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MANOVA: POWER ANALYSIS OF MODELS OF SUDOKU SQUARE DESIGNS

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ABSTRACT

This paper assesses the performance of multivariate treatment tests (Wilk's Lambda, Hoteling-lawley, Roy's largest root and Pillai) on multivariate Sudoku square design models in terms of power analysis. Monte carlo simulation was conducted to compare the power of these four tests for the four multivariate Sudoku square design models. This study used ± 0.062 as interval value for Power difference between two tests of the same sample size. The test is considered powerful or having advantage, if the difference between the powers of the tests is ± 0.062 . The results of Power test show that Hoteling-lawley has advantage over three other tests at P=2 while at P=3 Wilk's lambda test has power advantage over other tests in all the multivariate Sudoku models.

Keywords: Multivariate, Sudoku, test, power.

INTRODUCTION

Multivariate analysis of variance takes cognizance in hypotheses relating group differences on a set of variables, instead of, on individual variables. Multivariate hypothesis prompts a researcher to a multivariate analysis, because it is most suitable for assessing group differences on the set of variables simultaneously (Huberty and Olejnik, 2006). Specifically, multivariate analysis of variance (MANOVA) is appropriate for testing hypotheses about differences between groups (Hair, et al. 1987). Hancock et al. (2001) stated that "MANOVA evaluates group differences on a linear composite of observed variables constructed so as to maximally differentiate the groups in multivariate space".

Alternatively, multiple independent ANOVA techniques may be employed to the models to determine if there are significant differences among group means on each of several outcome measures of interest, but the result may lead to an unwarranted conclusion due to high rate of type I error from individual univariate tests. Multiple independent ANOVA may be popular and easy to many researchers, but the outcome of its results is that, the corresponding Type I error rate is inflated which eventually causes decrease in power, when the response is indeed multivariate data.

Sylvan and Jean-Francois (2007) viewed that estimation of power in multivariate analysis is one of the areas that has been neglected owning to the fact that is difficult to estimate. These authors considered three test statistics for the estimation of power of non-central distribution namely; Hoteling –lawley trace, Pillai and Wilk's lambda. Gatti and Harwell (1998) defined power of a statistical test as the probability of rejecting false null hypothesis. Cohen (1977) explained power of a test as when type II error is subtracted from one (type II error is the probability of accepting false null hypothesis). Cohen (1977) also defined power of a statistical test as $1-\boldsymbol{\beta}$ where $\boldsymbol{\beta}$ is the probability of Type II error. He explained further that when type II error rate is small the power is large and the sensitivity of the test is also high. Pearson and Hartley (1997), Cohen (1977), Montgomery (2008) and Faul et al. (2007) considered power of a test theoretically, that can be expressed as a function of effect size, sample size, non-central parameter and nominal alpha.

Finch and French (2013) compare robust MANOVA test statistics using Monte Carlo with respect to power. The study used four MANOVA test statistics namely: Wilk's Lambda, Hoteling-Lawley, Roy's largest root and Pillai trace.

Sudoku square design is a modification of Latin square with an additional square effect in the model and analysis Hui-Dong and Ru-Gen (2008). Subramani and Ponnuswamy (2009) added two different effects to the models by Hui-Dong and Ru-Gen (2008) which are row-blocks and column-blocks.

Subramani and Ponnuswamy (2009) proposed four Sudoku square design models, these models were proposed on the basis of measuring one observation per plot in the design of an experiment and design was analyzed by ANOVA..

Shehu et al. (2017) extended the work of Subramani and Ponnuswamy (2009) to a multivariate case with the aim of measuring more than one variable in a plot in the design. Four multivariate Sudoku square design models were proposed, MANOVA technique was used for the estimation of sum of squares and products for each of the effects in the models.

From these authors none has made comparison on models like multivariate Sudoku square design and some of the authors only made comparison using three multivariate test. Using ANOVA test on the each of independent variables of the multivariate variables of Sudoku square design data poses high inflation of type I error rates that lowered the power of a test.

In this research, Monte Carlo Simulation was performed to obtained power for the multivariate Sudoku square design models with number of dependent variables P=2 and P=3.

Multivariate Sudoku Square design Models

The following multivariate Sudoku square design models are used, in conjunction with test statistics to obtain the powers.

MANOVA Sudoku Model Type I

We will assume that row, column, and treatments effect as in the Latin square, also in addition to the assumption, row- block, column-block and square effects.

$$\boldsymbol{y_{ij(k,l,p,q)}} = \boldsymbol{\mu} + \boldsymbol{\alpha_i} + \boldsymbol{\beta_j} + \boldsymbol{\tau_k} + \boldsymbol{C_p} + \boldsymbol{\gamma_l} + \boldsymbol{s_q} + \boldsymbol{e_{i,j(k,l,p,q)}} \quad (1)$$

where $i = 1 \cdots, m, \quad j = 1 \cdots, m \quad k = 1 \cdots, m^2 \quad l = 1 \cdots, m^2 \quad p = 1 \cdots, m^2$, $q = 1 \cdots, m^2$

$\boldsymbol{\mu}$ = Grand mean

$\boldsymbol{\alpha_i}$ = $\boldsymbol{i}th$ Row block effect

$\boldsymbol{\beta_j} = \boldsymbol{j}th$ Column block effect

$\boldsymbol{\tau_k} = \boldsymbol{k}th$ treatment effect

$\boldsymbol{C_p}$= pth column effect

$\boldsymbol{\gamma_l}$= **l**th row effect

$\boldsymbol{s_q}$ =qth square effect

$\boldsymbol{e_{i,j(k,l,p,q,r)}}$ = is the error component assumed to have vector mean zero and constant variance - covariance Σ

where $Y_{ij(k,l,p,q)}$ is a p-vector valued observations and $\boldsymbol{e_{i,j(k,l,p,q)}}$ is independent ramdon $N_P(0, \Sigma)$

Note: All effects are of p-vector values including grand mean and error term

Shehu et al. (2017)

MANOVA Sudoku Model Type II

The model assumed that row effects are nested in the row block effect and the column effects are nested in the column block effects.

$$\boldsymbol{y_{ij(k,l,p,q)}} = \boldsymbol{\mu} + \boldsymbol{\alpha_i} + \boldsymbol{\beta_j} + \boldsymbol{\tau_k} + \boldsymbol{\gamma(\alpha)_{l(i)}} + \boldsymbol{C(\beta)_{p(j)}} + \boldsymbol{s_q} + \boldsymbol{e_{i,j(k,l,p,q)}} \quad (2)$$

Where $i = 1 \cdots, m, j = 1 \cdots, m, l = 1 \cdots, m, p = 1 \cdots, m, k = 1 \cdots, m^2$, $q = 1 \cdots, m^2$

$\boldsymbol{\mu}$ = general mean

$\boldsymbol{\alpha_i}$ = $\boldsymbol{i}th$ Row block effect

$\boldsymbol{\beta_j} = \boldsymbol{j}th$ Column block effect

$\boldsymbol{\tau_k} = \boldsymbol{k}th$ treatment effect

$\boldsymbol{s_q}$ =qth square effect

$\boldsymbol{\gamma(\alpha)_{l(i)}}$ = lth Row effect nested in ith row block effect

$\boldsymbol{c(\beta)_{p(j)}}$= pth Column effect nested in jth column block effect

$\boldsymbol{e_{i,j(k,l,p,q,r)}}$ = is the error component assumed to have vector mean zero and constant variance-covariance Σ

where $Y_{ij(k,l,p,q)}$ is a p-vector valued observations and $\boldsymbol{e_{i,j(k,l,p,q)}}$ is independent ramdon $N_P(0, \Sigma)$

Note: All effects are of p-vector values including grand mean and error term

Shehu et al. (2017)

MANOVA Sudoku Model Type III

The model assumes that the horizontal square effects are nested in the row block and vertical Square effects are nested in the column block effects.

$$\mathbf{y_{ij(k,l,p,q,r)}} = \boldsymbol{\mu} + \boldsymbol{\alpha_i} + \boldsymbol{\beta_j} + \boldsymbol{\tau_k} + \boldsymbol{\gamma_l} + \boldsymbol{c_p} + \boldsymbol{s(\alpha)_{q(i)}} + \boldsymbol{\pi(\beta)_{r(j)}} + \boldsymbol{e_{i,j(k,l,p,q,r)}} \quad (3)$$

where $i = 1 \cdots, m, j = 1 \cdots, m, q = 1 \cdots, m, r = 1 \cdots, m, p = 1 \cdots, m^2 \quad k = 1 \cdots, m^2, l = 1 \cdots, m^2$

$\boldsymbol{\mu}$ = Grand mean

$\boldsymbol{\alpha_i}$ = $\boldsymbol{i}th$ Row block effect

$\boldsymbol{\beta_j} = \boldsymbol{j}th$ Column block effect

$\boldsymbol{\tau_k} = \boldsymbol{k}th$ treatment effect

$\boldsymbol{C_p}$= pth column effect

$\boldsymbol{\gamma_l}$= **l**th row effect

$\boldsymbol{s(\alpha)_{q(i)}} = qth$ Horizontal square effect nested in ith Row block effect
$\boldsymbol{\pi(\beta)_{r(j)}} = rth$ vertical square effect nested in the jth column block effect
$\boldsymbol{e_{i,j(k,l,p,q,r)}}$ = is the error component assumed to have vector mean zero and constant variance-covariance Σ
where $Y_{ij(k,l,p,q)}$ is a p-vector valued observations and $\boldsymbol{e_{i,j(k,l,p,q)}}$ is independent ramdon $N_P(0, \Sigma)$
Note: All effects are of p-vector values including grand mean and error term

Shehu et al. (2017)

MANOVA Sudoku Model Type IV

In the model below, it is assumed that the row effects and horizontal square effects are nested in the row block and the column effects and the vertical square effects are nested in the column block effects.

$$\boldsymbol{y_{ij(k,l,p,q,r)}} = \boldsymbol{\mu} + \boldsymbol{\alpha_i} + \boldsymbol{\beta_j} + \boldsymbol{\tau_k} + \boldsymbol{\gamma(\alpha)_{l(i)}} + \boldsymbol{c(\beta)_{p(j)}} + \boldsymbol{s(\alpha)_{q(i)}} + \boldsymbol{\pi(\beta)_{r(j)}} + \boldsymbol{e_{i,j(k,l,p,q,r)}} \quad (4)$$

Where $i = 1 \cdots, m,\ j = 1 \cdots, m, l = 1 \cdots, m, p = 1 \cdots, m, \quad r = 1 \cdots, m$
$q = 1 \cdots, m, \quad k = 1 \cdots, m^2$
$\boldsymbol{\mu}$ = Grand mean
$\boldsymbol{\alpha_i}$ = $\boldsymbol{i}th$ Row block effect
$\boldsymbol{\beta_j} = \boldsymbol{j}th$ Column block effect
$\boldsymbol{\tau_k} = \boldsymbol{k}th$ treatment effect
$\boldsymbol{C_p}$= pth column effect
$\boldsymbol{\gamma_l}$= **l**th row effect
$\boldsymbol{s_q}$ =qth square effect
$\boldsymbol{\gamma(\alpha)_{l(i)}}$ = lth Row effect nested in ith row block effect
$\boldsymbol{c(\beta)_{p(j)}}$= pth Column effect nested in jth column block effect
$\boldsymbol{s(\alpha)_{q(i)}} = qth$ Horizontal square effect nested in ith Row block effect
$\boldsymbol{\pi(\beta)_{r(j)}} = rth$ vertical square effect nested in the jth column block effect
$\boldsymbol{e_{i,j(k,l,p,q,r)}}$ = is the error component assumed to have vector mean zero and constant variance -covariance Σ
where $Y_{ij(k,l,p,q)}$ is a p-vector valued observations and $\boldsymbol{e_{i,j(k,l,p,q)}}$ is a p-vector independent ramdon $N_P(0, \Sigma)$
Note: All effects are of p-vector values including error term and grand mean

Shehu et al. (2017)

Let M_T, $M_{\boldsymbol{k}}$, M_i, M_j represent sum of squares and product matrices for total, treatments, row-blocks and column-blocks respectively. While M_l, M_p, M_q and M_E represents sum of squares and product matrices for rows, column, squares and error respectively.

$$M_{\mathbf{k}} = \sum_{\boldsymbol{k}=1}^{m^2} \frac{y_{\boldsymbol{k}\cdots}y'_{\boldsymbol{k}\cdots}}{k} - \frac{y_{\cdots}y'_{\cdots}}{k^2} \quad (5a)$$
$$M_{\boldsymbol{q}} = \sum_{q=1}^{m^2} \frac{y_{q\cdots}y'_{q\cdots}}{k} - \frac{y_{\cdots}y'_{\cdots}}{k^2} \quad (5b)$$
$$M_l = \sum_{l=1}^{m^2} \frac{y_{\cdots l}y'_{\cdots l}}{k} - \frac{y_{\cdots}y'_{\cdots}}{k^2} \quad (5c)$$
$$M_{\boldsymbol{p}} = \sum_{p=1}^{m^2} \frac{y_{\boldsymbol{p}\cdots}y'_{\boldsymbol{p}\cdots}}{k} - \frac{y_{\cdots}y'_{\cdots}}{k^2} \quad (5d)$$
$$M_{\boldsymbol{E}} = \sum_{l=1}^{m^2} y_{ijlm}y'_{ijlm} - \frac{y_{i\cdots}y'_{i\cdots}}{k^2} \quad (5e)$$
$$M_T = \sum_{l=1}^{k} y_{ij(kl)}y'_{ij(kl)} - \frac{y_{\cdots}y'_{\cdots}}{k^2} \quad (5f)$$

Let $D_i$be the row-box (or row block) total and D_{j} be the column-box (or column block) totals. The respective sum of squares and product matrices for row-block and column-block are

$$\left.\begin{aligned} M_i &= \sum_{i=1}^{m} \frac{y_{\boldsymbol{i}\cdots}y'_{i\cdots}}{m^2} - \frac{y_{\cdots}y'_{\cdots}}{k^2} \\ M_{\mathrm{j}} &= \sum_{j=1}^{m} \frac{y_{j\cdots}y'_{j\cdots}}{m^2} - \frac{y_{\cdots}y'_{\cdots}}{k^2} \end{aligned}\right\} \quad (6)$$

The respective sum of squares and product matrices for rows within row-block and column within column-block are

$$\left.\begin{aligned} M_{l(i)} &= \sum_{i=1}^{k} \frac{y_{\,l(i)\cdots} y'_{l(i)\cdots}}{m^2} - \sum_m \frac{y_{\ldots i} y'_{..i}}{m^3} \\ M_{p(j)} &= \sum_{j=1}^{k} \frac{y_{p(j)\cdots} y'_{p(j)\cdots}}{m^2} - \sum_m \frac{y_{..j} y'_{..j}}{m^3} \end{aligned}\right\} \quad (7)$$

The respective sum of squares and product matrices for horizontal boxes within row-block and vertical boxes within column-block are

$$\left.\begin{aligned} M_{r(j)} &= \sum_{i=1}^{k} \frac{y_{r(j)\cdots} y'_{r(j)\cdots}}{m^2} - \sum_m \frac{y_{\ldots j} y'_{..j}}{m^3} \\ M_{q(i)} &= \sum_{j=1}^{k} \frac{y_{q(i)\cdots} y'_{q(i)\cdots}}{m^2} - \sum_m \frac{y_{..i} y'_{..i}}{m^3} \end{aligned}\right\} \quad (8)$$

$$M_E = M_T - M_k - M_i - M_j - M_l - M_p - M_q$$

NOTE: For Sudoku design to be square the relationship between k and m exists as $k = m^2$(both k and m are positive integers) where k is the number of rows or columns or square boxes or treatments while m is the number of row-blocks or column-blocks for the detail see Shehu et al. (2017).

Multivariate test statistics

This study use the following test statistics for comparison in terms of the power of the test which are Wilk's lambda Λ , Pillai trace, Lawley-Hotelling and Roy's largest root. The tests are used in conjunction with the multivariate Sudoku square models 1-4 with their respective sum of squares and products.

(i)Wilks'lambda: $\Lambda = \frac{|M_E|}{|M_E+M_T|}$ (9)

$$a = k^2 - k - \frac{p-k+2}{2}$$

$$b = \sqrt{\frac{p^2(k-1)^2-4}{p^2+(k-1)^2-5}}$$

$$c = \frac{p(k-1)-2}{2}$$

$$F = \left(\frac{1-\Lambda^b}{\Lambda^b}\right)\left(\frac{ab-c}{p(k-1)}\right) \sim F_{P(k-1),ab-c} \quad (10)$$

(ii) Lawley-Hotelling:

$$T^{(p)} = trace\left(M_T M_E^{\;-1}\right) \quad (11)$$

$$s = min(p, k-1)$$

$$t = \frac{|p-k-1|-1}{2}$$

$$u = \frac{k^2-p-k-1}{2}$$

$$F = \frac{2(su+1)}{s^2(2t+s+1)} * T \sim F_{s(2t+s+1),2(su+1)} \quad (12)$$

(iii) Pillai trace

$$V^{(p)} = trace(M_T(M_T + M_E)^{-1}) \quad (13)$$

$$F = \left(\frac{2u+s+1}{(2t+s+1)}\right)\left(\frac{V}{s-V}\right) \sim F_{2(2t+s+1),S(2u+S+1)}$$

(iv) Roy's largest root:

$$\vartheta = \max((M_T(M_T + M_E)^{-1})) \quad (14)$$

$$v_1 = \frac{|p-k-1|-1}{2}$$

$$v_2 = \frac{k^2-p-k-1}{2}$$

$$F = \left(\frac{2v_2+2}{2v_1+2}\right)\vartheta \sim F_{(2v_1+2),\ (2v_2+2)} \quad (15)$$

(Timm,1975)

p =number of variables
k = number of groups
M_T= Sum of squares and product for treatment effect
M_E= Error sum of squares and products

Estimation of power

The power for a particular MANOVA procedure is similar to that of ANOVA and can be estimated by Monte Carlo Simulation with the following steps.

(i) fill the $y_{(ij)lkpq}$ from the multivariate Sudoku design data

(ii) Compute the statistics for the appropriate statistical procedure which power is to be estimated.

Let $H_x = \begin{cases} 0, if\ the\ false\ null\ hypothesis\ is\ accepted \\ 1\ if\ the\ false\ null\ hypothesis\ is\ rejected \end{cases}$

(iii) Perform steps (i) and (ii) for $x = 1,2,\cdots N$

(iv) Estimate the corresponding Power of the test using $\frac{\sum_{x=1}^{N} H_x}{N}$

The procedures are repeated as N = 1000, as normality and positive definite of sigma of data were not violated.

Manipulated factors used

This Monte Carlo study manipulated five factors with 1000 replications per combination of conditions using a R software program written by the authors. Manipulated factors included Variance-covariance matrix homogeneity/heterogeneity, distribution of dependent variables, group mean differences, correlations among the dependent variables, and the number of dependent variables. Two levels were employed: 2 and 3 dependent variables. The data were simulated under two conditions for correlation among the dependent variables, including (0.3), smallest and largest (0.6).The correlation was small to moderate. Normality of the dependent variables is another assumption of the standard statistical tests used in MANOVA for this study.

Parameter used for the study

This study make used of the following parameters.

Grand mean $\boldsymbol{\mu_2} = \begin{pmatrix} \mu_{21} \\ \mu_{22} \end{pmatrix}, \boldsymbol{\mu_3} = \begin{pmatrix} \mu_{31} \\ \mu_{32} \\ \mu_{33} \end{pmatrix}$

$\mu_{21} = \mu_{22} = 0,\ \mu_{31} = \mu_{32} = \mu_{33} = 0$

Σ_1 and Σ_2 are the variance-covariance matrices , the diagonals of the matrice are variance of each of the dependent variable $Y_{ij(klpq)}$ and the off diagonal components are the covariance between each of the dependent variable $Y_{ij(klpq)}$. So if there are p dependent variables then there would be $p \times p$ variance-covariance matrix.

The sigma $\Sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\Sigma_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ were used for the random normal data for the number of dependent variables p=2 and p=3 respectively.

i.e $\sigma_{11} = \sigma_{22} = \sigma_{33} = 1\ and\ \ \sigma_{ij} = 0,\ i \neq j$
The study also considered when the variances are equal and covariance for each dependent variable differ from zero i.e $\sigma_{11} = \sigma_{22} = \sigma_{33} = 3\ \ \sigma_{ij} \neq 0,\ i \neq j$

$\Sigma_1 = \begin{pmatrix} 3 & 0.5 \\ 0.5 & 3 \end{pmatrix}$ and $\Sigma_2 = \begin{pmatrix} 3 & 0.5 & 1 \\ 0.5 & 3 & 0.5 \\ 1 & 0.5 & 3 \end{pmatrix}$

2.5 Nominal Alpha used

Nominal Alpha used for this study is from 0.01 to 0.1. Other parameters used for this study are listed below.
Number of rowblock $m = 3$
Number of columnblock $m = 3$
Number of treatments $k = m^2 = 9$
Number of rows $m^2 = 9$
Number of Columns $m^2 = 9$
Number of Sub-blocks (squares) $m^2 = 9$

3.0 RESULTS AND DISCUSSION

3.1 POWER OF THE TEST ON THE TREATMENT EFFECT.

The result of the simulated data on power for the treatment effect for the models is presented in Tables 1 and 2, only results for $\alpha = 0.05$ is reported in this research, because the results for other α values were generally found to be similar. Power difference within $\pm$ 0.062 between power estimate of any two tests for the same sample is said to be have similar power performance or else one is more powerful than other (Lin and Myers, 2006). The simulated Sudoku square data used had met the conditions of normality and sigma having determinant greater than zero

(positive definite).

Table 1: Power Test for treatment effects when $\boldsymbol{\sigma_{11} = \sigma_{22} = \sigma_{33} = 1}$ and

$\sigma_{12} = \sigma_{21} = \sigma_{23} = \sigma_{32} = \sigma_{13} = \sigma_{31} = 0$ at $\alpha = 0.05$

Model	Test statistics	P=2	P=3
M. Sudoku model I	hoteling-lawley Wilk'slambda Roy Test Pillai's trace	0.4510 0.1940 0.2110 0.3500	NA 0.6770 0.6160 0.0020
M. Sudoku model II	hoteling-lawley Wilk'slambda Roy Test Pillai's trace	0.0000 0.0000 0.0000 0.0000	NA 0.4860 0.3490 0.0030
M. Sudoku model III	hoteling-lawley Wilk'slambda Roy Test Pillai's trace	1.0000 1.0000 NA 0.4390	NA 0.7540 0.3880 0.0020
M. Sudoku model IV	hoteling-lawley Wilk'slambda Roy Test Pillai's trace	0.7080 0.3630 NA 0.5950	NA 0.3470 0.2460 0.0040

NA: Means Not Applicable

Table 2: Power Test for treatment effects when when $\sigma_{11} = \sigma_{22} = \sigma_{33} = 3$ and $\sigma_{12} = \sigma_{21} = \sigma_{23} = \sigma_{32} = 0.5$ and $\sigma_{13} = \sigma_{31} = 1$ at $\alpha = 0.05$

Model	Test statistics	P=2	P=3
M. Sudoku model I	hoteling-lawley	0.4010	NA
	Wilk'slambda	0.1870	0.4840
	Roy Test	0.1930	0.3910
	Pillai's trace	0.3340	0.0030
M. Sudoku model II	hoteling-lawley	0.0000	NA
	Wilk'slambda	0.0000	0.2320
	Roy Test	0.0000	0.1450
	Pillai's trace	0.0000	0.0000
M. Sudoku model III	hoteling-lawley	1.0000	0.0000
	Wilk'slambda	1.0000	0.0000
	Roy Test	NA	0.3300
	Pillai's trace	0.3550	0.0020
M. Sudoku model IV	hoteling-lawley	0.6430	NA
	Wilk'slambda	0.3490	0.0660
	Roy Test	NA	0.0420
	Pillai's trace	0.5700	0.0000

NA means Not Applicable

Table 1 above shows the power of treatment effect when variances and covariances are chosen to be 1 and 0 respectively. For M. Sudoku model I, the results show that Hoteling-Lawley test is more powerful than the remaining tests at P=2, followed by Pillai's test. At P=3 Wilk's lambda test showed a power advantage over the remaining tests. In M. Sudoku model II, at P=2 none has a power advantage over the other. However the situation is different at P=3 with Wilk's lambda revealed a power advantage over three other tests.

For the M.Sudoku model III at P=2, Hoteling-Lawley and Wilk's lambda tests have the same power value having numerical advantage and more powerful than Pillai's test with an exception of Roy's test that reported NA. NA was also reported against Hoteling-Lawley at P=3, with Wilk's Lambda test having power advantage over the two other tests.

The result of M.Sudoku model IV showed that Hoteling-Lawley is the most powerful test of all the tests used at P=2 while at P=3, Wilk's Lambda test performed better than the remaining other tests. Across the models, it was observed at P=3 that Hoteling-Lawley revealed NA in all the models which might be an indication that the test statistics lacked the ability to investigate power. Similarly, the same situation was reported with Roy's test at P=2 for M.Sudoku models III anaIV. It was also observed that wilk's lambda and Roy's tests were found to be similar at P=3 for M.Sudoku model I. In general Wilk's Lambda and Hoteling-Lawley were the highest power observed at P=2 for M.Sudoku model III while the lowest power with zero through the four tests came from M.Sudoku model II at P=2.

Table 2 shows the power for treatment effect when variance and covariance are 3 and (0.5, 1.0) respectively at $= 0.05$. At P=2, the result shows that Hoteling-Lawley test has power advantage over remaining tests for M. Sudoku model I, III and IV while M. Sudoku model II reveal none of the tests has power advantage over the other. However at P=3, it has been Wilk's lambda that has the highest power and having advantage over the remaining tests in all M.Sudoku models with an exception of M. Sudoku model III, where Roy's test claim the advantage over the three other tests. Still on Table 2, at P=3, it is observed that NA is recorded against Hoteling-Lawley test in all four M.Sudoku models which was related to what we have in Table 1 that revealed NA against Roy's test at P=2 for M. Sudoku models III and IV.

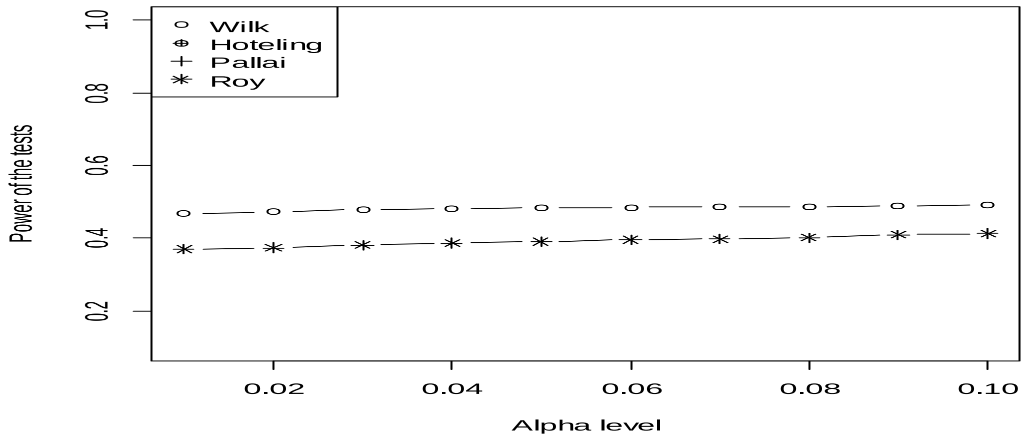

Fig 1. Power test for treatment effect when variance =3 and covariance =(0.5 &1) for M.Sudoku model 1 at P=3

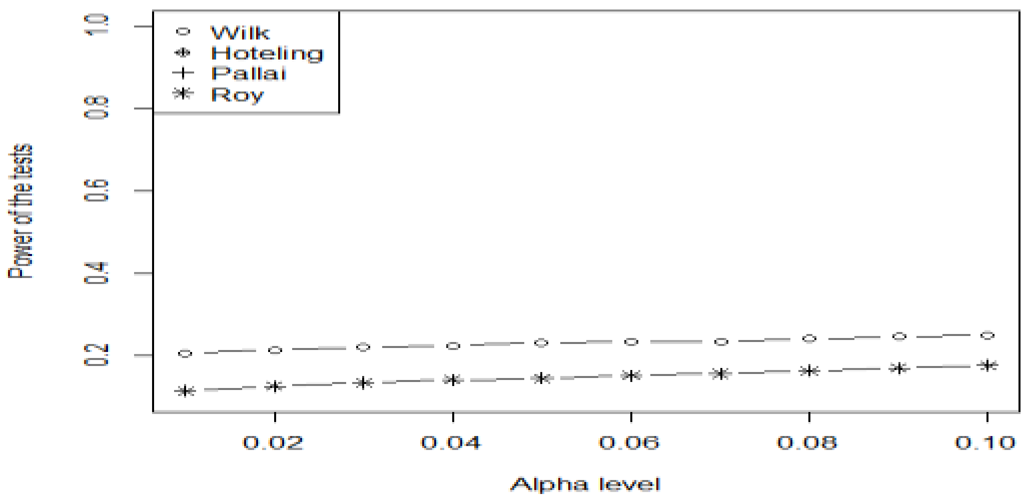

Fig 2. Power test for treatment effect when variance =3 and covariance =(0.5 &1) for M.Sudoku model II at P=3

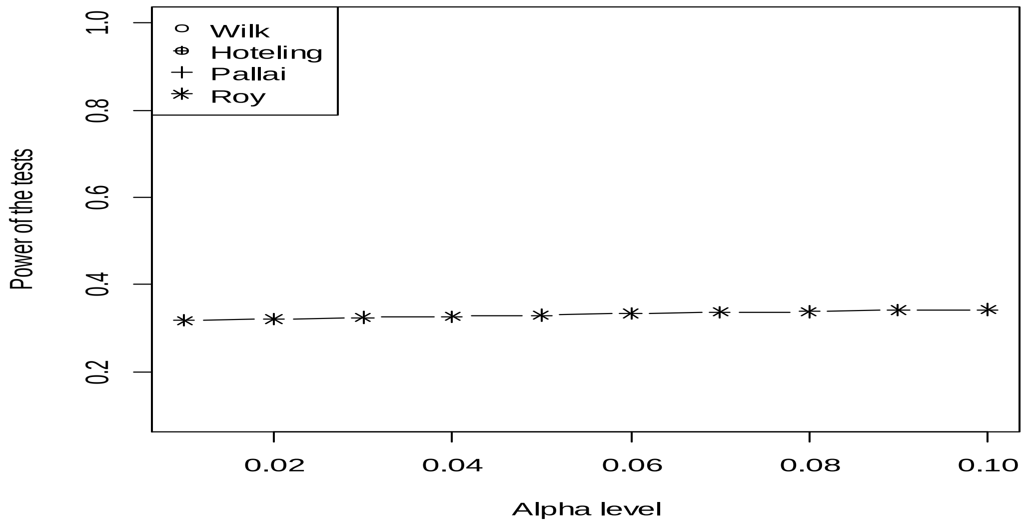

Fig 3. Power test for treatment effect when variance =3 and covariance = (0.5 &1) for M.Sudoku model III at P=3.

Figures 1, 2, 3 and 4 show the power against some level of significance (alpha) when variance and covariance are 3 and (0.5 & 1.0) respectively. At P=3 the results show that Wilk's lambda has highest power in Figures 1, 2 and 4 while in Figure 3 Roy's test, the only test appears on the graph has the highest power. From these figures there was little or no appreciable increase in the power of the tests as alpha level increases.

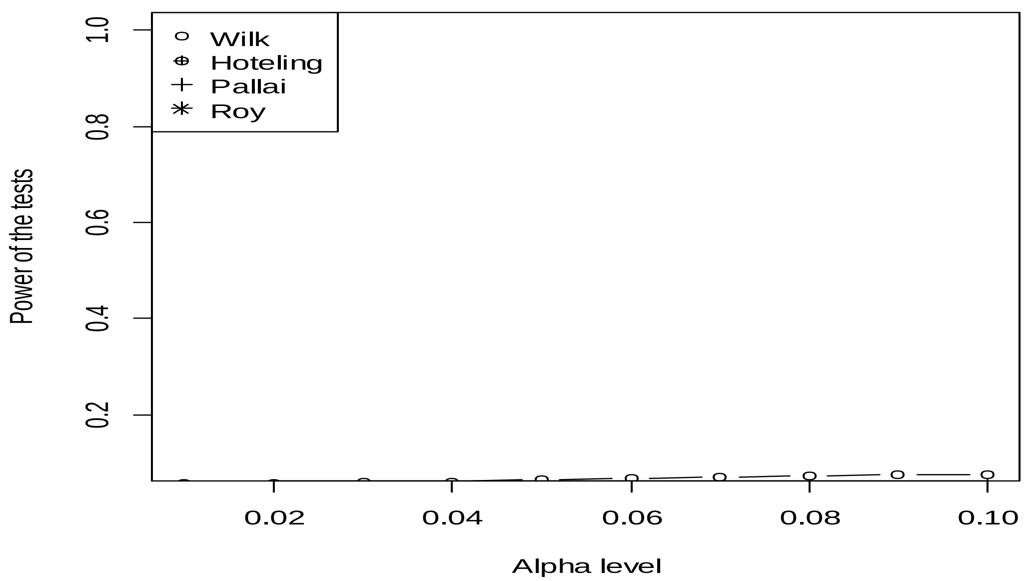

Fig 4. Power test for treatment effect when variance =3 and covariance =(0.5 &1) for M.Sudoku model 1V at P=3

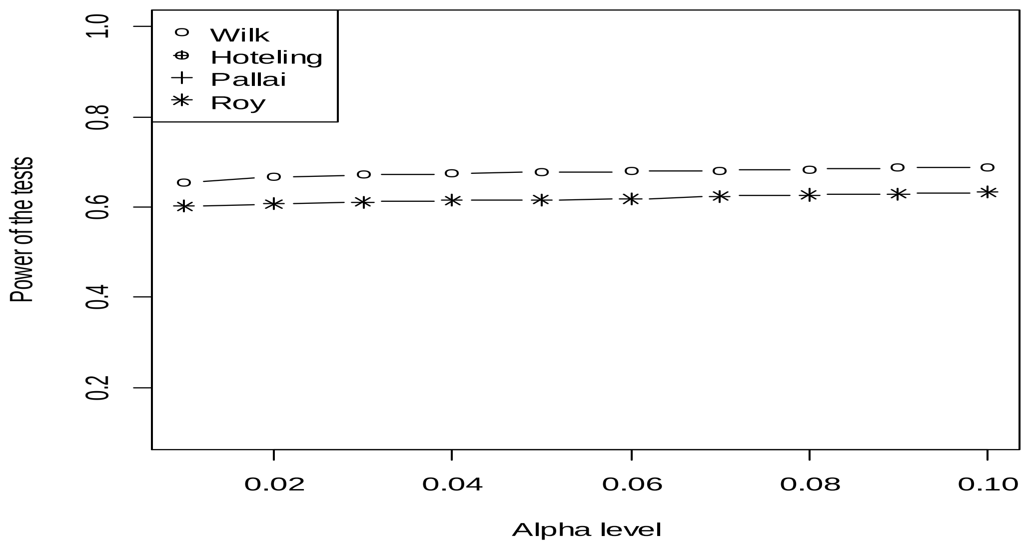

Fig 5.Power test for treatment effect when variance =1 and covariance =0 for M.Sudoku model I at P=3

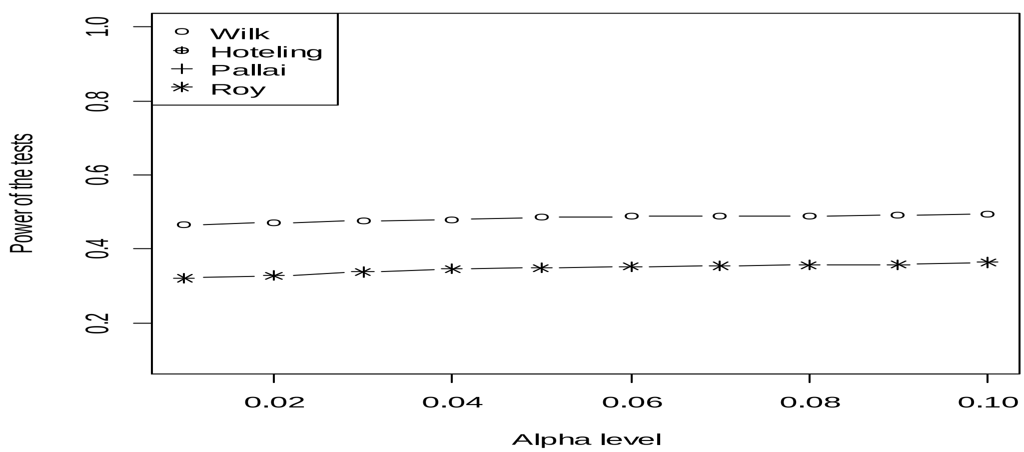

Fig 6. Power test for treatment effect when variance =1 and covariance =0 for M.Sudoku model II at P=3

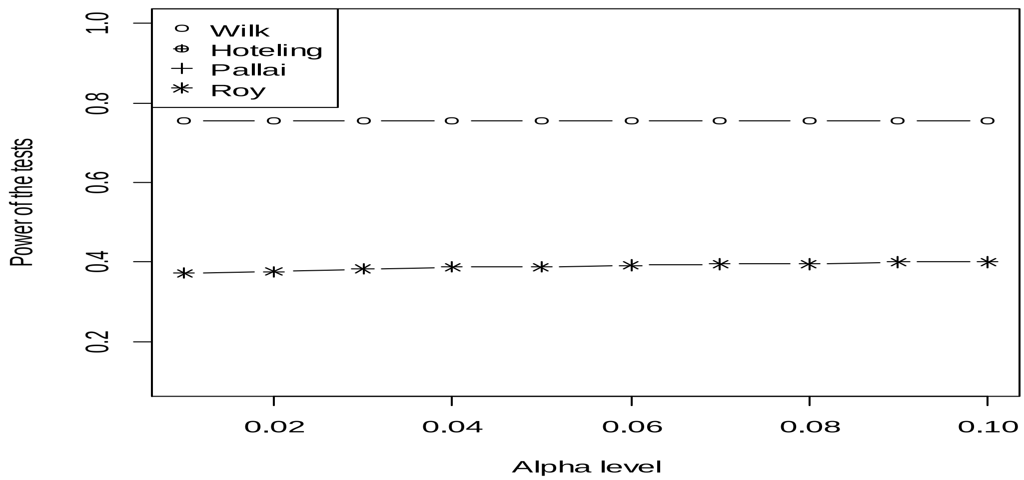

Fig 7. Power test for treatment effect when variance =1 and covariance =0 for M.Sudoku model III at P=3

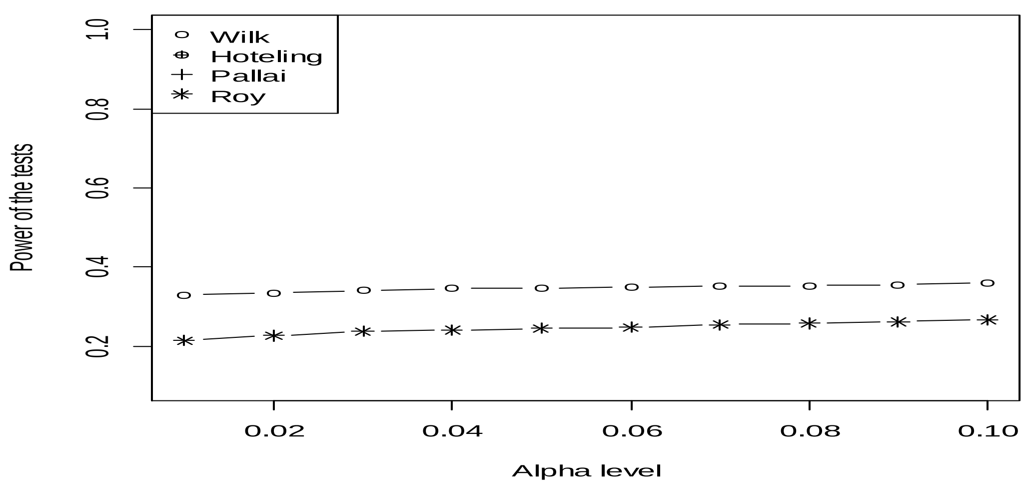

Fig 8. Power test for treatment effect when variance =1 and covariance =0 for M.Sudoku model IV at P=3.

Figures 5, 6, 7 and 8 show the power against some levels of alpha when variance and covariance are 1 and 0 respectively. At P=3 the results show that Wilk's Lambda has the highest power in all the models used followed by Roy's test. It was observed that there were little or no increase in power of the test as the alpha level increases. The graphs of other tests did not appear it might be due their poor performances in their type I error rates.

Figures 9, 10 and 11 show the power against some alpha levels when variances of the sigma are chosen to be 3 and covariances are 0.5 and 1.0. The results show that Hoteling-Lawley test has the highest power in M.Sudoku models I, III and IV.

In Figure 9, it revealed that there is slight increase in power as alpha level increases most especially for Pillai test, Roy's and Wilk's lambda tests are found to have similar power. In Figure 10 the increment was not seen as alpha level increases with Hoteling-Lawley test compare to Pillai test. Figure 11, despite Hoteling-Lawley the most powerful of all tests used, the rate of increment of power across the levels of alpha was not seen compare to Pillai and Wilk's lambda test.

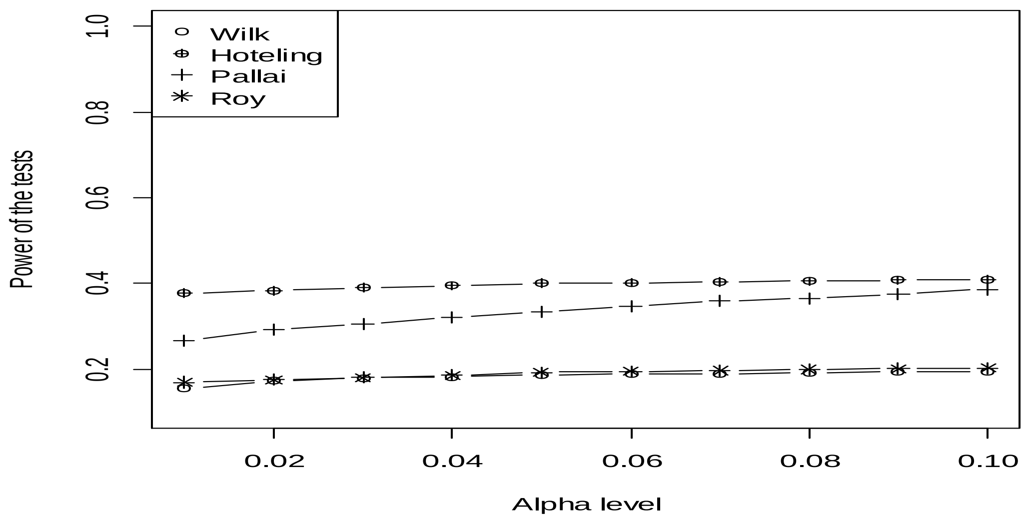

Fig 9. Power test for treatment effect when variance =3 and covariance =(0.5 &1) for M.Sudoku model I at P=2

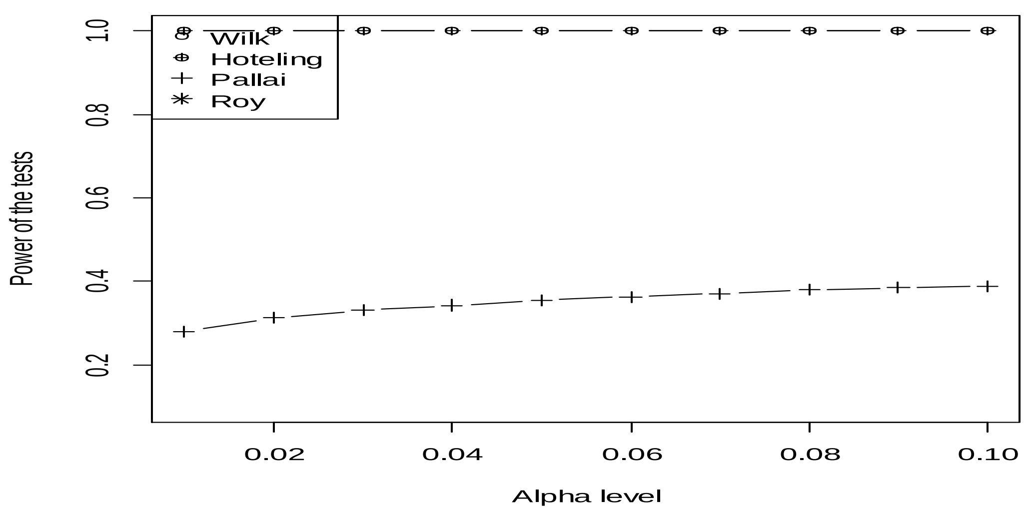

Fig 10. Power test for treatment effect when variance =3 and covariance =(0.5 &1) for M.Sudoku model III at P=2

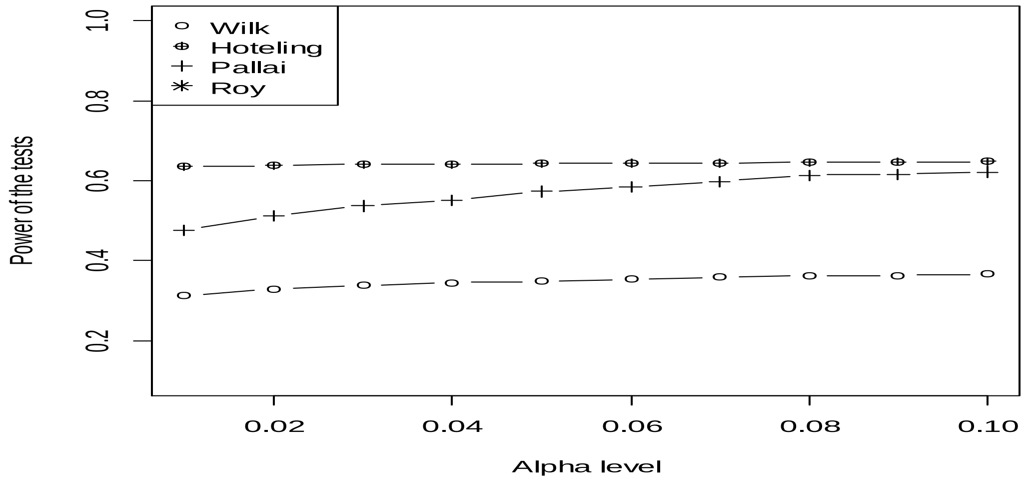

Fig 11. Power test for treatment effect when variance =3 and covariance = (0.5 &1) for M.Sudoku model IV at P=2

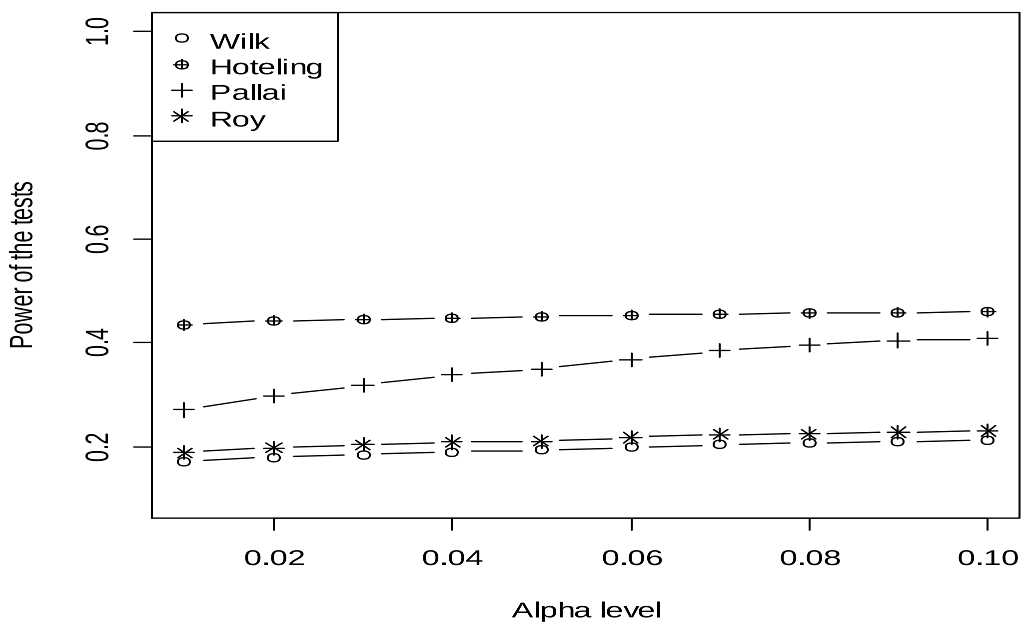

Fig 12. Power test for treatment effect when variance =1 and covariance = 0 for M.Sudoku model I at P=2

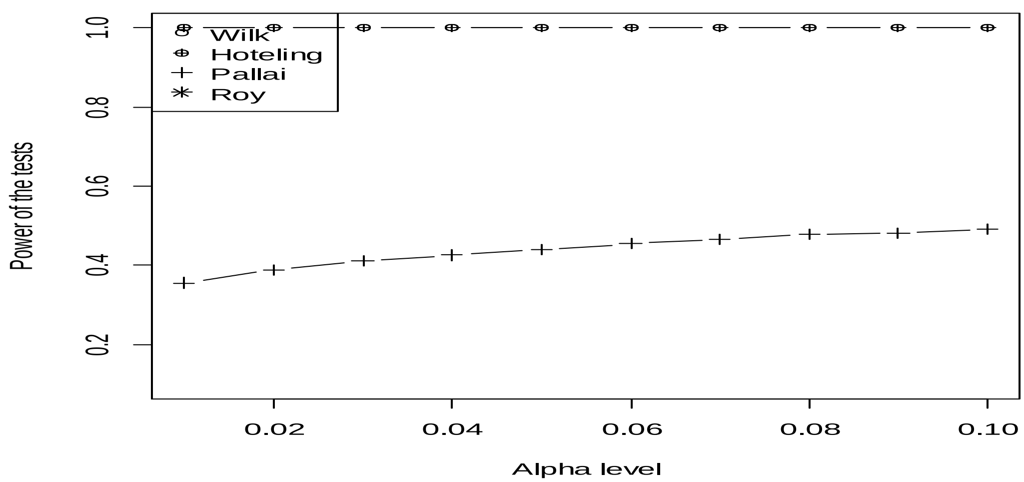

Fig 13. Power test for treatment effect when variance =1 and covariance = 0 for M.Sudoku model III at P=2

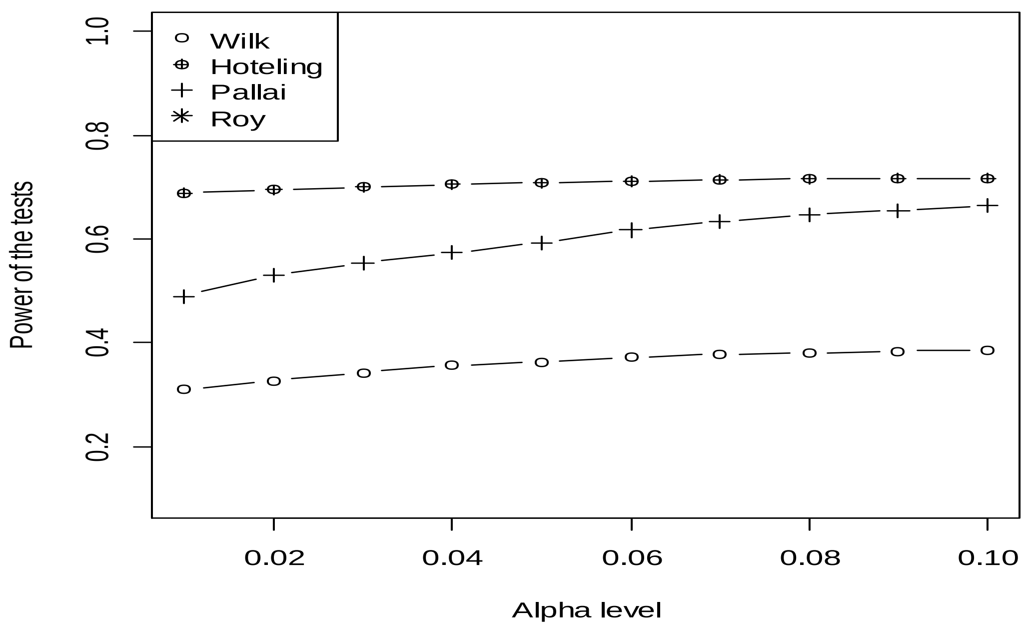

Fig 14. Power test for treatment effect when variance =1 and covariance = 0 for M.Sudoku model IV at P=2

Figures 12, 13 and 14 are the results of power test when variance and covariance are 1 and zero respectively. At P=2 the results show that Hoteling-Lawley test has the highest power value in M.Sudoku model I, III and IV. The value of power for Hoteling-Lawley almost constant as alpha level increases in three models plotted. However, there are slight increase in power with sother tests as the level of alpha increases.

CONCLUSION

This research estimates power of multivariate tests on multivariate Sudoku square design models, comparison of power of the these tests was made on treatment effect using Monte Carlo simulation for dependent variables P=2 and P=3. The results of Power test show that Hoteling-lawley has advantage over three other tests dependent variable P=2 while Wilk's Lambda test is the most powerful of all tests for the dependent variable P=3.

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