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MATHEMATICAL ANALYSIS OF RAYLEIGH BEAM WITH DAMPING COEFFICIENT SUBJECTED TO MOVING LOAD

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ABSTRACT

In this paper, the mathematical analysis of Rayleigh beam with damping coefficient subjected to moving load is investigated. The governing partial differential equation of order four was reduced to an ordinary differential equation using series solution. Numerical result was presented and it is found that the dynamic response of the beam increases as the length of the mass increases, the same result is also found for the length of the beam and the mass of the load but the dynamic response of the beam decreases as the length of the load. It also reduces as the speed at which the load moves increases. Also, the dynamic response of the beam is not affected by the damping coefficient.

Keywords: Dynamic Response, Damping coefficient, Rayleigh Beam, Series Solution, Moving Load.

INTRODUCTION

The study of beams has been a trending research for some time now. The study of beam practically deals with the bearing of loads. A beam is a structured member used for carrying loads. It is typically used for resisting vertical loads, shear forces and bending moments. It is also a component that is designed to support transverse loads (loads that act perpendicular to longitudinal axis of the beam). Nguyen *et al.*(2011) beam supports loads by bending only. Moving loads have a great effect on the bodies or structures over which they travel. It causes them to vibrate intensively, especially at high velocities. Modern means of transportation are very fast and heavier, while the structure they are moving over are slender and lighter. That is why the dynamic stresses they produce are larger than the static one. Load is the quantity that can be carried at one time by a specified means. Load also refers to the mass or weight that is well supported. It is the forces to which a structure is subjected due to superposed weight or to wind pressure on the vertical surfaces. Moving loads cause solid bodies to vibrate intensively, particularly at high temperatures. Thus, the study of the response of bodies subjected to moving loads has been of concern to several researchers. Emem *et al.*, Jang and Bert[2014, 1989] investigated the dynamic behavior of an elastic isotropic rectangular plate under travelling distributed loads. Their study centers on the flexural vibrations of a simply supported rectangular plate under travelling distributed loads. Both gravity and inertia effects of the distributed loads are taken into consideration. The solution technique is based on the two-dimensional finite Fourier Sine integral transformations and a modification of the Struble's asymptotic technique Chen, W.Q. Lu, C.F and Bian Z.G (2004). The closed form solutions are obtained and numerical analyses in plotted curves are presented. Results show that as the foundation moduli K and rotatory inertia correction factor R_0 increase, the response amplitudes of the dynamical system decrease Lu and Law (2009). Analyses further show that for the same natural frequency, the critical speed for the moving distributed mass problem is smaller than that of the moving distributed force problem. Hence resonance is reached earlier in the moving distributed mass problem Liu and Gurram (2009). Furthermore, it is clearly seen that the response amplitude of the moving distributed mass system is higher than that of the moving distributed force system for fixed values of rotatory inertia correction factor and foundation moduli Adamek (2008). Thus for the simply supported moving distributed load problem, it is established that the moving distributed force solution is not an upper bound for an accurate solution of the moving distributed mass problem. For the two-dimensional plate problem, the solution techniques is based on the double Fourier Finite Sine integral transformation, the expansion of the Dirac Delta function in series form, a modification of Struble's asymptotic method and the use of Fresnel sine and Fresnel cosine integrals Lee *et al.* (1994). Numerical analyses in plotted curves are presented. The analyses reveal interesting results on the effect of structural parameters such as foundation moduli, rotatory inertia correction factor and prestressing forces on the dynamic behavior of isotropic rectangular plate under the actions of concentrated masses moving at variable velocity. Jaworski *et al.* (2008) in particular it is found that the critical velocity of the travelling load which brings about the occurrence of a resonance state increases as the values of these structural parameters increase. Laura (1983) investigated the Response of a Prestressed Beam-type Structure subjected to Partially Distributed Load Moving at Non-Uniform Velocities. The transverse vibration of a prismatic Rayleigh beam resting on bi-parametric Vlasov foundation and continuously acted upon by partially distributed masses moving at varying velocities Lujuin *et al.*(2018). The solution of the fourth order partial differential equation with singular and variable coefficients, use is made of the technique based on the Generalized Finite Fourier Integral Transform, Struble's asymptotic technique and the use of Fresnel sine and cosine identities. Bapat (1987) Numerical results in plotted curves are presented. The results show that the response amplitude of the beam traversed by a distributed load moving with variable velocity decrease with an increase in the value of foundation modulus, Other structural parameters such as axial force, rotatory inertia and shear modulus are also found to reduce the displacement response of the beam as their values are increased in the dynamical system. The results also show that the critical speed for the system traversed by a moving distributed force is found to be greater than that traversed by moving mass Arched Mehmood (2015). This confirms that the inertia effect of the moving distributed load must be considered for accurate and safe assessment of the response to moving distributed load of elastic structural members. Also, Emem *et al* worked on the flexural motions under a transversing partially distributed load of a uniform Rayleigh beam with general boundary conditions. Firouz-Abadi, *et al.* (2007) the dynamic analysis of a uniform Rayleigh beam resting on Winkler-type foundation and under uniform distributed moving masses is investigated in this paper Kacar (2014), Kim *et al.* (2001). A procedure involving the generalized integral transformation with beam function as kernel, the use of properties of Heaviside function to express it in series form and a modification of the Struble's asymptotic technique was used to obtain an analytical solution valid for all variants of classical boundary conditions to the dynamical problem. Yeih *et al.*(1999) the analytical solution and numerical analysis show that the critical speed for the moving distributed mass problem is reached earlier than that of the moving distributed force problem for both illustration examples considered. The results further show that an upward variations of rotatory inertia correction factor and foundation stiffness decrease the response amplitudes of the uniform Rayleigh beam whether the beam is traversed by moving distributed force or moving distributed mass Rao and Naidu (1983).

Nguyen (2017) investigated Comparative Spectral Analysis of Flexible Structure Models: the Euler-Bernoulli Beam model, the Rayleigh Beam model, and the Timoshenko Beam Model. The approximate spectra for three different models of transversely vibrating beams is derived. Each model consists of a system of partial differential equations (PDEs) with various boundary conditions. The three models that we consider are the Euler-Bernoulli model, the Rayleigh model, and the Timoshenko model. Lastly, the asymptotic approximations of some of the various spectral equations found from each model is presented Amiri and Onayango (2010).

Ratnadeep (2015) investigated Dynamic Response of Uniform Rayleigh Beams on Variable Biparametric Elastic Foundation under Partially Distributed Loads. Paul F. Doyle and Milija N. Pavlovic (1982) presents the dynamic response of pre-stressed Rayleigh beam resting on variable bi-parametric elastic foundation under moving distributed masses. The system is governed by fourth order partial differential equation with variable and singular coefficients. De Rosa (1994) aim of the study was to obtain the dynamic deflections of the bi-parametric elastic subgrade having shear layer under moving distributed force and moving distributed mass, respectively. Generalized Galerkin Method (GGM) was employed to reduce the governing equation to second order ordinary differential equations and a modification of Struble's asymptotic technique was used to solve the reduced equations BalaSubramanian and Subramanian (1985). From the obtained results, it was observed that the deflection profile of moving distributed mass was higher than the moving distributed force for the boundary conditions considered in this new study. From this study, the moving distributed force is not a safe approximation to the moving distributed mass problem. Thus, safety not guaranteed for a design based on the moving distributed force solution Hsu *et al.*(2008). Wang and Lin(1996) investigated Isospectrals of non-uniform Rayleigh beams with respect to their uniform counterparts. In this paper, the non-uniform Rayleigh beams isospectral to a given uniform Rayleigh beam is found. Isospectral systems are those that have the same spectral properties, i.e. the same free vibration natural frequencies for a given boundary condition. Maurizi *et al.* (1976) proposed the fourth-order governing differential equation of non-uniform Rayleigh beam into a uniform Rayleigh beam. If the coefficients of the transformed equation match with those of the uniform beam equation, then the non-uniform beam is isospectral to the given uniform beam De Rosa (1982). The boundary-condition configuration should be preserved under this transformation. P.B. Ojih et al investigated the dynamic response under moving concentrated loads of uniform Rayleigh beam resting on Pasternak foundation. The dynamic response under moving concentrated masses on Uniform Rayleigh beam resting on Pasternak foundation, with simply supported boundary condition is investigated in this paper Bert *et al.* (1989). The governing equation is a fourth order partial differential equation, a technique based on the generalized integral transform (GIT) is used to reduce the governing equation to a sequence of second order differential equation Zhou (2013).

MATHEMATICAL FORMULATION

Consider a non-prismatic Rayleigh beam with damping coefficient of length L and transverse by uniform partially distributed moving mass. The resulting vibrational behavior of this system is described by the following partial differential equation:

$$\frac{\partial^2}{\partial x^2}\left(EI\left(\frac{\partial^2 V(x,t)}{\partial x^2}\right)\right)+\mathcal{N}\frac{\partial^2 V(x,t)}{\partial t^2}-\mathcal{N}R_0\frac{\partial^4 V(x,t)}{\partial x^2\partial t^2}=p(x,t) \quad (1)$$

where

$$\mathrm{P(x,t)}=\left(\frac{1}{\epsilon}\right)\left[-Mg-M\frac{d^2v}{dt^2}\right]\left[H\left(x-\epsilon+\frac{\epsilon}{2}\right)-H\left(x-\epsilon-\frac{\epsilon}{2}\right)\right] \quad (2)$$

$\frac{d^2}{dt^2}$ is the acceleration operator defined as follows $\frac{d^2}{dt^2}=\frac{d^2}{dt^2}+2v\frac{d^2}{dxdt}+v^2\frac{d^2}{dx^2}$

Where

$\frac{d}{dx}H(x) = \delta(x)$

V	=	is the transverse displacement
M	=	is the mass of the beam
x	=	spatial coordinate
t	=	Time
E	=	Young's modulus
I	=	moment of inertia
m	=	Mass per unit length of the beam
$H(x)$	=	Heaviside function

R_0 is the measure of rotating inertia With the boundary conditions

$$V(0,t)=0=V(l,t) \quad (3)$$

$$\frac{\partial^2 V(0,t)}{\partial x^2}=0=\frac{\partial^2 V(l,t)}{\partial x^2} \quad (4)$$

Without loss of generality, one can consider the initial conditions of the form

$$V(x,0)=0=\frac{\partial V(x,0)}{\partial t} \quad (5)$$

Method of Solution

In this section, in order to compute the response of the Rayleigh beam due to the moving load the non-homogeneous partial differential equation of order four is solved by using a series solution method defined by $V(x,t)=\sum_{j=1}^{\infty} sin\frac{j\pi x}{L}T_j(t)$

$$EI(\pi/L)^4\sum_{j=1}^{\infty}j^4\sin\frac{j\pi x}{L}T_j(t)+\mathcal{N}\sum_{j=1}^{\infty}\sin\frac{j\pi x}{L}\ddot{T}_j(t)$$
$$-\mathcal{N}R_0\left(-(\pi/L)^2\right)\sum_{j=1}^{\infty}j^2 sin\frac{j\pi x}{L}\ddot{T}_j(t)$$
$$\sum_{j=1}^{\infty}\sin\frac{j\pi x}{L}T_j(t)=\left(\frac{1}{\epsilon}\right)\left(-Mg-M\left(\frac{\partial^2 V}{\partial t^2}\right)\left(H\left(x-\epsilon+\frac{\epsilon}{2}\right)-H\left(x-\epsilon-\frac{\epsilon}{2}\right)\right)\right)$$
$$-M\sum_{j=1}^{\infty}\ddot{T}_j(t)\sin\frac{j\pi x}{L}\left[H\left(x-\epsilon+\frac{\epsilon}{2}\right)-H\left(x-\epsilon-\frac{\epsilon}{2}\right)\right]-2MV\sum_{j=1}^{\infty}\dot{T}_j(t)j\pi/L$$
$$\cos\frac{j\pi x}{L}\left[H\left(x-\epsilon+\frac{\epsilon}{2}\right)-H\left(x-\epsilon-\frac{\epsilon}{2}\right)\right]-MV^2\frac{\pi^2}{L^2}\sum_{j=1}^{\infty}j^2T_j(t)\sin\frac{j\pi x}{L}$$
$$\left[H\left(x-\epsilon+\frac{\epsilon}{2}\right)-H\left(x-\epsilon-\frac{\epsilon}{2}\right)\right] \quad (6)$$

Now multiplying equation 3.8 by $\sin\frac{j\pi x}{L}$ and integrating the resulting expression with respect to x over an interval (0,*L*), we have

$$\sum_{j=1}^{\infty}\sin\frac{j\pi x}{L}\sin\frac{i\pi x}{L}T_j(t)=\frac{1}{\mathcal{E}}\{-Mg\sin\frac{i\pi x}{L}H\left(x-\epsilon+\frac{\epsilon}{2}\right)-H\left(x-\epsilon-\frac{\epsilon}{2}\right)\}$$
$$-M\sum_{j=1}^{\infty}\ddot{T}_j(t)\sin\frac{j\pi x}{L}\sin\frac{i\pi x}{L}\{H\left(x-\epsilon+\frac{\epsilon}{2}\right)-H\left(x-\epsilon-\frac{\epsilon}{2}\right)\}-2MV\sum_{j=1}^{\infty}\dot{T}_j(t)j\frac{\pi}{L}\sin\frac{i\pi x}{L}$$
$$\cos\frac{j\pi x}{L}\{H\left(x-\epsilon+\frac{\epsilon}{2}\right)-H\left(x-\epsilon-\frac{\epsilon}{2}\right)\}-MV^2\frac{\pi^2}{L^2}\sum_{j=1}^{\infty}j^2\ddot{T}_j(t)\sin\frac{j\pi x}{L}\sin\frac{i\pi x}{L}\left[H\left(x-\epsilon+\frac{\epsilon}{2}\right)-H\left(x-\epsilon-\frac{\epsilon}{2}\right)\right]$$

i.e

$C15 = C11 + C12 + C13 + C14$ (7)

where

$$C_{11}=\frac{-Mg}{\mathcal{E}}\int\sin\frac{i\pi x}{L}\left(H\left(x-\epsilon+\frac{\epsilon}{2}\right)-H\left(x-\epsilon-\frac{\epsilon}{2}\right)\right)dx \quad (8)$$

$$C_{12}=\frac{-M}{\mathcal{E}}\sum_{j=1}^{\infty}\ddot{T}_j(t)\int\sin\frac{i\pi x}{L}\sin\frac{j\pi x}{L}\left(H\left(x-\epsilon+\frac{\epsilon}{2}\right)-H\left(x-\epsilon-\frac{\epsilon}{2}\right)\right)dx \quad (9)$$

$$C_{13}=\frac{-2MV}{\mathcal{E}}\sum_{j=1}^{\infty}\dot{T}_j(t)j\frac{\pi}{L}\int\sin\frac{i\pi x}{L}\cos\frac{j\pi x}{L}\left(H\left(x-\epsilon+\frac{\epsilon}{2}\right)-H\left(x-\epsilon-\frac{\epsilon}{2}\right)\right)dx \quad (10)$$

$$C_{14}=\frac{MV^2}{\mathcal{E}}\frac{\pi^2}{L^2}\sum_{j=1}^{\infty}j^2\ddot{T}_j(t)\int\sin\frac{i\pi x}{L}\sin\frac{j\pi x}{L}\left(H\left(x-\epsilon+\frac{\epsilon}{2}\right)-H\left(x-\epsilon-\frac{\epsilon}{2}\right)\right)dx \quad (11)$$

and by orthogonality condition

$$C_{15}=\sum_{j=1}^{\infty}T_{fj}(t)\int_0^1\sin\frac{i\pi x}{L}\sin\frac{j\pi x}{L}dx \quad (12)$$

Evaluating the integrals in equation 6 - 10, we have

$$C_{11}=\frac{-MgL}{j\pi\epsilon}\sin\frac{j\pi\epsilon}{L}\sin\frac{i\pi\epsilon}{2L} \quad (13)$$

$$C_{12}=\frac{-2M}{\pi\epsilon}\sum_{j=1}^{\infty}\ddot{T}_j(t)\{\frac{1}{j-1}\{\sin\frac{\pi\epsilon(j-i)}{2L}\cos\frac{\pi\epsilon(j-i)}{L}-\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}\}$$
$$C_{13}=\frac{-2MV}{\mathcal{E}}\sum_{j=1}^{\infty}\dot{T}_j(t)j\{\sin\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}+\frac{i}{i-j}\sin\frac{\pi\epsilon(j-i)}{L}\sin\frac{\pi\epsilon(j-i)}{2L}\} \quad (14)$$

$$C_{14}=\frac{-\pi^2}{L^2}\frac{MV^2}{\mathcal{E}}\sum_{j=1}^{\infty}j^2T_j(t)\frac{1}{i+j}\{\sin\frac{\pi\epsilon(j+i)}{L}\cos\frac{\pi\epsilon(j+i)}{2L}+\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}$$

and

$C_{15} = T_{fj}(t)$.......

Combining equations 11 -15, we finally obtained

$$T_{fj}(t)=\frac{-2MgL}{j\pi\epsilon}\sin\frac{j\pi\epsilon}{L}\sin\frac{i\pi\epsilon}{2L} \quad (15)$$
$$-\frac{2M}{\pi\epsilon}\sum_{j=1}^{\infty}\ddot{T}_j(t)\{\frac{1}{j-i}\{\sin\frac{\pi\epsilon(j-i)}{2L}\cos\frac{\pi\epsilon(j-i)}{L}\}$$
$$-\frac{1}{j+1}\{\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}\}$$

$$\frac{-2MV}{\mathcal{E}}\sum_{j=1}^{\infty}\dot{T}_j(t)\,j\{\sin\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}$$
$$+\frac{i}{j-i}\sin\frac{\pi\epsilon(j-i)}{L}\sin\frac{\pi\epsilon(j-i)}{2L}\}$$
$$+\frac{MV^2\pi^2}{\mathcal{E}L^2}\sum_{j=1}^{\infty}j^2T_j(t)\frac{1}{i+j}\{\sin\frac{\pi\epsilon(j+i)}{L}\cos\frac{\pi\epsilon(j+i)}{2L}$$
$$+\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}$$
$$EI(\pi/L)^4\sum_{j=1}^{\infty}j^4\sin\frac{j\pi x}{L}T_j(t)+\mathcal{N}\sum_{j=1}^{\infty}\sin\frac{j\pi x}{L}\ddot{T}_j(t) \tag{16}$$
$$-\mathcal{N}R_0\left(-(\pi/L)^2\right)\sum_{j=1}^{\infty}j^2sin\frac{j\pi x}{L}\ddot{T}_j(t)=\sum_{j=1}^{\infty}\sin\frac{j\pi x}{L}T_{fj}(t)$$

where

$$\sum_{j=1}^{\infty}\sin\frac{j\pi x}{L}T_{fj}(t)=EI(\pi/L)^4\sum_{j=1}^{\infty}j^4\sin\frac{j\pi x}{L}T_j(t)+\mathcal{N}\sum_{j=1}^{\infty}\sin\frac{j\pi x}{L}\ddot{T}_j(t)$$
$$-\mathcal{N}R_0\left(-(\pi/L)^2\right)\sum_{j=1}^{\infty}j^2sin\frac{j\pi x}{L}\ddot{T}_j(t) \tag{17}$$

Numerical Analysis

The numerical method alluded to is the central difference technique applying the central difference formula to the derivative in equation (17), we obtain

$$\ddot{T}_j(t)=\frac{T_{j+1}-2T_j+T_{j-1}}{h^2} \tag{18}$$

Substituting equation (18) into equation (17) ,

$$\sum_{j=1}^{\infty}\sin\frac{j\pi x}{L}\Bigg[\frac{-2MgL}{j\pi\epsilon}\sin\frac{j\pi\epsilon}{L}\sin\frac{i\pi\epsilon}{2L}$$
$$-\frac{2M}{\pi\epsilon}\sum_{j=1}^{\infty}\ddot{T}_j(t)\{\frac{1}{j-i}\{\sin\frac{\pi\epsilon(j-i)}{2L}\cos\frac{\pi\epsilon(j-i)}{L}\}$$
$$-\frac{1}{j+1}\{\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}\}$$
$$\frac{-2MV}{\mathcal{E}}\sum_{j=1}^{\infty}\dot{T}_j(t)\,j\{\sin\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}$$
$$+\frac{i}{j-i}\sin\frac{\pi\epsilon(j-i)}{L}\sin\frac{\pi\epsilon(j-i)}{2L}\}$$
$$+\frac{MV^2\pi^2}{\mathcal{E}L^2}\sum_{j=1}^{\infty}j^2T_j(t)\frac{1}{i+j}\{\sin\frac{\pi\epsilon(j+i)}{L}\cos\frac{\pi\epsilon(j+i)}{2L}$$
$$+\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}=0 \quad (19)$$

$$\sum_{j=1}^{\infty}EI(\pi/L)^4j^4\sin\frac{j\pi x}{L}T_j(t)+\mathcal{N}\sin\frac{j\pi x}{L}\ddot{T}_j(t)$$
$$-\mathcal{N}R_0\left(-(\pi/L)^2\right)j^2sin\frac{j\pi x}{L}\ddot{T}_j(t)-\sin\frac{j\pi x}{L}\Bigg[\frac{-2MgL}{j\pi\epsilon}\sin\frac{j\pi\epsilon}{L}\sin\frac{i\pi\epsilon}{2L}$$
$$-\frac{2M}{\pi\epsilon}\ddot{T}_j(t)\{\frac{1}{j-i}\{\sin\frac{\pi\epsilon(j-i)}{2L}\cos\frac{\pi\epsilon(j-i)}{L}\}$$
$$+\frac{1}{j+1}\{\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}\}$$

$$+\frac{2MV}{\mathcal{E}}\dot{T}_j(t)\,j\{\sin\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}$$
$$+\frac{i}{j-i}\sin\frac{\pi\epsilon(j-i)}{L}\sin\frac{\pi\epsilon(j-i)}{2L}\}$$
$$+\frac{MV^2\pi^2}{\mathcal{E}L^2}j^2T_j(t)\frac{1}{i+j}\{\sin\frac{\pi\epsilon(j+i)}{L}\cos\frac{\pi\epsilon(j+i)}{2L}$$
$$+\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}=0 \quad (20)$$

$$\sum_{j=1}^{\infty}\sin\frac{j\pi x}{L}EI(\pi/L)^4j^4T_j(t)+\mathcal{N}\ddot{T}_j(t)$$
$$-\mathcal{N}R_0\left(-(\pi/L)^2\right)j^2\ddot{T}_j(t)+\left[\frac{2MgL}{j\pi\epsilon}\sin\frac{j\pi\epsilon}{L}\sin\frac{i\pi\epsilon}{2L}\right.$$
$$+\frac{2M}{\pi\epsilon}\ddot{T}_j(t)\{\frac{1}{j-i}\{\sin\frac{\pi\epsilon(j-i)}{2L}\cos\frac{\pi\epsilon(j-i)}{L}\}$$
$$+\frac{1}{j+1}\{\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}\}$$
$$+\frac{2MV}{\mathcal{E}}\dot{T}_f(t)\,j\{\sin\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}$$
$$+\frac{i}{j-i}\sin\frac{\pi\epsilon(j-i)}{L}\sin\frac{\pi\epsilon(j-i)}{2L}\}$$
$$-\frac{MV^2\pi^2}{\mathcal{E}L^2}j^2T_j(t)\frac{1}{i+j}\{\sin\frac{\pi\epsilon(j+i)}{L}\cos\frac{\pi\epsilon(j+i)}{2L}$$
$$+\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}=0 \quad (21)$$

$\sum_{j=1}^{\infty}\sin\frac{j\pi x}{L}$= 0 provided that

$EI(\pi/L)^4j^4T_j(t)+\mathrm{N}\ddot{T}_j(t)$

$$-\mathcal{N}R_0\left(-(\pi/L)^2\right)j^2\ddot{T}_j(t)=-\frac{2MgL}{j\pi\epsilon}\sin\frac{j\pi\epsilon}{L}\sin\frac{i\pi\epsilon}{2L}$$
$$-\frac{2M}{\pi\epsilon}\ddot{T}_j(t)\{\frac{1}{j-i}\{\sin\frac{\pi\epsilon(j-i)}{2L}\cos\frac{\pi\epsilon(j-i)}{L}\}$$
$$-\frac{1}{j+1}\{\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}\}$$
$$-\frac{2MV}{\mathcal{E}}\dot{T}_f(t)\,j\{\sin\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}$$
$$+\frac{i}{j-i}\sin\frac{\pi\epsilon(j-i)}{L}\sin\frac{\pi\epsilon(j-i)}{2L}\} \quad (22)$$
$$+\frac{MV^2\pi^2}{\mathcal{E}L^2}j^2T_j(t)\frac{1}{i+j}\{\sin\frac{\pi\epsilon(j+i)}{L}\cos\frac{\pi\epsilon(j+i)}{2L}$$
$$+\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\} \quad (2$$

$$EI(\pi/L)^4j^4T_j(t)+\ddot{T}_j(t)\{\mathcal{N}+\mathcal{N}R_0\left(-(\pi/L)^2\right)j^2\}=-\frac{2MgL}{j\pi\epsilon}\sin\frac{j\pi\epsilon}{L}\sin\frac{i\pi\epsilon}{2L}$$
$$-\ddot{T}_j(t)\frac{2M}{\pi\epsilon}\frac{1}{j-i}\{\sin\frac{\pi\epsilon(j-i)}{2L}\cos\frac{\pi\epsilon(j-i)}{L}\}-\frac{1}{j+1}\{\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}$$
$$-\dot{T}_f(t)\,j\frac{2MV}{\mathcal{E}}\{\sin\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}+\frac{i}{j-i}\sin\frac{\pi\epsilon(j-i)}{L}\sin\frac{\pi\epsilon(j-i)}{2L}\}$$

$$+T_j(t)\frac{MV^2\pi^2}{\mathcal{E}L^2}j^2\frac{1}{i+j}\{\sin\frac{\pi\epsilon(j+i)}{L}\cos\frac{\pi\epsilon(j+i)}{2L}+\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\} \quad (23)$$

When

$$\ddot{T}_j(t)=\frac{T_{j+1}-2T_j-T_{j-1}}{h^2}$$

and

$$\dot{T}_j(t)=\frac{T_{j+1}-T_{j-1}}{2h}$$

$$EI(\pi/L)^4j^4T_j(t)2h^2+\left(\frac{T_{j+1}-2T_j-T_{j-1}}{h^2}\right)\{\mathcal{N}+\mathcal{N}R_0\left(-(\pi/L)^2\right)j^2\}2h^2=$$

$$-\frac{2MgL}{j\pi\epsilon}\sin\frac{j\pi\epsilon}{L}\sin\frac{i\pi\epsilon}{2L}2h^2-\frac{2M}{\pi\epsilon}\left(\frac{T_{j+1}-2T_j-T_{j-1}}{h^2}\right)2h^2\frac{1}{j-i}\{\sin\frac{\pi\epsilon(j-i)}{2L}\cos\frac{\pi\epsilon(j-i)}{L}\}$$

$$-\frac{1}{j+1}\{\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}$$

$$-\frac{2MV}{\mathcal{E}}\left(\frac{T_{j+1}-T_{j-1}}{2h}\right)2h^2\{\sin\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}+\frac{i}{j-i}\sin\frac{\pi\epsilon(j-i)}{L}\sin\frac{\pi\epsilon(j-i)}{2L}\}$$

$$+T_j(t)\frac{MV^2\pi^2}{\mathcal{E}L^2}j^2\frac{1}{i+j}\{\sin\frac{\pi\epsilon(j+i)}{L}\cos\frac{\pi\epsilon(j+i)}{2L}+\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\} \quad (24)$$

$$\Bigg(2\{\mathcal{N}+\mathcal{N}R_0\left(-(\pi/L)^2\right)j^2\}+\frac{4M}{\pi\epsilon}\frac{1}{j-1}\{\sin\frac{\pi\xi(j-i)}{2L}\cos\frac{\pi\epsilon(j-i)}{L}\}$$
$$-\frac{1}{j+1}\{\cos\frac{\pi\xi(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}$$
$$+\frac{2hMV}{\mathcal{E}}\{\sin\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}+\frac{i}{j-i}\sin\frac{\pi\epsilon(j-i)}{L}\sin\frac{\pi\epsilon(j-i)}{2L}\}T_{j+1}-T_j$$

$$\left(\left(2\left(EI\left(\frac{\pi}{L}\right)^4h^2\right)\right)-4\{\mathcal{N}+\mathcal{N}R_0\left(-(\pi/L)^2\right)j^2\}+\frac{8M}{\pi\epsilon}\frac{1}{j-1}\{\sin\frac{\pi\epsilon(j-i)}{2L}\cos\frac{\pi\epsilon(j-i)}{L}\}\right.$$

$$-\frac{1}{j+1}\{\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}-\frac{MV^2\pi^2}{\mathcal{E}L^2}\{\sin\frac{\pi\epsilon(j+i)}{L}\cos\frac{\pi\epsilon(j+i)}{2L}$$

$$+\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}T_j-\left(2\{\mathcal{N}+\mathcal{N}R_0\left(-(\pi/L)^2\right)j^2\}\right.$$

$$+\frac{4M}{\pi\epsilon}\frac{1}{j-1}\{\sin\frac{\pi\epsilon(j-i)}{2L}\cos\frac{\pi\epsilon(j-i)}{L}\}-\frac{1}{j+1}\{\cos\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}\}$$
$$+\frac{2hMV}{\mathcal{E}}\{\sin\frac{\pi\epsilon(j+i)}{L}\sin\frac{\pi\epsilon(j+i)}{2L}+\frac{i}{j-i}\sin\frac{\pi\epsilon(j-i)}{L}\sin\frac{\pi\epsilon(j-i)}{2L}\}T_{j-1}=$$
$$-\frac{4MgLh^2}{j\pi\epsilon}\sin\frac{j\pi\epsilon}{L}\sin\frac{i\pi\epsilon}{2L} \quad (25)$$

RESULTS AND DISCUSSION

In order to validate our model in the previous section, the following beam dimension and specification were used:

Table 1: Beam Specification and Dimension.

The beam was made of steel E	$2.10\times10^{11}N$
Length(L)	2m, 6m, 10 m
Breadth	0.05 m
Height	0.15 m
Flexural Rigidity EI	2910937.5
Mass of the beam (M)	50kg,70kg,100kg
Length of the mass ()	1,3,5
Length of the load (ξ)	1,2,3
Foundation Modulus K	0, 1, 2, 3, 5N/m^3
Mass per unit length of the beam m	7.04

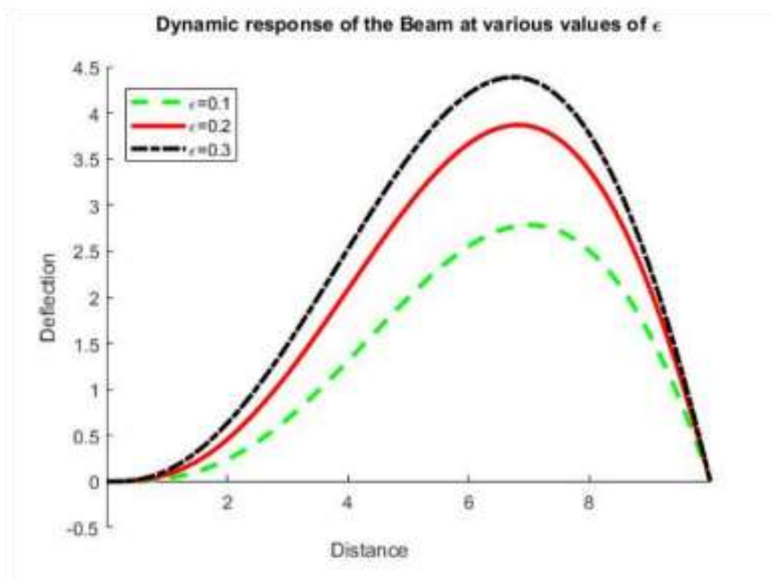


Fig. 1: Dynamic response of Beam for various values of ϵ

Figure 1 Shows the dynamic response of the Beam at various values of. It is observed that the dynamic response of the beam increases as the length of the mass increases.

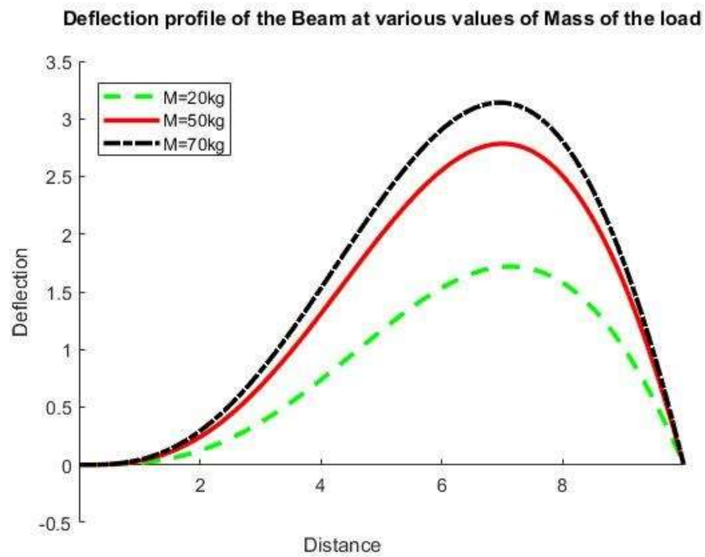
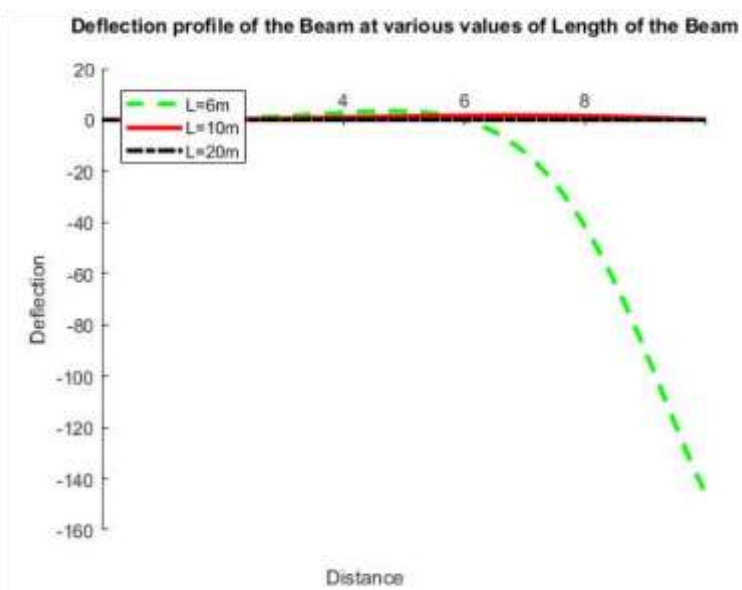


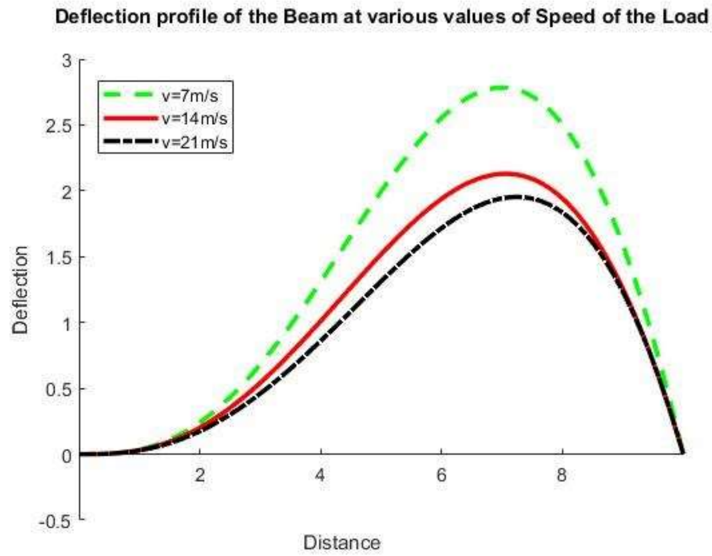
Fig. 2: Dynamic response of Beam for various values of M

Figure 2 Shows the dynamic response of the beam at various values of *M*. It is also observed that the dynamic response of the beam increases as the mass of the load increases.



Fig, 3: Dynamic response of Beam for various values of L

Figure 3 Displays the dynamic response profile for the length of the beam L. It is observed that the dynamic response of the beam decreases with an increase in the length of the beam.



Fig, 4: Dynamic response of Beam for *v*

Figure 4 Shows the dynamic response profile for the speed at which the load is moves. It is observed that the dynamic response of the beam decreases as the value of the speed of the increases.

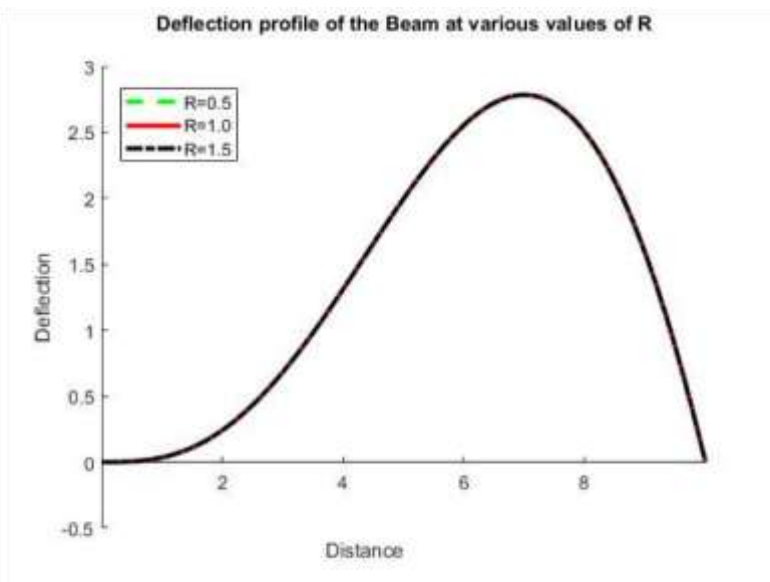


Fig. 5: Dynamic response of Beam for various values of *R*

Figure 5 Displays the dynamic beam's response variation profile for damping coefficient. It is observed that the dynamic response of the beam is the same for all values of the damping coefficient.

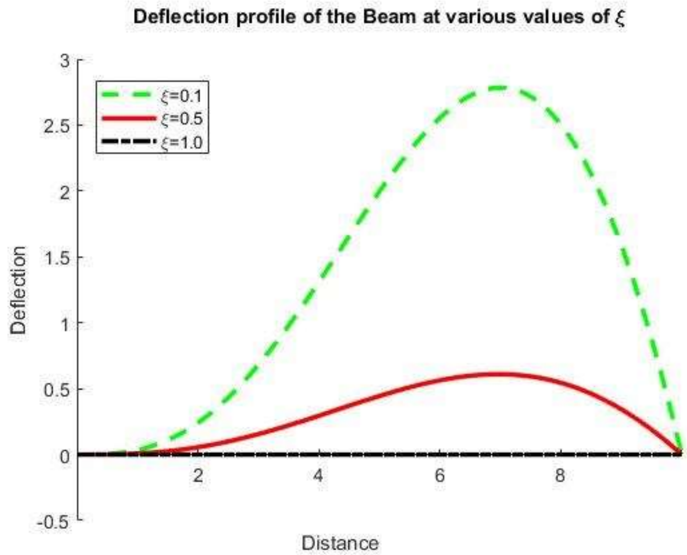


Fig. 6: Dynamic response of Beam for various values of ξ

Figure 6 Displays the dynamic beam's response variation profile for the length of the load. It is observed that the dynamic response of the beam decreases with an increase in the mass of the load.

CONCLUSION

Mathematical analysis of Rayleigh beam with damping coefficients subjected to moving load was considered in this study. The governing partial differential equation of fourth order was reduced to an ordinary differential equation using series solution. The frequency of the oscillation was found and substituted back into the assumed series solution which is the solution of the governing equation. Numerical result was presented and plotted against x for various parameters using a computer program (MATLAB).

From the numerical results, this study concludes that the dynamic response of the beam increases as the length of the mass increases, the same result was also found for the length of the beam and the mass of the load but the dynamic response of the beam decreases as the length of the load. It also reduces as the speed at which the load moves increases. Also, the dynamic response of the beam is not affected by the damping coefficient.

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