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SIMULATION OF RELIABILITY, RELIABILITY INDEX, PROBABILITY DENSITY FUNCTION AND FAILURE FUNCTIONS FROM WEIBULL DISTRIBUTION FOR ENGINEERING APPLICATIONS

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ABSTRACT

In modelling and simulating future rainfall for a selected location, the probability distributions have been established to be an effective tool. In this study, the different methods utilised in the estimation of the probability distributions' parameters were evaluated and presented using Weibull's two parameters. Different estimator methods (mean rank, median rank, symmetric, graphical, least square, empirical, maximum likelihood, general probability, modified maximum likelihood, Mabchour, alternative maximum likelihood, equivalent energy, moment expression, Lysen and Moment methods) were used to determine probability density function, reliability, reliability index and failure functions of rainfall data from Maiduguri. The performances of these different methods were compared probability density function, reliability, reliability index and failure functions of Weibull two parameters. The study revealed that the values of probability distribution dimensionless shape variables were between 1.0193 and 4.205, and probability distribution scale factor constants were between 0.302 and 7.254. These values are all positive (non-negative values or less than zero) values. It was established that there were significant differences ($F_{108,\ 1728}$ was 162.1976 and the probability (p) was zero) between the individual reliabilities and Weibull estimators ($F_{15,\ 1620}$ was 14928.98 and probability was zero) at a 95 % confidence level (p less than 0.05). It was concluded that caution must be taken in the utilization of general probability, equivalent energy, Alternative Maximum Likelihood Method and moment expression methods in any engineering applications to prevent failure of devices or infrastructure.

Keywords: Probability distribution, Probability density function, Reliability, Reliability index, Failures, Weibull distribution

INTRODUCTION

The probability distribution functions are commonly used as a statistical tool for analysing, understanding and forecasting floods, strength failure and persistence life in engineering mechanisms and materials (Abaza et al., 2020; Alizadeh and Jarrahiferiz, 2018). The relevance of these probability distribution functions is established in aerospace, electronics, hydrology, materials design, Water Resources data, and automotive industries. Several studies on the applications of probability distribution in several areas of engineering and Sciences are presented in the literature such as Oke (2008); Razali et al. (2009); Reza and Ali (2017); Alizadeh and Jarrahiferiz (2018); Casas et al. (2010a); Chaurasiya et al.(2018; 2017); Gokarna and Chris (2011); Baseer et al. (2017); Casas et al. (2010b); Kopala et al. (2016); Parajuli (2016); Jamali et al. (2020); Jean-Francos and Ben (2013); Kang et al. (2018); Kasara et al. (2016); Khahro et al. (2014); Santra et al. (2016; 2017); Kidmo et al. (2015); Klimenko (2020); Martel et al. (2021); Vivekanandan (2015); Tsukuma and Kubokawa (2011); Shaban et al (2019); Sohoni et al. (2016); Teimouri et al. (2013); Nathan et al. (2016); Noughabi et al. (2020). The probability distributions frequently used to describe these environmental and engineering events include the Weibull distribution (two- and three-parameter), gamma distribution, Humbel distribution of extremes, lognormal and logarithmical gamma distributions, Kritskii and Menkel's three-parameter gamma distribution, and others. In the last four or five decades, advances in Weibull theory have generated numerous expert Weibull applications. Computing technologies have prepared and developed many of these techniques and made them accessible across the engineering variety. In addition, the popularity of these distributions is attributable to the fact that these distributions provide useful descriptions for many kinds of data, especially in emerging engineering and non-engineering applications which include wind speed and finance (stock prices and actuarial data) in addition to its traditional engineering applications (Nwobi and Ugomma, 2014). With the increase in economic development and highly accelerated urbanization, water consumption, water resources and supply are of a notable increase in demand (Wang et al., 2015), which necessitates further applications of probability density function, reliability, reliability index and failure functions in the areas of environmental science and engineering, civil engineering works, flood control and drainage design. Figures 1 to 4 show the actions and effects of floods in selected communities in Nigeria. The probability density function [f (y)] and the cumulative distribution function [F(y)] of the two-parameters Weibull distributions are expressed as follows (Pichugina, 2008; Pobočíková *et al*., 2014; 2017; Lukman et al., 2023), respectively:

$$f(y) = \frac{a}{b} y^{a-1} \, exp\left(\left(-\frac{y}{b}\right)^{a}\right) \qquad (1)$$

$$F(y) = 1 - exp\left(\left(-\frac{y}{b}\right)^{a}\right) \qquad (2)$$

Where; y is greater than zero, "a" is the probability distribution dimensionless shape variable, constant and parameter is greater than zero and "b" is the probability distribution scale factor constant and parameter in units of rainfall is greater than zero. A high value of "a" implies a narrow distribution with a variable concentrated around a value, whereas a low α value involves a widely dispersed variable. The shape parameter is large when there is low variation in variables. It shows the constancy of the variable for that location. "b" is the scale factor. It determines the quality of the variable. The scale factor parameter in the Weibull distributions indicates how deep the variable the location is. It depends on the average variable. When the average variable is high, the scale parameter is large (Tizgui *et al.*, 2016). The probability density function f(x) and the cumulative distribution function F(x) of the three-parameter Weibull distribution are expressed as follows (Pobočíková *et al*., 2017; Jones *et al*., 2020; Lukman et al., 2023):

Figure 1: Effects of Typical Flood situation in Gombe (Source: Bing.com, 2024)

Figure 2: The effect of Flood in Jigawa State (Source: Bing.com, 2024)

Figure 3: A Flood scenario in Northern Nigeria (Source: Bing.com, 2024)

Figure 4: A sample of Flood Menace in Plateau State (Source: Bing.com, 2024)

$$f(y) = \frac{a}{b^a}(y-c)^{a-1}\,exp\left(\left(-\frac{y-c}{b}\right)^a\right) \quad (3)$$

$$F(y) = 1 - exp\left(\left(-\frac{y-c}{b}\right)^a\right) \quad (4)$$

For y is greater than zero, "a" is greater than zero and "b" is greater than zero. "a" is the dimensionless shape parameter "c" is the dimensionless location parameter and "b" is the scale parameter in units of rainfall intensity. The Weibull probability density function satisfies the following properties (Nwobi and Ugomma, 2014):

i. If zero is less than "b" is less than 1.0, f (y) is decreasing with f (y) turning to infinity as y turns to zero.
ii. If "b" is equal to 1.0, f (y) is decreasing with f (y) is turning to 1.0 as y is turning to zero.
iii. If "b" is greater than 1.0, f (y) at first increases and then decreases, with a maximum value at the mode

$$y = \left(1 - \frac{1}{a}\right)^{\frac{1}{a}}. \quad (5)$$

iv. For all "b" is greater than zero, f (y) is turning to zero as y is turning to infinity.

With the great usefulness of Weibull distributions, probability density function, reliability, reliability index and failure functions from Weibull distributions to prevent environmental, engineering and economic calamities are limited in the literature. The main aim of this study is to estimate the Weibull distribution parameters with particular attention to the Weibull two-parameter distribution and to present probability density function, reliability, reliability index and failure functions to support its applications in the areas of environmental science and engineering, civil engineering works, flood control and drainage design.

MATERIALS AND METHODS

Rainfall data between 1915 and 2018 for Maiduguri, Nigeria was obtained from Akintola (1986) and Idi et al. (2020). The parameters in the Weibull probability distributions (the two-parameter Weibull) were estimated using selected Weibull distribution estimator methods. The parameters of the probability distributions were calculated using the sixteen (16) estimators and methods through Microsoft Excel Solver. The calculated parameters (detailed presented in another paper) were utilized to compute two-parameter Weibull in terms of probability density function, reliability, reliability index and failure functions. In total sixteen (16) estimators and methods were used to establish probability density function, reliability, reliability index and failure functions through Microsoft Excel Solver. Figure 5 presents the summary of the method used and Microsoft Excel Solver (MES) procedures. MES was utilized in the determination and estimation of the numerically derived two-Weibull parameters based on accessibility, availability and easy use at no further cost. The reliability of any system refers to the survival, consistency or stability of measurement of the system (Mohammed and Ahmed, 2019). A function or device with competent reliability indicates that the system or device will acquire the same performance on repeated function as long as no other environmental or extraneous factors influence the achievement (Mohammed and Ahmed, 2019; Rodica, 2022; 2023; Trainor-Guitton et al., 2011; Silverson et al., 2018; Nawafleh et al., 2013; Gupta et al., 2023). Reliability and reliability Index are extremely essential because confirmation of the reliability of a device or technique is necessarily the fundamental step in establishing the methodical acceptance and efficacy of a technique (Mohammed and Ahmed, 2019; Almazah and Ismail, 2021; Rodica, 2022; 2023). Reliability (R(x)), reliability index (RI) and failure function (h(y)) were calculated as follows (equations 6, 7 and 8):

$$R(y) = exp\left(\left(-\frac{y}{a}\right)^{b}\right) \quad (6)$$

$$RI(y) = -\,log_e\left(exp\left(\left(-\frac{y}{a}\right)^{b}\right)\right) = \left(\left(\frac{y}{a}\right)^{b}\right) = \frac{1}{R(y)} \quad (7)$$

$$h(y) = \frac{f(y)}{R(y)} = b \times a^{-b} \times y^{(b-1)} \quad (8)$$

Figure 5 presents the summary of the method used and Microsoft Excel Solver (MES) procedures. MES was applied in the determination and estimation of the numerically derived two-Weibull parameters probability density function, reliability, reliability index and failure functions to support its applications in the areas of environmental science and engineering, civil engineering works, flood control and drainage design based on accessibility, availability and easy use at no further cost.

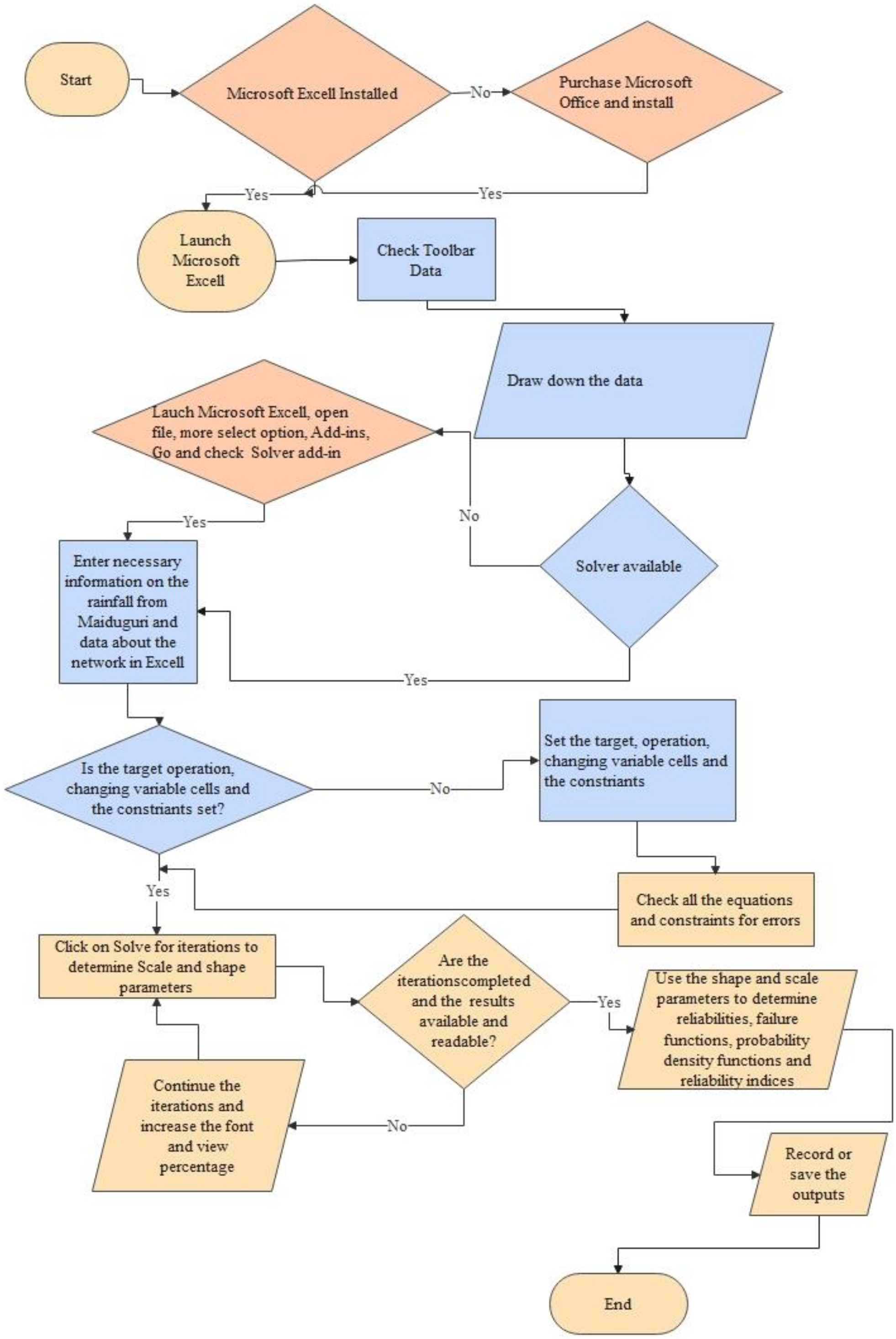

Figure 5: A flow chart of the methodology and technique for utilizing Microsoft Excel Solver in the computation of the Weibull Parameters, probability density function, reliability, reliability index and failure functions

RESULTS AND DISCUSSION

The study revealed that the values of the probability distribution dimensionless shape variable ("a") and the values of the probability distribution scale factor constant ("b"). The study revealed that the values of "a" were between 1.0193 and 4.205. This study and result revealed that the mean rank estimator provided the lowest value of "a" while equivalent energy provided the highest. The values "a" are all positive (non-negative values or less than zero), which agrees with the literature (Mohammed and Ahmed, 2019; Silverson et al., 2018; Kayid and Alshehri, 2023; Almazah and Ismail, 2021; Rodica, 2022; 2023; Gupta et al., 2023). These values of "a" revealed that it is expected that probability density functions from these estimators are expected to be exponential. In addition, the study establishes the values of "b" to be between 0.302 (from the equivalent energy estimator) and 7.254 (from the alternative maximum likelihood). These values of "b" revealed that these estimators can be categorized into two groups as follows:

i. Estimators with "b" less than 1.0, which are MLM, Lysen Method, Moment Method, Empirical Method, Energy Pattern Method, Modified Maximum Likelihood Method, Mabchour formula, General Probability, Least Square, Graphical, Symmetric, median rank, mean rank, equivalent energy method and Moment Expression, which indicates that probability density functions increase with increasing variables, and
ii. Estimator with "b" greater than 1.0, which is the Alternative Maximum Likelihood Method, which indicates probability density functions at first are expected to increase and then decrease with increasing variables.

These values of "a" and "b" are in agreement with previous studies and research (Idi et al., 2024). Figure 6 shows the reliability of the rainfall data using Weibull distribution and the scale and shape parameters from the estimators. The Figure revealed that reliability from an estimator (Alternative Maximum Likelihood Method) was of linear regression or fixed rather than non-linear regression as stated or expected from the literature (Mohammed and Ahmed, 2019; Silverson et al., 2018; Kayid and Alshehri, 2023; Almazah and Ismail, 2021; Rodica, 2022; 2023; Gupta et al., 2023). This characteristic of the estimator can be attributed to the higher value of "b" from the estimator and the linear function of the f(x). The specific character of the estimators indicates that the estimators are not accurate and reliable (Mohammed and Ahmed, 2019; Silverson et al., 2018; Kayid and Alshehri, 2023; Almazah and Ismail, 2021; Rodica, 2022; 2023; Gupta et al., 2023). The figure further established that the reliability of the rainfall data using Weibull distribution from the other estimators (MLM, Moment Method, Empirical Method, Energy Pattern Method, Modified Maximum Likelihood Method, Mabchour formula, Moment Expression, Least Square, Graphical, Symmetric, Median Rank and mean rank) were non-linear progression, which decreases with increasing value of the rainfall. The observations or transformation agreed with observations made in previous research or studies such as Shimizu et al. (2010); Mohammed and Ahmed (2019); Silverson et al. (2018); Kayid and Alshehri, (2023); Almazah and Ismail (2021); Rodica (2022; 2023) and Gupta et al. (2023). on the reliability of the rainfall data using Weibull distribution. In addition, these results of the reliability of the rainfall data using Weibull distribution indicate that reliability from these estimators performed better than other estimators and the level of their accuracies are reliable. Table 1 shows the result of the ANOVA of the estimators and the reliabilities from Weibull estimators. From the Table, the $F_{108,\ 1728}$ was 162.1976 and the probability (p) was 0.00 for analysis of the reliabilities. These results revealed that there were significant differences between the individual reliabilities at a 95 % confidence level (p less than 0.05).

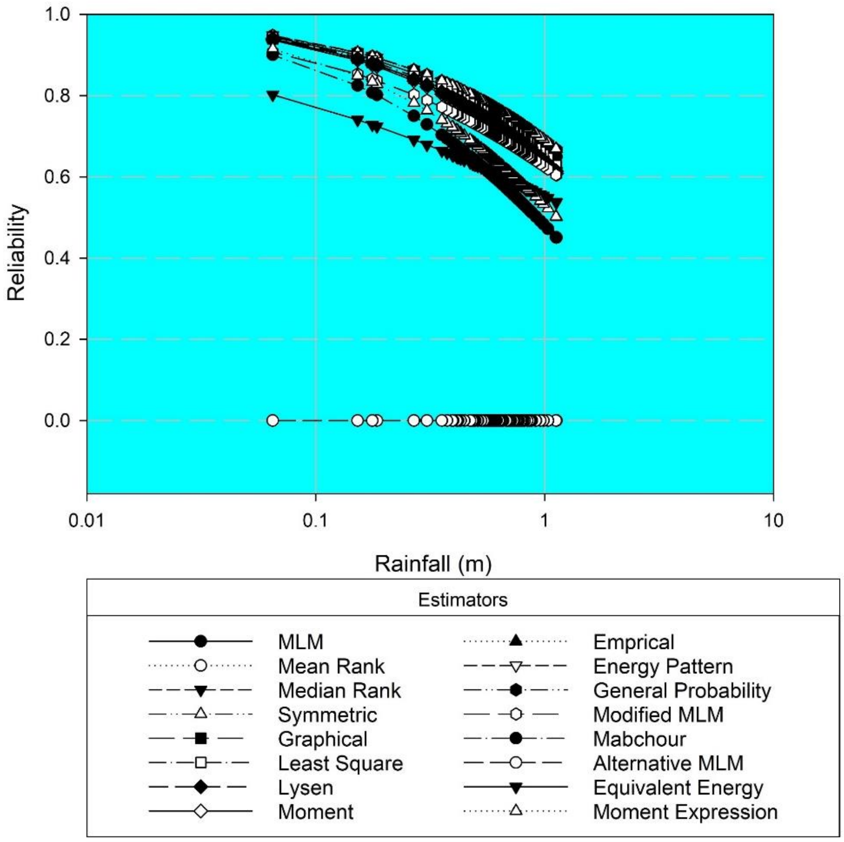

Figure 6: The relationships between reliability and 16 computed Weibull 2-probability

Table 1: The result of ANOVA of the reliability and the Weibull estimators

Source of Variation	Sum of Square	Degree of freedom	Mean Sum of Square	F-Value	P-value	F critical values
Within the reliabilities from the Weibull estimators	4.425957	108	0.040981	162.1976	0.000	1.24414
Between the Weibull estimators	56.57968	15	3.771979	14928.98	0.000	1.67256
Error	0.409312	1620	0.000253			
Total	61.41495	1743				

The result established that reliabilities are functions of events and actions. In addition, Table 1 presents the results of ANOVA with respect to the Weibull estimators. From the Table, the $F_{15, 1620}$ was 14928.98 and probability was 0.000 for analysis of the Weibull estimators. These results revealed that there were significant differences between the Weibull estimators at a 95 % confidence level (p less than 0.05). Figure 7 shows the reliability index of the rainfall data using the Weibull distribution and the scale and shape parameters from the estimators. The Figure revealed that reliability from an estimator (Alternative Maximum Likelihood Method) was of linear regression or fixed rather than non-linear regression as stated or expected from the literature such as Shimizu et al. (2010); Mohammed and Ahmed (2019); Silverson et al. (2018); Kayid and Alshehri, (2023); Almazah and Ismail (2021); Rodica (2022; 2023) and Gupta et al. (2023).

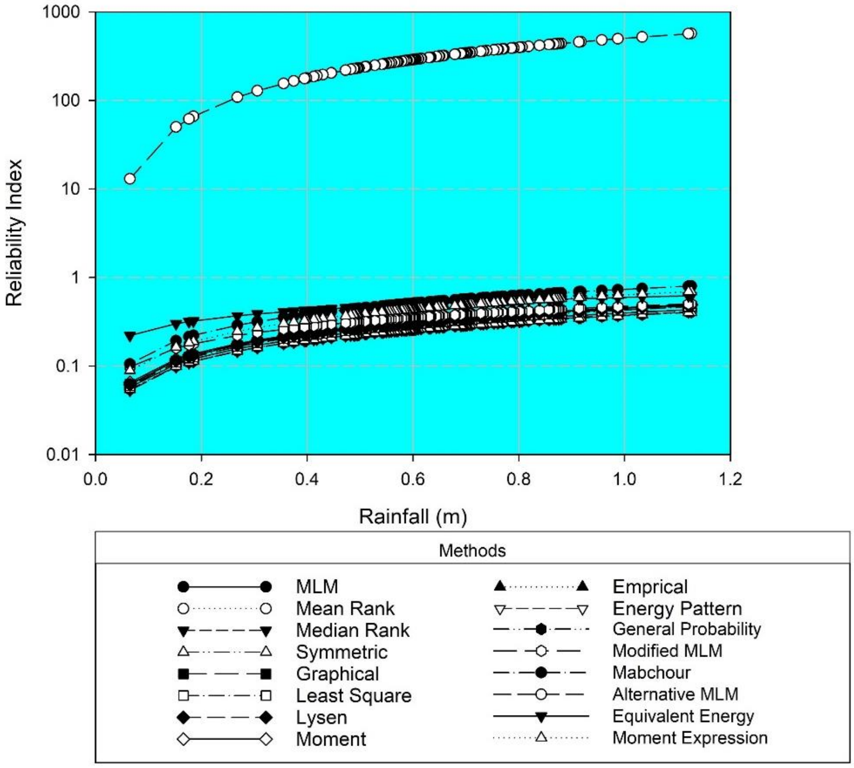

Figure 7: The relationships between reliability index and 16 computed Weibull 2-probability

Table 2: The result of ANOVA of the reliability index and the Weibull estimators

Source of Variation	Sum of Square	Degree of freedom	Mean Sum of Square	F-Value	P-value	F critical value
Within the reliability indices from Weibull estimators	75214.816	108	696.43348	1.022795	0.4200377	1.2441402
Between the Weibull estimators	9832683.4	15	655512.22	962.69729	0.000	1.6725599
Error	1103077.6	1620	680.91209			
Total	11010976	1743				

This performance of the estimator can be attributed to the higher value of "b" from the estimators, the linear function of the f(x) and reliability. The performance of the estimators indicates that the estimators are not accurate and reliable (Mohammed and Ahmed, 2019; Silverson et al., 2018; Kayid and Alshehri, (2023); Almazah and Ismail, 2021; Rodica, 2022; 2023; Gupta et al., 2023). The figure further established that the reliability index of the rainfall data using Weibull distribution from the other estimators (MLM, Moment Method, Empirical Method, Energy Pattern Method, Modified Maximum Likelihood Method, Mabchour formula, Moment Expression, Least Square, Graphical, Symmetric, Median Rank and mean rank) were non-linear progression, which decreases with increasing value of the rainfall. The observations or transformation agreed with observations made in previous research or studies such as Shimizu et al. (2010); Mohammed and Ahmed (2019); Silverson et al. (2018); Kayid and Alshehri,. (2023); Almazah and Ismail

(2021); Rodica (2022; 2023) and Gupta et al. (2023) on the reliability index of the rainfall data using Weibull distribution. In addition, these results of the reliability index of the rainfall data using Weibull distribution indicate that the reliability index from these estimators performed better than other estimators and the level of their accuracies is reliable. Table 2 shows the result of the ANOVA of the estimators and the reliability indices from Weibull estimators. From the Table, the $F_{108, 1620}$ was 1.022795 and the probability (p) was 0.4200 for analysis of the reliability indices. These results revealed that there were no significant differences between the individual reliabilities at a 95 % confidence level (p greater than 0.05). The same Table 5 also presents the results of ANOVA with respect to the Weibull estimators. From the Table, the $F_{15, 1620}$ was 962.91209 and probability was zero for analysis of the Weibull estimators. These results revealed that there were significant differences between the Weibull estimators at a 95 % confidence level (p less than 0.05). Figure 8 shows the probability density function (f(x)) of the rainfall data using Weibull distribution and the scale and shape parameters from the estimators. The Figure revealed that f(x) from three estimators (General Probability, equivalent energy and moment expression methods) were of non-normal distribution curves rather than normal distribution curves as stated or expected from the literature such as Shimizu et al. (2010); Mohammed and Ahmed (2019); Silverson et al. (2018); Kayid and Alshehri (2023); Almazah and Ismail (2021); Rodica (2022; 2023) and Gupta et al. (2023). This conduct of the three estimators can be attributed to the linear function of the f(x) from the three estimators. The conduct of the three estimators indicates that these three estimators are not accurate and reliable (Mohammed and Ahmed, 2019; Silverson et al., 2018; Kayid and Alshehri (2023); Almazah and Ismail, 2021; Rodica, 2022; 2023; Gupta et al., 2023). The figure further established that the values of cumulative probability from the other estimators (MLM, Moment Method, Empirical Method, Energy Pattern Method, Modified Maximum Likelihood Method, Mabchour formula, Least Square, Graphical, Symmetric, Median Rank and mean rank) were normal curves, which increases with increasing value of the Weibull parameters. The observations or transformation agreed with observations made in the previous research or studies such as on probability density function. In addition, these results of the f(x) indicate that the probability density function from these estimators performed better than other estimators and the level of their accuracies is reliable. The shape of the probability density function revealed that these estimators can be classified into two categories as follows:

i. Estimators with normal distribution curve probability density function, which are MLM, Moment Method, Empirical Method, Energy Pattern Method, Modified Maximum Likelihood Method, Mabchour formula, Least Square, Graphical, Symmetric, Median Rank and mean rank methods.
ii. Estimators with non- normal distribution curve probability density function, which are general probability, equivalent energy and moment expression methods.

The figure revealed that the probability density function from MLM was sharper at the peak than any other probability density functions, which are flat in shape at the peak. Table 3a shows the mode of annual rainfall with probability density functions. The table revealed that the minimum mode was 0.297 m per annual which occurred with an alternative MLM estimator and “b” value of 7.254 and the maximum was 0.937 m per annual which occurred with an equivalent energy estimator and “b” value of 0.362. These results indicate that the mode of probability density function of a particular event is related to the value of “b”. Tables 3b and 3c present the results of ANOVA in relation to the values of “a” and “b”.

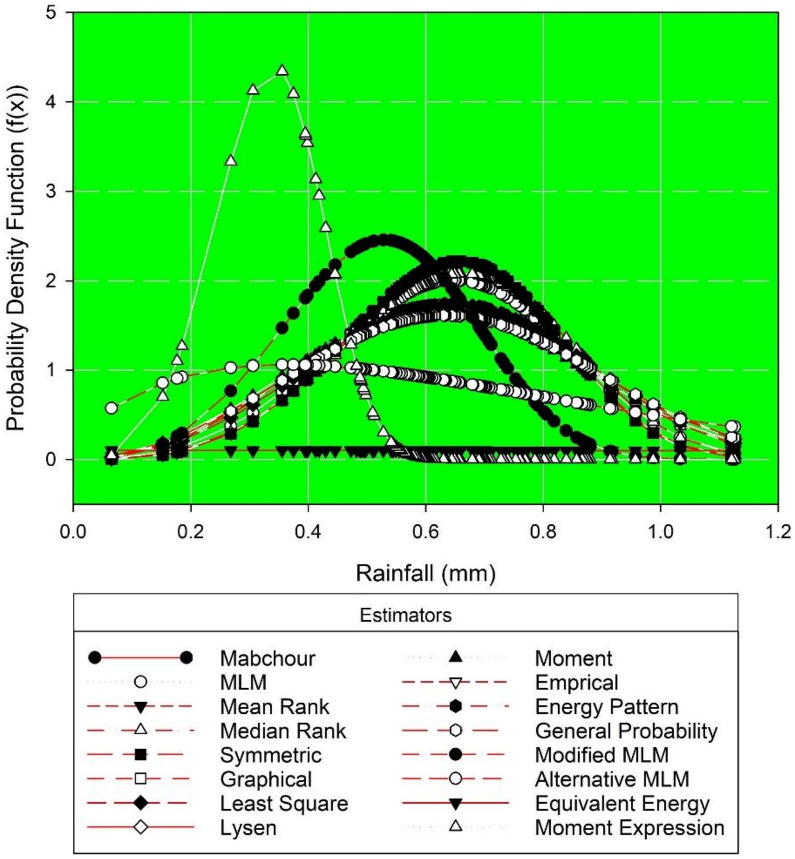

Figure 8: The relationships between the probability density function and 16 computed Weibull two-probability

Table 3a: Results of annual rainfall mode, values of "a" and "b" for the estimators evaluated

Methods	MLM	Mean Rank	Median Rank	Symmetric	Graphical	Least Square	Lysen Method	Moment Method	Empirical Method	Energy Pattern Method
Mode of Rainfall (m)	0.918	0.932	0.927	0.934	0.888	0.892	0.876	0.875	0.876	0.876
a	3.697	4.050	3.912	4.106	3.182	3.232	3.033	3.016	3.033	3.033
b	0.707	0.704	0.717	0.703	0.714	0.723	0.725	0.711	0.725	0.725

Table 3b: Results of ANOVA of the annual rainfall mode in relation to the value of "a"

Source of Variation	Sum of Square	Degree of freedom	Mean Sum of Square	F-Value	P-value
Between Estimators	8.306766	15	0.553784	0.19253	0.998679
Within modes	46.02166	16	2.876354		
Total	54.32842	31			

Table 3c: Results of ANOVA of the annual rainfall mode in relation to the value of "b"

Source of Variation	Sum of Square	Degree of freedom	Mean Sum of Square	F-Value	P-value
Between Estimators	17.04256	15	1.136171	0.738089	0.719173
Within modes	24.62947	16	1.539342		
Total	41.67203	31			

Table 4 shows the result of ANOVA of the probability density functions and the Weibull estimators. From the Table, the $F_{108, 1728}$ was 12.98167 and the probability (p) was 2.00 x 10^{-152} for analysis of the probability density functions. These results revealed that there was a significant difference between the probability density functions at a 95 % confidence level (p less than 0.05). In the same, Table 4 the results of ANOVA with respect to probability density functions Weibull estimators. From the Table, the $F_{15, 1620}$ was 112.8336 and probability was 1.40 X 10^{-238} for analysis of the Weibull estimators. These results revealed that there was a significant difference between the Weibull estimators at a 95 % confidence level (p less than 0.05).

Figure 9 shows the failures of the rainfall data using the Weibull distribution and the scale and shape parameters from the estimators. The Figure revealed that failure from an estimator (Alternative Maximum Likelihood Method) was of non-linear regression or exponential with increasing failure with rainfall rather than non-linear regression with decreasing failure with increasing rainfall as stated or expected from the equation and the literature such as Shimizu et al. (2010); Mohammed and Ahmed (2019); Silverson et al. (2018); Kayid and Alshehri, (2023); Almazah and Ismail (2021); Rodica (2022; 2023) and Gupta et al. (2023). These activities of the failure can be attributed to the higher value of "b"(greater than 1.0) from the estimator (Alternative Maximum Likelihood Method) and the non-linear function of the failure. The behaviour of the Alternative Maximum Likelihood Method indicates that the estimator is neither accurate nor reliable (Mohammed and Ahmed, 2019; Silverson et al., 2018; Kayid and Alshehri, 2023; Almazah and Ismail, 2021; Rodica, 2022; 2023; Gupta et al., 2023). The figure further established that the failure of the rainfall data using Weibull distribution from the other estimators (MLM, Moment Method, Empirical Method, Energy Pattern Method, Modified Maximum Likelihood Method, Mabchour formula, Moment Expression, Least Square, Graphical, Symmetric, Median Rank and mean rank) were non-linear progression, which decreases with increasing value of the rainfall. The explanations or changes agreed with explanations made in previous research or studies such as Shimizu et al. (2010); Mohammed and Ahmed (2019); Silverson et al. (2018); Kayid and Alshehri (2023); Almazah and Ismail (2021); Rodica (2022; 2023) and Gupta et al. (2023) on the failures of the rainfall data using Weibull distribution. In addition, these results of the failures of the rainfall data using Weibull distribution indicate that the failure from these estimators performed better than the Alternative Maximum Likelihood Method, which translated to a higher level of their accuracy are reliability.

Table 5 shows the result of the ANOVA of the estimators and the failures from Weibull estimators. From the Table, the $F_{108, 1620}$ was 0.9911894 and the probability (p) was 0.5087067 for analysis of the failures. These results revealed that there were no significant differences between the individual failure at a 95 % confidence level (p greater than 0.05). The same Table 5 also presents the results of ANOVA with respect to the Weibull estimators. From the Table, the $F_{15, 1620}$ was 122.6079 and probability was 0.000 for analysis of the Weibull estimators. These results revealed that there were significant differences between the Weibull estimators at a 95 % confidence level (p less than 0.05).

Table 4: The result of ANOVA of the probability density function and the Weibull estimators

Source of Variation	Sum of Square	Degree of freedom	Mean Sum of Square	F-Value	P-value
Within the probability density functions and rainfall.	229.1664	108	2.121911	12.98167	2.0×10^{-152}
Between the performance of the Weibull estimators	276.6473	15	18.44315	112.8336	$1.4.0 \times 10^{-238}$
Error	264.7962	1620	0.163454		
Total	770.6099	1743			

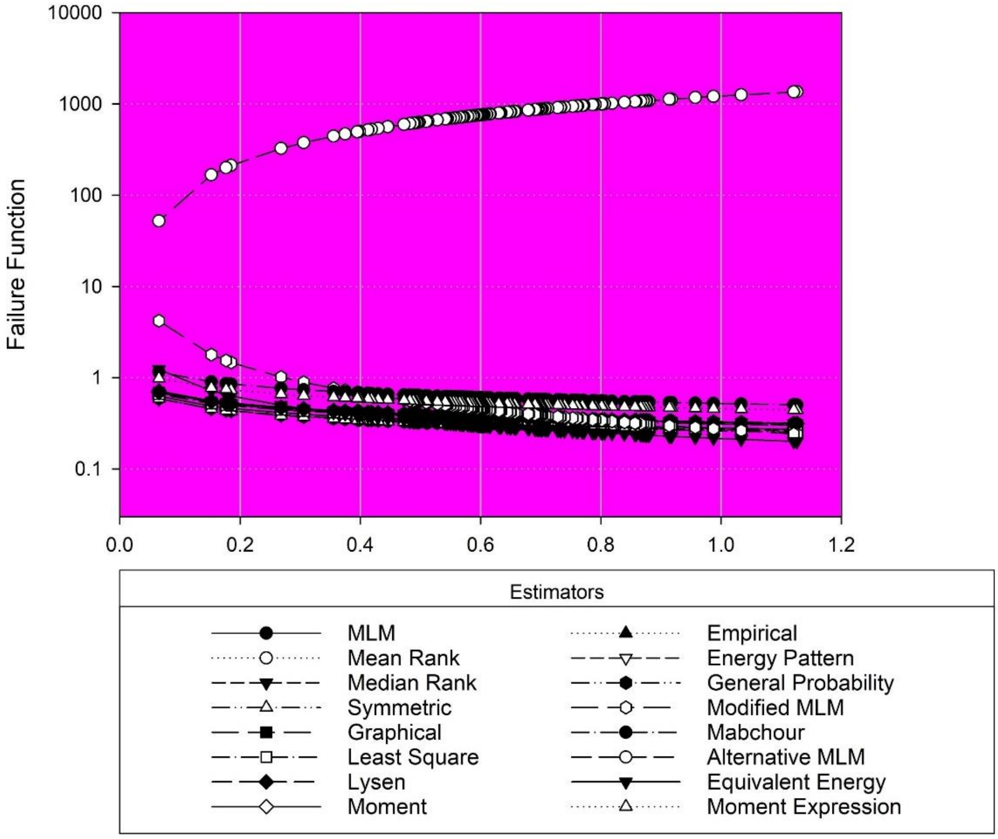

Figure 9: The relationships between the failure function and sixteen computed Weibull two-probability

Table 5: The result of ANOVA of the failure function and the Weibull estimators.

Source of Variation	Sum of Square	Degree of freedom	Mean Sum of Square	F-Value	P-value
Within the probability density functions and rainfall.	378380.27	108	3503.521034	0.9911894	0.5087067
Between the performance of the Weibull estimators	64822613	15	4321507.537	1222.6079	0.00000000
Error	5726154.9	1620	3534.663521		
Total	70927148	1743			

CONCLUSION

In this study, sixteen estimators and methods for estimating Weibull parameters were utilised to compute reliability, reliability index, probability density function and failure of rainfall data from Maiduguri. The study concluded that the values of “a” and “b” were positive with “a” greater than a unit (1.0) and “b” of two categories, first group the value of “b” is less than a unit and the second group “b” greater than a unit. The values of “a” and “b” revealed that Weibull probability density function are expected to the exponential in nature, which agrees with previous studies. MLM, Empirical Method, Energy Pattern Method, Modified Maximum Likelihood Method, Mabchour formula, Least Square, Graphical, Symmetric, Median Rank and mean rank are the estimators that performed better based on the reliability, reliability index, probability density function and failure values. MLM, Empirical Method, Energy Pattern Method, Modified Maximum Likelihood Method, Mabchour formula, Least Square, Graphical, Symmetric, Median Rank and mean rank are the estimators with the most accurate for all topographical conditions among the sixteen methods. These estimators, general probability, equivalent energy, Alternative Maximum Likelihood Method and moment expression methods were ranked next to the best estimators based on the values of reliability, reliability index, probability density function and failure. It was highlighted that cautions must be taken in the utilization of general probability, equivalent energy, Alternative Maximum Likelihood Method and moment expression methods in any engineering applications so as to reduce failure of device or infrastructure (due to lower reliability, reliability index, probability density function and failure than other estimators).

REFERENCES

Abaza, M. A., El-Maghlany, W. M., Hassab, M., and Abulfotuh, F. (2020). 10 MW Concentrated Solar Power (CSP) plant operated by 100% solar energy: Sizing and techno-economic optimization. Alexandria Engineering Journal, 59(1), 39–47. https://doi.org/10.1016/j.aej.2019.12.005

Akintola, J.O (1986).: Rainfall Distribution in Nigeria 1892–1983, 1st edn. Impact Publishers Nigeria Ltd, Ibadan.

Alizadeh N., H., and Jarrahiferiz, J. (2018). Moments of nonparametric probability density functions of entropy estimators applied to testing the inverse Gaussian distribution. Journal of Statistical Computation and Simulation, 88(16), 3217–3229.

Almazah , M.A and Ismail, M. (2021). Selection of Efficient Parameter Estimation Method forTwo-Parameter Weibull Distribution. Mathematical Problems in Engineering. 2021, Article ID 5806068, 8 pages. https://doi.org/10.1155/2021/5806068

Baseer, M.A., Meyer, J.P., Rehman, S. and Alam, M.M. (2017), “Wind power characteristics of seven data collection sites in Jubail, Saudi Arabia using Weibull parameters”, Renewable Energy, 102, 35-49.

Casas M.C, Rodr´ıguez R, and Reda˜no A. (2010a). Analysis of extreme rainfall in Barcelona using a microscale rain gauge network. Meteorological Applications 17(1): 117–123.

Casas, M.C, Rodr´ıguez, R., Prohom, M, Gazquez, G and Redano, A. (2010b). Estimation of the probable maximum precipitation in Barcelona (Spain). International Journal Of Climatology 17(1): 117–123

Chaurasiya PK, Ahmed S and Warudkar V (2018) Study of different parameters estimation methods of Weibull distribution to determine wind power density using ground based Doppler SODAR instrument. Alexandria Engineering Journal 57(4): 2299–2311.

Chaurasiya, P. K., Ahmed, S., and Warudkar, V. (2017). Study of different parameters estimation methods of Weibull distribution to determine wind power density using ground based Doppler SODAR instrument. Alexandria Engineering Journal. https://doi.org/10.1016/j.aej.2017.08.008

Gokarna R. A., and Chris P. T. (2011) Transmuted Weibull Distribution: A Generalization of the Weibull Probability Distribution European Journal of Pure And Applied Mathematics. 4(2), 89-102

Gupta, S.; Kanwar, S. and Sharma, N. (2023) Materials Today: Proceedings reliability. 3rd Biennial International Conference on Future Learning Aspects of Mechanical Engineering (FLAME 2022) https://www.sciencedirect.com/science/article/pii/S2214785323005199

Idi, M. D. Aladesanmi, T. A. Lukman, S. Asani, M. A, Usman, Y. M and Oke, I.A. (2020). A Numerical Formula And Cost Benefit Of Groundwater Recharge In Maiduguri, Nigeria . (FUTA Journal of Technology, Akure). FUTAJEET, 14(2), 126 -138 https://futajeet.com/index.php/jeet/article/view/158

Jamali, H., Ebrahimi, A., Ardestani, E. G., and Pordel, F. (2020). Evaluation of plotless density estimators in different plant density intensities and distribution patterns. Global Ecology and Conservation, e01114. https://doi.org/10.1016/j.gecco.2020.e01114

Jean-François, A. and Ben, S. (2013) The Poisson quasi-maximum likelihood estimator: a solution to the ‘adding up’ problem in gravity models, Applied Economics Letters, 20:6, 515-519,

Jones, L.D.; Vandeperrea, L.J. Haynes, , T.A. and Wenman, M.R.(2020). Theory and application of Weibull distributions to 1D peridynamics for brittle solids. Comput. Methods Appl. Mech. Engrg. 363, 112903

Kang, D, Kyungnam, K and Jongchul H.(2018). Comparative Study of Different Methods for Estimating Weibull Parameters: A Case Study on Jeju Island, South Korea. Energies, 11, 356; https://doi.org/10.3390/en11020356 www.mdpi.com/journal/energies

Kasara M., Omid, A., Ali, M.; Navid, G. and Mahdi, J. (2016). Assessing different parameters estimation methods of Weibull distribution to compute wind power density. Energy Conversion and Management 108, 322–335

Kayid, M.; and Alshehri, M.A. (2023). Modified Maximum Likelihood Estimation of the Inverse Weibull Model. Axioms 2023, 12, 961. https://doi.org/10.3390/axioms12100961

Khahro, S.F., Tabbassum, K., Soomro, A.M., Dong, L. and Liao, X. (2014), “Evaluation of wind power production prospective and Weibull parameter estimation methods for Babaurband, Sindh Pakistan”, Energy Conversion and Management, 78, 956-967

Kidmo, D.K., Danwe, R., Doka, S.Y. and Djongyang, N. (2015), “Statistical analysis of wind speed distribution based on six Weibull methods for wind power evaluation in Garoua, Cameroon”, Revue des Energies Renouvelables, . 18 (1), 105-125.

Klimenko D. Y. (2020). Estimating the Probable Maximum Precipitation by Physical Methods Using Satellite and Radiolocation Observation Data: Case Study of the Middle Urals Water Resources, 47(4), 641–650.

Kopala, I , Dana, B., Pavel, K. , Zora, J., Jan, V. , and Marta, H. (2016). Weibull distribution application on temperature dependence of polyurethane storage modulus. Int. J. Mater. Res. (formerly Z. Metallkd.) 107 (2016) 5, 472-476

Laurence, T. A. and Chromy, B. A. Efficient Maximum Likelihood Estimator Fitting of Histograms. Nat. Methods 2010, 7 (5), 338-339.

Lukman S., Sani S.B., Asani A.M., Ojo S.O. and Oke A.I. (2023). Performance Evaluation of Nine Methods for Estimation of Weibull Distribution Parameters, FUOYE Journal of Engineering and Technology (FUOYEJET), 8(1), 100-108. http://doi.org/10.46792/fuoyejet.v8i1.947

Martel, J.L; Brissette, F. P.; Lucas-Picher, P.; Troin, M.; and Arsenault, R. (2021). Climate Change and Rainfall Intensity–Duration–Frequency Curves: Overview of Science and Guidelines for Adaptation. ASCE J. Hydrol. Eng.,26(10): 03121001

Mohammed, M. and Ahmed A. (2019). Efficiency of the Reliability Function and Scheduling Method in Determining the Optimal Preventive Maintenance Time for Digital X-Ray Devices. International Journal of Statistics and Applications 2019, 9(2): 53-58 https://doi.org/10.5923/j.statistics.20190902.02

Nathan, R., Jordan, P., Scorah, M., Lang, S., Kuczera, G., Schaefer, M., and Weinmann, E. (2016). Estimating the exceedance probability of extreme rainfalls up to the probable maximum precipitation. Journal of Hydrology, 543, 706–720.

Nawafleh, N. A., Mack, F., Evans, J., Mackay, J., and Hatamleh, M. M. (2013). Accuracy and Reliability of Methods to Measure Marginal Adaptation of Crowns and FDPs: A Literature Review. Journal of Prosthodontics, 22(5), 419–428.

Noughabi, H. A., and Jarrahiferiz, J. (2020). Nonparametric probability density functions of entropy estimators applied to testing the Rayleigh distribution. Journal of Statistical Computation and Simulation, 1–15.

Nwobi, F. N and Ugomma, C.A (2014). A Comparison of Methods for the Estimation of Weibull Distribution Parameters. Metodološki zvezki, 11(1), 65-78.

Oke, I. A (2008) Utilization of Weibull Techniques For Short-Term Data In Environmental Engineering. Environmental Engineering and Sciences. 25 (7),1099- 1107.

Parajuli, A. (2016), “A statistical analysis of wind speed and power density based on Weibull and Rayleigh models of Jumla, Nepal”, Energy and Power Engineering, 8(7), 271-282.

Pichugina, S. V. (2008). Application of the Theory of Truncated Probability Distributions to Studying Minimal River Runoff: Normal and Gamma Distributions. Water Resources, 35(1), 23–29.

Pisarenko, V. F. Bolgov, M. V. Osipova, N. V. and Rukavishnikova, T. A. (2002). Application of the Theory of Extreme Events to Problems of Approximating Probability Distributions of Water Flow Peaks. Water Resources, 29(6), 593–604.

Pobocikova I, and Sedliackova Z (2014) Comparison of four methods for estimating the Weibull distribution parameters. Appl Math Sci 8(83):4137–4149

Pobočíková, I.; Sedliačková, Z. and Michalková, M (2017). Application of four probability distributions for wind speed modelling. TRANSCOM 2017: International scientific conference on sustainable, modern and safe transport. Procedia Engineering 192, 713 – 718

Razali; A. M; Ali, A.S and Mahdi, A.A (2009). Estimation Accuracy of Weibull Distribution Parameters. Journal of Applied Sciences Research . 5(7), 790-795.

Reza G. and Ali Z. H., (2017)," Application of the mixed Weibull distribution in machine reliability analysis for a cell formation problem ", International Journal of Quality and Reliability Management, 34(1). 128 – 142

Rodica, D (2022) Weilbull Distribution In Predictible Failure Analysis Annals of 'Constantin Brancusi' University of Targu-Jiu. Engineering Series / Analele Universităţii Constantin Brâncuşi din Târgu-Jiu. Seria Inginerie.3(5), 84-88

Rodica, D (2023) Weibull / Rayleigh Distribution Used To Model Wind Speeds In Order To Predict Wind Energy Production. Annals of 'Constantin Brancusi' University of Targu-Jiu. Engineering Series / Analele Universităţii Constantin Brâncuşi din Târgu-Jiu. Seria Inginerie .1, 20-23

Santra, K., Emily A. Smith, Jacob W. Petrich, and Xueyu S. (2017). Photon Counting Data Analysis: Application of the Maximum Likelihood and Related Methods for the Determination of Lifetimes in Mixtures of Rose Bengal and Rhodamine B J. Phys. Chem. A, 121, 122-132

Santra, K.; Zhan, J.; Song, X.; Smith, E. A.; Vaswani, N.; Petrich, J. W. (2016). What Is the Best Method to Fit Time-Resolved Data? A Comparison of the Residual Minimization and the Maximum Likelihood Techniques as Applied to Experimental Time-Correlated, Single-Photon Counting Data. J. Phys. Chem. B, 120 (9), 2484-2490.

Shaban, A. H., Resen, A. K., & Bassil, N. (2019). *Weibull parameters evaluation by different methods for windmills farms. Energy Reports.* https://doi.org/10.1016/j.egyr.2019.10.037

Shimizu, S., Tosha, K., and Tsuchiya, K. (2010). New data analysis of probabilistic stress-life (P–S–N) curve and its application for structural materials. International Journal of Fatigue, 32(3), 565–575

Silverson, O., Cascia, N., Hettrich, C. M., Hoch, M., and Uhl, T. (2018). Reliability of Clinical Assessment Methods to Measure Scapular Upward Rotation: A Critically Appraised Topic. Journal of Sport Rehabilitation, 1–21. https://doi.org/10.1123/jsr.2018-0012

Sohoni, V., Gupta, S. and Nema, R. (2016), "A comparative analysis of wind speed probability distributions for wind power assessment of four sites", Turkish Journal of Electrical Engineering and Computer Sciences, 24, 4724-4735.

Teimouri, M., Hoseini, S. M., and Nadarajah, S. (2013). Comparison of estimation methods for the Weibull distribution. Statistics, 47(1), 93–109.

Tizgui, I., Bouzahir, H., El Guezar, F. and Benaid, B. (2016), "Estimation of electricity production for a Moroccan wind farm", IEEE Digital Library, 1-4

Tizgui, I., El Guezar, F., Bouzahir, H., and Benaid,B. (2017) "Comparison of methods in estimating Weibull parameters for wind energy applications", International Journal of Energy Sector Management, https://doi.org/10.1108/IJESM-06-2017-0002

Trainor-Guitton, W. J., Caers, J. K., and Mukerji, T. (2011). A Methodology for Establishing a Data Reliability Measure for Value of Spatial Information Problems. Mathematical Geosciences, 43(8), 929–949.

Tsukuma, H., and Kubokawa, T. (2011). Modifying estimators of ordered positive parameters under the Stein loss. Journal of Multivariate Analysis, 102(1), 164–181. https://doi.org/10.1016/j.jmva.2010.08.011

Vivekanandan, V (2015). Estimation of Probable Maximum Precipitation Using Statistical Methods. World Journal of Research and Review (WJRR) 1(2), 13-16

Wang, L. N., Zhu, Q. K., Zhao, W. J., & Zhao, X. K. (2015). *The drought trend and its relationship with rainfall intensity in the Loess Plateau of China. Natural Hazards, 77(1), 479–495.* https://doi.org/10.1007/s11069-015-1594-0

WWW. Bing.com (2024). Flood in Nigeria available at https://www.bing.com/images/search?q=flood+in+nigeria+2023andform=HDRSC3andfirst=1 accessed on 7th January, 2024