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VOLATILITY ANALYSIS OF CRUDE OIL PRICES IN NIGERIA

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ABSTRACT

This study investigates the symmetric and asymmetric characteristics as well as the persistence of shocks in the Nigerian crude oil returns, utilizing monthly and daily crude oil prices spanning from January 2006 to September 2022 and November 3, 2009, to November 4, 2022, respectively. Descriptive statistics, normality measures, time plots, and the Dickey-Fuller Generalized Least Squares unit root test were employed to analyze the series properties. Symmetric ARMA (1,1)-GARCH (2,1) and asymmetric ARMA (1,1)-TARCH (2,1) models for monthly and daily returns, with varying innovation densities, were utilized, alongside symmetric GARCH (1,1) and asymmetric TARCH (1,1) models. Model selection criteria including AIC, SIC, HQC, and log likelihood guided the order and error distribution selection. Results revealed non-normal distributions for both monthly and daily prices and returns, non-stationarity in prices, and weak stationarity in log returns with ARCH effects detected in both returns. Symmetric models exhibited volatility clustering, high shocks persistence, mean-reverting behaviour, and predictability in both returns. Asymmetric models identified asymmetry with leverage effects in both returns, indicating that negative shocks induce greater volatility than positive shocks of the same magnitude. Mean reversion and volatility half-life findings suggested that crude oil prices tend to revert to their long-run averages. The study recommended promoting market information flow and aggressive trading to enhance market depth and mitigate the volatile nature of the Nigerian crude oil market.

Keywords: Crude Oil Price, GARCH Variants, Half-Life, Mean Reversion, Volatility, Nigeria

INTRODUCTION

Nigeria is a major oil-producing country, and the prices of crude oil are highly susceptible to various domestic and global factors (Thomas, 2015). Some of the key factors influencing the fluctuation of crude oil prices in Nigeria are: The most fundamental factor affecting crude oil prices globally is the balance between demand and supply. Any disruptions in major oil-producing regions, changes in global economic conditions, or geopolitical events can impact the supply and demand dynamics, consequently affecting prices. Nigeria is a member of OPEC, and decisions made by the organization regarding oil production quotas can have a significant impact on oil prices. OPEC's decisions to increase or decrease production levels can influence the global supply of oil and, consequently, its price. Political instability or conflicts in oil-producing regions, including the Niger Delta in Nigeria, can disrupt oil production and transportation, leading to fluctuations in oil prices. Any geopolitical tensions in major oil-producing areas can create uncertainty and impact oil prices (Thomas *et al*., 2016).

Since oil is priced in U.S. dollars, fluctuations in currency exchange rates can affect the purchasing power of oil-producing countries, including Nigeria. Changes in the strength of the U.S. dollar can influence the revenue generated from oil exports (Usoro *et al*., 2020). Nigeria's economic policies, including taxation, subsidies, and regulatory frameworks, can influence the country's oil sector. Changes in these policies may impact oil production and investment in the sector. The overall health of the global economy can affect oil prices. Economic growth or contraction in major economies can influence oil demand, and hence, its price (Kuhe, 2019).

Crude oil price volatility refers to the degree of variation or fluctuation in the market prices of crude oil over a specific period. It is a measure of the extent to which the prices of crude oil change, reflecting the uncertainty, risk, and dynamic nature of the oil market. Higher volatility indicates larger and more frequent price movements, while lower volatility suggests more stable and predictable prices (Thomas, 2015).

Modeling the volatility of crude oil prices is crucial for several reasons, as it helps market participants, policymakers, and researchers to understand and manage risks, make informed decisions, and develop effective strategies (Kuhe, 2019). Some of the key reasons for modeling crude oil price volatility are:

Volatility modeling aids in assessing the level of risk associated with crude oil price movements. This is particularly important for market participants, such as traders, investors, and companies in the energy sector, who need to manage and hedge against price fluctuations (Kuhe, 2019). Investors use volatility models to make informed decisions about allocating resources and constructing portfolios. Understanding the volatility of crude oil prices is essential for optimizing investment strategies and minimizing potential losses. Policymakers and government officials use volatility models to assess the potential impact of oil price movements on the economy. This information is valuable for designing effective policies to mitigate economic risks and promote stability. Companies involved in the production, transportation, and distribution of oil and oil-related products use volatility models to optimize their supply chain management (Usoro *et al*., 2020). This includes making decisions related to inventory levels, production planning, and logistics. Volatility models contribute to academic research and forecasting efforts. Researchers use these models to better understand the underlying factors influencing oil price movements and to develop predictive models for future price trends (Usoro *et al*., 2020).

The use of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models in modeling crude oil prices is well-suited due to their ability to capture key features of crude oil price dynamics (Sujoy and Arshad, 2018). These models effectively address volatility clustering, a phenomenon observed in crude oil markets where periods of high volatility tend to cluster together. GARCH models also accommodate asymmetry in volatility, allowing for a differential impact of positive and negative shocks. Their flexibility in modeling time-varying volatility aligns with the dynamic nature of crude oil markets, and they can handle sudden jumps and extreme events that characterize the oil industry (Sujoy and Arshad, 2018).. GARCH models are valuable for forecasting future volatility, aiding in risk management and option pricing. Their simplicity, diagnostic tools, and adaptability make them accessible for both academic research and practical applications, though researchers often explore variations to enhance accuracy and address model limitations (Thomas *et al*., 2016). The aim of this study is therefore to investigate the symmetric and asymmetric characteristics, as well as the persistence of shocks in the returns of Nigerian crude oil, using both monthly and daily recent crude oil prices.

Literature Review

Several documented evidence on the volatility modeling of crude oil prices and returns abound in literature. For example, Omur *et al*. (2016) employed GARCH variants to analyze the volatility of crude oil and natural gas return and to determine their accuracy. Asymmetric and integrated GARCH models performed relatively better than other competing models, with FIGARCH-BBM (SST) and EGARCH (GED) identified as minimum loss models for specific periods. Ham *et al*. (2016) evaluated volatility models' performance on daily crude oil returns, highlighting the impact of the global financial crisis on crude oil prices. APGARCH and FIAPGARCH models with Student-t and Skewed Student-t distributions were found to best fit oil prices, indicating high volatilities and long memory effects during the crisis.

Ijeoma *et al*. (2016) Examined the effect of oil prices on food price volatility in Nigeria, the study found no long-run relationship between oil prices and individual food price volatility. However, a positive and significant short-run relationship was identified through a VAR model, suggesting unidirectional causality from oil prices to maize, soya bean, and sorghum price volatilities. Mehesh and Prasad (2016) Analyzed crude oil price return volatility patterns; the study used symmetric and asymmetric GARCH family models. GARCH (1,1) and EGARCH (1,1) models with a student's t distribution were found to better analyze symmetric and asymmetric volatility estimates of near month expiry futures contract crude oil price returns.

Bahar *et al.* (2017) used West Texas Intermediate daily data; the study identified structural breaks in crude oil prices. Geometric Brownian motion outperformed the mean-reverting Ornstein-Uhlenbeck process for short-term forecasting. Thomas *et al*. (2016) employed Markov-switching multifractal (MSM) models and GARCH-type models to model and forecast oil price volatility. The new MSM model consistently outperformed other models in forecasting horizons and subsamples, demonstrating its superiority. Abduchakeem and Kilishi (2016) analyzed oil price-macroeconomic volatility in Nigeria, GARCH models and variants reveal high volatility in macroeconomic variables. Asymmetric models suggest oil prices as a major source of economic volatility in Nigeria.

Olugbenga and Ogunsola (2017) examined the impact of oil price volatility on investment decision making in marginal fields development in Nigeria, the study found a significant positive relationship between oil price volatility and crude oil production. Deebom and Isaac (2017) modeled price volatility and risk-return in the Nigerian crude oil market, the study favoured symmetric GARCH models over asymmetric ones. Positive risk premiums suggested that investors were rewarded for holding risky assets.

Ayeni (2018) investigated the short and long-run effects of oil price shocks and exchange rate volatility on investment in Nigeria, the study found significant impacts of exchange rate volatility on investment. Onyeka-Ubaka *et al.* (2018) analyzed crude oil price return volatility in Nigeria; the study concluded that GARCH (1, 1) and ARIMA (1, 1, 0) models performed well in capturing the features of high-frequency crude oil prices. Bashir (2018) investigated the relevance of GARCH-family models in forecasting Nigerian crude oil prices, the study found that the symmetric GARCH (1, 1)-GED model performed better than other competing GARCH models. Jawadi and Fhiti (2019) focused on oil price volatility and uncertainty; the study proposed stochastic oil volatility models and concluded that the standard stochastic volatility model outperformed other competing models in forecasting oil price uncertainty.

Awidan (2019) introduced a hybrid Bayesian Network method for short-term forecasting of crude oil prices, finding it effective in capturing volatility characteristics. Yue-Jun *et al.* (2019) estimated and forecasting crude oil price volatility, the study finds limited significance in incorporating regime-switching, with single-regime GARCH models performing well. Kuhe (2019) investigated the dynamic relationship between crude oil prices and stock market volatility in Nigeria, the study identified no long-run stable relationship. Crude oil prices and stock market prices had positive and significant impacts on each other. Lu-Tao *et al*. (2019) used fractional GARCH models, the study improved crude oil price risk measurement, emphasizing the importance of considering long memory, asymmetry, and fat tails. The current study attempts to extend the existing literature and contributes to the existing body of knowledge by modeling the volatility of crude oil prices in Nigeria using symmetric and asymmetric GARCH models and more recent data.

MATERIALS AND METHODS

Source of Data and Data Transformation

The data utilize in this study are the secondary monthly and daily time series data on crude oil price in Nigeria from January, 2006 to September, 2022 and 3rd November, 2009 to 4th November, 2022 obtained from Central Bank of Nigeria (CBN, 2022) website. The crude oil prices P_t are converted to log return series r_t through the following equation:

$$r_t = 100.\ln \nabla P_t \quad (1)$$

where $\nabla \mathrm{P_t} = \ln(P_t - P_{t-1})$, r_t denotes the log return series and P_t denotes the closing crude oil price index at the current month t.

Methods of Data Analysis

For the purpose of data analysis in this work, the following statistical tools were utilized.

Dickey-Fuller generalized least squares (DF GLS) unit root test

The Dickey-Fuller Generalized Least Squares (DF GLS) unit root test has been employed to investigate the unit root property and order of integration of oil prices and returns in Nigeria. The DFGLS test involves estimating the standard ADF test equation:

$$\Delta r_t = \alpha r_{t-1} + X_t'\delta + \beta_1 \Delta r_{t-1} + \beta_2 \Delta r_{t-2} + \cdots + \beta_p \Delta r_{t-p} + \varepsilon_t \quad (2)$$

After substituting the DFGLS detrended r_t^d for the original r_t, we have

$$\Delta r_t^d = \alpha r_{t-1}^d + \beta_1 \Delta r_{t-1}^d + \cdots + \beta_p \Delta r_{t-p}^d + \varepsilon_t \quad (3)$$

As with the ADF test, we consider the t-ratio for $\hat{\alpha}$ from this test equation and evaluate

$$t_\alpha = \frac{\hat{\alpha}}{(se(\hat{\alpha}))} \quad (4)$$

where $\hat{\alpha}$ is the estimate of α, and $se(\hat{\alpha})$ is the coefficient standard error. The null and alternative hypotheses may be written as: H_0: $\alpha = 0$ against H_1: $\alpha < 0$. The test rejects the null hypothesis of unit root if the DFGLS test statistic is less than the test critical values at the designated test sizes (Elliot *et al*., 1996).

Heteroskedasticity test

The Lagrange Multiplier (LM) test due to Engle (1982) has been applied to test for heteroskedasticity or ARCH effect in the residuals of returns. The procedure of performing the Engle's LM test is to first obtain the residuals e_t from an ordinary least squares regression of the conditional mean equation which could be an AR, MA or ARMA model that best fit the data. For instance, in an ARMA (1,1) model, the conditional mean equation is specified as:

$$r_t = \alpha_1 r_{t-1} + \varepsilon_t + \beta_1 \varepsilon_{t-1} \quad (5)$$

where r_t is the return series, $\alpha_1$and β_1 are the coefficients of the AR and MA terms while ε_t is the random error term. Having obtained the residuals e_t, we then regress the squared residuals on a constant and q lags such as in the following equation:

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \alpha_3 e_{t-3}^2 + \cdots + \alpha_q e_{t-q}^2 + v_t \quad (6)$$

The null hypothesis of no ARCH effect up to lag q is then formulated as follows:

H_0: $\alpha_1 = \alpha_2 = \alpha_3 = \cdots = \alpha_q$ versus the alternative H_1: $\alpha_i > 0$ for at least one $i = 1,2,3, \dots, q$.

There are two test statistics for the joint significance of the q-lagged squared residuals. The F-statistic and the number of observations times R-squared (nR^2) from the regression. The F-statistic is estimated as:

$$F = \frac{SSR_0 - SSR_1/q}{SSR_1(n-2q-1)} \quad (7)$$

where

$SSR_1 = \sum_{t=q+1}^{T} e_t^2, SSR_0 = \sum_{t=q+1}^{T} (r_t^2 - \bar{r})^2$

and $\bar{r} = \frac{1}{n}\sum_{t=1}^{T} r_t^2$

$\hat{e}_t$ is the residual obtained from least squares linear regression, $\bar{r}$ is the sample mean of r_t^2. The nR^2 is evaluated against $\chi^2(q)$ distribution with q degrees of freedom under H_0. The decision is to reject the null hypothesis of no ARCH effect in the residuals of returns if the p-values of the F-statistic and nR^2 statistic are less than $\alpha = 0.05$.

Model Specifications

The following models have been specified in this study to capture the time-varying volatility in the crude oil returns:

Autoregressive moving average (ARMA) process

A stochastic process resulting from the combination of autoregressive and moving average models is called an Autoregressive Moving Average (ARMA) model. An ARMA model of order one written ARMA (1,1) is specified as:

$$r_t = \mu + \alpha_1 r_{t-1} + \varepsilon_t + \beta_1 \varepsilon_{t-1} \quad (8)$$

where μ is a constant term, α_1 is the autoregressive parameter, β_1 is the moving average parameter.

The generalized ARCH (GARCH) model

The ARCH model of Engle (1982) was generalized to GARCH model by Bollerslev (1986). A Generalized Autoregressive Conditional Heteroskedasticity process is said to be a GARCH (1, 1) process if:

$$r_t = \mu + \varepsilon_t \quad (9)$$

$$\varepsilon_t = \sigma_t e_t; \quad e_t \sim N(0,1) \quad (10)$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (11)$$

where ε_t^2 is the ARCH term σ_t^2 is the GARCH term. The above model is variance and covariance stationary if the following necessary conditions are satisfied: $\omega > 0$; $\alpha_1 > 0$, $\beta_1 > 0$, and $\alpha_1 + \beta_1 < 1$. Bollerslev *et al*. (1992) showed that basic GARCH (1,1) model is sufficient in capturing all the volatility in any financial time series.

The symmetric ARMA (1,1)-GARCH (2,1) model is expressed as:

$$r_t = \mu + \alpha_1 r_{t-1} + \varepsilon_t + \beta_1 \varepsilon_{t-1} \quad (12)$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_1 \varepsilon_{t-2}^2 + \beta_1 \sigma_{t-1}^2 \quad (13)$$

Equation (12) is the mean equation while Equation (13) is called the conditional variance equation. The ARMA (1,1)-GARCH (2,1) model is stationary if the sum of ARCH and GARCH parameters is less than unity.

The threshold GARCH (TGARCH) model

The Threshold GARCH (TARCH) model was introduced independently by Glosten *et al*. (1993) and Zakoian (1994). This model allows for asymmetric shocks to volatility. The conditional variance for the simple TARCH (1,1) model is defined by:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 d_{t-1} \quad (14)$$

where $d_t = 1$ if ε_t is negative and 0 otherwise. In the TGARCH (1,1) model, volatility tends to increase with bad news ($\varepsilon_{t-1} < 0$) and decreases with good news ($\varepsilon_{t-1} > 0$). Good news has an impact of $\alpha_1$whereas bad news has an impact of $\alpha_1 + \gamma$. If leverage effect parameter $\gamma > 0$ and statistically significant then the leverage effect exists. If $\gamma \neq 0$, the shock is asymmetric, and if $\gamma = 0$, the shock is symmetric. The persistence of shocks to volatility is measured by $\alpha_1 + \beta_1 + \gamma/2$.

The ARMA (1,1)-TARCH (2,1) model is expressed as follows:

$$r_t = \mu + \alpha_1 r_{t-1} + \varepsilon_t + \beta_1 \varepsilon_{t-1} \quad (15)$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_1 \varepsilon_{t-2}^2 + \beta_1 \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 d_{t-1} \quad (16)$$

Model Selection Criteria

To select the best fitting ARMA-GARCH model, Akaike Information Criteria (AIC) due to (Akaike, 1974), Schwarz Information Criterion (SIC) due to (Schwarz, 1978) and Hannan-Quinn Information Criterion (HQC) due to (Hannan, 1980) and Log likelihood are the most commonly used model selection criteria. These criteria are used in this study and are computed as follows:

$$AIC(K) = -2\log \mathrm{L} + 2K \quad (17)$$

$$SIC(K) = -2\log \mathrm{L} + K \log T \quad (18)$$

$$HQC(K) = 2\log[\log \mathrm{T}]\, K - 2\log \mathrm{L} \quad (19)$$

where K is the number of independently estimated parameters in the model, T is the number of observations; L is the maximized value of the Log-Likelihood for the estimated model defined as follows:

$$L = \prod_{t=0}^{n} \left(\frac{1}{2\pi\sigma_t^2}\right)^{1/2} exp\left[-\sum_{t=1}^{n} \frac{(r_t-\mu)^2}{2\sigma_t^2}\right] \quad (20)$$

$$\ln(L) = In\left[\prod_{t=1}^{n} \left(\frac{1}{2\pi\sigma_t^2}\right)^{1/2}\right] - \frac{1}{2}\sum_{t=1}^{n} \frac{(r_t-\mu)^2}{\sigma_t^2} \quad (21)$$

Thus given a set of estimated ARMA-GARCH models for a given set of data, the preferred model is the one with the minimum information criteria and largest log likelihood value.

Estimation of ARMA-GARCH Models and Error Distributions

When modeling returns series for high frequency time series data, the estimates of ARMA-GARCH process are obtained by maximizing the following likelihood function:

$$L\theta_t = -\,{}^{1}\!/_{2} \sum_{t-1}^{T} \left(\ln 2\pi + ln\sigma_t^2 + \frac{\varepsilon_t^2}{\sigma_t^2} \right) \quad (22)$$

The three error distributions are defined as follows:

(1)The normal (Gaussian) distribution is given by:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, -\infty < z < \infty \quad (23)$$

(2) The Student-t distribution is defined as:

$$f(z) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)} \left(1 + \frac{z^2}{v}\right)^{-\left(\frac{v+1}{2}\right)}, -\infty < z < \infty \quad (24)$$

where v denotes the number of degrees of freedom and Γ denotes the Gamma function. The degree of freedom $v > 2$ controls the tail behaviour. The t −distribution approaches the normal distribution as $v \to \infty$.

(3) The Generalized Error Distribution (GED) is given as:

$$f(z, \mu, \sigma, v) = \frac{\sigma^{-1} v e^{\left(-\frac{1}{2}\left|\frac{\left(\frac{z-\mu}{\sigma}\right)}{\lambda}\right|^{v}\right)}}{\lambda 2^{(1+(1/v))}\Gamma\left(\frac{1}{v}\right)}, 1 < z < \infty \quad (25)$$

$v > 0$ is the degrees of freedom or tail -thickness parameter and

$$\lambda = \sqrt{2^{(-2/v)}\Gamma\left(\frac{1}{v}\right) \Big/ \Gamma\left(\frac{3}{v}\right)}$$

If $v = 2$, the GED yields the normal distribution. If $v < 1$, the density function has thicker tails than the normal density function, whereas for $v > 2$ it has thinner tails.

Model Diagnostic Checking

When a time series model such as GARCH models has been fitted to a given data set, it is advisable to check that the model does really give an adequate description of the data. In doing so, we employed Lagrange Multiplier Engle Heteroskedasticity test for ARCH effects earlier discussed in section 3.2.2.

Volatility Mean Reversion and Half-Life

Mean reversion in volatility refers to the tendency of a financial instrument's volatility to revert to its historical average level over time. In stationary GARCH-type models, the volatility mean reverts to its long run level, at a rate given by the sum of ARCH and GARCH coefficients, which is usually close to one (1) for financial time series. The average number of time periods for the volatility to revert to its long run level is measured by the half-life of the volatility shock. The mean reverting form of the basic GARCH (1, 1) model is given by:

$$(\varepsilon_t^2 - \bar{\sigma}^2) = (\alpha_1 + \beta_1)(\varepsilon_{t-1}^2 - \bar{\sigma}^2) + \mu_t + \beta_1\mu_{t-1} \quad (26)$$

where $\bar{\sigma}^2 = \frac{\omega}{(1-\alpha_1-\beta_1)}$, the conditional long-run volatility level and $\mu_t = (\varepsilon_t^2 - \bar{\sigma}^2)$. The magnitude of mean reverting rate $(\alpha_1 + \beta_1)$ controls the speed of mean reversion.

The average number of time periods for the volatility to revert to its long run level is measured by the half-life of the volatility shock. Engle and Bollerslev (1986) defined half-life of volatility as the time taken by the volatility shock to cover half the distance back towards its mean volatility after a deviation from it. Half-life volatility measures the speed of mean reversion (average time) of a stock price or returns. The volatility half-life is computed as

$$H_{Life} = 1 - \frac{\log(2)}{\log(\sum \alpha_i + \beta_i)} \quad (27)$$

RESULTS AND DISCUSSION

Summary Statistics and Normality Measures

The descriptive statistics and normality measures for both daily and monthly crude oil prices and returns are computed and presented in Table 1.

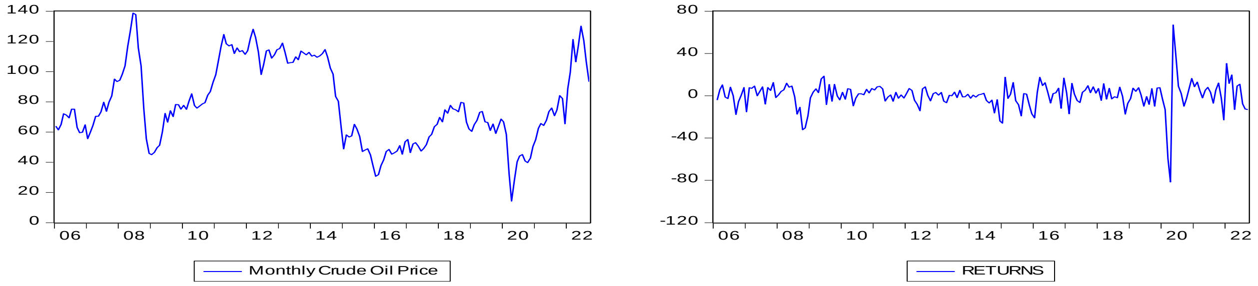
Table 1: Summary Statistics of Crude Oil Prices and Returns

Variable	Monthly		Daily	
	Prices	Returns	Prices	Returns
Mean	78.1723	0.1894	78.6217	0.0072
Median	73.6532	1.3894	74.2536	0.0591
Maximum	138.7400	66.9767	139.413	58.8928
Minimum	14.2800	-81.5898	7.15000	-66.0451
Standard Dev.	26.5285	12.9212	27.9037	3.5176
Skewness	0.2609	-1.1699	0.0854	-2.0811
Kurtosis	2.0874	15.4473	1.8616	114.1962
Jarque-Bera	9.2546	1336.75	165.64	1547224
P-value	0.0099	0.00000	0.00000	0.00000
No. of Obs.	201	200	3000	2999

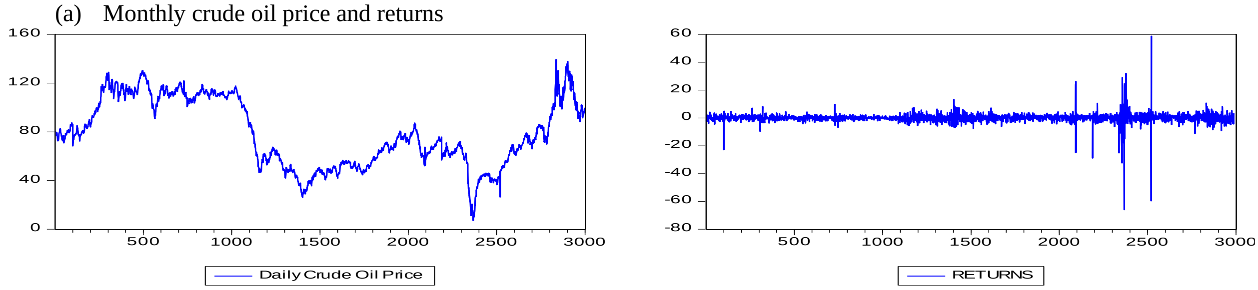
The summary statistics presented in Table 1 for crude oil prices in Nigeria reveal monthly and daily mean values of 78.1723 and 78.6217 US Dollars per barrel, respectively, with corresponding positive means for crude oil returns (0.1894% monthly and 0.0072% daily). These positive means indicate overall gains in both prices and returns during the analyzed period. However, high dispersions from the means are evident, as reflected in the substantial standard deviations for both prices (monthly: 26.5285, daily: 27.9023 US Dollars per barrel) and returns (monthly: 12.9212%, daily: 3.5176%). The wide gaps between the maximum and minimum prices and returns underscore the considerable variability in oil price changes in the Nigerian market, implying high volatility and associated risk. Positive skewness in monthly and daily crude oil prices suggests more frequent price rises than falls, while negative skewness in returns indicates a higher frequency of falls than rises. Additionally, the excess kurtosis and rejection of the normality hypothesis through the Jarque-Bera test emphasize the non-Gaussian nature of crude oil returns in the Nigerian market during the examined period.

Graphical Examination of Crude Oil Prices and Return Series

To examine the characteristics of the series, the raw crude oil price and return data are graphically represented over time, and the resulting time series plots are illustrated in Figure 1.



(a) Monthly crude oil price and returns



(b) Daily crude oil price and returns

Figure 1: Time Plots of Crude Oil Prices and Returns

The time plots of monthly and daily crude oil prices in Nigeria, as depicted in Figure 1 (left), reveal non-smooth trend movements, indicating heteroskedastic means and variances and suggesting non-stationarity in the series. To address this, a transformation to natural log returns is applied. The resulting time series plots of monthly and daily log returns, presented in Figure 1 (right), exhibit a smoother trend, indicating covariance stationarity with varying amplitudes over time. Noteworthy is the observation that large changes in returns are succeeded by similarly large changes, and vice versa for small changes, suggesting a common driving force behind the returns. The presence of both volatility clustering and shock persistence in the crude oil price log returns is evident, signifying frequent changes in oil prices and either constant oil price stability or persistent oil price shocks in the Nigerian economy.

Unit Root and Heteroskedasticity Tests of Returns

To examine the presence of a unit root in the monthly and daily crude oil prices and returns, the Dickey-Fuller Generalized Least Squares Elliott, Rothenberg, and Stock (DF GLS (ERS)) unit root test has been utilized, and the outcomes are outlined in the upper panel of Table 2. Additionally, the Engle's Lagrange Multiplier (LM) test for ARCH effect has been applied to assess heteroskedasticity in the residuals of the monthly and daily crude oil prices and returns series, with the results presented in the lower panel of Table 2.

Table 2: Unit Root and Test and Heteroskedascity Test for ARCH Effects

DF GLS (ERS) Unit Root and Test				
Variable	**Option**	**DF GLS Test Statistic**	**1% Critical Value**	**5% critical value**
Monthly Oil Price	Intercept only	-1.4969	-2.5766	-1.9424
	Intercept & trend	-1.6126	-3.4612	-2.9310
Monthly Oil Returns	Intercept only	-10.6912	-2.5766	-1.9424
	Intercept & trend	-11.1231	-3.4612	-2.9310
Daily Oil Price	Intercept only	-1.5931	-2.5966	-1.9409
	Intercept & trend	-1.5799	-3.4612	-2.8900
Daily Oil Returns	Intercept only	-4.1987	-2.5966	-1.9409
	Intercept & trend	-7.1429	-3.4612	-2.8900

Heteroskedascity Test for ARCH Effects				
Variable	F-Statistic	P-value	nR^2	P-value
Monthly Crude Oil Returns	87.05500	0.0000	59.11943	0.0000
Daily Crude Oil Returns	193.2009	0.0000	180.8883	0.0000

The results of the DF GLS unit root test, as presented in Table 2, indicate that the monthly and daily crude oil prices during the investigated period are non-stationary in levels, evidenced by the DF GLS test statistics surpassing the corresponding critical values at 1% and 5% significance levels. In contrast, the DF GLS unit root test conducted on the monthly and daily crude oil returns series reveals that both the monthly and daily returns are stationary, with the test statistics falling below the critical values at the 1% and 5% significance levels. Consequently, it is inferred that the monthly and daily crude oil prices in Nigeria lack stationarity, while their respective returns exhibit stationarity. Furthermore, the Engle's LM test for ARCH effect, reported in the lower panel of Table 2, strongly rejects the null hypothesis of no ARCH effect in the residuals of crude oil returns. This implies that the monthly and daily crude oil returns in Nigeria, under review, display heteroskedasticity, indicating a time-varying conditional variance, and are best modeled using ARCH or GARCH family models.

Model Order Selection and Error Distribution

The study employs AIC, SIC, HQC, and log likelihood to select the optimal model order and error distribution for both monthly and daily crude oil returns. Lower and upper symmetric, as well as asymmetric GARCH models, are considered for their volatility-capturing capabilities. The model with the lowest information criteria is chosen. Model order and error distribution selection results for monthly and daily crude oil returns are presented in Tables 3 and 4, respectively.

Table 3: Model Order Selection for Symmetric and Asymmetric GARCH Models (Monthly Crude Oil Returns)

Distribution	Model	LogL	AIC	SIC	HQC
Symmetric ARMA-GARCH Model					
Normal	ARMA (1,1)-GARCH (1,1)	-727.38	7.3706	7.4699	7.4108
	ARMA (1,1)-GARCH (1,2)	-744.85	7.5663	7.6721	7.6032
	ARMA (1,1)-GARCH (2,1)	-725.34	7.3602	7.4760	7.4071
	ARMA (1,1)-GARCH (2,2)	-746.81	7.5860	7.7184	7.6396
Sudent's-t	ARMA (1,1)-GARCH (1,1)	-722.84	7.3375	7.4698	7.3919
	ARMA (1,1)-GARCH (1,2)	-734.74	7.4647	7.5971	7.5183
	ARMA (1,1)-GARCH (2,1)	-724.62	7.3630	7.4954	7.4166
	ARMA (1,1)-GARCH (2,2)	-731.21	7.4393	7.5883	7.4996
GED	ARMA (1,1)-GARCH (1,1)	-723.93	7.3459	7.4618	7.3929
	ARMA (1,1)-GARCH (1,2)	-735.90	7.4764	7.6088	7.5299
	ARMA (1,1)-GARCH (2,1)*	**-722.05**	**7.3371**	**7.4695**	**7.3907**
	ARMA (1,1)-GARCH (2,2)	-740.28	7.5304	7.6794	7.5907
Asymmetric ARMA-TARCH Model					
Normal	ARMA (1,1)-TARCH (1,1)	-721.78	7.3244	7.4403	7.3713
	ARMA (1,1)-TARCH (1,2)	-737.19	7.4893	7.6217	7.5429
	ARMA (,1)-TARCH (2,1)	-732.52	7.4424	7.5748	7.4960
	ARMA (1,1)-TARCH (2,2)	-743.50	7.5628	7.7117	7.6231
Student's-t	ARMA (1,1)-TARCH (1,1)	-720.01	7.3167	7.4491	7.3703
	ARMA (1,1)-TARCH (1,2)	-733.36	7.4609	7.6099	7.5212
	ARMA (1,1)-TARCH (2,1)*	**-716.19**	**7.2883**	**7.4373**	**7.3486**
	ARMA (1,1)-TARCH (2,2)	-731.91	7.4563	7.6218	7.5233
GED	ARMA (1,1)-TARCH (1,1)	-720.48	7.3214	7.4538	7.3750
	ARMA (1,1)-TARCH (1,2)	-740.27	7.5304	7.6793	7.5907
	ARMA (1,1)-TARCH (2,1)	-722.52	7.3519	7.5009	7.4123
	ARMA (1,1)-TARCH (2,2)	-732.81	7.3781	7.6012	7.4462

Table 4: Model Order Selection for Symmetric and Asymmetric GARCH Models (Daily Crude Oil Returns)

Distribution	Model	LogL	AIC	SIC	HQC
Symmetric GARCH Model					
Normal	GARCH (1,1)	-6818.61	4.5499	4.5579	4.5528
	GARCH (1,2)	-6889.63	4.5980	4.6080	4.6016
	GARCH (2,1)	-6790.40	4.5318	4.5418	4.5354
	GARCH (2,2)	-6764.68	4.5153	4.5273	4.5196
Student's-t	**GARCH (1,1)***	**-6287.17**	**4.1962**	**4.2062**	**4.1998**
	GARCH (1,2)	-6707.88	4.4774	4.4894	4.4817
	GARCH (2,1)	-6287.11	4.1968	4.2088	4.2011
	GARCH (2,2)	-6882.90	4.5948	4.6088	4.5998
GED	GARCH (1,1)	-6351.03	4.2388	4.2488	4.2424
	GARCH (1,2)	-6350.98	4.2394	4.2514	4.2437
	GARCH (2,1)	-6350.93	4.2394	4.2514	4.2437
	GARCH (2,2)	-6349.78	4.2393	4.2533	4.2443
Asymmetric TARCH Model					
Normal	TARCH (1,1)	-6812.70	4.5466	4.5567	4.5503
	TARCH (1,2)	-6919.73	4.6187	4.6307	4.6230
	TARCH (2,1)	-6788.03	4.5309	4.5429	4.5352
	TARCH (2,2)	-6758.51	4.5118	4.5259	4.5169
Student's-t	**TARCH (1,1)***	**-6281.05**	**4.1928**	**4.2048**	**4.1971**
	TARCH (1,2)	-6723.73	4.4887	4.5027	4.4937
	TARCH (2,1)	-6281.17	4.1934	4.2074	4.1984
	TARCH (2,2)	-6889.43	4.5998	4.6158	4.6056
GED	TARCH (1,1)	-6348.27	4.2376	4.2496	4.2419
	TARCH (1,2)	-6348.26	4.2383	4.2523	4.2433
	TARCH (2,1)	-6347.93	4.2380	4.2521	4.2431
	TARCH (2,2)	-6345.37	4.2370	4.2530	4.2428

The model order and error distribution selection results in Tables 3 and 4 reveal three considered error distributions (normal, student-t, and generalized error distribution) for both monthly and daily crude oil return series. For monthly crude oil returns, the chosen models are the symmetric ARMA (1,1)-GARCH (2,1) model with GED and the asymmetric ARMA (1,1)-GARCH (2,1) model with student's-t distribution. These selections are based on minimizing information criteria and maximizing log likelihoods.

For daily crude oil returns, the selected models are the symmetric GARCH (1,1) model with student's-t distribution (STD) and the asymmetric TARCH (1,1) model with student's-t distributions. These choices are determined by minimizing information criteria and maximizing log likelihoods. The error distributions selections, which include only heavy-tailed distributions (student-t and generalized error distribution), indicate that the Nigerian crude oil return series exhibit fat-tailed characteristics in their volatility modeling.

Results of Parameter Estimation of Volatility Models

The study employs symmetric ARMA-GARCH models for monthly returns and symmetric GARCH models for daily returns to investigate their symmetric features, while asymmetric ARMA-TARCH models for monthly returns and asymmetric TARCH models for daily returns are employed to examine the asymmetric and leverage effects properties of the monthly and daily crude oil returns. The results of these analyses are presented in Tables 5 and 6, respectively.

Table 5: Parameter Estimates for Symmetric GARCH Models

Symmetric ARMA (1,1)-GARCH (2,1) Model for Monthly Crude Oil Returns				
		Mean Equation		
Variable	Coefficient	Std. Error	z-Statistic	P-value
μ	1.439279	0.720866	1.996596	0.0459
AR(1)	0.197477	0.069755	2.831009	0.0028
MA(1)	0.627840	0.189352	3.315729	0.0000
		Variance Equation		
ω	2.502873	0.412145	6.072797	0.0000
α_1	0.095177	0.020304	4.687599	0.0000
α_2	0.224397	0.110640	2.028181	0.0425
β_1	0.596368	0.186026	3.205837	0.0013
v	1.412880	0.188055	7.513136	0.0000
$\alpha_1+\alpha_2+\beta_1$	0.915942			
Symmetric GARCH (1,1) Model for Daily Crude Oil Returns				
		Mean Equation		
μ	0.065681	0.028469	2.307072	0.0211
		Variance Equation		
ω	0.248337	0.040713	6.099675	0.0000
α_1	0.178130	0.021423	8.315063	0.0000
β_1	0.793226	0.017648	44.94737	0.0000
v	4.254656	0.250423	16.98989	0.0000
$\alpha_1+\beta_1$	0.971356			

Table 6: Parameter Estimates for Asymmetric TARCH Models

Asymmetric ARMA (1,1)-TARCH (2,1) Model for Monthly Crude Oil Returns				
		Mean Equation		
Variable	Coefficient	Std. Error	z-Statistic	P-value
μ	0.638457	0.773785	0.825109	0.4093
AR(1)	-0.613279	0.294270	-2.084069	0.0420
MA(1)	0.245402	0.069165	3.548066	0.0000
		Variance Equation		
ω	4.698682	1.112250	4.224484	0.0009
α_1	0.232651	0.065452	3.554529	0.0000
γ	0.941848	0.352761	2.669933	0.0076
α_2	0.186773	0.085002	2.197292	0.0280
β_1	0.482110	0.057437	8.393718	0.0000
v	8.379826	1.828056	4.583508	0.0000
$\alpha_1+\alpha_2+\beta_1$	0.901534			
Asymmetric TARCH (1,1) Model for Daily Crude Oil Returns				
		Mean Equation		
μ	0.053305	0.028699	1.857389	0.0633
		Variance Equation		
ω	0.246576	0.038608	6.386622	0.0000
α_1	0.115543	0.023311	4.956665	0.0000
γ	0.111970	0.033128	3.379898	0.0007
β_1	0.796341	0.016915	47.07919	0.0000
v	4.307197	0.250691	17.18129	0.0000
$\alpha_1+\beta_1$	0.911884			

The volatility estimates, as shown in Table 5, outline the coefficients of both the mean and conditional variance equations for the symmetric ARMA (1,1)-GARCH (2,1) and GARCH (1,1) models applied to monthly and daily crude oil returns, respectively. The results of the mean equation reveal a positive and statistically significant relationship between the intercept (μ) and monthly crude oil log returns at a 5% significance level, implying that the predicted value of monthly crude oil log returns will be approximately 1.4% when other variables are held constant. The AR and MA slope coefficients are also statistically significant at 5% significance levels, satisfying the stationarity condition with the sum of AR and MA terms being less than unity.

In the conditional variance equations, all estimated parameters are highly statistically significant at 1% marginal significance levels, meeting the non-negativity restrictions of the models. The significance of ARCH parameters (α_i) indicates that past volatilities have explanatory power on current volatilities, suggesting volatility clustering in both monthly and daily returns. In the same way, the statistical significance of the GARCH parameters (β_i) does not only indicate that news about volatilities from previous periods have explanatory powers on current volatilities but also suggest volatility clustering in the monthly and daily returns of the crude oil series. The conditional variance equations for both models exhibit mean-reverting stability, indicating stationary and predictable variance processes. However, the high volatility persistence coefficients suggest slow decay of conditional variance due to the effects of volatility shocks.

The results of the asymmetric ARMA (1,1)-TARCH (1,1) model for monthly crude oil returns and TARCH (1,1) model for daily crude oil returns, as presented in Table 6, indicate that all parameters in the variance equations are statistically significant at 5% levels. The significance of the ARCH and GARCH terms implies that past squared error terms significantly influence volatility, and previous volatility of crude oil returns affects current volatilities. The models are stationary, with the sums of ARCH and GARCH terms being less than unity, indicating persistent conditional variances and stable volatility shocks, making crude oil log returns predictable in the market. The positive and statistically significant values of the leverage effect parameter (γ) in the asymmetric models provide evidence for the presence of asymmetry and leverage effects in both monthly and daily crude oil returns in Nigeria. This suggests that negative shocks increase volatility more than positive shocks of the same magnitude, confirming empirical evidence for asymmetry and leverage effects.

In estimating GARCH family models with heavy-tailed distributions, such as the student's-t distribution (STD), the shape parameter (ν) needs to be greater than 2 for fat-tailed distributions. Conversely, when estimating GARCH models with the generalized error distribution (GED), the shape parameter (ν) needs to be less than 2 for fat-tailed distributions. The results in Tables 5 and 6 reveal that the shape parameter ($\nu > 2$) for all GARCH models estimated with STD, indicating fat-tailed distributions, while ($\nu < 2$) for all GARCH models estimated using GED, signifying leptokurtic characteristics in the crude oil returns during the investigated period.

Model Diagnostic Checking

To validate the estimated volatility models for both monthly and daily crude oil returns, we employed the Engle's LM test and the results are presented in Table 7.

Table 7: Heteroskedasticity Test for ARCH Effects for the Estimated Models

Model	F-statistic	P-value	nR^2	P-value
Monthly Crude Oil Returns				
ARMA (1,1)-GARCH (2,1)	0.395857	0.5300	0.399091	0.5276
ARMA (1,1)-TARCH (2,1)	0.050530	0.8224	0.051033	0.8213
Daily Crude Oil Returns				
GARCH (1,1)	0.026781	0.8700	0.026799	0.8700
TARCH (1,1)	0.005584	0.9404	0.005588	0.9404

The results of the heteroskedasticity test for ARCH effects, presented in Table 7 for the GARCH (1,1), ARMA (1,1)-GARCH (2,1), TARCH (1,1), and ARMA (1,1)-TARCH (2,1) models applied to both monthly and daily crude oil returns, demonstrate that the GARCH family models effectively capture all the ARCH effects in the residuals of the crude oil return series. This conclusion is supported by the highly statistically insignificant p-values of the ARCH LM test statistics. The findings suggest that the estimated GARCH-type models are robust, suitable, valid, and accurate in characterizing the volatility of crude oil returns in Nigeria.

Volatility Mean Reversion and Half-Life

Two tests were conducted to assess mean reversion in volatility for the monthly and daily crude oil return series. The first test utilized the DF GLS (ERS) unit root test, reported in Table 2, indicating that the under-review series are stationary, implying mean-reverting behaviour, as stationary series eventually revert to their long-run averages. The second test employed symmetric ARMA-GARCH models for monthly returns and symmetric GARCH models for daily crude oil returns. In a stationary ARMA (1,1)-GARCH (1,1) and GARCH (1,1) models, the volatility mean reversion rate is represented by the sum ($\alpha_1 + \beta_1$), typically close to unity for financial data. The estimates of mean reversion rates and volatility half-lives for both monthly and daily crude oil returns are detailed in Table 8. The results affirm the mean-reverting nature of volatility in the crude oil return series, providing insights into the time it takes for volatility to revert to its long-run average.

Table 8: Results of Volatility Half-Lives from Symmetric GARCH Models

	Log(2)	$\alpha_i + \beta_i$	$\log(\alpha_i + \beta_i)$	$\frac{\log(2)}{\log(\alpha_i + \beta_i)}$	$1 - \frac{\log(2)}{\log(\alpha_i + \beta_i)}$
Monthly Crude Oil Returns					
ARMA-GARCH	0.30103	0.915942	-0.03813	-7.89441	8.894414
ARMA-TARCH	0.30103	0.901534	-0.04502	-6.6869	7.686897
Daily Crude Oil Returns					
GARCH (1,1)	0.30103	0.971358	-0.01262	-23.8521	24.85212
TARCH (1,1)	0.30103	0.911881	-0.04006	-7.51413	8.514134

The volatility half-life, indicating the average time for volatility shocks to decrease by half to their original values, is examined in this study. The results, as presented in Table 8, reveal that monthly crude oil returns exhibit volatility half-lives of around 9 months and 8 months when modeled by symmetric ARMA (1,1)-GARCH (2,1) and asymmetric ARMA (1,1)-TARCH (2,1) models, respectively. Daily crude oil returns, when modeled by symmetric GARCH (1,1) and asymmetric TARCH (1,1) models, demonstrate volatility half-lives of approximately 25 days and 9 days, respectively. Both monthly and daily crude oil returns, under various volatility models, exhibit mean-reverting behaviour, implying a return to their long-run average values. This characteristic of mean reversion in oil prices and stocks presents favourable short-term investment opportunities for both local and foreign investors.

CONCLUSION

This study investigates the symmetric and asymmetric properties, as well as shock persistence in Nigerian crude oil returns, utilizing monthly and daily crude oil prices from the Central Bank of Nigeria (CBN) spanning from January 2006 to September 2022 and from November 3, 2009, to November 4, 2022, respectively. Employing descriptive statistics, normality measures, time plots, and Dickey-Fuller Generalized Least Squares unit root tests, the study explores distributional and stationarity properties. Heteroskedasticity is modeled using various specifications of symmetric ARMA-GARCH and asymmetric ARMA-TARCH models, with model selection based on information criteria. The findings reveal non-Gaussian distributions for both monthly and daily crude oil prices and returns, non-stationary prices, and weak or covariance stationarity in log returns. The presence of ARCH effects in log returns indicates heteroskedasticity. Volatility clustering, high shock persistence, stationarity, mean-reverting, and predictable behaviour are observed in symmetric models. Asymmetric models reveal asymmetry and leverage effects, suggesting that negative shocks induce more volatility than positive shocks of the same magnitude. The study recommends measures to reduce crude oil price volatility, the use of alternative heavy-tailed error distributions in modeling crude oil price volatility in Nigeria, and highlights investment opportunities in mean-reverting oil prices for long-term traders.

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