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APPLICATION OF GENERALIZED EXTREME VALUE DISTRIBUTION TO ANNUAL MAXIMUM RAINFALL

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ABSTRACT
Generalized Extreme Value Distributions was used to model annual maximum rainfall data in Akure, Ondo State, Nigeria from 1981-2019. The parameters of the distribution are estimated using the maximum likelihood estimation method. The model fit indicated that the shape α parameter is negative this suggests that Fréchet distribution is the appropriate model for describing annual maximum rainfall in Ondo-state Nigeria. The estimated return levels for different return periods revealed an increase in the value over the years.

Keywords: Generalized Extreme Value Distribution (GEV), Maximum rainfall measurement, Return level, Maximum likelihood (ML) method

INTRODUCTION
Climate change is a long-term change in the average of weather patterns. Two of the indicators of climate change are extreme rainfall and temperature but extreme rainfall is said to be the most significant effect of the climate change. The world has increasingly seen the effect of extreme rainfall. For instance, the joint survey by the National Emergency Management Agency (NEMA) a federal government body in the year 2013 reported that heavy rainfall prompted severe floods in 33 states across Nigeria which affected millions of people and caused fatalities. It also destroyed farmlands, houses, and schools and caused outbreaks of cholera and other diseases. It is therefore important to know about occurrences of such extreme events and their chances of occurring. Extreme value theory has found application in hydrology, environmental research and meteorology. Fisher and Tippet (1928) showed that extreme limit distributions can only be one of three types, namely the Gumbel (type I), Frechet (type II) and Weibull (type III) distributions. The Generalized extreme value (GEV) distribution is also known as Fisher-Tippet distribution. It is a three parameter model that combines the Gumbel, Frechet and Weibull maximum extreme value distributions. it is the limiting distribution of maxima of a sequence of independent and identically distributed random variables. GEV has great importance in hydrology. It is widely used for modelling extremes of natural phenomena. Recently, researchers in Nigeria have worked on the applications of extreme values in hydrology such as maximum rainfall and floods, for instance, Nashwan et al (2019) compared four probability distributions namely, Generalized Pareto, Gumbel, GEV, and Exponential. the finding showed that GEV distribution was the best for extreme rainfall in Peninsular Malaysia. Sharma and Mujamdar (2019) also employed the generalized extreme value distribution to model extreme precipitation events which is a common in hydrology and climatology for analysing rare and extreme events. It was determined that local mean temperature was a significant covariate of nonstationary GEV distribution for extreme rainfall in the studied regions. Jasmine and syafrina (2020) model rainfall data using Generalized extreme value distribution with cyclic covariate. The results showed the capability of the nonstationary GEV with cyclic covariates in the capturing the extreme rainfall events. Ologunorisa and Tersoo (2006) conducted analysis of recent changes in the characteristics of extreme rainfall and their implication on flood frequency in Makurdi, they employed data on extreme daily rainfall, evapotranspiration and flood occurrences. They analyzed the annual rainfall for trends using spearman rank correlation coefficient and annual rainfall variability using standardized rainfall anomaly index while recurrence intervals were analyzed using Gumbel Extreme probability theory. Their results show among other things that there was a remarkable continuous downward trend in annual rainfall amounts; that the period between 1996 and 2001 witnessed the highest frequencies of extreme rainfall events and flood frequencies; that major floods were associated with high recurrence intervals, and that the seasonality of flooding in Makurdi occurs between May and October. Ekpoh and Nsa (2011) examined Some aspects of the climate of north-western Nigeria, focusing more on rainfall, its inter- and intra-annual variability and patterns of distribution. The authors adopted some statistical tools commonly used to describe climatic conditions. Their study found that climatic conditions in north-western Nigeria have altered substantially as four drought episodes took place within the last three decades of the 20th Century. Globally, many researches were done on rainfall analysis to understand the characteristics of precipitation so that the appropriate measure can be taken. Nadarajah and Shiau (2005) employed the Gumbel and GEV distributions as models of extreme values. They showed that the Gumbel distribution can be considered over the GEV distribution for both flood volume and flood peak. According to Hirose (1994) "Weibull distribution" was found to be the suitable distribution for describing annual maximal rainfall in Japan. Also Deka, Borah and Kakaty (2009) considered "five extreme value distributions and derived the best distribution to describe the annual series of maximum rainfall data of nine distantly located stations in north east India for the period 1966 to 2007. Varathan, Perera and Nalin (2010) found the "Gumbel distribution to be the best fitting model for describing the annual maximum rainfall in Colombo district". Koutsoyiannis (2004) compared two types of extreme value distributions, Gumbel (EV1), and Frechet (EV2), by applying them to a collection of 169 gauges of the available maximum rainfall records worldwide, with each record having 100-154

years of data. He showed that the (EV2) distribution is more appropriate than (EV1) for a longer-record of maximum rainfall data. The most studies on the application of extreme value models in hydrology have always centered on a particular state, region or city in a given country and little attention has been placed on hydrology in Nigeria. The purpose of this work is to identify suitable distribution for describing annual maximum rainfall in Ondo state, Nigeria by using generalized extreme value distributions.

MATERIALS AND METHODS

In this work, annual maximum rainfall data was collected from Nigeria Meteorological Agency, Akure, Ondo State for a period of 1981 to 2019. Identification of extremeties in the data distribution was carried out using block maxima method. The data was partitioned into a set of group or line which is based on the months and the maximum observations were taken. These selected observations constitute the extreme events. The GEV distribution was fitted to the observed maximum rainfall data by means of maximum likelihood and the second order N-R technique embedded in (Hessian matrix) method. The execution was carried out using R package.

Generalized Extreme Value Distribution

The generalized extreme-value (GEV) distribution, introduced by Jenkinson (1955) has been used to model a wide variety of natural extremes in many applications, including floods, rainfall, wind speeds, wave height, and other situations involving extremities. Its limiting maximum cumulative distribution function (CDF) is of the form

$$F(x) = exp^{-\left[1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right]^{\frac{1}{\alpha}}} \quad (1)$$

Its probability density function (PDF) is given as

$$f(x) = \left[1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right]^{\frac{1}{\alpha}-1} exp^{-\left[1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right]^{\frac{1}{\alpha}}} \quad (2)$$

Where $\sigma > 0$ is the scale parameter, $-\infty < \alpha < \infty$ is the shape parameter and $\mu > 0$ is the location parameter. The value of α dictates the tail behavior of CDF. For $\alpha < 0$, $\alpha > 0$ and $\alpha = 0$. The GEV distribution function reduces to Frechet, Weibull, and Gumbel distributions, respectively.

Parameter Estimation of the GEV Distribution

The maximum likelihood estimation technique was used to estimate the parameters of the GEV distribution. The maximum likelihood (ML) estimator of the given random sample of extreme rainfall measurements was obtained by seeking the likelihood function of the GEV distribution as follows;

$$L(\mu,\sigma,\alpha) = f(x_1, x_2, \dots, x_n / \mu,\sigma,\alpha) = \prod_{i=1}^{n} f(x_i / \mu,\sigma,\alpha)$$

$$= \left(\frac{1}{\sigma}\right)^n e^{-\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\sigma}}} \prod_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\sigma}-1} \quad (3)$$

The log-likelihood function is given as

$$\ln(L) = -n ln(\sigma) - \left(1-\frac{1}{\sigma}\right)\sum_{i=1}^{n} ln\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right) - \sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\sigma}} \quad (4)$$

The classical approach to the ML method requires the computation of the first-order partial derivatives of the log-likelihood function with respect to each of its parameters, equating them equal to zero and then solving the resulting system of equations. The first-order partial derivatives are obtained as follows

$$\frac{\partial l}{\partial \mu} = -\frac{\alpha}{\sigma}\left(1-\frac{1}{\sigma}\right)\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-1} - \left(\frac{1}{\alpha}\right)\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-1} = 0 \quad (5)$$

$$\frac{\partial l}{\partial \sigma} = -\frac{n}{\sigma} + \left(1-\frac{1}{\sigma}\right)\left(-\frac{\alpha}{\sigma^2}\right)\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-1}(x-\mu) +$$

$$\left(\frac{1}{\alpha}\right)\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-1}\left(\frac{\alpha}{\sigma^2}\right)(x-\mu)\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}} = 0 \quad (6)$$

$$\frac{\partial l}{\partial \alpha} = \frac{1}{\sigma}\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-1}\left(\frac{x-\mu}{\sigma}\right) + \left(1-\frac{1}{\alpha}\right)\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-1}\left(\frac{x-\mu}{\sigma}\right)$$

$$-\left(\frac{1}{\alpha^2}\right)\sum_{i=1}^{n} ln\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right) + \left(\frac{1}{\alpha^2}\right)\sum_{i=1}^{n} ln\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}} = 0 \quad (7)$$

There is no analytical solution to the eqn (5) (6) (7) Then the location, scale, and shape parameters can be obtained using second order Newton Raphson method. It requires to obtain hessian matrix H given by:

$$H = \begin{bmatrix} \frac{\partial^2 l}{\partial \mu^2} & \frac{\partial^2 l}{\partial \mu \partial \sigma} & \frac{\partial^2 l}{\partial \mu \partial \alpha} \\ \frac{\partial^2 l}{\partial \sigma \partial \mu} & \frac{\partial^2 l}{\partial \sigma^2} & \frac{\partial^2 l}{\partial \sigma \partial \alpha} \\ \frac{\partial^2 l}{\partial \alpha \partial \mu} & \frac{\partial^2 l}{\partial \sigma \partial \alpha} & \frac{\partial^2 l}{\partial \alpha^2} \end{bmatrix}$$

$$\frac{\partial^2 l}{\partial \mu^2} = \frac{\alpha^2}{\sigma^2}\left(1-\frac{1}{\alpha}\right)\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-2} - \left(\frac{\alpha}{\sigma^2}\right)\left(\frac{1}{\alpha}-1\right)\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-2} = 0 \quad (8)$$

$$\frac{\partial^2 l}{\partial \mu \partial \sigma} = \frac{\alpha}{\sigma^2}\left(1-\frac{1}{\alpha}\right)\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-1} + \frac{\alpha^2}{\sigma^3}\left(1-\frac{1}{\alpha}\right)\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-2}(x-\mu)$$

$$+\frac{1}{\partial^2}\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-1} - \frac{\alpha}{\sigma^3}\left(\frac{1}{\alpha}-1\right)\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-2}(x-\mu) = 0 \quad (9)$$

$$\frac{\partial^2 l}{\partial \mu \partial \alpha} = -\frac{1}{\sigma\alpha}\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-1} - \frac{1}{\sigma}\left(1-\frac{1}{\alpha}\right)\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-1}$$

$$-\frac{\alpha}{\sigma}\left(1-\frac{1}{\alpha}\right)\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)\left(\frac{x-\mu}{\sigma}\right)+\frac{1}{\alpha^2\sigma}\sum_{i=1}^{n} ln\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)$$
$$\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-1}+\frac{1}{\sigma}\left(\frac{1}{\alpha}-1\right)\sum_{i=1}^{n}\left(\frac{x-\mu}{\sigma}\right)\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-2} =0 \quad (10)$$

$$\frac{\partial^2 l}{\partial\sigma^2}=\frac{n}{\sigma^2}-\frac{\alpha}{\sigma^2}\left(\frac{1}{\alpha}-1\right)\sum_{i=1}^{n}(x-\mu)\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-1}+\frac{2\alpha}{\sigma^3}\sum_{i=1}^{n}(x-\mu)\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-1}=0 \quad (11)$$

$$\frac{\partial^2 l}{\partial\sigma\partial\alpha}=-\frac{1}{\alpha}\sum_{i=1}^{n}\left(\frac{x-\mu}{\sigma}\right)\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-1}-\frac{1}{\sigma}\left(1-\frac{1}{\alpha}\right)\sum_{i=1}^{n}\left(\frac{x-\mu}{\sigma}\right)\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-1}$$
$$-\frac{\alpha}{\sigma}\left(1-\frac{1}{\alpha}\right)\sum_{i=1}^{n}\left(\frac{x-\mu}{\sigma}\right)^2\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^2+\frac{1}{\alpha^2\sigma}\sum_{i=1}^{n}\left(\frac{x-\mu}{\sigma}\right) ln\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)$$
$$\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-1}+\frac{1}{\sigma}\left(\frac{1}{\alpha}-1\right)\sum_{i=1}^{n}\left(\frac{x-\mu}{\sigma}\right)^2\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-2}=0 \quad (12)$$

$$\frac{\partial^2 l}{\sigma\alpha^2}=\frac{2}{\alpha^3}\sum_{i=1}^{n} ln\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)+\frac{1}{\alpha^2}\sum_{i=1}^{n}\left(\frac{x-\mu}{\sigma}\right)\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-1}$$
$$+\frac{2}{\alpha^2}\sum_{i=1}^{n}\left(\frac{x-\mu}{\sigma}\right)\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-1}+\left(1-\frac{1}{\alpha}\right)\sum_{i=1}^{n}\left(\frac{x-\mu}{\sigma}\right)^2\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{-2}$$
$$-\frac{2}{\alpha^2}\sum_{i=1}^{n} ln\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}}-\frac{1}{\alpha^4}\sum_{i=1}^{n}\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}}$$
$$ln\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^2-\frac{1}{\alpha^3}\sum_{i=1}^{n}\left(\frac{x-\mu}{\sigma}\right) ln\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-1}$$
$$-\frac{2}{\alpha^2}\sum_{i=1}^{n}\left(\frac{x-\mu}{\sigma}\right)\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-1}-\frac{1}{\alpha}\left(\frac{1}{\alpha}-1\right)\sum_{i=1}^{n}\left(\frac{x-\mu}{\sigma}\right)^2\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-2}$$
$$-\frac{1}{\alpha^4}\sum_{i=1}^{n}\left(\frac{x-\mu}{\sigma}\right) ln\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)\left(1-\alpha\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\alpha}-1}=0 \quad (13)$$

To obtain the parameter μ , σ and α, we proceed with the Newton-Raphson Algorithm (second order),

$$\begin{bmatrix}\mu^{(1)}\\ \sigma^{(1)}\\ \alpha^{(1)}\end{bmatrix}=\begin{bmatrix}\mu^{(0)}\\ \sigma^{(0)}\\ \alpha^{(0)}\end{bmatrix}-H^{-1}(\mu^{(0)},\quad \sigma^{(0)},\quad \alpha^{(0)})f(\mu^{(1)},\quad \sigma^{(1)},\quad \alpha^{(1)})^{-1} \quad (14)$$

The iteration converges when,

$$\left[\begin{bmatrix}\mu^{(1)}\\ \sigma^{(1)}\\ \alpha^{(1)}\end{bmatrix}-\begin{bmatrix}\mu^{(0)}\\ \sigma^{(0)}\\ \alpha^{(0)}\end{bmatrix}\right]^1\left[\begin{bmatrix}\mu^{(1)}\\ \sigma^{(1)}\\ \alpha^{(1)}\end{bmatrix}-\begin{bmatrix}\mu^{(0)}\\ \sigma^{(0)}\\ \alpha^{(0)}\end{bmatrix}\right]\leq k \quad (15)$$

Where, $\mu^{(0)}$, $\sigma^{(0)}$and $\alpha^{(0)}$ are the initial values, $\mu^{(1)}$, $\sigma^{(1)}$and $\alpha^{(1)}$ are the iterative values and k is the tolerance level. We use R statistical software to run the functions in the hessian matrix to obtain the final estimate.

Return Level Estimation for GEV model

A return level is defined as the precipitation amount that is exceeded by the annual maximum in any particular year with probability $\frac{1}{T}$, or in other words, the amount that on average occurs every T years (Coles et al 2001). The average interval between each occurrence is referred to as return period T, or recurrence interval. In extreme values studies. It is important to know with which probability a rare event can occur in the next period of time. Return levels, is used in predicting that annual maximum rainfall exceeding the maximum observations in the data. In this study, return levels for 5, 10, 15, 20 and 25 return period are estimated. The return level can be obtained by inverting the GEV cumulative distribution in equation

$$x_T=\mu-\frac{\sigma}{\alpha}\left[1-\left(-\log\left(1-\frac{1}{T}\right)\right)^{\alpha}\right] \quad (16)$$

Probability of Exceedance and Return Periods

The probability of exceedance is explained as the chance of occurrence of some events over a given time period. The probability of exceedance is used in predicting how often rainfall amount will be exceeded the periods after the period of recorded data. It is given by

$$P_e=P(X>x)=1-P(X\leq x)=1-p_e \quad (17)$$

Where x is the reference point in the sample space of X

Return Periods

The return period which is also known as recurrence interval, is an estimation of the likelihood of an event, such as rainfall, flood, storms or a river discharge flow to occur. The return period (T)is expressed as $\frac{1}{p_e}$ where p_e represents the probability of exceedance.

RESULTS AND DISCUSSION

Fitting GEV Distribution to Annual Maximum Rainfall Data

The GEV distribution was fitted to annual maximum rainfall data using second order Newton-Raphson method. This was executed (implemented) in R package. The output shows the maximum likelihood estimate of each of the parameters (location, scale and shape), standard errors and their corresponding 95% confidence intervals

Table 1: Maximum likelihood estimates for the GEV model with standard errors

Parameter	MLE (SE)	95% Confidence Interval
Location (μ)	169.74 (15.216)	(137.96, 201.52)
Scale (∂)	22.89(2.723)	(17.56, 28.22)
Shape (α)	-0.41 (0.003)	(-0.416, -0.404)

The result in Table 1 shows that, the shape parameter is negative this suggests that the underlying distribution belongs to the "Frechet distribution".

Prediction of Return Level for Different Return Periods and Confidence Limits for GEV
the corresponding maximum likelihood confidence intervals (CI) for the return periods; 5, 10, 15, 20, 25 and the quantiles; 0.2, 0.1, 0.07,0.05 and 0.04 of the GEVD at 5% level of significance is given in the Table 2.

Table 2: The confidence intervals for the 0.2, 0.1, 0.07, 0.05 and 0.04 quantiles of the GEVD at 5% level of significant:

	P	$\hat{x}_p$	CI
$\alpha = 0.05$	0.2	217.173	(205.95,228.39)
	0.1	254.374	(243.16,265.59)
	0.07	277.574	(266.36,288.79)
	0.05	302.596	(291.38,313.82)
	0.04	321.113	(309.89,332.33)

The results show that for every increase in the return period, there exist a slight positive increase in the return level at 5% level of significant which indicate that, rainfall frequency will keep on increasing in the value over years.

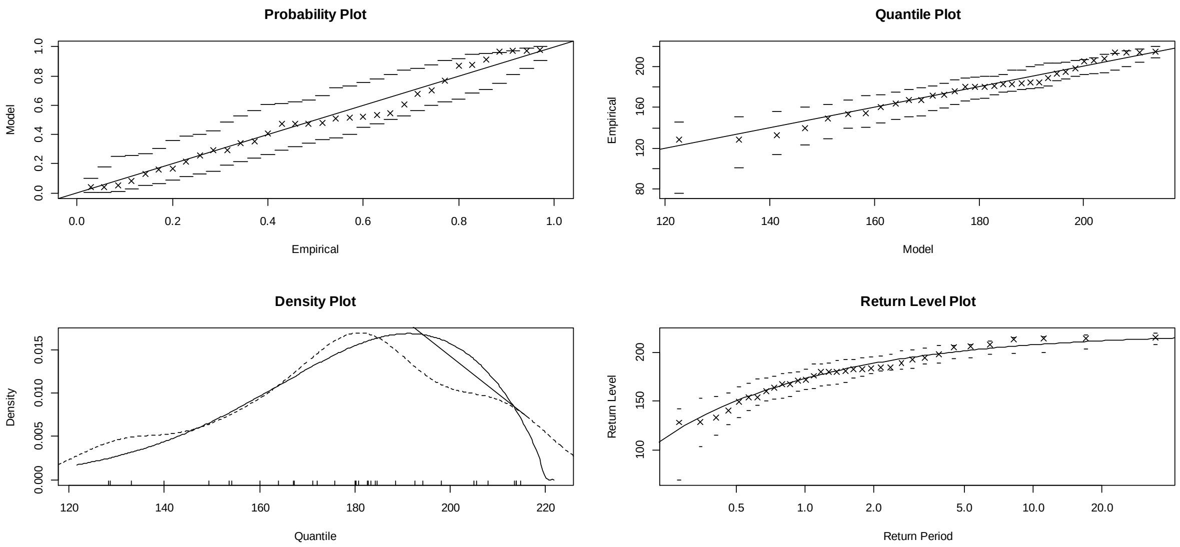

Figure 1: The diagnostic plots

CONCLUSION

In this work, the Generalized Extreme Value Distributions was used to model annual maximum rainfall data in Akure, Ondo State, Nigeria from 1981-2019. The maximum likelihood estimates of the parameters of the distribution is given in Table- 1. The shape parameter gives a negative value which indicated that the Fréchet distribution family is the appropriate model for describing annual maximum rainfall data set in Akure, Ondo state. The least maximum extreme value of rainfall which will occur after 25 years is given in Table-2. It was also observed that for every increase in the return period, there exists a positive increase in the return levels, which led to the conclusion that, rainfall frequency will keep on increasing in the value over years.

REFERENCES

Coles, S, Bawa, J., Trenner, L and Dorazio, P (2001) An introduction to statistical modeling of extreme values, Vol. 208.

Deka, S. Borah M. and Kakaty, S. C. (2009). Distributions of annual maximum rainfall series of north-east India. *European Water, 27(28)*, 3-14.

Ekpoh I.J and Nsa E. (2011) Extreme Climatic Variability in North-western Nigeria: An Analysis of Rainfall Trends and Patterns, Journal of Geography and Geology Vol. 3, No. 1

Fisher, A. R. and Tippett, L. C. (1928). Limiting forms of the frequency distribution of the largest or smallest member of a sample. *Proceedings of the Cambridge Philosophical Society 24*, 180-190.

Hirose, H. I. D. E. O. (1994). Parameter estimation in the extreme-value distributions using the continuation method. *Transactions of Information Processing Society of Japan*, *35*(9).

Jasmine Lee J., M, and Syafrina A.,H , (2020). Rainfall Modelling using Generalized Extreme Value Distribution with Cyclic Covariate. Mathematics and Statistics, 8(6), 762-772. DOI: 10.13189/ms.2020.080617.

Jenkinson, A. F., (1955). The frequency distribution of the annual maximum (or minimum) values of meteorological elements, Q. J. R. *Meteorol. Soc.*, 81, 158–171.

Koutsoyiannis.D. (2004). Statistics of extremes and estimation of extreme rainfall: (i)Empirical investigation of long rainfall records. ii. empirical investigatio. Hydrological Sciences Journal, 49(4).

Nadarajah S. and Shiau J.T. (2005). Analysis of extreme ood events for the Pachang river, Taiwan. Water resources management, 19(4):363-374, 20

Nashwan, M. S, Ismail, T and Ahmed, K. (2019) Non-stationary analysis of extreme rainfall in peninsular Malaysia, J. Sustain. Sci. Manag, Vol. 14, 17–34.

Ologunorisa T. E. and Tersoo T (2006) The Changing Rainfall Pattern and Its Implication for Flood Frequency in Makurdi, Northern Nigeria J. Appl. Sci. Environ. Mgt. September, 2006 Vol. 10 (3) 97 - 102

Sharma, S., and Mujumdar, (2019) On the relationship of daily rainfall extremes and local mean temperature, Journal of Hydrology, Vol. 572,179–191, 2019.

Varathan, N., Perera, K., & Nalin. (2010). *Statistical modeling of extreme daily rainfall in Colombo.* Sri Lanka: M.Sc thesis, Board of Study in Statistics and Computer Science of the postgraduate institute of science, university of Peradeniya.