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A COMPARATIVE ANALYSIS OF GROWTH MODELS ON NIGERIA POPULATION

***Esosa G. Idemudia and Oluwadare O. Ojo**

Federal University of Technology Akure, Nigeria

*Corresponding authors' email: gift9121@gmail.com

ABSTRACT

Growth models have been applied over time to track and forecast changes in variables such as population, body height, biomass, fungal growth, and other aspects of numerous fields of study. This research focuses on modelling the growth of Nigeria's population from the year 1981 to 2021 and determining the best fit model to represent Nigeria's population growth (male, female and total). Seven growth models were considered in this research which includes: the linear, the exponential (Malthusian), Logistic (Verhulst), Gompertz, Hyperbolic, Brody and the Von Bertalanffy growth models. The criteria used for comparison of best fitted model were the coefficient of determination (R^2), Akaike Information Criterion (AIC), Mean Square Error (MSE), and Bayesian Information Criterion (BIC). The R^2 showed that the exponential, the logistic and the Gompertz growth models were all better fits for Nigeria's population (male, female and total) having the highest R^2 (0.999). Further comparison with the MSE, AIC and BIC revealed that the exponential growth model best represented Nigeria's population growth (male, female and total) having the least MSE, AIC and BIC. Hence the exponential growth model should be considered by researchers in Nigeria population projection.

Keywords: Nigeria, MSE, growth model, population, exponential

INTRODUCTION

Population studies are as old as the world itself, and the importance of studying demography or population cannot be overstated. 211 million people are estimated to live in Nigeria in 2021, making it the most populous country in Africa and the seventh most populous country in the world as stated by the data World Bank. The country's population is growing at one of the fastest rates in the world, at 2.6% per year. According to National Population Commission, (2021), Nigeria's population could double in the next 25 to 30 years if current trends continues. Understanding the factors that affect the size, expansion, makeup, and dispersion of the human population is made easier by population studies. By examining birth and death rates, immigration trends, and life expectancy tables, it makes an effort to explain historical patterns and make precise predictions about the future. For the purpose of calculating population size, many nations conduct censuses. The process is quite expensive because it calls for numerous resources and advances in technology. Censuses cannot be completed quickly due to the associated expenditures, hence projection models must be used to forecast future population. This makes it possible to plan well and use resources and money wisely.

According to Pambegua (2022), the first census in Nigeria was conducted in 1866, and then more were conducted in 1871, 1881, 1891, and 1901. However, they were only present in Lagos Colony and the surrounding area. According to Nigeria – Census History, (1991), despite several estimations of the population of Nigeria during the colonial era, the first attempt at a nationwide census was carried out in 1952–1953, . This effort produced a total population figure of 31.6 million inside the nation's present borders. This census was widely viewed as an undercount due to worries that it was used to collect taxes, political unrest at the time in eastern Nigeria, logistical challenges in getting to many distant regions, and insufficient training of enumerators in some places. Estimates of undercounting have ranged from 10% to less, with regional variations in accuracy expected by region. Despite its challenges, the 1952–1953 census was thought to be less challenging than any of its successors. Attempts to carry out an accurate post-independence census were dogged by controversy, with just one being formally recognized. The initial attempt, which took place in the middle of 1962, was abandoned due to intense debate and claims of massive overcounting. A second attempt, which was formally adopted in 1963, was also dogged by allegations of accuracy issues and political manipulation on the municipal and regional levels. Because it projected a practically impossibly high yearly growth rate of 5.8 percent, the official total national population count of 55.6 million in 1963 was incongruent with the census taken a decade earlier. Significant intraregional inconsistencies as well as possible aggregate figure exaggeration are shown by a close comparison of the 1953 and 1963 numbers. The two sets of statistics, for instance, show that in the southeast, some nonurban local government areas (LGAs) increased at a pace of roughly 13 percent annually, whilst other nearby areas only had a 0.5 percent annual growth. The findings of the 1963 census were eventually accepted notwithstanding the debate.

A 1973 census was attempted after the civil war of 1967–1970, but the findings were canceled due to persistent disagreement. As of 1990, there has not been another nationwide census, despite numerous attempts to estimate the population at the state or local level. The majority of officially released estimates of the country's population are based on forecasts from the 1963 census.

The 1991 census was a success because of major advancements in most sectors in terms of accessibility, technology, and educational attainment, as well as wide acceptance of the legitimacy and coherence of the nation. It was to be conducted by the National Population Commission, which had offices in each of the country's LGAs, in around 250,000 enumeration districts. State results were confirmed by supervisors from outside the state, and religious and ethnic identification were eliminated from census forms to minimize any potential dispute. "The most accurate and respectable census up until the 2006 Population and Housing Census was that of 1991". (2022, Pambegua). The project was divided into phases. For the first time, Geo-referenced EAs were created using Satellite Imagery and the Global Positioning System.

Information from respondents was also gathered through the use of Machine-readable forms (OMR/OCR/ICR). Another census was planned for 2016, but it was postponed because of the economic downturn in 2017. The corona virus pandemic prevented the exercise from being conducted in 2020 as planned.

In order to better manage farm animals, growth curve methodologies have recently been employed to examine the growth of those animals. In their study, Behzadi et al. (2014) used growth models to characterize the growth curve of Baluchi sheep based on their live weight from birth to yearling using five nonlinear growth functions such as the Von Bertalanffy, Gompertz, Brody, Logistic and Richards growth models. The residual mean square (MSE) and Akaike's Information Criterion (AIC) were used to compare the growth models. Out of the five growth functions, the Brody growth model gave the best fit having the lowest AIC and MSE.

In order to characterize the growth of maize leaves, Karadavut (2010) compared five different growth models. The growth of the maize leaves was modeled using the Richards, Logistic, Weibull, MMF, and Gompertz models. The coefficient of determination (R^2), sum of square error (SSE), root mean square error (RMSE), and mean relative error were used as comparative metrics. The findings showed that the Richards, Logistic, and Gompertz models were the best fits for maize leaf growth.

Most articles that had been published on population growth models especially in Nigeria context, were limited to few growth models. The most recent paper.Olanrewaju et al., (2020) used various growth models to model Nigeria's population growth considering the population as a whole and also considering the male and female population of Nigeria.They only considered limited growth models in their study while Brody and the Von Bertalanffy growth models were not taken into account. The Brody model has been used widely in population study and had showed to have better performance than most growth models (see Hojjati and Hossein-Zadeh (2018) &Madal*et al.* (2021) for more details). Also, Von Bertalanffy growth model, that is a special case of generalised logistic model, has been discovered to have so many advantages in growth modelling of population study (Liu, et al. 2021).

In this work, we will examine Nigeria population from the year 1981 to 2021 with the aid of seven growth models namely; the linear, the exponential (Malthusian), Logistic (Verhulst), Gompertz, Hyperbolic, Brody and the Von Bertalanffy while the model that gives the best fit will be determined. The male, female and total population will be considered. Also, the trend of the Nigeria population will also be determined.

The rest of this work is organized as follows. Section 2 gave an overview of several growth models and criteria for comparison were provided. The data are presented in Section 3 for analysis. In Section 4, data analysis and a discussion of the results were offered. Section 5 gives the conclussion of the study.

MATERIALS AND METHODS

Growth models

Growth models are mathematical models that are used to describe and predict the growth of a population over time. These models can be used to study the growth of a variety of different types of populations, including humans, animals, and microorganisms. In this research, Nigeria's population growth is modelled using linear growth model, Malthusian growth model, Gompertz growth model, Logistic growth model, Hyperbolic growth model, Brody growth model, and the Von Bertalanffy growth model.

Linear Growth Model

$\triangle P \propto \triangle t$

$\triangle P = b.\triangle t$ (1)

where b = proportionality constant

$\frac{\triangle P}{\triangle t} = b$

Making Δt as the smallest possible value i.e. Δt → 0

$\text{Lm } \Delta t \rightarrow 0 \;\; \frac{\triangle P}{\triangle t} = b$

i.e.

$\frac{dP}{dt} = b$

$dP = b.dt$ (2)

Integrating both sides of equation (2), we have:

$\int dP = \int b.dt$

$P = bt + a$

$P = a + bt$ (3)

where P is Population at time and a is the Initial population, b= common difference

Malthusian Growth Model

The simple exponential growth model is another name for the Malthusian growth theory. Based on the notion that the function is proportional to the rate of growth of the function, it is essentially exponential growth based. After Thomas Robert Malthus, who published "An Essay on the Principle of Population" in 1798, one of the first and most important publications on population, the Malthusian model was named. It has two basic assumptions:

i. Change in population growth is directly proportional to change in time. i.e. ΔP α Δt.

a. Change in population growth is directly proportional to growth up to that stage i.e. ΔP α P

Combining these two assumptions,

$\triangle P \propto \triangle t$

$\triangle P = b.P \triangle t$ (4)

where b = proportionality constant

$\frac{\triangle P}{\triangle t} = b.P$ (5)

Making Δt as the smallest possible value i.e. Δt → 0

$\text{Lm } \Delta t \rightarrow 0 \; \frac{\triangle P}{\triangle t} = b.P$

i.e.

$\frac{dP}{dt} = b.P$

$\frac{dP}{P} = b.dt$ (6)

Integrating both sides of equation (6), we have

$\int \frac{dP}{P} = \int b.dt$

$log_e P = bt + c$ (7)

Taking the exponential of both sides of equation (7),

$e^{log_e P} = e^{bt+c}$

$P = e^{bt}.e^{c}$

$P = P_o e^{bt}$ (8)

where P is the Population at time t and P_o is the Initial population

Logistic Growth model

Belgian mathematician Pierre Verhulst developed the logistic growth model, sometimes known as the Verhulst model, in 1838. By modifying the exponential growth model, he developed it as a population growth model. The growth is roughly exponential in the early stages, slows to linear growth as saturation sets in, and finally ends at maturity. Malthusian model was updated by Pierre Verhulst by adding the term

$$\frac{a-bP(t)}{a} \tag{9}$$

where a and b are population coefficients. This term refers to the population's separation from its limiting value. using the aforementioned equation as a coefficient in the Malthusian model, we obtain:

$$\frac{dP(t)}{dt} = \frac{aP(t)(a-bP(t))}{a} \tag{10}$$

Simplifying equation (2) yields the logistic differential equation, which is denoted by the following equation:

$$\frac{dP(t)}{dt} = aP(t)\left(1 - \frac{P(t)}{K}\right) \tag{11}$$

where $K = \frac{a}{b}$

K is the carrying capacity, P(t) is the population function at time t, and a is the proportionality constant in equation (3). By using the separation of variables method to solve equation (3), we obtain

$$\int \frac{dP(t)}{P(t)\left(1-\frac{P(t)}{K}\right)} = \int a dt \tag{12}$$

After analyzing equation (12's left side), we have:

$$\frac{1}{P(1-P/K)} = \frac{K}{P(K-P)} = \frac{1}{P} + \frac{1}{K-P}$$

Hence, equation (12) becomes:

$$\int \frac{dP}{P} + \int \frac{dP}{K-P} = \int a dt \tag{13}$$

$$\ln|P| - \ln|K-P| = at + c$$

$$\ln\left|\frac{K-P}{P}\right| = -at - \mathrm{c}$$

$$\left|\frac{K-P}{P}\right| = e^{-at-c}$$

$$\frac{K-P}{P} = Ae^{-at}$$

where $A = \pm e^{-c}$

Thus the model becomes;

$$P = \frac{K}{1+Ae^{-at}} \tag{14}$$

where the constant

$$A = \frac{K-P_o}{P_o}$$

Hence,

$$P(t) = \frac{K}{1+(\frac{K-P_o}{P_o})e^{-at}} \tag{15}$$

Gompertz Growth Model

A sort of mathematical time series model is called the Gompertz growth model, after the actuary, Benjamin Gompertz. According to a sigmoid function, growth is slowest at the beginning and end of a given time period. The curve approaches the function's future value asymptote considerably more gradually than it does with the lower valued asymptote. This is contrast with the logistic growth model, where the curve approaches both asymptotes symmetrically. The Gompertz growth model is a solution of the differential function:

$$\frac{dP}{dt} = c\ ln(\frac{M}{P})P \tag{16}$$

where M is the carrying capacity and c is a constant.

Solving the differential equation:

$$\frac{dP}{dt} = c\ln(\frac{M}{P})P$$

$$\frac{1}{P\ln(\frac{M}{P})}dP = c\ dt \tag{17}$$

Integrating both sides of equation (17), we have;

$$\int \frac{1}{P\ln(\frac{M}{P})}dP = \int c\ dt \tag{18}$$

Let $u = \ln(\frac{M}{P})$,so $du = \frac{1}{\frac{M}{P}} . -\frac{M}{P^2}dP = \frac{P}{M}.-\frac{M}{P^2}dP = -\frac{1}{P}\ dP$

Hence equation (18) becomes;

$$-\int -\frac{1}{P\ln\left(\frac{M}{P}\right)}dP = \int c\ dt \tag{19}$$

$$-\int \frac{1}{u}du = \int c\ dt \tag{20}$$

$$\int \frac{1}{u}du = -\int c\ dt \tag{21}$$

$$\ln|u| = -[ct + C_1]$$

$$\ln|u| = -ct + C_2$$

$$|u| = e^{-ct+C_2}$$

$$|u| = e^{-ct}.e^{C_2}$$

$$|u| = C_3e^{-ct}$$

And because $|u| = C_3e^{-ct} \Longrightarrow u \pm C_3e^{-ct}$, we have

$$u = C_4e^{-ct}$$

$$\ln\left(\frac{M}{P}\right) = C_4e^{-ct} \tag{22}$$

Now, at t=0, $P = P_o$, and

$$\ln\left(\frac{M}{P_o}\right) = C_4e^{-c.0}$$

$$\ln\left(\frac{M}{P_o}\right) = C_4 \tag{23}$$

So equation (23) becomes;

$$\ln\left(\frac{M}{P}\right) = \ln\left(\frac{M}{P_o}\right)e^{-ct}$$

$$\frac{M}{P} = e^{\ln\left(\frac{M}{P_o}\right)e^{-ct}}$$

$$P = Me^{-\ln\left(\frac{M}{P_o}\right)e^{-ct}} \tag{24}$$

Hyperbolic Growth model

when a quantity approaches a finite-time singularity (singularity with finite variation). It supposedly experiences hyperbolic growth. The differential equation is derived from the exponential growth model as:

$$\frac{dP}{dt} = bP \tag{25}$$

where it is assumed that b is constant through the relative growth rate. We now want to investigate changing this presumption so that, generally speaking, b becomes a function of P, i.e.

$$\frac{dP}{dt} = b(P)P \tag{26}$$

The assumption that b is proportional to P is the most basic function in the hyperbolic growth. In order to retain the dimensions in equation (26), We can state that,

$$\frac{dP}{dt} = \frac{bP}{P_o} \tag{27}$$

Whereas in equation (27), b is now a constant multiplied by P, which fluctuates over time and P is a constant. The differential equation now reads as follows:

$$\frac{dP}{dt} = b(P)P = \frac{bP}{P_o}P = \frac{bP^2}{P_o} \tag{28}$$

Solving the differential equation in equation (28),

$$\frac{dP}{dt} = \frac{bP^2}{P_o}$$

$$\frac{dP}{P^2} = \left(\frac{b}{P_o}\right)dt \tag{29}$$

Integrating both sides of equation (29), we have;

$$\int \frac{dP}{P^2} = \int \left(\frac{b}{P_o}\right)dt$$

This gives us,

$$-\frac{1}{P} = \left(\frac{b}{P_o}\right)t + C$$

At time t = 0, we can say that

$$C = -\frac{1}{P_o}$$

$$-\frac{1}{P} = -\frac{b}{P_o}t - \frac{1}{P_o}$$

$$-P = \frac{P_o}{bt-1}$$

$$P = -\frac{P_o}{bt-1}$$

So we have our hyperbolic growth model,

$$P = \frac{P_o}{1-bt} \quad (30)$$

Brody Growth Model

The Brody model is a mathematical model that describes the relationship between a population's rate of population growth and rate of resource consumption. It was developed in the 1940s by economist Solomon Brody. The model is predicated on the notion that resource availability limits the rate of population expansion. When resources are plentiful, the population can grow rapidly. The rate of population growth, however, slows as resources become scarce. The model describes this relationship using a mathematical equation that takes into account resource availability as well as resource efficiency.The Brody model has been used to study a wide range of populations, including human populations, animal populations, and even microorganism growth. It has also been used to understand the effects of resource consumption on population growth and to develop population management strategies. Despite its popularity, the model has been chastised for its simplicity and assumptions about resource availability and consumption. The model can be represented as

$$P = a * (1 - b * e^{-kt}) \quad (31)$$

Where P = Population at time t
a = Initial population
b = constant
k = Carrying capacity

Von Bertalanffy Growth Model

The Von Bertalanffy growth model is a mathematical model that describes growth over time. It was developed in the 1930s by Ludwig von Bertalanffy, a biophysicist and systems theorist. The model is based on the idea that an organism's rate of growth slows as it ages, eventually reaching a maximum size, or "asymptote."The model has been applied to the growth of many organisms, including fish, birds, and mammals. It has also been used to study the growth of populations and community of organisms. The model can be represented as:

$$P = a * (1 - b * e^{-kt})^3 \quad (32)$$

Where P = Population at time t
a = Initial population
b = constant
k = Carrying capacity

Criteria for Model Comparison

Criteria for model comparison refers to the tools used in comparing the growth models to determine the best fit. In this research, the criteria used for comparison the growth model to fit Nigeria's population (male, female and total population) are:

Coefficient of Determination (R^2)

The percentage of the variance in the dependent variable that is explained by the independent variable(s) in a regression model is shown by the coefficient of determination, also referred to as the R-squared value, in statistics. It is an indicator of how well the model fits the data and ranges from 0 to 1, with 1 indicating a perfect fit. The formula to calculate the coefficient of determination (R^2):

$$1 - \frac{RSS}{TSS} = 1 - \frac{\sum(Y_i - \hat{Y})^2}{\sum(Y_i)^2} \quad (33)$$

where RSS = Residual sum of squares
TSS = Total sum of squares
Y_i = observed values
$\hat{Y}_i$ = predicted values

Akaike Information Criterion (AIC)

The Akaike Information Criterion (AIC), based on the trade-off between the model's goodness of fit and the number of parameters, assesses the relative quality of statistical models. It is used to evaluate various models and choose the one that fits a particular data set the best. The AIC is described as:

$$AIC = 2K - 2\ln(L) \quad (34)$$

where k = the model's total number of parameters.
L = the model's highest level of likelihood.
It can also be defined as:

$$AIC = nln(RSS) + 2p \quad (35)$$

where n = number of observation
p = number of parameters of the model
RSS = Residual sum of squares
The AIC can only be used to compare models fitted to the same data set. By calculating the AIC for each model and choosing the one with the lowest AIC, multiple models can be compared.

Bayesian Information Criterion (BIC)

The Bayesian information criterion (BIC), which compares and chooses the best model from a group of models, is a measure of the relative quality of statistical models. The BIC can be used to compare several models with various parameter and various data fits. Because it optimally balances model fit and complexity, the model with the lowest BIC is recommended. The BIC is defined as:

$$BIC = k * \ln(n) - 2 * \ln(L) \quad (36)$$

where k = number of parameters in the model
n = is the sample size
L is the model's likelihood function's greatest value, which indicates how well the model fits the data.
It can also be expressed as:

$$BIC = nln\left(\frac{RSS}{n}\right) + kln(n) \quad (37)$$

where RSS = Residual sum of squares

Mean Square Error (MSE)

A measure of the discrepancy between a sample's predicted and actual values is the mean squared error (MSE). It is widely used to assess the precision of a prediction or estimate in the context of statistical modeling and machine learning. The MSE is determined by adding up the squared differences between the anticipated and actual values, dividing by the sample size, and then adding the result. A measure of the average squared difference between the expected and actual values is the result. The model or forecast is better with the lowest the MSE. Because it is easy to understand and can be used to compare the performance of various models, MSE is a well-popular metric for assessing prediction accuracy. It may also be calculated rather easily since all that is needed are the true and anticipated values for each sample. The formula to find the MSE of a model is defined by:

$$MSE = \frac{1}{n}\sum(Y_i - \hat{Y}_i)^2 \quad (38)$$

where n = number of observations
Y_i = observed values
$\hat{Y}_i$ = predicted values

Data Presentation

The population for this research consists of the total population of all 36 states in Nigeria, which consists of the male and female populations from the years 1981 to 2021.

Data Analysis and Interpretation

The data was analyzed by fitting the annual population into different growth models and comparing the growth models using the Akaike Information Criterion (AIC), the Mean Square Error (MSE), the Bayesian Information Criterion (BIC), and the coefficient of determination (R^2).

Table 1 Descriptive analysis of Nigeria's Population

	Female population	Male population	Total population
Mean	65300331	66599015	131899348
Range	66738015	69222184	135960199
Minimum	37512694	37927811	75440505
Maximum	104250709	107149995	211400704
Observation	41	41	41
Standard Deviation	19810930.84	20598248.72	40409103.54

Table 1 reveals that the least population in Nigeria for the period of 41 years (1981-2021) was 75440505 and the highest population was 211400704 with an average value of 131899348. The standard deviation for the total population (40409103.54) is large which implies that the mean value is far from the observed value of the total population.

The least female population in Nigeria for the period of 41 years (1981-2021) was 37512694 and the highest female population was 104250709 with an average value of 65300331. The standard deviation for the female population (19810930.84) is large which implies that the mean value is far from the observed value of the female population.

The least male population in Nigeria for the period of 41 years (1981-2021) was 37927811 and the highest female population was 107149995 with an average value of 66599015. The standard deviation for the female population (20598248.72) is large which implies that the mean value is far from the observed value of the female population.

Table 2 Goodness of fit for Nigeria's female population

Growth model	R^2	AIC	BIC	MSE
Linear	0.980	1372.61	1223.78	8.05E+6
Hyperbolic	0.986	1356.72	1207.89	5.47E+6
Malthusian	0.999	1161.02	1012.19	4.6E+4
Logistic	0.999	1163.02	1015.91	4.7E+4
Brody	0.980	1374.66	1227.54	8.28E+6
Gompertz	0.999	1229.56	1082.45	2.4E+5
Von Bertalanffy	0.997	1296.10	1148.99	1.22E+6

From Table 2 above, the R^2 gotten for the growth models of Nigeria's female population (0.999) shows that the Malthusian model, Logistic model, and Gompertz model are the best fitted model for Nigeria's female population. The Malthusian growth model is the model with the least AIC, BIC, and MSE (1161.0167, 1012.1928, 4.6E+4). Hence, we can say that the Malthusian model produces the best fit for Nigeria's female population growth.

Table 3 Goodness of fit for Nigeria's male population

Growth model	R^2	AIC	BIC	MSE
Linear	0.979	1377.73	1525.99	9.13E+6
Hyperbolic	0.986	1361.32	1212.49	6.12 E+6
Malthusian	0.999	1165.62	1016.79	5.2 E+4
Logistic	0.999	1167.62	1020.50	5.3 E+4
Brody	0.979	1380.09	1232.97	9.45 E+6
Gompertz	0.999	1249.51	1102.39	3.91 E+5
Von Bertalanffy	0.979	1380.14	1233.02	9.459 E+6

From Table 3 above, the R^2 gotten for the growth models of Nigeria's male population (0.999) shows that the Malthusian model, Logistic model, and Gompertz model are the best fitted model for Nigeria's male population. The Malthusian growth model is the model with the least AIC, BIC, and MSE (1165.6176, 1016.7882, 5.2E+4). Hence, we can say that the Malthusian model produces the best fit for Nigeria's male population growth.

Table 4 Goodness of fit for Nigeria's total population

Growth model	R^2	AIC	BIC	MSE
Linear	0.980	1432.05	1283.22	34.33E+6
Hyperbolic	0.986	1415.87	1267.04	23.13E+6
Malthusian	0.999	1219.85	1071.02	1.94E+5
Logistic	0.999	1221.85	1074.73	1.99E+5
Brody	0.979	1434.42	1287.31	35.55E+6
Gompertz	0.999	1299.33	1152.22	6.05E+6
Von Bertalanffy	0.996	1361.80	1214.68	1.32E+6

According to the Table 4, the Malthusian growth model, the logistic growth model, and the Gompertz growth model best describe the population growth of Nigeria's total population over the 41 years observed, i.e. from 1981 to 2021 when the R^2 is taken into consideration with an R^2 of 0.999 each. When the (AIC) and the (BIC) are applied, it reveals that the Malthusian growth model performs better than the other growth models for Nigeria's total population growth with AIC and BIC of 1219.8475 and 1071.0182 respectively. Also, by comparing the mean square errors (MSE) of the population, Malthusian growth model also performed best having the least MSE 1.94E+5.

CONCLUSION

In this research, different growth models were applied to Nigeria population from the year 1981 to 2021 and determining the best fit model to represent Nigeria's population growth. In Nigeria population, Male, female and total population were examined. The growth models considered namely; considered in this research which includes: the linear, the exponential (Malthusian), Logistic (Verhulst), Gompertz, Hyperbolic, Brody and the Von Bertalanffy. Four criteria were used for comparison in order to know the best fitted model. The criteria were; coefficient of determination (R^2), Mean Square Error (MSE), Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The R^2 showed that the exponential, the logistic and the Gompertz growth models were all better fits for Nigeria's population (male, female and total). The Mean squared error, Akaike Information and Bayesian Information Criteria revealed that the exponential growth model is the best model that represented Nigeria's population growth (male, female and total). The study also revealed that Nigeria's population is rapidly growing. This has a number of implications. One of the most serious consequences is that it may lead to overpopulation, putting a strain on the country's resources and infrastructure. This can lead to issues such as insufficient access to healthcare, education, and basic necessities such as food and clean water. With more people to feed, clothe, and provide basic necessities for, demand for goods and services may rise, putting pressure on the country's production capabilities. This can lead to inflation and a decrease in many people's standard of living. In addition, as the population grows, so does the demand for natural resources such as water, timber, and fossil fuels. This can result in deforestation, water scarcity, and air and water pollution.

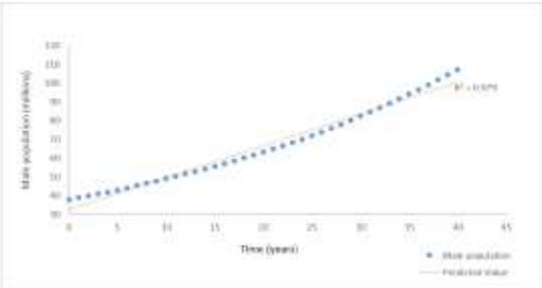

Figure 1: Graph of linear male population growth against time for a period of 41 years

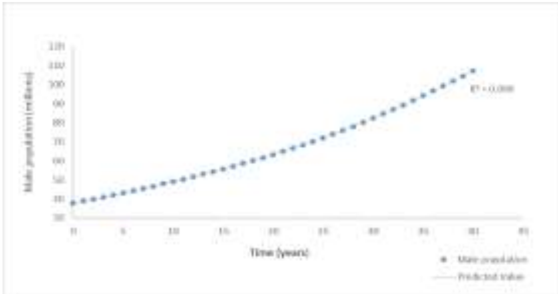

Figure 2: Graph of exponential male population growth against time for a period of 41 years

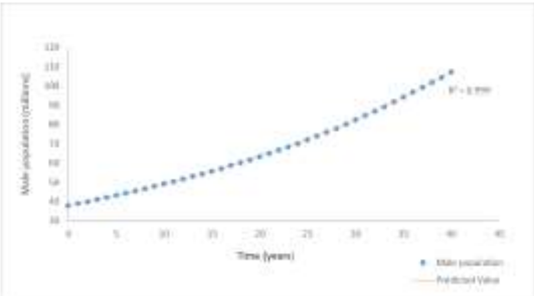

Figure 3: Graph of logistic male population growth against time for a period of 41 years

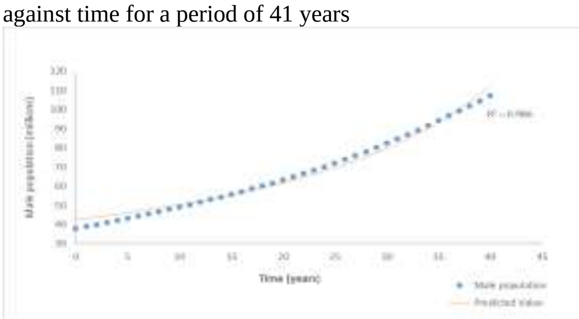

Figure 4: Graph of hyperbolic male population growth against time for a period of 41 years

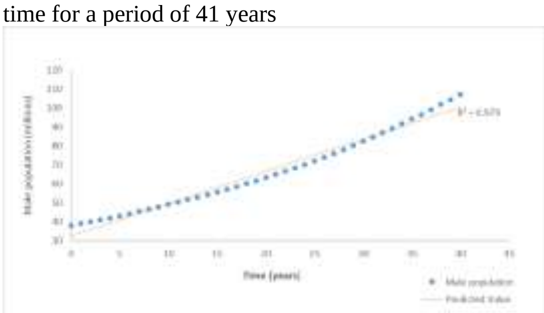

Figure 5: Graph of Brody male population growth against time for a period of 41 years

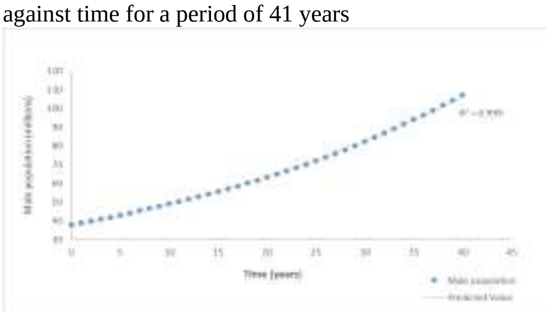

Figure 6: Graph of Gompertz male population growth against time for a period of 41 years

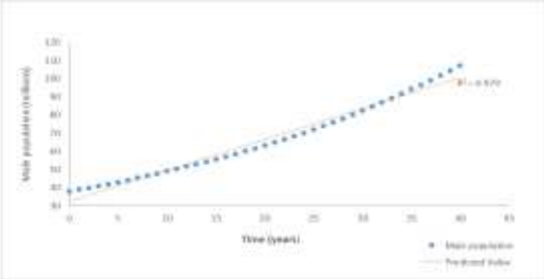

Figure 7: Graph of Von Bertalanffy male population growth against time for a period of 41 years

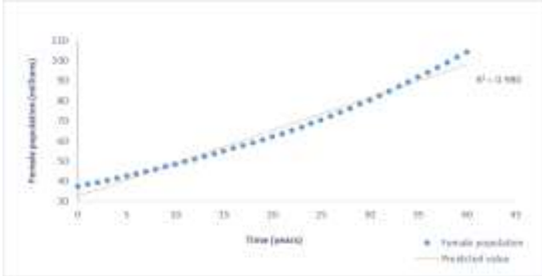

Figure 8: Graph of linear female population growth against time for a period of 41 years

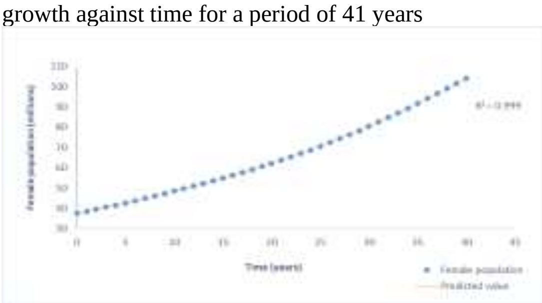

Figure 9: Graph of exponential female population growth against time for a period of 41 years

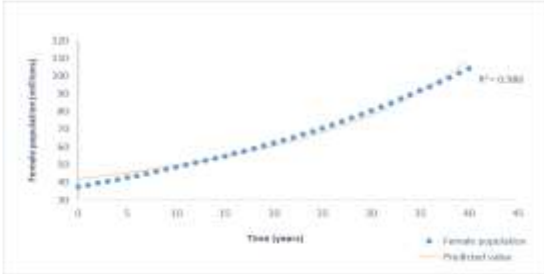

Figure 10: Graph of hyperbolic female population growth against time for a period of 41 years

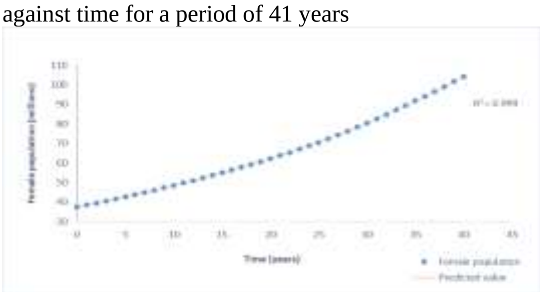

Figure 11: Graph of logistic female population growth against time for a period of 41 years

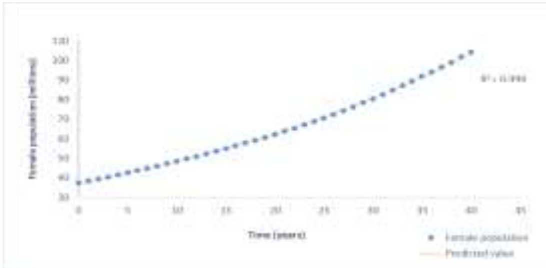

Figure 12: Graph of Gompertz female population growth against time for a period of 41 years

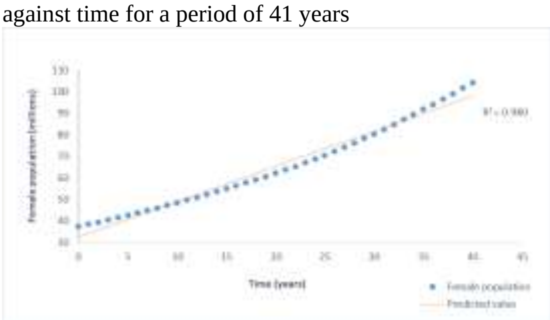

Figure 13: Graph of Brody female population growth against time for a period of 41 years

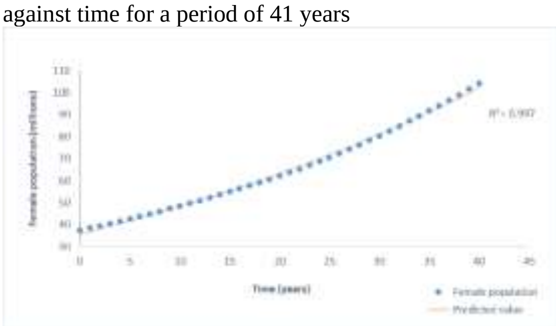

Figure 14: Graph of Von Bertalanffy female population growth against time for a period of 41 years

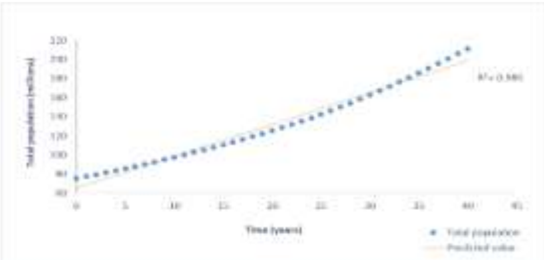

Figure 15: Graph of total linear population growth against time for a period of 41 years

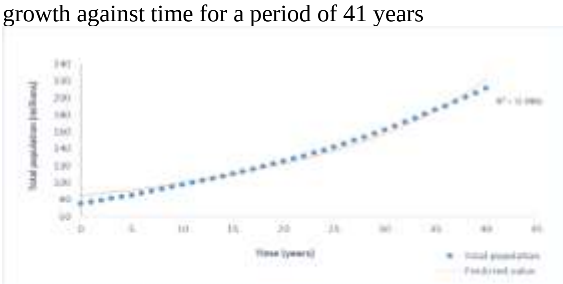

Figure 16: Graph of hyperbolic total population growth against time for a period of 41 years

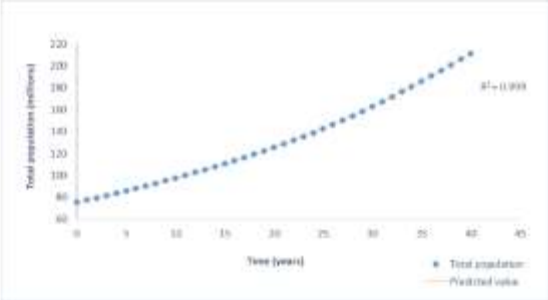

Figure 17: Graph of exponential total population growth against time for a period of 41 years

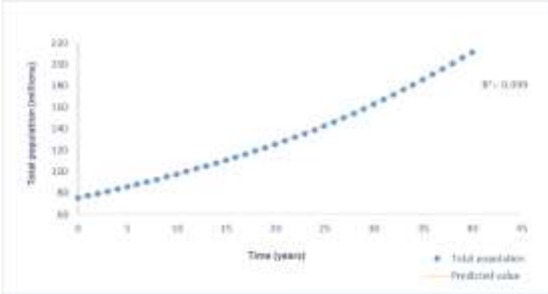

Figure 18: Graph of logistic total population growth against time for a period of 41 years

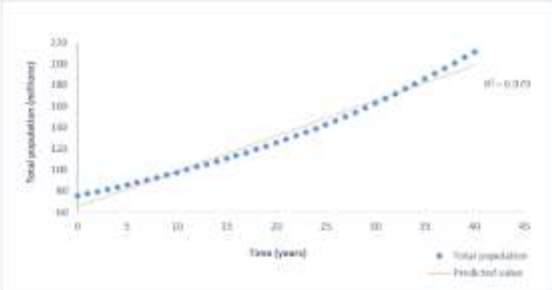

Figure 19: Graph of Brody total population growth against time for a period of 41 years

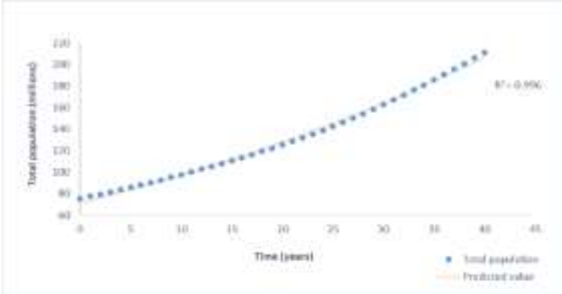

Figure 20: Graph of von Bertalanffy total population growth against time for a period of 41 years

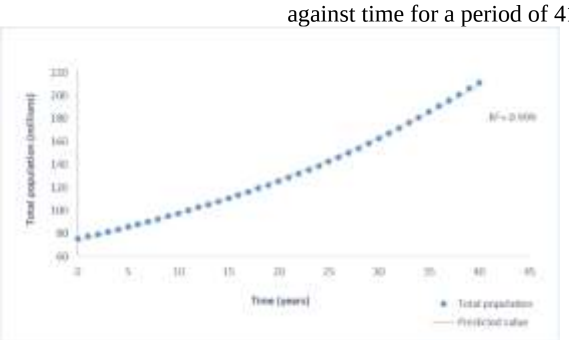

Figure 21: Graph of Gompertz total population growth against time for a period of 41 years

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