - \left(\frac{\alpha_3\theta}{2} + \frac{1}{4}\right)e_1 + \left(\frac{(\alpha_3^2+\alpha_3)\theta^2}{4} + \frac{3}{16}\right)e_1^2 - \left(\frac{\alpha_3\theta}{2} + \frac{1}{4}\right)e_0e_1\right] \quad (34)$$
$$\bar{y}_{M4} = \bar{Y}\left[1 + e_0 + \left(\frac{\alpha_4}{2} - \frac{1}{2}\right)e_1 + \left(\frac{\alpha_4^2-\alpha_4}{4} + \frac{1}{2}\right)e_1^2 + \left(\frac{\alpha_4}{2} - \frac{1}{2}\right)e_0e_1\right] \quad (35)$$
$$\bar{y}_{M5} = \bar{Y}\left[1 + e_0 - \left(\frac{\alpha_5\theta}{2} + \frac{1}{2}\right)e_1 + \left(\frac{(\alpha_5^2-\alpha_5)\theta^2}{4} + \frac{1}{2}\right)e_1^2 - \left(\frac{\alpha_5\theta}{2} + \frac{1}{2}\right)e_0e_1\right] \quad (36)$$

$$\bar{y}_{M6} = \bar{Y}\left[1 + e_0 - \left(\frac{\alpha_6}{2} + \frac{1}{2}\right)e_1 + \left(\frac{\alpha_6^2+\alpha_6}{4} + \frac{1}{2}\right)e_1^2 - \left(\frac{\alpha_6}{2} + \frac{1}{2}\right)e_0e_1\right] \quad (37)$$

$$\bar{y}_{M7} = \bar{Y}\left[1 + e_0 + \left(\frac{\alpha_7\theta}{2} - \frac{1}{2}\right)e_1 + \left(\frac{(\alpha_7^2-\alpha_7)\theta^2}{4} + \frac{1}{2}\right)e_1^2 + \left(\frac{\alpha_7\theta}{2} - \frac{1}{2}\right)e_0e_1\right] \quad (38)$$

Where, $\theta = \frac{\bar{M}}{\bar{M}+n}$

By subtracting $\bar{Y}$ from both sides of equations (32) to (38) , we have,

$$\bar{y}_{M1} - \bar{Y} = \left[e_0 + \alpha_1\theta e_1 + \frac{(\alpha_1^2-\alpha_1)\theta^2}{2}e_1^2 + \alpha_1\theta e_1 e_0 e_1\right] \quad (39)$$

$$\bar{y}_{M2} - \bar{Y} = \left[e_0 + \left(\frac{\alpha_2}{2} - \frac{1}{4}\right)e_1 + \left(\frac{\alpha_2^2-\alpha_2}{4} + \frac{3}{16}\right)e_1^2 + \left(\frac{\alpha_2}{2} - \frac{1}{4}\right)e_0e_1\right] \quad (40)$$

$$\bar{y}_{M3} - \bar{Y} = \left[e_0 - \left(\frac{\alpha_3\theta}{2} + \frac{1}{4}\right)e_1 + \left(\frac{(\alpha_3^2+\alpha_3)\theta^2}{4} + \frac{3}{16}\right)e_1^2 - \left(\frac{\alpha_3\theta}{2} + \frac{1}{4}\right)e_0e_1\right] \quad (41)$$

$$\bar{y}_{M4} - \bar{Y} = \left[e_0 + \left(\frac{\alpha_4}{2} - \frac{1}{2}\right)e_1 + \left(\frac{\alpha_4^2-\alpha_4}{4} + \frac{1}{2}\right)e_1^2 + \left(\frac{\alpha_4}{2} - \frac{1}{2}\right)e_0e_1\right] \quad (42)$$

$$\bar{y}_{M5} - \bar{Y} = \left[e_0 - \left(\frac{\alpha_5\theta}{2} + \frac{1}{2}\right)e_1 + \left(\frac{(\alpha_5^2-\alpha_5)\theta^2}{4} + \frac{1}{2}\right)e_1^2 - \left(\frac{\alpha_5\theta}{2} + \frac{1}{2}\right)e_0e_1\right] \quad (43)$$

$$\bar{y}_{M6} - \bar{Y} = \left[e_0 - \left(\frac{\alpha_6}{2} + \frac{1}{2}\right)e_1 + \left(\frac{\alpha_6^2+\alpha_6}{4} + \frac{1}{2}\right)e_1^2 - \left(\frac{\alpha_6}{2} + \frac{1}{2}\right)e_0e_1\right] \quad (49)$$

$$\bar{y}_{M7} - \bar{Y} = \left[e_0 + \left(\frac{\alpha_7\theta}{2} - \frac{1}{2}\right)e_1 + \left(\frac{(\alpha_7^2-\alpha_7)\theta^2}{4} + \frac{1}{2}\right)e_1^2 + \left(\frac{\alpha_7\theta}{2} - \frac{1}{2}\right)e_0e_1\right] \quad (50)$$

By squaring and taking expectation of equations (39) to (50), we obtained the mean squared errors (MSEs) of the proposed estimator to first order of approximation as

$$MSE(\bar{y}_{M1}) = V(\bar{y}) + \alpha_1^2\left(\frac{\bar{Y}}{\bar{M}}\right)^2\theta^2V(m) + 2\alpha_1\theta\left(\frac{\bar{Y}}{\bar{M}}\right)Cov(\bar{y},m) \quad (51)$$

$$MSE(\bar{y}_{M2}) = V(\bar{y}) + \left(\frac{\alpha_2}{2} - \frac{1}{4}\right)^2\left(\frac{\bar{Y}}{\bar{M}}\right)^2V(m) + 2\left(\frac{\alpha_2}{2} - \frac{1}{4}\right)\left(\frac{\bar{Y}}{\bar{M}}\right)Cov(\bar{y},m) \quad (52)$$

$$MSE(\bar{y}_{M3}) = V(\bar{y}) + \left(\frac{\alpha_3\theta}{2} + \frac{1}{4}\right)^2\left(\frac{\bar{Y}}{\bar{M}}\right)^2V(m) - 2\left(\frac{\alpha_3\theta}{2} + \frac{1}{4}\right)\left(\frac{\bar{Y}}{\bar{M}}\right)Cov(\bar{y},m) \quad (53)$$

$$MSE(\bar{y}_{M4}) = V(\bar{y}) + \left(\frac{\alpha_4}{2} - \frac{1}{2}\right)^2\left(\frac{\bar{Y}}{\bar{M}}\right)^2V(m) + 2\left(\frac{\alpha_4}{2} - \frac{1}{2}\right)\left(\frac{\bar{Y}}{\bar{M}}\right)Cov(\bar{y},m) \quad (54)$$

$$MSE(\bar{y}_{M5}) = V(\bar{y}) + \left(\frac{\alpha_5\theta}{2} + \frac{1}{2}\right)^2\left(\frac{\bar{Y}}{\bar{M}}\right)^2V(m) - 2\left(\frac{\alpha_5\theta}{2} + \frac{1}{2}\right)\left(\frac{\bar{Y}}{\bar{M}}\right)Cov(\bar{y},m) \quad (55)$$

$$MSE(\bar{y}_{M6}) = V(\bar{y}) + \left(\frac{\alpha_6}{2} + \frac{1}{2}\right)^2\left(\frac{\bar{Y}}{\bar{M}}\right)^2V(m) - 2\left(\frac{\alpha_6}{2} + \frac{1}{2}\right)\left(\frac{\bar{Y}}{\bar{M}}\right)Cov(\bar{y},m) \quad (56)$$

$$MSE(\bar{y}_{M7}) = V(\bar{y}) + \left(\frac{\alpha_7\theta}{2} - \frac{1}{2}\right)^2\left(\frac{\bar{Y}}{\bar{M}}\right)^2V(m) + 2\left(\frac{\alpha_7\theta}{2} - \frac{1}{2}\right)\left(\frac{\bar{Y}}{\bar{M}}\right)Cov(\bar{y},m) \quad (57)$$

Differentiating equations (51) to (57) partially with respect to $\alpha_i(i = 1,2,3,4,5,6,7)$and equate to zero and solve to obtain the optimum values of $\alpha_i(i = 1,2,3,4,5,6,7)$as:

$\alpha_1 = \frac{-\bar{M}Cov(\bar{y},m)}{\theta\bar{Y}V(m)}$, $\alpha_2 = \frac{1}{2} - \frac{2\bar{M}Cov(\bar{y},m)}{\bar{Y}V(m)}$, $\alpha_3 = \frac{-\alpha_2}{\theta}$, $\alpha_4 = \frac{1}{2} + \alpha_2$,

$\alpha_5 = \frac{-1}{\theta}\left(1 + \frac{2\bar{M}Cov(\bar{y},m)}{\bar{Y}V(m)}\right)$, $\alpha_6 = -\alpha_4$, $\alpha_7 = \frac{\alpha_4}{\theta}$

Substituting the optimum values of $\alpha_i(i = 1,2,3,4,5,6,7)$into equation (51) to (57) , to obtain the minimum MSEs of the proposed estimators $\bar{y}_{Mi}(i = 1,2,3,4,5,6,7)$as:

$$MSE(\bar{y}_{Mi})(\bar{y})\frac{(Cov(\bar{y},m))^2}{V(m)}_{min} \quad (58)$$

Empirical Study

In this section, we carry out an empirical study to elucidate the performance of the proposed estimators with respect to some existing related estimators using population data sets at different sample sizes.

Po**pulation 1: [Source: Singh and Mangat (1996)]**

X: Number of fish caught in year 1993, Y: Number of fish caught in year 1995

Po**pulation 2: [Source: Hafeez et al. (2020)]**

X: Auxiliary variable, Y: Study variable

Table 1: Summary statistics for different sample sizes

Parameters	Population 1		Population 2	
	n=3	n=5	n=3	n=5
N	24	24	27	27
$\bar{Y}$	19.1667	19.1667	725.37	725.37
$\bar{M}$	19.4575	19.5895	731.92	734.08
$\bar{X}$	16.7917	16.7917	644.48	.644.48
$V(\bar{y})$	4.7620	2.5839	9651.10	5306.35
$V(m)$	7.1272	4.1052	16765.87	10880.47
$V(\bar{x})$	6.6477	3.6070	5214.85	2867.22
$Cov(\bar{y},m)$	5.0051	2.6678	11276.64	6582.336
$Cov(\bar{y},\bar{x})$	3.0429	1.6511	5330.98	2931.071
ρ	0.5408	0.5408	0.7515	0.7515
ρ_{ym}	0.8591	0.8191	0.8865	0.8663

Table 2: MSEs of proposed and existing estimators for different sample sizes

Estimators	Population 1		Population 2	
	MSE (n=3)	MSE (n=5)	MSE (n=3)	MSE (n=5)
$\bar{y}$	4.762	2.5839	9651.1	5306.35
$\bar{y}_R$	6.476605	3.514139	16239.4	11894.65
$\bar{y}_P$	20.36974	11.05266	16524.69	12179.94
$\bar{y}_{BT}$	3.454009	1.874145	11262.51	6917.762
$\bar{y}_j$	3.369283	1.828201	4200.62	2309.577
$\bar{y}_Z$	3.415017	1.85299	9650.655	5305.905
$\bar{y}_H$	1.560642	0.9561559	2592.159	1458.066
$\bar{y}_g$	3.369283	1.828201	4200.62	2309.577
$\bar{y}_{M1}$	**1.247152**	**0.8502069**	**2066.489**	**1324.247**
$\bar{y}_{M2}$	**1.247152**	**0.8502069**	**2066.489**	**1324.247**
$\bar{y}_{M3}$	**1.247152**	**0.8502069**	**2066.489**	**1324.247**
$\bar{y}_{M4}$	**1.247152**	**0.8502069**	**2066.489**	**1324.247**
$\bar{y}_{M5}$	**1.247152**	**0.8502069**	**2066.489**	**1324.247**
$\bar{y}_{M6}$	**1.247152**	**0.8502069**	**2066.489**	**1324.247**
$\bar{y}_{M7}$	**1.247152**	**0.8502069**	**2066.489**	**1324.247**

Table 3: PREs of proposed and existing estimators for different sample sizes

Estimators	Population 1		Population 2	
	PRE (n=3)	PRE (n=5)	PRE (n=3)	PRE (n=5)
$\bar{y}$	100	100	100	100
$\bar{y}_R$	73.52617	73.52869	59.43017	44.61125
$\bar{y}_P$	23.37781	23.37809	58.40411	43.5663
$\bar{y}_{BT}$	137.8688	137.8709	85.69225	76.70617
$\bar{y}_j$	141.3357	141.3357	229.7542	229.7542
$\bar{y}_Z$	139.443	139.4449	100.0046	100.0084
$\bar{y}_H$	305.1309	270.2383	372.319	363.9308
$\bar{y}_g$	141.3357	141.3357	229.7542	229.7542
$\bar{y}_{M1}$	**381.83**	**303.9143**	**467.029**	**400.7068**
$\bar{y}_{M2}$	**381.83**	**303.9143**	**467.029**	**400.7068**
$\bar{y}_{M3}$	**381.83**	**303.9143**	**467.029**	**400.7068**
$\bar{y}_{M4}$	**381.83**	**303.9143**	**467.029**	**400.7068**
$\bar{y}_{M5}$	**381.83**	**303.9143**	**467.029**	**400.7068**
$\bar{y}_{M6}$	**381.83**	**303.9143**	**467.029**	**400.7068**
$\bar{y}_{M7}$	**381.83**	**303.9143**	**467.029**	**400.7068**

RESULTS AND DISCUSSION

Table 1 to Table 3 shows the MSEs and PREs of the proposed and other estimators considered in the study using information of the two populations 1 and 2. Results obtained from each category revealed that proposed estimators $\vec{y}_{Mi}(i = 1,2,3,4,5,6,7)$ have minimum MSEs and higher PREs compared to other existing estimators. This implies that the suggested estimators are more efficient than existing estimators considered in the study.

CONCLUSION

In this study, we have suggested new median based estimators for the estimation of the population mean of the study variable. From the empirical study, the results showed that the estimators are more efficient than the existing ones considered in this study. Hence, we strongly recommend that the proposed estimators should be used in both theoretical and real life applications.

REFERENCES

Audu A, Yunusa M. A., Zoramawa A. B, Buda S., Singh, R. V. K. (2021) Exponential-ratio-type imputation class of estimators using nonconventional robust measures of dispersions. *Asian Journal of Probability and Statistics*, 15(2):59-74.

Audu A, and Singh, R. V. K. (2021) Exponential-type regression compromised imputation class of estimators. *Journal of Statistics and Management Systems*, 24(6):1253-1266

Cochran, W. G.,(1940).The Estimation of Yields of the cereal Experiments by Sampling for the Ratio of Grain to Total Produce. *The Journal of Agric. Science*, 30,262-275.

Hassan, M. Z., Hossain, M. A., Sultana, M., Fatema, K., and Hossain, M. M. (2019) A new modified product estimator for estimation of population mean when median of the auxiliary variable is known. *International Journal of scientific research in Mathematical and Statistical sciences,* 6,108-113.

Hassan, M. Z., Sultana, M., Fatema, K., Hossain, Md. A., and Hossain, M. M. (2020) A new regression type estimator and its application in survey sampling. *Open Journal of Statistics*, 10, 1010-1019.

Hafeez, W., Shabbir, J., Shah, M., T., and Ahmed, S. (2020). Some median type estimators to estimate the finite population mean. *Asian Journal of Probability and Statistics*, 7(4), 48-58.

Kadilar C. and Cingi H. (2005). A new estimator using two auxiliary variables. *Applied Mathematics and Computation*, 16(2):901–908.

Muili, J. O., Agwamba, E. N., Erinola, Y. A., Yunusa, M. A., Audu, A, and Hamzat, M. A. (2020). Modified ratio cum product estimators of population mean using two auxiliary variables. *Asian Journal of research in computer*, 6(1), 55-65.

Muili, J. O. and Audu, A. (2019). A modification of ratio estimator for population mean. *Annals computer science series*, 17(2), 74-78.

Murthy, M. N. (1967) Sampling Theory and Methods.

Perri P. F. (2007). Improved ratio-cum-product type estimators. *Statistics in Transition- New Series*, 8(2):51-69

Singh, S. (2003) Advanced Sampling Theory with Applications, Springer Science and Business, Media Dordrecht.

Singh, R., and Mangat N., S. (1996) Elements of Survey Sampling, volume 15, Springer.

Singh M. P. (1965). On the estimation of ratio and product of the population parameters. Sankhya B, 2(7): 231-328.

Singh M. P. (1967) Ratio cum product method of estimation. Metrika. 1(2):34–42

Singh R, Kumar M. (2011) A note on transformations on auxiliary variable in survey sampling. MASA, 6(1):17-19.

Sisodia, B. V. S. and Dwivedi, V. K. (1981). Modified ratio estimator using coefficient of variation. *Journal Indian Society of Agricultural Statistics*, 33, 13-18

Subramani, J. and Kumaranpandiyan, G. (2013) New Modified Ratio Estimator for Estimation of population mean when median of the Auxiliary variable is known. *Pakistan Journal of Statistics and Operation Research*, 9, 137-145.

Upadhyaya, L. N., Singh, H. P., Chatterjee, S., andYadav, R. (2011) Improved ratio and product exponential type estimators. *Journal of Statistical theory and practice*, 5, 285-302.

Yunusa M. A., Audu A., Ishaq O., O., Beki D. O. (2021). An efficient Exponential Type Estimator for Population mean under Simple random sampling. *Annals Computer Science Series*, 19(1), 46-51.