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ODD GOMPERTZ-G FAMILY OF DISTRIBUTION, ITS PROPERTIES AND APPLICATIONS

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ABSTRACT

In this research paper, we introduced a novel generator derived from the continuous Gompertz distribution, known as the odd Gompertz-G distribution family. We conducted an in-depth analysis of the statistical characteristics of this family, including moments, moment-generating functions, quantile functions, survival functions, hazard functions, entropies, and order statistics. Within this family, we also derived a specific distribution called the odd Gompertz-Exponential distribution. To evaluate the reliability of the distribution's parameters, we employed Monte Carlo simulations. Furthermore, we assessed the applicability of this newly proposed distribution family by examining its performance on real-world data and the results demonstrate that the new model (OG-E) outperformed its comparators under consideration.

Keywords: Gompertz distribution, odd Gompertz-G Family, exponential distribution, maximum likelihood, Simulation

INTRODUCTION

Recent studies revealed that the theory and application of probability distribution have received significant achievement with the introduction of new generalized families of distributions. Lifetime data can be represented through several statistical distributions, like Weibull, Gompertz, Frechet, exponential, Rayleigh, and others. However, this attracts the demand for more comprehensive kinds of these conventional distributions because, in numerous real-world scenarios, these conventional distributions do not sufficiently align with the data being modeled. Hence, there is a motivation to introduce asymmetry and adaptability into established probability distributions, especially in the case of the Gompertz distribution.

Various extensions of these distributions were made by some authors, including the generalized Gompertz by El-Gohary et al., (2013), the beta Gompertz by Jafari et al., (2014), the odd generalized Exponential-Gompertz by El-Damcese et al., (2015), and the Power Gompertz distribution by Ieren *et al.*, (2019).

Several developed families of distributions have been thoroughly investigated in a number of fields, and it has been found that they produce improved adaptability. Examples include the development of the exponentiated-G (E-G) class by Gupta *et al*. in 1998, beta-G class by Eugene *et al*. (2002), Marshall-Olkin-G class in 1997, the gamma-G distributions by Zografos and Balakrishnan (2009), and the Kumaraswamy Weibull-G by Cordeiro et al. in 2010. Alternative Gamma-G was proposed by Ristic and Balakrishnan in 2011, Kumaraswamy-G by Cordeiro and Castro in 2011, Kumaraswamy beta generalized by Cordeiro *et al*. in 2012, Type II Half-Logistic Exponentiated-G by Bello *et al.*, (2021) and transform-transformer by Alzaatreh *et al*, (2013). The aim of this paper is to develop and explore the Odd Gompertz-G (OG-G) family of distribution for analyzing lifetime data that can accommodate a wide array of behavior patterns such as increasing and decreasing failure rates, as well as cases where failure rates remain constant or exhibit a bathtub-shaped pattern in practical applications.

MATERIAL AND METHOD

The new family

Consider the probability density function (pdf) and cumulative distribution function (cdf) of the Gompertz distribution as defined by lanert (2012) with θ as scale parameter and γ as shape parameter are respectively known as:

$$G(t;\theta,\gamma) = 1 - e^{-\frac{\theta}{\gamma}\left(e^{\gamma t}-1\right)}; 0 < t < \infty, \theta, \gamma > 0 \quad (1)$$

$$g(t;\theta,\gamma) = \theta e^{\gamma t} e^{-\frac{\theta}{\gamma}\left(e^{\gamma t}-1\right)}; 0 < t < \infty, \theta, \gamma > 0 \quad (2)$$

The Cumulative distribution function $G(t, \Phi)$ with survival function $\overline{G}(t;\Phi) = 1 - G(t;\Phi)$ of the parent distribution depends on a parameter vector Φ and assuming a random variable T relates to a system having a baseline G distribution. So the odd T that a system will not work at particular time interval is given by $G(t)/\overline{G}(t)$. The random variable T of the odds using Gompertz model is given by

$$p(X \le x) = p(X \le G(t)/\overline{G}(t)) = F\left[\frac{G(t)}{\overline{G}(t)}\right]$$

Replacing x in the Gompertz cdf by the odd ratio $G(t)/\overline{G}(t)$, the cdf of the novel family, OG-G is as follows

$$F_{OG-G}(x;\theta,\gamma,\Phi) = \int_0^{\frac{G(x,\Phi)}{1-G(x,\Phi)}} q(t)dt = 1 - e^{-\frac{\theta}{\gamma}\left(e^{\gamma\frac{G(x;\Phi)}{1-G(x;\Phi)}}-1\right)} \quad (3)$$

The corresponding pdf to (3) is given by

$$f_{OG-G}(x;\theta,\gamma,\Phi) = \frac{\theta g(x;\Phi)}{\left(1-G(x;\Phi)\right)^2} e^{\gamma\frac{G(x;\Phi)}{1-G(x;\Phi)}} e^{-\frac{\theta}{\gamma}\left(e^{\gamma\frac{G(x;\Phi)}{1-G(x;\Phi)}}-1\right)} \quad (4)$$

where $g(x,\Phi)$ is the pdf of any parent distribution and Φ is the parameter vector, therefore a random variable X with density function and distribution function in equations (3) and (4) is denoted by $X \sim OG - G(\theta, \gamma, \Phi)$.

Validity Check of the OG-G family of distributions

It is significant to ascertain whether the pdf of OG-G family of distributions as given in equation (4) establishes a valid probability density and this can be realized by ensuring that its integral over the domain of X equals to unity. i.e

$$\int_0^\infty f_{OG-G}(x;\theta,\gamma,\Phi)dx = 1$$

$$f_{OG-G}(x;\theta,\gamma,\Phi) = \frac{\theta g(x;\Phi)}{\left(1-G(x;\Phi)\right)^2} e^{\gamma\frac{G(x;\Phi)}{1-G(x;\Phi)}} e^{-\frac{\theta}{\gamma}\left(e^{\gamma\frac{G(x;\Phi)}{1-G(x;\Phi)}}-1\right)} dx \tag{5}$$

Supposed that $y = \frac{\theta}{\gamma}\left(e^{\gamma\frac{G(x;\Phi)}{1-G(x;\Phi)}}-1\right)$ and $u = \frac{G(x,\Phi)}{1-G(x,\Phi)}$

Then $dx = \frac{(1-G(x,\Phi))^2 dy}{\theta g(x,\Phi)e^{\gamma\frac{G(x,\Phi)}{1-G(x,\Phi)}}}$ and as $x \to 0; y \to 0$and as $x \to \infty; y \to \infty$

Now equation (5) can be written as

$$\int_0^\infty \frac{\theta g(x,\Phi)}{(1-G(x,\Phi))^2} e^{\gamma\frac{G(x,\Phi)}{1-G(x,\Phi)}} e^{-\frac{\theta}{\gamma}(e^{\gamma\frac{G(x,\Phi)}{1-G(x,\Phi)}}-1)} \frac{(1-G(x,\Phi))^2 dy}{\theta g(x,\Phi)e^{\gamma\frac{G(x,\Phi)}{1-G(x,\Phi)}}} = 1 \Rightarrow \int_0^\infty e^{-y}dy = 1$$

Hence, the pdf of the OG-G family of distributions be valid as required.

Useful expansion

In this section, we will explore a valuable expansion of the distribution functions for the OG-G family.

Proposition: The expression that provides a linear representation of the OG-G family of distributions is as follows:

$$F(x) = 1 - \sum_{k=0}^\infty \sum_{l=0}^\infty Z_{k,l} H_{(k+l)}(x)$$

Proof

$$F(x;\theta,\gamma,\Phi) = 1 - e^{-\frac{\theta}{\gamma}\left(e^{\gamma\frac{G(x;\Phi)}{1-G(x;\Phi)}}-1\right)}$$

From the power series expansion

$$e^{-\frac{\theta}{\gamma}(e^{\gamma\frac{G(x)}{1-G(x)}}-1)} = \frac{\sum_{i=0}^\infty (-1)^i \left(\frac{\theta}{\gamma}\right)^i \left(e^{\gamma\frac{G(x)}{1-G(x)}}-1\right)^i}{i!}$$

$$F(x) = 1 - \frac{\sum_{i=0}^\infty (-1)^i \left(\frac{\theta}{\gamma}\right)^i \left(e^{\gamma\frac{G(x)}{1-G(x)}}-1\right)^i}{i!} \tag{6}$$

But $\left(e^{\gamma\left(\frac{G(x)}{1-G(x)}\right)}-1\right)^i = \left(\frac{1}{e^{-\gamma\left(\frac{G(x)}{1-G(x)}\right)}}-1\right)^i$

$$\Rightarrow \left(\frac{1-e^{-\gamma\left(\frac{G(x)}{1-G(x)}\right)}}{e^{-\gamma\left(\frac{G(x)}{1-G(x)}\right)}}\right)^i = \frac{\left(1-e^{-\gamma\left(\frac{G(x)}{1-G(x)}\right)}\right)^i}{\left(e^{-\gamma\left(\frac{G(x)}{1-G(x)}\right)}\right)^i}$$

$$F(x) = 1 - \sum_{i=0}^\infty \frac{(-1)^i}{i!}\left(\frac{\theta}{\gamma}\right)^i \left(1-e^{-\gamma\left(\frac{G(x)}{1-G(x)}\right)}\right)^i \left(e^{\gamma\left(\frac{G(x)}{1-G(x)}\right)}\right)^i \tag{7}$$

By binomial expansion,

$$\left(1-e^{-\gamma\left(\frac{G(x)}{1-G(x)}\right)}\right)^i = \sum_{j=0}^\infty (-1)^j \binom{i}{j} e^{-\gamma\left(\frac{G(x)}{1-G(x)}\right)^j}$$

$$F(x) = 1 - \sum_{i=0}^\infty \frac{(-1)^i}{i!}\left(\frac{\theta}{\gamma}\right)^i \sum_{j=0}^\infty (-1)^j \binom{i}{j} \left(e^{\gamma\left(\frac{G(x)}{1-G(x)}\right)}\right)^i \left(e^{-\gamma\left(\frac{G(x)}{1-G(x)}\right)}\right)^j \tag{8}$$

$$F(x) = 1 - \sum_{i=0}^\infty \sum_{j=0}^\infty \frac{(-1)^{i+j}}{i!}\left(\frac{\theta}{\gamma}\right)^i \binom{i}{j} e^{(i-j)\gamma\left(\frac{G(x)}{1-G(x)}\right)}$$

By power series expansion,

$$e^{(i-j)\gamma\left(\frac{G(x)}{1-G(x)}\right)} = \sum_{k=0}^{\infty} \frac{(i-j)^k \gamma^k \left(\frac{G(x)}{1-G(x)}\right)^k}{k!}$$

$$F(x) = 1 - \theta^i \gamma^{k-i} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=o}^{\infty} \frac{(-1)^{i+j}(i-j)^k}{i!k!} \binom{i}{j} \left(\frac{G(x)}{1-G(x)}\right)^k \tag{9}$$

$$F(x) = 1 - \theta^i \gamma^{k-i} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=o}^{\infty} \frac{(-1)^{i+j}(i-j)^k}{i!k!} \binom{i}{j} \left(G(x)\right)^k \left(1-G(x)\right)^{-k} \tag{10}$$

Using generalize binomial theorem, $\left(1-G(x)\right)^{-k} = \sum_{l=0}^{\infty} \frac{\Gamma k + l(G(x))^l}{l!\Gamma k}$

$$F(x) = 1 - \theta^i \gamma^{k-i} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=o}^{\infty}\sum_{l=0}^{\infty} \frac{(-1)^{i+j}(i-j)^k}{i!k!l!} \binom{i}{j} \frac{\Gamma k + l}{\Gamma k} G(x)^{(k+l)} \tag{11}$$

$$F(x) = 1 - \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} Z_{k,l} G(x)^{(k+l)} \tag{12}$$

$$F(x) = 1 - \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} Z_{k,l} H_{(k+l)}(x) \tag{13}$$

Where $Z_{k,l} = \theta^i \gamma^{k-i} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \frac{(-1)^{i+j}(i-j)^k}{i!k!l!} \binom{i}{j} \frac{\Gamma k + l}{\Gamma k}$ and $H_{(k+l)}(x) = G(x)^{(k+l)}$ denotes the cdf of the Exponentiated-G distribution with power parameter $(k+l) > 0$

By differentiating (13), the pdf of x can be given in the mixture form as;

$$f(x) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} Z_{k,l} h_{(k+l)}(x) \tag{14}$$

Where $h_{(k+l)}(x) = (k+l)g(x)G(x)^{(k+l)-1}$ denotes the Exponentiated-G density function with power parameter $(k+l)$.

Statistical properties of OG-G Family of distribution

Moments of the OG-G Family

The rth moment of a random variable X that follows the Odd Gompertz-G (OG-G) family is given as:

$$E(X^r) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} Z_{k,l} E(X_{k,l}^r) \tag{15}$$

Where, $E(X_{k,l}^r) = \int_0^{\infty} x^r (k+l) g(x) G(x)^{(k+l)-1} dx$

Moment-Generating Function of the OG-G Family

The moment-generating function of a random variable X that follows the Odd Gompertz-G (OG-G) family is given as:

$$M_x^{OG-G}(t) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} Z_{k,l} E(e^{tx_{k,l}}) \tag{16}$$

Where, $E(e^{tx_{k,l}}) = \int_0^{\infty} e^{tx_{k,l}} h_{(k+l)}(x) dx$

Quantile Function of OG-G Family

The quantile function of the OG-G family is obtained by inverting the CDF in equation (3). Say $Q(u) = F^{-1}(u)$ of x be given via

$$x = Q(u) = G^{-1}\left[\frac{\frac{1}{\gamma}\log\left(1-\frac{\gamma}{\theta}\log(1-u)\right)}{\left[1+\left[\frac{1}{\gamma}\log\left(1-\frac{\gamma}{\theta}\log(1-u)\right)\right]\right]}\right] \tag{17}$$

Where, G^{-1} is the quantile function of any continuous parent distribution and u is considered as a uniform random variable on the interval (0, 1).

Entropies of the OG-G Family

Entropy is a measure of variation or uncertainty of a random variable X (Renyi 1961). The entropy of the OG-G family is defined statistically as follow:

$$I_z(x) = \frac{1}{1-Z} \log \int_0^{\infty} f_{OG-G}(x;\theta,\gamma,\Phi)^z dx = \frac{1}{1-Z} \log\left(\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} Z_{k,l}\, h_{(k+l)}(x;\Phi)\right)^z dx$$

$$I_z(x) = \frac{1}{1-Z} \log\left(\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} Z_{k,l}\right)^z \int_0^{\infty} \left(h_{(k+l)}(x;\Phi)\right)^z dx \tag{18}$$

Where $Z > 0$ and $Z \neq 1$

Order Statistics of the OG-G Family

Let $X_1, X_2, \ldots, X_n$ be a random sample from the OG-G distribution and $X_{1:n} \leq X_{2:n} \leq \ldots X_{n:n}$ denote the corresponding order statistics, then the ith order statistic is given as:

$$f_{i:n}(x)=\frac{n!}{(i-1)(n-i)!}f(x;\theta,\gamma,\Phi)F(x;\theta,\gamma,\Phi)^{i-1}\left[1-F(x;\theta,\gamma,\Phi)\right]^{n-i}$$

$$f_{i:n}(x)=\frac{n!}{(i-1)!(n-i)!}\sum_{j=0}^{i-1}(-1)^{j}\binom{n-i}{j}\left(\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}Z_{k,l}h_{(k+l)}(x)\right)\left(1-\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}Z_{k,l}h_{(k+l)}(x)\right)^{i+j-1} \quad (19)$$

Survival and Hazard Rate Function of the OG-G Family

The survival function and hazard function are respectively given as:

$$S_{OG-G}(x;\theta,\gamma,\Phi)=e^{-\frac{\theta}{\gamma}\left(e^{\gamma\frac{G(x;\Phi)}{1-G(x;\Phi)}}-1\right)} \quad (20)$$

$$h_{OG-G}(x;\theta,\gamma,\Phi)=\frac{\frac{\theta g(x;\Phi)}{\left(1-G(x;\Phi)\right)^{2}}e^{\gamma\frac{G(x;\Phi)}{1-G(x;\Phi)}}e^{-\frac{\theta}{\gamma}\left(e^{\gamma\frac{G(x;\Phi)}{1-G(x;\Phi)}}-1\right)}}{e^{-\frac{\theta}{\gamma}\left(e^{\gamma\frac{G(x;\Phi)}{1-G(x;\Phi)}}-1\right)}} \quad (21)$$

Estimation of Parameters of the OG-G Family

Suppose that $x_1,\ x_2,\ x_3,\ \dots\ ,\ x_n$ are the observed values from the proposed OG-G family with parameters θ,γ. Suppose that $\Phi\ =\ [\theta,\ \gamma]^T$ is the $[m\ \times\ 1]$ vector of the parameter. The log-likelihood function $\Phi$is expressed by

$$\ell_n=L(\Omega)=n\log\theta+\sum_{i=1}^{n}\log(g(x;\Phi))-2\sum_{i=1}^{n}\log(1-G(x;\Phi))$$
$$+\gamma\sum_{i=1}^{n}\left[\frac{G(x;\Phi)}{1-G(x;\Phi)}\right]-\frac{\theta}{\gamma}\sum_{i=1}^{n}\left[e^{\gamma\frac{G(x;\Phi)}{1-G(x;\Phi)}}-1\right] \quad (22)$$

Taking the partial derivative of equation (22) w.r.t the parameters $(\theta,\ \gamma,\Phi)$ are respectively given as

$$\frac{\partial L(\Omega)}{\partial\theta}=\frac{n}{\theta}-\frac{1}{\gamma}\sum_{i=1}^{n}\left[e^{\gamma\frac{G(x;\Phi)}{1-G(x;\Phi)}}-1\right] \quad (23)$$

$$\frac{\partial L(\Omega)}{\partial\gamma}=\sum_{i=1}^{n}\left[\frac{G(x;\Phi)}{1-G(x;\Phi)}\right]+\frac{\theta}{\gamma^{2}}\sum_{i=1}^{n}\left[e^{\gamma\frac{G(x;\Phi)}{1-G(x;\Phi)}}-1\right]-\frac{\theta}{\gamma}\sum_{i=1}^{n}\frac{G(x;\Phi)}{1-G(x;\Phi)}e^{\gamma\frac{G(x;\Phi)}{1-G(x;\Phi)}} \quad (24)$$

$$\frac{\partial L(\Omega)}{\partial L(\Phi)}=\sum_{i=1}^{n}\frac{g'(x;\Phi)}{g(x;\Phi)}+2\sum_{i=1}^{n}\frac{g(x;\Phi)}{1-G(x;\Phi)}+\gamma\sum_{i=1}^{n}\frac{g(x;\Phi)}{(1-G(x;\Phi))^{2}}-\theta\sum_{i=1}^{n}\frac{g(x;\Phi)}{(1-G(x;\Phi))^{2}}e^{\gamma\frac{g(x;\Phi)}{1-G(x;\Phi)}} \quad (25)$$

The MLEs of the parameters$(\theta,\ \gamma,\Phi)$, says $(\hat{\theta},\ \hat{\gamma},\hat{\Phi})$are the simultaneous solution of equations (23), (24), and (25) equating them to zero, i.e $\frac{\partial(\Omega)}{\partial\theta}=0;\ \ \frac{\partial(\Omega)}{\partial\gamma}=0;\frac{\partial(\Omega)}{\partial\Phi}=0$. These equations are intractable and can only be solved using a numerical iterative method.

Sub-Model of the OG-G Family

By substituting an Exponential distribution into the OG-G family, a new distribution is formed. The cdf and pdf of Exponential distribution which serves as the baseline distribution with parameter σ is given as;

$$K(x;\sigma)=1-e^{-\sigma x} \quad (26)$$
$$k(x;\sigma)=\sigma e^{-\sigma x}x>0,\sigma>0 \quad (27)$$

Inducing equation (26) and (27) into equation (3) and (4), the cdf and pdf of the Odd Gompertz- Exponential (OG-E) distribution is given as;

$$F(x;\theta,\gamma,\sigma)=1-e^{-\frac{\theta}{\gamma}\left(e^{\gamma\frac{(1-e^{-\sigma x})}{(1-(1-e^{-\sigma x}))}}-1\right)} \quad (28)$$

$$f(x;\theta,\gamma,\sigma)=\frac{\theta\sigma e^{-\sigma x}}{(1-(1-e^{-\sigma x}))^{2}}e^{\gamma\frac{(1-e^{-\sigma x})}{(1-(1-e^{-\sigma x}))}}e^{-\frac{\theta}{\gamma}\left(e^{\gamma\frac{(1-e^{-\sigma x})}{(1-(1-e^{-\sigma x}))}}-1\right)} \quad (29)$$

More so, the following are the survival, hazard and the quantile functions of the OG-E distribution respectively.

$$s(x;\theta,\gamma,\sigma)=e^{-\frac{\theta}{\gamma}\left(e^{\gamma\frac{(1-e^{-\sigma x})}{(1-(1-e^{-\sigma x}))}}-1\right)} \quad (30)$$

$$h(x;\theta,\gamma,\sigma)=\frac{\frac{\theta\sigma e^{-\sigma x}}{(1-(1-e^{-\sigma x}))^{2}}e^{\gamma\frac{(1-e^{-\sigma x})}{(1-(1-e^{-\sigma x}))}}e^{-\frac{\theta}{\gamma}\left(e^{\gamma\frac{(1-e^{-\sigma x})}{(1-(1-e^{-\sigma x}))}}-1\right)}}{e^{-\frac{\theta}{\gamma}\left(e^{\gamma\frac{(1-e^{-\sigma x})}{(1-(1-e^{-\sigma x}))}}-1\right)}} \quad (31)$$

$$x=\frac{-1}{\lambda}\left[\log\left(1-\left[\frac{\frac{1}{\gamma}\log\left(1-\frac{\gamma}{\theta}\log(1-u)\right)}{\left[1+\left[\frac{1}{\gamma}\log\left(1-\frac{\gamma}{\theta}\log(1-u)\right)\right]\right]}\right]\right)\right] \quad (32)$$

Graph of the Special Sub-Model of the OG-G Family

The plot of the probability density function, hazard function, survival function and cumulative distribution function of the Odd Gompertz- Exponential (OG-E) distribution is given as;

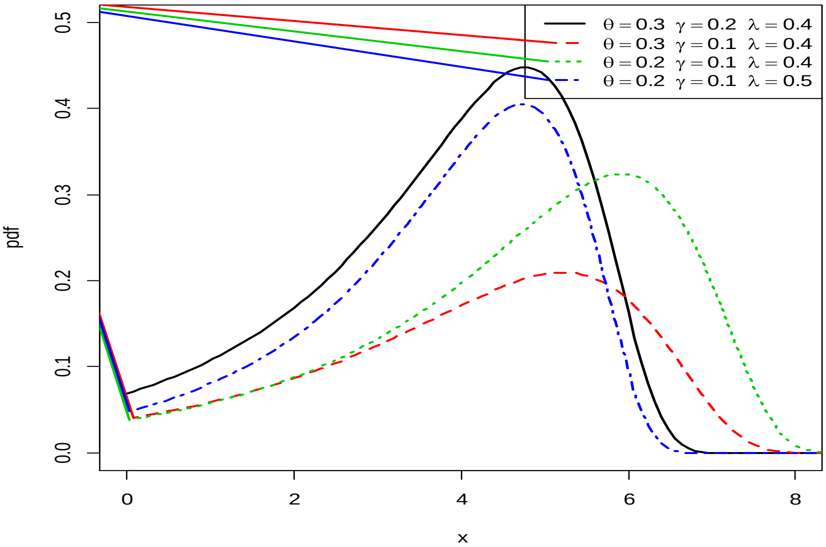

Figure 1: pdf of the Odd Gompertz-Exponential Distribution

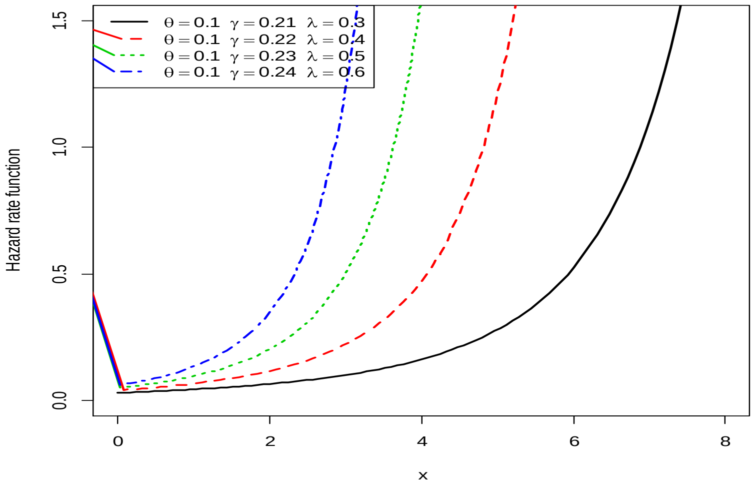

Figure 2: hazard function of the Odd Gompertz-Exponential Distribution

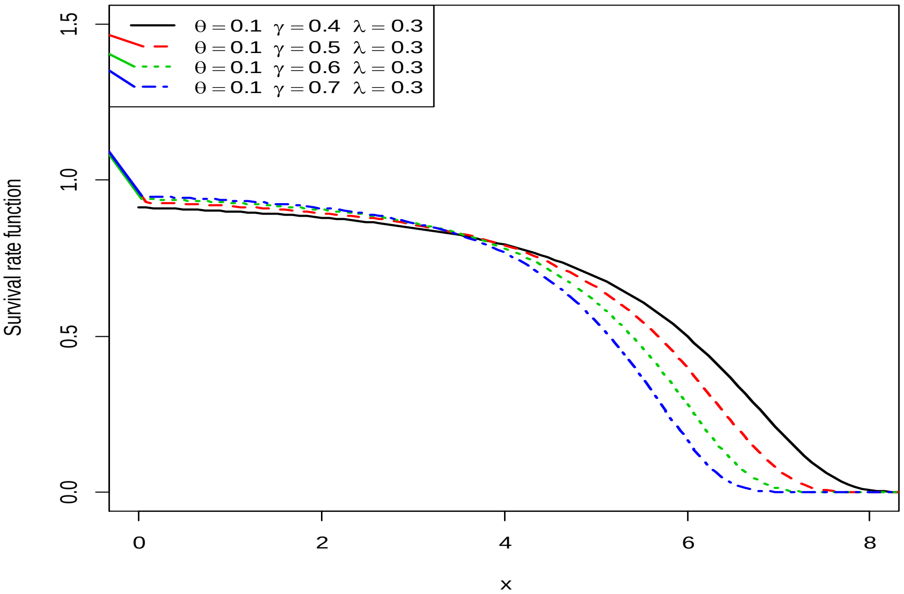

Figure 3: Survival function of the Odd Gompertz-Exponential Distribution

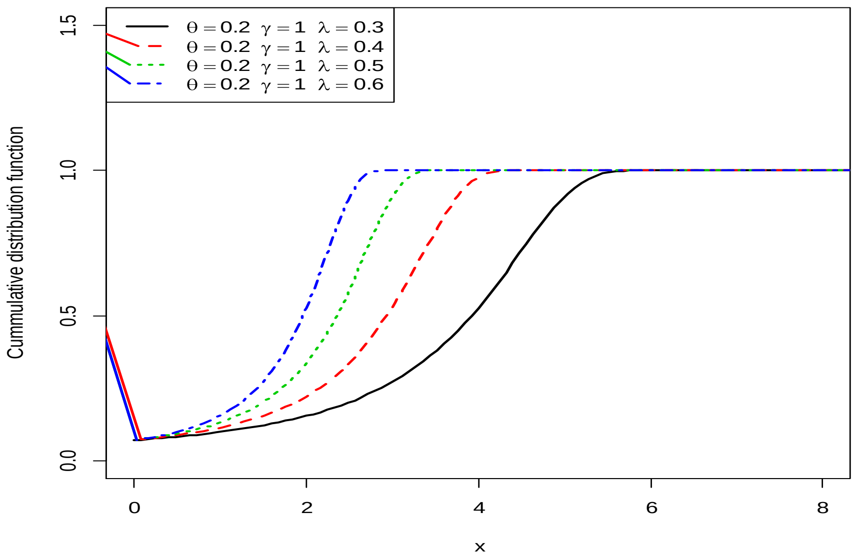

Figure 4: cdf of the Odd Gompertz-Exponential Distribution

Monte Carlo Simulation and Application

Monte Carlo Simulation

The well-known class of computational algorithms known as "Monte Carlo simulation" is applied to a replicated random sample in order to produce numerical results so as to address the problem of risk in modeling lifetime data.

Simulation Study

To appraise the consistency of the OG-ED model, simulation training was conceded out using Monte Carlo Simulation method. This study aimed to calculate mean, bias, and root mean square error of the estimated model parameters obtained through maximum likelihood estimation. Simulated data was generated using the quantile function described in equation (18), and this process was repeated 1,000 times for various sample sizes: n = 50, 100, 250, 500, and 1,000. The parameters were held constant at a specific value for each of these simulation runs.

Table 1: Average Values of the MLEs, Biases and RMSEs of the OG-ED for $\theta = 0.75, \gamma = 4, \lambda = 0.7$

Sample size	Parameter	Estimates	Bias	RMSE
50	θ	0.7682	0.0182	0.2862
	γ	4.1509	0.1509	0.5723
	λ	0.7031	0.0031	0.0726
100	θ	0.7657	0.0157	0.2100
	γ	4.1385	0.1385	0.4704
	λ	0.6950	-0.0050	0.0586
250	θ	0.7601	0.0101	0.1366
	γ	4.1251	0.1251	0.3752
	λ	0.6916	-0.0084	0.0467
500	θ	0.7632	0.0132	0.0959
	γ	4.1122	0.1122	0.3114
	λ	0.6897	-0.0103	0.0391
1000	θ	0.7609	0.0109	0.0710
	γ	4.0726	0.0726	0.2476
	λ	0.6928	-0.0072	0.0312

Table 1 above indicates that biases and RMSEs tend to approach zero as the sample size rises. This trend suggests that the estimates become more accurate and reliable, converging towards the initial (true) values, it demonstrates that the estimates are both efficient and consistent as the sample size grows.

Applications

Here, we exhibit the potentiality of the Odd Gompertz-Exponential Distribution (OG-ED) using a real data set from a previous studies, see Arslan *et al.* (2019). The maximum likelihood estimates, as well as goodness-of-fit measures, were computed via R software and compared with Weibull Exponential (WE), Gompertz Exponential (GE), Kumaraswammy Exponential (KE), Exponentiated Weibull-Exponential (EW-E) and Exponential (E) distribution.

We employ the Akaike Information Criterion (AIC), which has the following mathematical expression in order to identify which of the competing models is the best:

AIC $= -2L + 2K$. Where L stands for log-likelihood function, k is the number of model parameters.
The data set used for the analysis is obtained from the work of Arslan *et al.* (2019) and it represents the time to failure (10^3/h) of turbocharger of one engine as seen below:

1.6, 3.5, 4.8, 5.4, 6.0, 6.5, 7.0, 7.3, 7.7, 8.0, 8.4, 2.0, 3.9, 5.0, 5.6, 6.1, 6.5, 7.1, 7.3, 7.8, 8.1, 8.4, 2.6, 4.5, 5.1, 5.8, 6.3, 6.7, 7.3, 7.7, 7.9, 8.3, 8.5, 3.0, 4.6, 5.3, 6.0, 8.7, 8.8, 9.0.

Table 2: Parameters Estimates and Goodness of fit Measures for the above Data set.

Model	Parameter Estimates and Goodness of Fit					
	$\hat{\theta}$	$\hat{\lambda}$	$\hat{\gamma}$	$\hat{\phi}$	LL	AIC
WE	0.0114	0.9065	0.6930	-	80.0063	166.0126
GE	0.0156	1.3257	0.4716	-	79.9550	165.9101
E	0.1599	-	-	-	113.3193	228.6385
OGE	0.0279	0.4380	0.0349	-	**78.9991**	**163.9983**
KE	5.7438	7.4707	0.1757	-	86.9518	177.1954
EWE	0.8325	0.0081	1.0174	0.6471	79.9895	167.9791

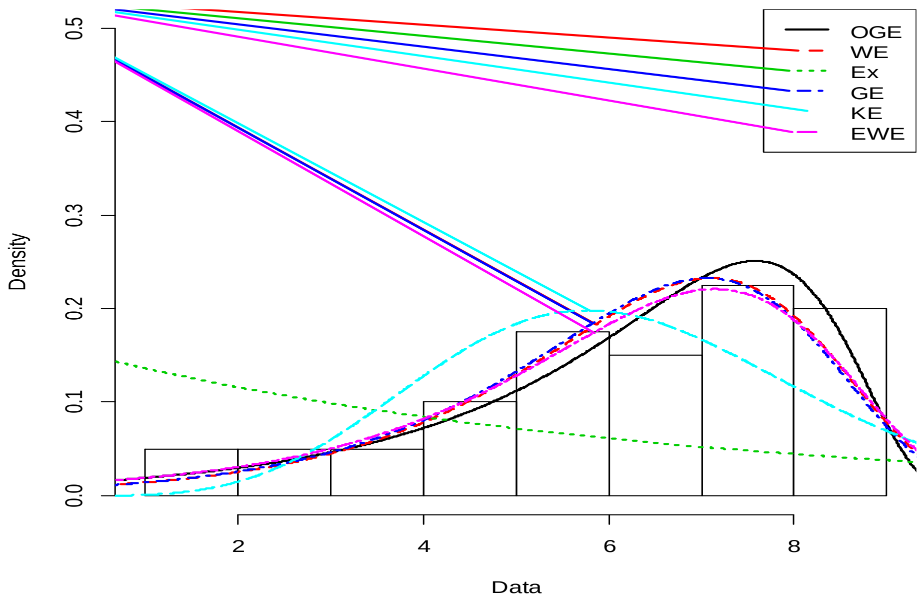

Figure 5: Histogram Plots of the Distribution of time to failure of turbocharger of one engine Data.

Table 2 displays the outcomes of the maximum likelihood estimation regarding the parameters of the new distribution and five other reference distributions. When assessing the goodness of fit, it was observed that the proposed distribution exhibited the lowest AIC value, with GE coming in a close second. A visual examination of the fit, as depicted in Figure 5, further validates that the proposed distribution outperformed its comparator distributions. Consequently, among the various distributions under consideration, the proposed distribution is deemed the most suitable for modeling the failure time of turbocharger of one engine dataset.

CONCLUSION

We define the odd Gompertz-G as a new family of continuous distribution. Some statistical characteristics of the new family, like the explicit quantile function, moments, moment-generating functions, survival function, hazard function, entropies, and distribution of order statistics, are investigated. Additionally, specific sub-model within this novel family was deliberated. The technique of maximum likelihood is employed to estimate the parameters of the model. Simulation training was conducted in order to assess the effectiveness of the offered distribution. To showcase the significance and adaptability of the sub-model, a real-life dataset is used in analyzing and comparing the well-known competing models. The results demonstrate that the new model (OG-E) outperformed the existing ones under consideration, suggesting its utility as a new distribution for modeling data in a wide range of applications.

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