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NEW MODELS FOR ESTIMATION OF DIFFUSE SOLAR RADIATION USING METEOROLOGICAL PARAMETERS FOR BENIN, NIGERIA

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ABSTRACT

In this comprehensive study, an extensive 22-year dataset (2001-2022) for Benin (Latitude 6.32 ºN, Longitude 5.10 ºE and 77.80 m above sea level) were obtained from the National Aeronautic Space Administration (NASA) website. The datasets comprises of the monthly average daily global solar radiation, diffuse solar radiation, relative humidity, atmospheric pressure, wind speed, and mean temperature, was utilized to develop 19 new models for estimating diffuse solar radiation. These models were categorized into five distinct groups: modified Page, Liu and Jordan models; clearness index and one-variable models; two-variable models; three-variable models, and a four-variable model. These models were statistically evaluated using a set of five validation indices—Mean Bias Error (MBE), Root Mean Square Error (RMSE), Mean Percentage Error (MPE), t-test, and the coefficient of determination (R^2). The study identified the most effective models in each category. Equation 28b from the modified Page, Liu and Jordan category, Equation 28f from the clearness index and one-variable models, Equation 28j from the two-variable models, and Equation 28o from the three-variable models category were found to be the best-performing models. A comparative assessment of these performed models revealed that the quadratic regression model (Equation 28b) stood out as the most suitable for accurately estimating diffuse solar radiation in Benin. This implies that the developed model equation 28b can be used to estimate the diffuse solar radiation for Benin and locations with similar climatic conditions.

Keywords: Benin, Diffuse Solar radiation, NASA, Quadratic regression model, Validation indices

INTRODUCTION

The availability of solar energy, or solar radiation, at the Earth's surface is influenced by a complex interplay of factors that span astronomy, physics, geography, and meteorology. These factors include the atmospheric transmittance, the level of extraterrestrial radiation, the geographical latitude, the varying relative distance between the Earth and the Sun, the actual amount of sunshine hours, the sunset hour angle, along with local atmospheric conditions like relative humidity, ambient temperature, and the degree of cloud cover (Duzen and Aydin, 2012; Adaramola, 2012).

Diffuse solar radiation, a significant component of solar energy reaching the Earth's surface, is affected by the interaction of solar rays with atmospheric matter. Understanding these interactions is vital for accurately modeling and predicting diffuse solar radiation in various geographical settings (Ogbulezie *et al*., 2017). On clear days, diffuse radiation accounts for about 10% of total solar energy, but under cloudy conditions, its contribution can soar to 90%, as clouds scatter and redirect sunlight (Iqbal, 1983; Klucher, 1991). Rayleigh scattering by air molecules, particularly affecting shorter wavelengths, is a key process in this phenomenon (Klucher, 1991).

Measuring diffuse solar energy typically involves instruments like pyranometers and solarimeters, but their high cost limits widespread installation. These devices, while essential, have limitations in accuracy and can produce erroneous data. To overcome this, solar energy models are developed using measured data, correlating solar energy with meteorological variables like temperature and humidity. These models are crucial for predicting solar radiation in areas without measurement devices (Kreider and Kreith, 1981).

Despite the importance of such measurements, many countries, especially developing ones, lack comprehensive solar radiation data. Global solar radiation data is more commonly available than diffuse radiation data, which is harder and costlier to measure and hence recorded at fewer stations (Li *et al*., 2012; Bakirci, 2015). This scarcity underscores the necessity of developing empirical correlations to estimate diffuse radiation in locations lacking measured data, a significant challenge in the field of solar radiation research.

Numerous models have been introduced for calculating diffuse solar radiation. In 1961, Liu and Jordan proposed a theoretical approach to determine the mean hourly solar radiation from the average daily total radiation, based on the premise that atmospheric transmission remains constant all day and does not vary with the solar altitude. Page, in 1964, established a linear correlation between the clearness index and the ratio of diffuse to global solar radiation. Subsequently, Iqbal in 1979 and Lam-Li in 1996 suggested a linear relation using the clearness index to estimate the monthly mean diffuse solar radiation. Collares-Pereira and Rabl, in 1979, utilizing data from five US stations along with Liu and Jordan's curve, formulated an analytical expression for the ratio of hourly to daily solar radiation, which is a function of the sunset hour angle. Later developments by Erbs *et al*. in 1982 and Muneer *et al*. in 1984 further enriched the field of solar radiation estimation models. Akpootu *et al*., (2015); Akpootu and Mustapha (2015) estimated diffuse solar radiation for Yola, Nigeria. More recent, Berrizbeitia *et al*. (2020) and Salhi *et al*. (2020) have both significantly contributed to the field of solar radiation estimation through their respective studies. Berrizbeitia *et al*. (2020) embarked on a comprehensive analysis of regression models, focusing on the relationship between the diffuse ratio (k) and clearness index (K_T) across various time frames, including annual, monthly, daily, and hourly intervals. A notable gap identified in existing literature was the lack of regressions based on averaged data, specifically in the context of monthly-averaged daily global irradiation and hourly averaged diffuse irradiation. Addressing this, the study presented new regression equations for estimating hourly averaged diffuse irradiation, utilizing data from 19 locations worldwide. The researchers developed three latitude-dependent regression models that connect the monthly-averaged hourly diffuse ratio with the clearness index, uncovering a robust correlation between these two parameters at the hourly level. The resulting regression equations stand out as instrumental tools for calculating averaged diffuse irradiation values, a significant advance given the limited availability of direct hourly diffuse irradiation measurements. This research fills a critical knowledge gap in estimating averaged diffuse irradiation on an hourly basis, enhancing the understanding of solar radiation patterns and providing practical utility for locations without direct measurements.

Similarly, Salhi *et al*. (2020) undertook a detailed assessment of the diffuse fraction and diffusion coefficient using statistical analysis. Their study involved the development of eighty models to predict these parameters, taking into account the sunshine ratio and clearness index. The research utilized monthly average global and diffuse solar radiation data, alongside sunshine duration data from Tamanrasset station, covering the period from 1995 to 2017. Employing a comprehensive set of nine statistical indicators, including mean bias error (MBE), mean absolute error (MAE), and root mean squared error (RMSE), among others, and a Global Performance Indicator, the study rigorously compared and evaluated the models. The outcome identified the cubic model, which incorporates both sunshine ratio and clearness index, as the most precise for estimating diffuse solar radiation on a horizontal surface in Tamanrasset. This model stands out for its accuracy and relevance in solar radiation estimation, marking a significant advancement in the field. The Page, Lui and Jordan models were found suitable for estimating diffuse solar radiation globally, hence the reason their models were adopted for this research. This study aims to achieve these specific objectives: (i) to identify the most suitable models for estimating diffuse solar radiation in Benin, utilizing modified versions of the Page, Liu, and Jordan models. (ii) to develop correlation models incorporating two to four variables for the estimation of diffuse solar radiation. (iii) to rank these models to ascertain the most appropriate one for accurate estimation and Lastly (iv), to analyze the correlations between the estimated or predicted values and the measured diffuse solar radiation values, providing a comprehensive evaluation of the model performance.

MATERIALS AND METHODS

Study Area

Benin, situated at latitude 6.32°N and longitude 5.10°E, with an elevation of 77.80 meters above sea level, serves as the capital of Edo State, Nigeria. Nestled in the southern region of Nigeria, Benin enjoys a strategic position in the coastal climatic zone, where the Atlantic Ocean influences its weather patterns. The city's coastal location bestows upon it a tropical climate characterized by warm temperatures and high humidity levels throughout the year.

Figure 1: Map of Nigeria, showing Study Area

Data collection

For this study, twenty-two (22) years' worth (2001-2022) of monthly average climatic data for was acquired. The data include measurements of all-sky surface shortwave diffuse irradiance, global solar radiation, wind speed, mean temperatures, surface pressure, and relative humidity. The data was obtained from the National Aeronautics and Space Administration (NASA) website.

The mean daily extraterrestrial radiation on a horizontal surface, denoted by H_O and measured in $MJm^{-2}day^{-1}$, can be estimated for each day of a month by averaging the daily values over that month (Iqbal, 1983; Zekai, 2008; Saidur *et* *al.*, 2009). This calculation is based on the equations presented by Iqbal (1983) and Zekai (2008) as reported by Akpootu and Abdullahi (2022)

$$H_o = \left(\frac{24}{\pi}\right) I_{sc}\left[1 + 0.033\cos\left(\frac{360n}{365}\right)\right]\left[\cos\phi\cos\delta\sin\omega_s + \left(\frac{2\pi\omega_s}{360}\right)\sin\phi\sin\delta\right] \quad (1)$$

where $I_{SC} = 1367Wm^{-2}$

The parameters I_{sc}, ϕ, δ, and ω_s, representing the solar constant, site latitude, solar declination, and mean sunrise hour angle, respectively, are utilized in the equation to determine H_O. Additionally, n, the number of days in a year from January 1st to December 31st, is incorporated into the calculation. The solar declination and mean sunrise hour angle are determined using the methods presented by Iqbal (1983) and Zekai (2008) as reported by Akpootu and Abdullahi (2022) are given by:

$$\delta = 23.45\sin\left\{360\left(\frac{284+n}{365}\right)\right\} \quad (2)$$

$$\omega_s = \cos^{-1}(-\tan\phi\tan\delta) \quad (3)$$

The clearness index, K_T, provides valuable information about the availability of solar radiation at a specific location. K_T value of one (1) indicates that the sky is completely clear and that the maximum amount of solar radiation is reaching the Earth's surface. K_T value of zero (0) indicates that the sky is completely overcast and that no solar radiation is reaching the Earth's surface. K_T values typically range between 0.2 and 0.8, with higher values indicating clearer skies and more available solar radiation (Iqbal, 1983). Mathematically, the clearness index as reported as reported by Akpootu *et al*. (2023) is given by:

$$K_T = \frac{H_m}{H_o} \quad (4)$$

The total incoming solar radiation, denoted by H_m, is the measured global solar radiation in $MJm^{-2}day^{-1}$

Developed Diffuse Solar Radiation Models

The proposed models of diffuse solar radiation based on the modified Page; Liu and Jordan models are:

$$\frac{H_d}{H_m} = a + bK_T \quad (5)$$

$$\frac{H_d}{H_m} = a + bK_T + cK_T^2 \quad (6)$$

$$\frac{H_d}{H_m} = a + bK_T + cK_T^2 + dK_T^3 \quad (7)$$

$$\frac{H_d}{H_m} = a + bK_T + cK_T^2 + dK_T^3 + eK_T^4 \quad (8)$$

To ensure that no relevant parameters are omitted, multiple linear regression was employed, using the four meteorological parameters (WS,T_{mean}, RH,PS) as independent variables and $\frac{H_d}{H_m}$ as the dependent variable. These meteorological parameters represent, respectively, monthly average daily wind speed (ms^{-1}), monthly average mean temperature (^{O}C), monthly average daily relative humidity (%) and monthly average daily atmospheric pressure (hPa).

The other proposed diffuse solar radiation models, developed in this study, involves correlations of the linear Page model with meteorological parameters as follows:

$$\frac{H_d}{H_m} = a + b\frac{H}{H_o} + cWS \quad (9)$$

$$\frac{H_d}{H_m} = a + b\frac{H}{H_o} + cRH \quad (10)$$

$$\frac{H_d}{H_m} = a + b\frac{H}{H_o} + cPS \quad (11)$$

$$\frac{H_d}{H_m} = a + b\frac{H}{H_o} + cT_{mean} \quad (12)$$

The proposed two variables correlation models are:

$$H_d = a + bWS + cRH \quad (13)$$

$$H_d = a + bWS + cT_{mean} \quad (14)$$

$$H_d = a + bWS + cPS \quad (15)$$

$$H_d = a + bRH + cT_{mean} \quad (16)$$

$$H_d = a + bRH + cPS \quad (17)$$

$$H_d = a + bT_{mean} + cPS \quad (18)$$

The proposed three variables correlation models are:

$$H_d = a + bWS + cRH + dT_{mean} \quad (19)$$

$$H_d = a + bWS + cT_{mean} + dPS \quad (20)$$

$$H_d = a + bWS + cPS + dRH \quad (21)$$

$$H_d = a + bRH + cT_{mean} + dPS \quad (22)$$

The proposed four variables correlations model is:

$$H_d = a + bWS + cRH + dT_{mean} + ePS \quad (23)$$

From equations (5) to (23), the algebraic constants a,b,c,d, and e are known as empirical constants or coefficients.

The Minitab software (version 21.2) package was employed to assess the model parameters used in obtaining the empirical constants.

Accuracy of the Models

The effectiveness of each model was statistical tested employing the following indices Mean Bias Error (MBE), Root Mean Square Error (RMSE), Mean Percentage Error (MPE), and t-test. The equations for calculating MBE, RMSE, and MPE, based on the method proposed by El-Sebaii *et al.,* (2005) as proposed by Akpootu *et al*., (2015) are presented as follows:

$$MBE = \frac{1}{n}\sum_{i=1}^{n}\left(H_{d_{i,cal}} - H_{d_{i,meas}}\right) \quad (24)$$

$$RMSE = \left[\frac{1}{n}\sum_{i=1}^{n}\left(H_{d_{i,cal}} - H_{d_{i,meas}}\right)^2\right]^{\frac{1}{2}} \quad (25)$$

$$MPE = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{H_{d_{i,meas}} - H_{d_{i,cal}}}{H_{d_{i,meas}}}\right) \text{x } 100 \quad (26)$$

As described by Bevington (1969), the t-test, being one of the statistical methods employed to evaluate mean values, utilizes a random variable denoted as t, possessing $n-1$ degrees of freedom. Also, according to (Akpootu *et al*., 2019c, d; Akpootu *et al*., 2023) t –test is a non-dimensional parameter. It can be expressed as follows:

$$t = \left[\frac{(n-1)(MBE)^2}{(RMSE)^2 - (MBE)^2}\right]^{\frac{1}{2}} \quad (27)$$

As derived from equations (24), (25), and (26), the variables $H_{d_{i,meas}}$ and $H_{d_{i,cal}}$ represents the i^{th} measured and i^{th} calculated values of daily diffuse solar radiation, respectively, along with the total number of observations denoted by n. Lower values of MBE, RMSE, MPE, and t-test indicate superior model performance. These statistical measures assess the accuracy of a model's predictions compared to actual observed values. Positive values of MPE and MBE quantify the average degree of overestimation in the model's predictions, while negative values indicate underestimation (Akpootu *et al*., 2023). According to Merges *et al.*, (2006); Gana *et al*., (2013a; b); Akpootu *et al*, (2014); Akpootu and Sulu, (2015); Olomiyesan *et al*., (2021), for a model to perform better, a low value for MPE is desirable and the percentage error between –10 % and +10 % is considered acceptable. Also, Halouani *et al*., (1993); Almorox *et al*., (2005) and Chen *et al*., (2004) did recommend a zero value for MBE as ideal and a low value for RMSE as desirable.

To achieve a more accurate and reliable data modeling outcome, the coefficient of determination (R^2) should strive to approach a value of 1, ideally reaching 100% (Akpootu and Iliyasu 2015a, b; Akpootu *et al*., 2019a, b). This indicates a strong correlation between the predicted and observed values, suggesting a robust and well-fitting model.

RESULTS AND DISCUSSION

The results of the Page, Liu and Jordan models; clearness index and one variable models; two variables model; three variables models and four variables model for Benin based on equations (5) to (23) are:

$$\frac{H_d}{H_m} = 1.0692 - 1.034K_T \quad (28a)$$
$$\frac{H_d}{H_m} = 0.033 + 3.62K_T - 5.14{K_T}^2 \quad (28b)$$
$$\frac{H_d}{H_m} = 4.57 - 27.2K_T + 63.7{K_T}^2 - 50.8{K_T}^3 \quad (28c)$$
$$\frac{H_d}{H_m} = 18.3 - 151K_T + 479{K_T}^2 - 665{K_T}^3 + 339{K_T}^4 \quad (28d)$$
$$\frac{H_d}{H_m} = 1.0914 - 0.997\frac{H}{H_o} - 0.168WS \quad (28e)$$
$$\frac{H_d}{H_m} = 1.133 - 1.060\frac{H}{H_o} - 0.00057RH \quad (28f)$$
$$\frac{H_d}{H_m} = 21.33 - 1.3304\frac{H}{H_o} - 0.02004PS \quad (28g)$$
$$\frac{H_d}{H_m} = 0.549 - 1.1284\frac{H}{H_o} + 0.02211T_{mean} \quad (28h)$$
$$H_d = 47.5 - 53.4WS - 0.285RH \quad (28i)$$
$$H_d = -6.80 - 2.62WS + 0.668T_{mean} \quad (28j)$$
$$H_d = 364 - 13.07WS - 0.350PS \quad (28k)$$
$$H_d = -9.66 + 0.0180RH + 0.693T_{mean} \quad (28l)$$
$$H_d = 452 + 0.0969RH - 0.450PS \quad (28m)$$
$$H_d = -77 + 0.795T_{mean} + 0.066PS \quad (28n)$$
$$H_d = -27.4 + 17.6WS + 0.119RH + 0.872T_{mean} \quad (28o)$$
$$H_d = 10 - 3.10WS + 0.640T_{mean} - 0.016dPS \quad (28p)$$
$$H_d = 1090 + 102.0WS - 1.172PS + 0.808RH \quad (28q)$$
$$H_d = 157 + 0.046RH + 0.452T_{mean} - 0.162PS \quad (28r)$$
$$H_d = 1410 + 118.0WS + 0.957RH - 0.34T_{mean} - 1.50PS \quad (28s)$$

Modified Page, Liu and Jordan Models

Below are the statistical analysis summary of the models Eqn 28a to Eqn 28d

Table 1: Modified Page, Liu and Jordan Models Statistical Error indicators

Models	R^2	MBE	RMSE	MPE	t
Eqn 28a	87.97%	-0.0108	0.3685	-0.1452	0.0976
Eqn 28b	95.01%	-0.0011	0.2361	-0.0686	0.0148
Eqn 28c	95.65%	-0.3426	0.4184	3.5212	4.7318
Eqn 28d	95.76%	0.5133	0.6640	-5.4854	4.0425

Table 1 presents a condensed overview of the statistical validation test conducted on the modified Page, Liu and Jordan models for this study. Among the models, the modified equation 28d, a polynomial of degree 4, exhibits the highest R^2 value as 95.76 %. Notably, the quadratic regression model (Eqn 28b) demonstrates the lowest for MBE, MPE, RMSE and t-test values with underestimation of 0.0011 $MJm^{-2}day^{-1}$ and 0.0686 % in its estimated value, 0.2361 $MJm^{-2}day^{-1}$ and 0.0148 respectively. The results equally shows that MPE values for all the developed models excluding equation 28e are within the acceptable range of ±10 %. The t-test values for Eqn 28a and Eqn 28b models are significant at 95% and 99% while the other two models (Eqn 28c and Eqn 28d) aren't.

Table 2: Ranks obtained from the estimated modified Page, Liu and Jordan Models for Benin

Models	R^2	MBE	RMSE	MPE	t	Rank
Eqn 4.2a	4	2	2	2	2	12
Eqn 4.2b	3	1	1	1	1	**7**
Eqn 4.2c	2	3	3	3	4	15
Eqn 4.2d	1	4	4	4	3	16

Table 2 above provides a summary of the ranks derived from the estimated modified Page, Liu and Jordan models for Benin. It is evident from the table that the ranks achieved by each model range from 7 to 16. The comprehensive findings indicate that the quadratic regression model, as defined in equation 28b, proves to be more accurate in estimating diffuse solar radiation in Benin when compared to the other three models.

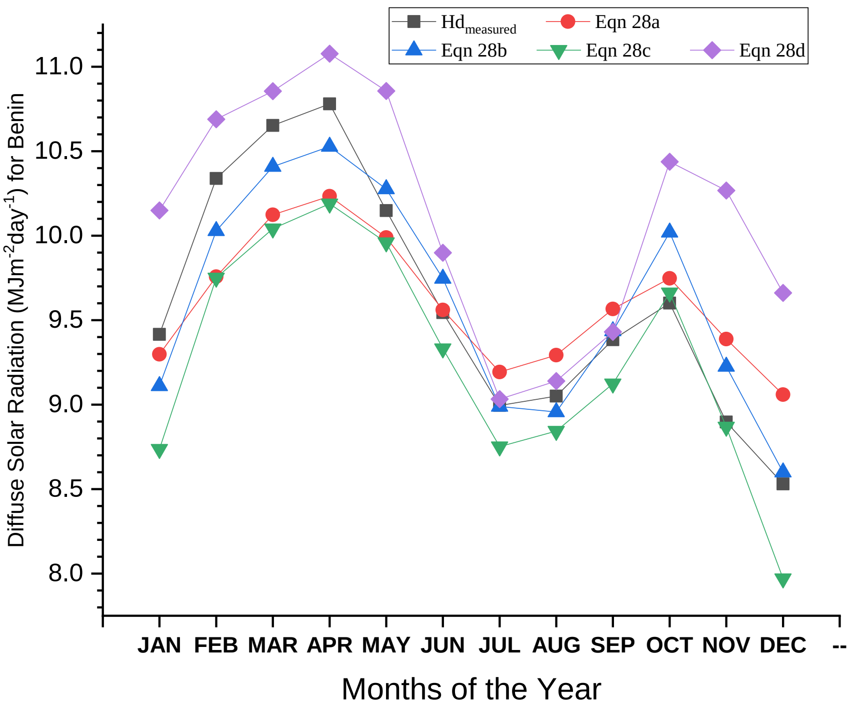

Figure 2: Comparison between the measured diffused solar radiation and estimated modified Page, Liu and Jordan models for Benin.

Figure 2 shows the comparison between the measured diffuse solar radiation and estimated models based on the modified Page, Liu and Jordan Models for Benin. It appears evidently from the figure that the developed polynomial model of degree 4 (Eqn 28d) overestimated the measured diffuse solar radiation and other developed models from the month of January to June and in October to December. The 3rd degree polynomial developed model (Eqn. 28c) underestimated the measured diffuse solar radiation and other developed models in the month of January, March to April, June to September and in December. The quadratic model (Eqn 28b) followed similar pattern of variation with the measured diffuse solar radiation and was reported to be the most suitable model for estimating diffuse solar radiation in Benin based on the modified Page, Liu and Jordan models as compared to other estimated models in this category.

Clearness Index and One Variable Correlation Models

Below are the statistical analysis summary of the models Eqn 28e to Eqn 28h

Table 3: Clearness Index and One Variable Models Statistical Error indicators

Models	R^2	MBE	RMSE	MPE	T
Eqn 28e	88.27%	-0.0065	0.3662	-0.1730	0.0592
Eqn 28f	88.03%	0.0023	0.3662	-0.2862	0.0211
Eqn 28g	96.33%	0.0388	0.2073	-0.4689	0.6323
Eqn 28h	96.04%	-0.0386	0.2731	0.3000	0.4729

Table 3 provides a summarized overview of the statistical validation tests conducted on the clearness index and a single variable model in this study. Among these models, modified equation 28g stands out with the highest R^2 value at 96.33 % and the lowest RSME value of 0.2073 $MJm^{-2}day^{-1}$. Notably, Equation 28f exhibits the lowest for both MBE and t-test values, recording an overestimation of 0.0023 $MJm^{-2}day^{-1}$ in its estimated value and 0.0211 respectively. Regarding MPE, equation 28e achieves the lowest value at 0.17301 %, with thus, an underestimation of 0.01730 % in its estimated value. Furthermore, the results indicate that the MPE values for all developed models fall within the acceptable range of ±10%. Additionally, the t-test values for all models are statistically significant at both 95% and 99%.

Table 4: Ranks obtained from the estimated Clearness Index and One Variable correlation Models for Benin

Models	R^2	MBE	RMSE	MPE	t	Rank
Eqn 28e	3	2	4	1	2	12
Eqn 28f	4	1	3	2	1	**11**
Eqn 28g	1	4	1	4	4	14
Eqn 28h	2	3	2	3	3	13

Table 4 presents a concise summary of the rankings obtained from the estimated clearness index and one variable models for Benin. The table clearly shows that the ranks attained by each model fall within the range of 11 to 14. The overall findings concludes that, in estimating diffuse solar radiation in Benin, model equation 28f demonstrates greater performance and accuracy compared to the other three models.

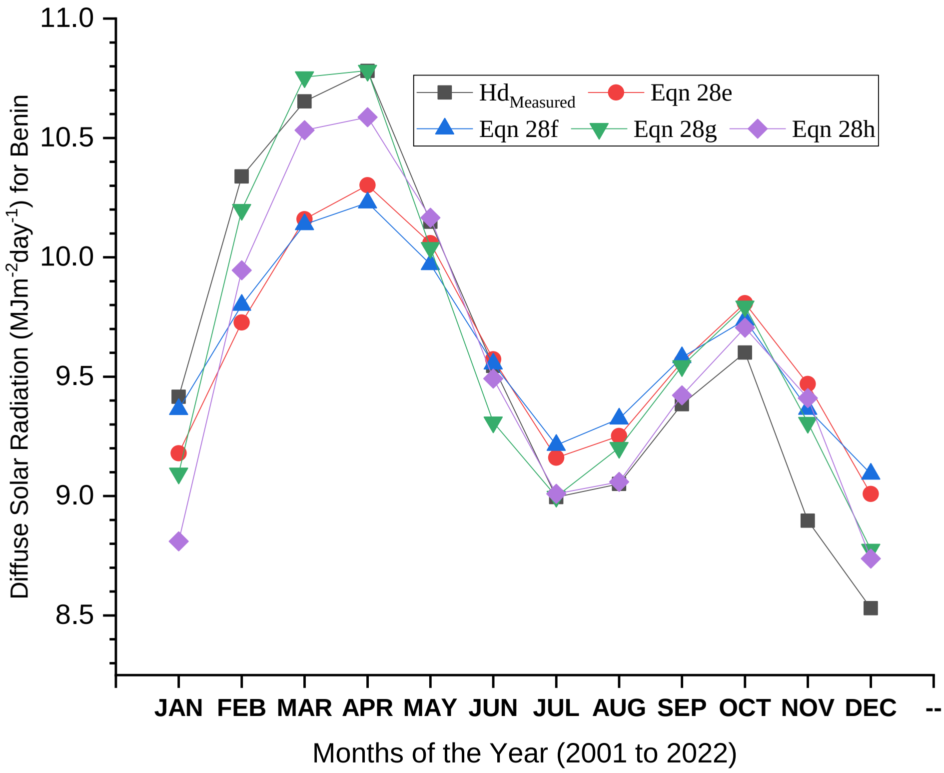

Figure 3: Comparison between the measured diffused solar radiation and estimated clearness index and a single variable models for Benin.

Figure 3 shows the comparison between the measured diffuse solar radiation and estimated models based on the clearness index and a single variable models for Benin. The model Eqn 28f followed similar pattern of variation with the measured diffuse solar radiation and was reported to be the most suitable model for estimating diffuse solar radiation in Benin based on the clearness index and a single variable models as compared to other estimated models.

Two Variables Correlation Models

Below are the statistical analysis summary of the models Eqn 28i to Eqn 28n

Table 5: Two variables Models Statistical Error indicators

Models	R^2	MBE	RMSE	MPE	t
Eqn 28i	44.95%	-0.0278	0.5137	0.0032	0.1797
Eqn 28j	59.14%	-0.0023	0.4419	-0.1936	0.0173
Eqn 28k	55.91%	-0.0991	0.4696	0.8035	0.7161
Eqn 28l	59.42%	-0.8008	0.9163	8.1546	5.9642
Eqn 28m	58.51%	0.2283	0.5996	-2.5974	1.3658
Eqn 28n	58.91%	-0.0860	0.4515	0.6799	0.6434

Table 5 provides a summarized overview of the statistical validation tests conducted on the two variable models in this study. Among these models, equation 28l stands out with the highest R^2 value at 59.42 %. The model equation 28i has the lowest MPE value of 0.0032 %, thus, with an overestimation of 0.0032 % in its estimated value. Notably, Equation 28j exhibits the lowest for MBE, RSME and t-test, recording an underestimation of 0.0023 $MJm^{-2}day^{-1}$ in its estimated value, 0.4419 $MJm^{-2}day^{-1}$ and 0.0173 respectively. Furthermore, the results indicate that the MPE values for all developed models fall within the acceptable range of ±10%. Additionally, the t-test values for all models are statistically significant at both 95% and 99%.

Table 6: Ranks obtained from the estimated Two Variables Correlation Models for Benin

Models	R^2	MBE	RMSE	MPE	t	Ranks
Eqn 28i	6	2	4	1	2	15
Eqn 28j	2	1	1	2	1	**7**
Eqn 28k	5	4	3	4	4	20
Eqn 28l	1	6	6	6	6	25
Eqn 28m	4	5	5	5	5	24
Eqn 28n	3	3	2	3	3	14

Table 6 provides a summary of the ranks derived from the estimated two variables models for Benin. It is evident from the table that the ranks achieved by each model range from 7 to 25. The comprehensive findings indicate that the model equation 28j, proves to be more accurate in estimating diffuse solar radiation in Benin when compared to the other five models in terms of the two variable correlation models above.

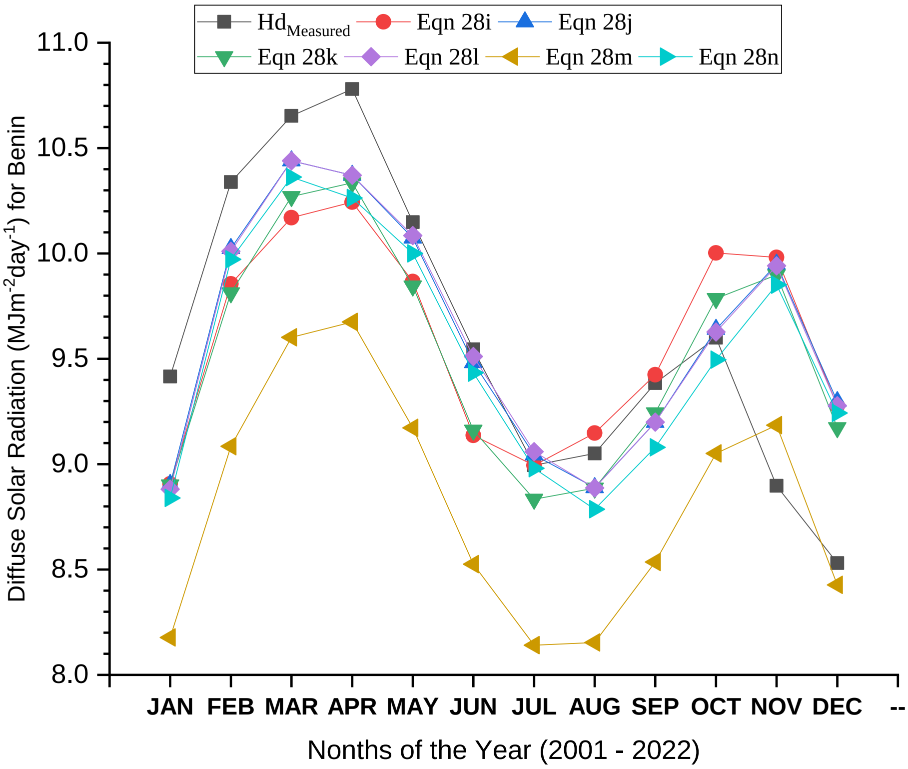

Figure 4: Comparison between the measured diffused solar radiation and estimated Two variable models for Benin.

Figure 4 shows the comparison between the measured diffuse solar radiation and estimated models based on the two variables Models for Benin. It appears evidently from the figure that the developed model equation 28m underestimated the measured diffuse solar radiation and other developed models from the month of January to October and then in December. The model Eqn 28j followed similar pattern of variation with the measured diffuse solar radiation and was reported to be the most suitable model for estimating diffuse solar radiation in Benin based on the two variable models as compared to other estimated models in this category.

Three Variables Correlation Models

Below are the statistical analysis summary of the models Eqn 28o to Eqn 28r

Table 7: Three Variable Models Statistical Error indicators

Models	R^2	MBE	RMSE	MPE	t
Eqn 28o	60.20%	-0.0532	0.4394	0.3442	0.4044
Eqn 28p	59.15%	1.3332	1.4125	-14.2048	9.4776
Eqn 28q	67.73%	-0.4377	0.5880	4.4045	3.6963
Eqn 28r	59.80%	0.3511	0.5616	-3.8858	2.6567

Table 7 presents a condensed overview of the statistical validation test conducted on the three variables correlation models for this study. Among the models, the equation 28q, exhibits the highest R^2 value as 67.73 %. Notably, the model equation Eqn 28o demonstrates the lowest for MBE, RMSE, MPE, and t-test values, with thus underestimation of 0.0532 $MJm^{-2}day^{-1}$ in its estimated value, 0.4394 $MJm^{-2}day^{-1}$, an overestimation of 0.3442 % in its estimated value and 0.4044 respectively. The results equally shows that MPE values for all the developed models excluding equation 28p are within the acceptable range of ± 10 %. The t-test values for Eqn 28o and Eqn 28r models are significant at 95% and 99% while the other two models aren't.

Table 8: Ranks obtained from the estimated Three Variables Correlation Models for Benin

Models	R^2	MBE	RMSE	MPE	t	Rank
Eqn 28o	2	1	1	1	1	**6**
Eqn 28p	4	4	4	4	4	20
Eqn 28q	1	3	3	3	3	13
Eqn 28r	3	2	2	2	2	11

Table 8 provides a summary of the ranks derived from the estimated three variable correlation models for Benin. It is evident from the table that the ranks achieved by each model range from 6 to 20. The comprehensive findings indicate that the model equation 28o, proves to be more accurate in estimating diffuse solar radiation in Benin when compared to the other three models.

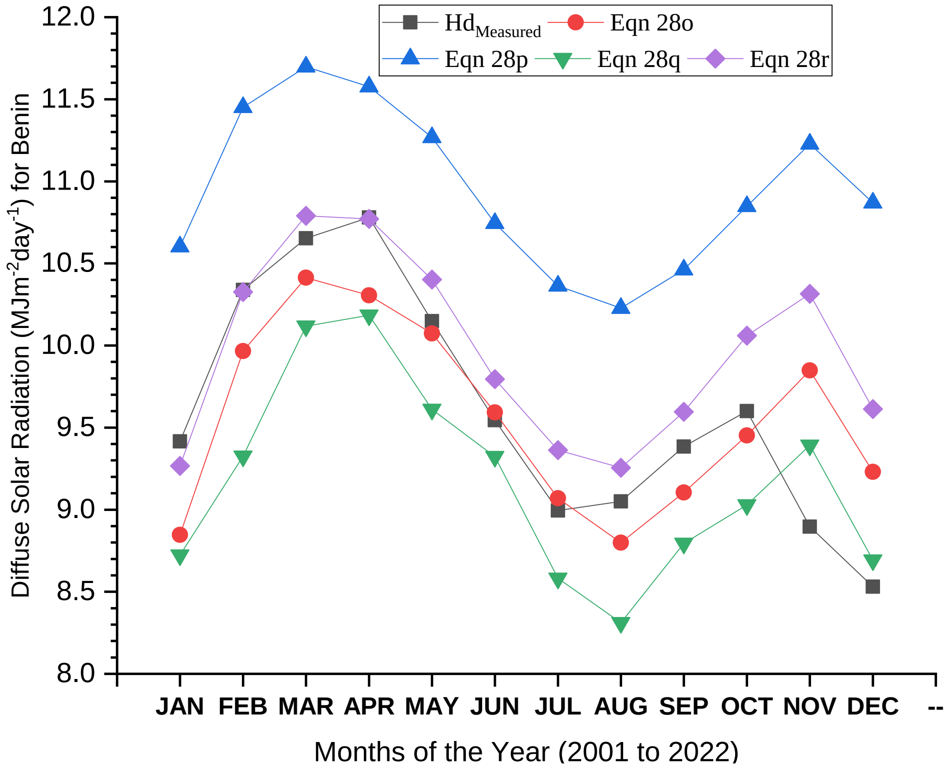

Figure 5: Comparison between the measured diffused solar radiation and estimated Three variable models for Benin.

Figure 5 shows the comparison between the measured diffuse solar radiation and estimated models based on the three variables models for Benin. It appears evidently from the figure that the developed model equation 28q underestimated the measured diffuse solar radiation and other developed models from the month of January to October. Also, the model equation 28p overestimated the measured diffuse solar radiation and other developed models from the month of January to December. The model equation 28o followed similar pattern of variation with the measured diffuse solar radiation and was reported to be the most suitable model for estimating diffuse solar radiation in Benin based on the three variable models as compared to other estimated models in this category.

Four Variables Correlations Model

Below are the statistical analysis summary of the model Eqn 28s

Table 9: Four variable model Statistical Error indicators for Benin

Model	R^2	MBE	RMSE	MPE	t
Eqn 28s	68.22%	-1.4031	1.4562	14.5020	11.9400

The four variables correlation model equation 28s as seen in table 9, has underestimation of 1.4031 MJm^{-2}day^{-1} and an overestimation of 14.5020 % in its estimated values respectively. The t-test value is not significant at 95% and 99%.

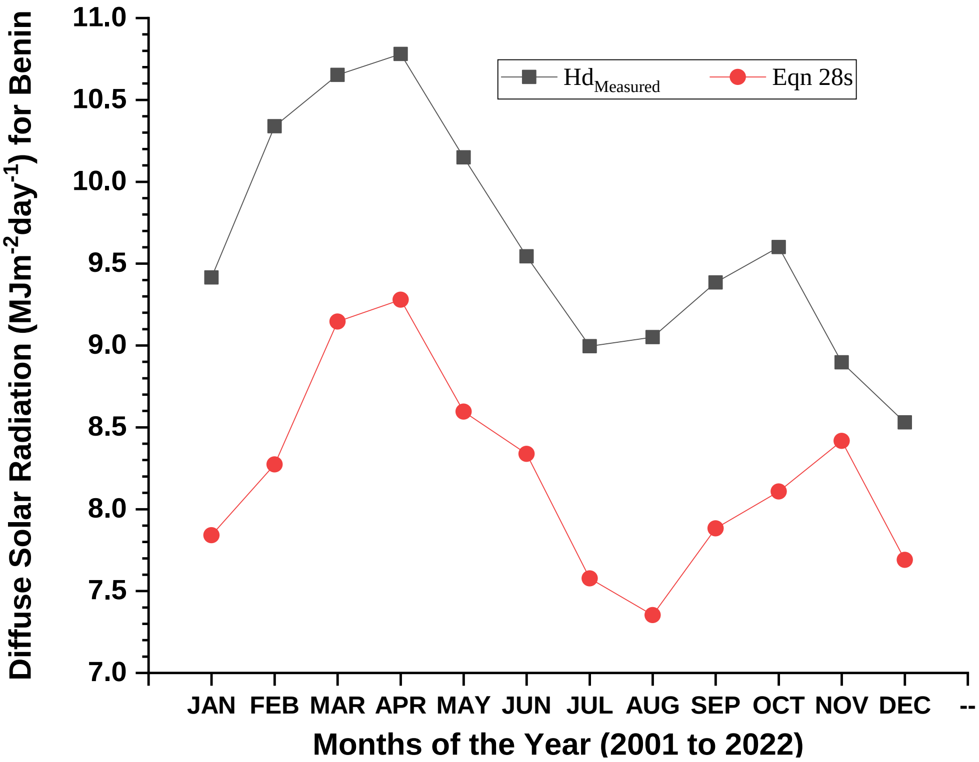

Figure 6: Comparison between the measured diffused solar radiation and estimated Four variable models for Benin.

Figure 6 shows the comparison between the measured diffuse solar radiation and estimated models based on the four variables models for Benin. It appears evidently from the figure that the developed model equation 28s underestimated the measured diffuse solar radiation and other developed models from the month of January to December.

Comparison of all Categories of Models

Table 10: Statistical summary of better performed models across each category for Benin

Models	R^2	MBE	RMSE	MPE	t
Eqn 28b	95.01%	-0.0011	0.2361	-0.0686	0.0148
Eqn 28f	88.03%	0.0023	0.3662	-0.2862	0.0211
Eqn 28j	59.14%	-0.0023	0.4419	-0.1936	0.0173
Eqn 28o	60.20%	-0.0532	0.4394	0.3442	0.4044
Eqn 28s	68.22%	-1.4031	1.4562	14.5020	11.9400

Table 11: Ranks obtained for the performed models across each category for Benin

Models	R^2	MBE	RMSE	MPE	t	Rank
Eqn 28b	1	1	1	1	1	**5**
Eqn 28f	2	2	2	3	3	12
Eqn 28j	5	2	4	2	2	15
Eqn 28o	4	4	3	4	4	19
Eqn 28s	3	5	5	5	5	23

From table 10 and 11, the quadratic regression model, equation 28b of the modified Page, Liu and Jordan model category was found to perform best in estimating diffuse solar radiation for Benin.

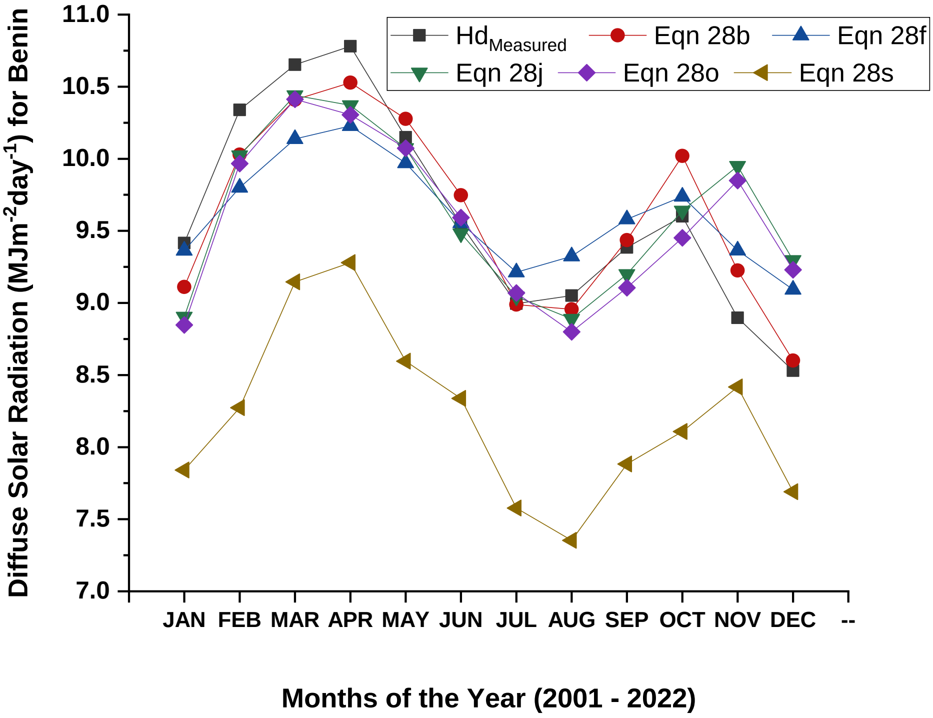

Figure 7: Comparison between the measured diffused solar radiation and best performed estimated models across the five categories for Benin.

Figure 6 shows the comparison between the measured diffuse solar radiation and the best performed estimated models across the categories. It appears that the measured diffuse solar radiation overestimated the developed models from the month of February to April while the model (equation 28s) underestimated the measured diffuse solar radiation and other developed models from the month of January to December. The model equation 28b, follows similar patterns as the measured diffuse solar radiation and was reported to be most suitable of all the categories developed models for estimation of diffuse solar radiation for Benin.

CONCLUSION

In this study the measured monthly average daily global solar radiation, diffuse solar radiation, relative humidity, atmospheric pressure, wind speed and mean temperature spanning a period of twenty-two years (2001 to 2022) dataset were utilized to develop new models for the estimation of diffuse solar radiation. A total of 19 models of five categories based on modified Page, Liu and Jordan models; clearness index and one variable models; two variable models; three variable models and four variable model were developed and tested using five validation indices of MBE, RMSE, MPE, t-test and R^2. The results obtained based on the models developed for Benin, in modified Page, Liu and Jordan models, equations 28b was found to be appropriate; equation 28f was found appropriate for the clearness index and one variable models category; equation 28j was found appropriate for the two variable models category; equation 28o was found appropriate for the three variable category. When all outstanding developed models from each categories were ranked, it was the quadratic regression model (Eqn.28b) that performed most appropriate for estimation of diffuse solar radiation for Benin located in the coastal climatic zones of Nigeria.

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REFERENCES

Adaramola, M. S. (2012). Estimating global solar radiation using common meteorological data in Akure, *Nigeria. Renewable Energy, 47*, 38–44.

Akpootu, D. O., & Abdullahi, Z. (2022). Development of Sunshine Based Models for Estimating Global Solar Radiation over Kano and Ikeja, Nigeria. *FUDMA Journal of Sciences, 6(3)*, 290-300. https://doi.org/10.33003/fjs-2022-0603-1001

Akpootu, D. O., & Gana, N. N. (2014). Comparative study of global solar radiation between Nguru and Abuja. Paper presented at the 24th Annual Congress and Colloquium of the Nigerian Association of Mathematical Physics, University of Benin, Benin City, Nigeria. 25th – 28th February, 2014.

Akpootu, D. O., & Iliyasu, M. I. (2015a). A comparative study of some meteorological parameters for predicting global solar radiation in Kano, Nigeria based on three variable correlations. *Advances in Physics Theories and Applications, 49*, 1–9.

Akpootu, D. O., & Iliyasu, M. I. (2015b). The impact of some meteorological variables on the estimation of global solar radiation in Kano, North Western, *Nigeria. Journal of Natural Sciences Research, 5(22)*, 1–13.

Akpootu, D. O., & Momoh, M. (2014). Empirical model for estimating global solar radiation in Makurdi, Benue State, North Central Nigeria. Paper presented at the 36th Annual

Nigerian Institute of Physics, National Conference, University of Uyo, Nigeria. 26th -29th May, 2014.

Akpootu, D. O., & Mustapha, W. (2015). Estimation of Diffuse Solar Radiation for Yola, Adamawa State, North-Eastern Nigeria. *International Research Journal of Engineering and Technology, 2(8)*, 77-82.

Akpootu, D. O., & Sulu, H. T. (2015). A comparative study of various sunshine-based models for estimating global solar radiation in Zaria, North-Western, Nigeria. *International Journal of Technology Enhancements and Emerging Engineering Research, 3(12)*, 1–5.

Akpootu, D. O., Alaiyemola, S. R., Abdulsalam, M. K., Bello, G., Umar, M., Aruna, S., Isah, A. K., Aminu, Z., Abdullahi, Z., & Badmus, T. O. (2023). Sunshine and Temperature Based Models for Estimating Global Solar Radiation in Maiduguri, Nigeria. *Saudi Journal of Engineering and Technology, 8(5)*, 82-90. https://doi.org/10.36348/sjet.2023.v08i05.001

Akpootu, D. O., Iliyasu, M. I., Mustapha, W & Aruna, S. (2015). Developing empirical models for predicting diffuse solar radiation over Yola, Adamawa State, North-Eastern, Nigeria.

Akpootu, D. O., Iliyasu, M. I., Olomiyesan, B. M., Fagbemi, S. A., Sharafa, S. B., Idris, M., Abdullahi, Z., Meseke, N. O. (2022). Multivariate Models for Estimating Global Solar Radiation in Jos, *Nigeria. Matrix Science Mathematic, 6(1)*, 05-12. http://doi.org/10.26480/mkmk.01.2022.05.12

Akpootu, D. O., Tijjani, B. I., & Gana, U. M. (2019a). Empirical models for predicting global solar radiation using meteorological parameters for Sokoto, Nigeria. I*nternational Journal of Physical Research, 7(2)*, 48–60. https://doi.org/10.14419/ijpr.v7i2.29160

Akpootu, D. O., Tijjani, B. I., & Gana, U. M. (2019b). Sunshine and temperature-dependent models for estimating global solar radiation across the Guinea savannah climatic zone of Nigeria. *American Journal of Physics and Applications, 7(5)*, 125-135. https://doi.org/10.11648/j.ajpa.20190705.15

Akpootu, D. O., Tijjani, B. I., & Gana, U. M. (2019c). New temperature-dependent models for estimating global solar radiation across the midland climatic zone of Nigeria. *International Journal of Physical Research, 7(2)*, 70–80. https://doi.org/10.14419/ijpr.v7i2.29214

Akpootu, D. O., Tijjani, B. I., & Gana, U. M. (2019d). New temperature-dependent models for estimating global solar radiation across the coastal climatic zone of Nigeria. *International Journal of Advances in Scientific Research and Engineering (IJASRE), 5(9)*, 126–141. https://doi.org/10.31695/IJASRE.2019.33523

Almorox, J., Benito, M., & Hontoria, C. (2005). Estimation of monthly Ångström-Prescott equation coefficients from measured daily data in Toledo, Spain. *Renewable Energy, 30*, 931-936.

Bakirci K. (2015). Models for the Estimation of Diffuse Solar Radiation for Typical Cities in Turkey. *Energy ; 82*:827–38. http://dx.doi.org/10.1016/j.energy.2015.01.093

Berrizbeitia S. E., Eulalia J. G. and Tariq M., (2020). Empirical Models for the Estimation of Solar Sky-Diffuse Radiation. A Review and Experimental Analysis. *Energies* ***2020***, 13, 701; doi:10.3390/en13030701

Bevington P. R. (1969) *Data Reduction and Error Analysis for the Physical Sciences*, first Edition McGraw Hill Book Co., New York.

Chen, R., Ersi, K., Yang, J., Lu, S., & Zhao, W. (2004). Validation of five global radiation models with measured daily data in China. *Energy Conversion and Management, 45*, 1759-1769.

Collares-Pereira, M., & Rabl, A. (1979). The average distribution of solar radiation correlations between diffuse and hemispherical and between daily and hourly insolation values. *Solar Energy, 22(2)*, 155-164.

Duzen, H., & Aydin, H. (2012). Sunshine-based estimation of global solar radiation on a horizontal surface at Lake Van region (Turkey). Energy Conversion and Management, 58, 35–46. https://doi.org/10.1016/S0306-2619(01)00012-5

El-Sebaii, A., & Trabea, A. (2005). Estimation of global solar radiation on horizontal surfaces over Egypt. *Egypt. J. Solids, 28(1)*, 163–175.

Erbs, D. G., Klein, S. A., & Duffie, J. A. (1982). Estimation of the diffuse radiation fraction for hourly, daily and monthly average global radiation. *Solar Energy, 28(4)*, 293-302.

Gana, N. N., & Akpootu, D. O. (2013a). Ångström type empirical correlation for estimating global solar radiation in North-Eastern Nigeria. *The International Journal of Engineering and Science, 2(11)*, 58-78.

Gana, N. N., & Akpootu, D. O. (2013b). Estimation of global solar radiation using four sunshine-based models in Kebbi, North-Western, Nigeria. *Pelagia Research Library, 4(5)*, 409-421.

Guermoui, M., Melgani, F., Gairaa, K., & Mekhalfi, M. L. (2020). A comprehensive review of hybrid models for solar radiation forecasting. *Journal of Cleaner Production, 258*, 120 - 357.

Halouani, N., Nguyen, C. T., & Vo-Ngoc, D. (1993). Calculation of monthly average solar radiation on horizontal surfaces using daily hours of bright sunshine. *Solar Energy, 50*, 247-248.

Iqbal, M. (1983). *An Introduction to Solar Radiation*. Academic Press.

Khatib, T., Mohamed, A., & Sopian, K. (2012). A review of solar energy modeling techniques. *Renewable and Sustainable Energy Reviews, 16(5)*, 2864-2869.

Klucher, T. M. (1991). *Estimating Solar Radiation*. Springer Science & Business Media.

Kreider J, Kreith F. (1981) *Solar Energy Handbook*. New York: McGraw-Hill

Lam, J. C., & Li, D. H. W. (1996). Correlation between global solar radiation and its direct and diffuse components. *Building and Environment, 31(6)*, 527-535.

Li, H., Bu, X., Long, Z., Zhao, L., & Ma, W. (2012). Calculating the Diffuse Solar Radiation in Regions Without Solar Radiation Measurements. *Energy*, *44*(1), 611–615. https://doi.org/10.1016/j.energy.2012.05.033

Liu B. H, & R. C. Jordan. (1960). The interrelationship and characteristics distribution of direct, diffuse and total solar radiation from meteorological data. *Solar Energy*, *4*, 1–9.

Merges, H. O., Ertekin, C., & Sonmete, M. H. (2006). Evaluation of global solar radiation models for Konya, Turkey. *Energy Conversion and Management*, *47*, 3149-3173.

Muneer, T., Hawas, M. M., & Sahili, K. (1984). Correlation between hourly diffuse and global radiation for New Delhi. *Energy Conversion and Management, 24(4)*, 265-267.

Myers, R. D. (2013). *Solar Radiation: Practical Modeling for Renewable Energy*. CRC Press.

Ogbulezie, J., Ushie, O., & Nwokolo, S. (2017). A Review of Regression Models Employed for Predicting Diffuse Solar Radiation in North-Western Africa. *Trends in Renewable Energy*, *3*(2), 160-206. DOI: 10.17737/tre.2017.3.2.0042

Olomiyesan, B. M., Akpootu, D. O., Oyedum, D. O., Olubusade, J. E., & Adebunmi, S. O. (2021). Evaluation of global solar radiation models performance using global performance indicator (GPI): A case study of Ado Ekiti, South West, Nigeria. Paper presented at the 43rd Annual Nigerian Institute of Physics, National Conference, Nnamdi Azikiwe University, Awka, Anambra State, May 26-29.

Page J. K. (1961). The estimation of monthly mean values of daily total short-wave radiation on vertical and inclined surfaces from sunshine records for latitudes 40°N – 40°S. Proceedings of the UN Conference on New Sources of Energy, 4, 378-390.

Saidur, R., Masjuki, H. H., & Hassanuzzaman, M. (2009). Performance of an improved solar car ventilator. *International Journal of Mechanical and Materials Engineering*, *4(1)*, 24–34.

Salhi Hicham, Lazhar Belkhiri and Ammar Tiri, (2020). Evaluation of Diffuse Fraction and Diffusion Coefficient Using Statistical Analysis. *Applied Water Science 10*:133 https://doi.org/10.1007/s13201-020-01216-0

Zekai, S. (2008). *Solar Energy Fundamentals and Modeling Techniques: Atmosphere, Environment, Climate Change, and Renewable Energy* (1st ed.). Springer, London.

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