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COMPARATIVE ANALYSIS OF GEOMETRIC BROWNIAN MOTION, ARTIFICIAL NEURAL NETWORK AND NAIVE BAYESIAN TECHNIQUES USING NIGERIA STOCK EXCHANGE DATA

[1]ThankGod Joshua, [2]Audu Isah, *[1]Abdulmudallib Ibrahim, [2]Usman Abdullahi, [3]Mohammed Lawan Danyaro

[1]Department of Mathematics and Statistics, Federal University Wukari, Taraba State, Nigeria.
[2]Department of Statistics, Federal University of Technology, Minna, Niger State, Nigeria
[3] Department of General Studies Yobe State College of Agriculture, Science and Technology Gujuba

*Corresponding authors' email: abdulmudallib@fuwukari.edu.ng

ABSTRACT

This paper presents a comparative analysis of three market price forecast models, namely Geometric Brownian Motion, Artificial Neural Network, and Naive Bayesian techniques, using data from the Nigeria Stock Exchange. The exploratory data analysis results indicate slight variations in the mean and median of the log stock price over a 5-year period. The data shows a relatively small spread from the mean and approximately symmetric distribution, as indicated by a skewness value close to zero. The normality test confirms that the log stock price data follows a normal distribution. The forecast using Artificial Neural Network (ANN) shows a minimal change in future stock price, suggesting low returns and moderate risk in the Nigerian stock market. The graphical representation of the ANN model demonstrates a constant path with little variation. Similarly, the Naive Bayesian technique provides a similar forecast to the ANN model, indicating limited profit potential. The Geometric Brownian Motion model also forecasts little variation in future stock prices, with 2023 showing slightly higher values. The accuracy of the forecast models is evaluated using the Mean Absolute Percentage Error (MAPE). The results indicate that the ANN model has an error of 4.60%, the Naive Bayesian model has an error of 9.29%, and the Geometric Brownian Motion model has an error of 12.67%. These findings suggest that the ANN model performs better in terms of accuracy compared to the other two models.

Keywords: Artificial Neural Network, Naïve Bayesian Classifiers, Geometric Brownian Motion, Stock Price

INTRODUCTION

The stock market is the meeting place of both buyers and sellers of stocks. It is a platform for investors to own shares of companies with the sole purpose of making profits. Forecasting is the best method to know the future price of a stock (Omar and Jaffar, 2014).

To forecast is to form an expectation of what will happen in the future and it is one of the most popular mathematical methods in many fields such as business, social science, engineering, and finance. The uncertainty property in stock price call for concern on the part of investors and financial managers, since the change in stock price occurs frequently, investors seek to know the future price of their investment and the risk associated with this investment which has been the motivation for many research in the quest for solution.

Two common approaches to predicting stock prices are those based on the theory of technical analysis and those based on the theory of fundamental analysis (Fama, 1995). Fundamental analysis assumes that the price of a stock depends on the intrinsic value and expected return on investment; while the technical analysis studies the price movement of a stock and predicts its future price movement. There are alternative approaches to the forecasting of stock prices, for instance, the Random Walk Theory. The Random Walk Theory is the idea that stocks prices take a random and unpredictable path, making it near impossible to outperform the market without assuming additional risk (Fama, 1970; 1991).

Neural network models have become more popular in forecasting over the last decade in business, economics, and finance (Avcı 2007). According to Khashei and Bijari (2010), Artificial Neural Network are distinguished and most effective for predictive modelling because of their data-driven self-adaptive nature and they are universal function approximators. The network can generalize, this means that once the network learns the data, it can predict the unseen or future part of the data even if the given data is not smooth (Islam and Nguyen, 2020).

Brownian motion is a special type of motion of molecular particles, first observed and described by the British-Scottish botanist Robert Brown in 1827. However, a French mathematician named this Brownian motion and proposed a model to predict stock prices using Brownian motion in (Bachelier,1900). According to the geometric Brownian motion model, the returns on a certain stock in successive, equal periods of time are independents and normally distributed (Dmouj 2006). Geometric Brownian motion has two components; a certain component and an uncertain component. The certain component represents the return that the stock will earn over a short period of time, also referred to as the drift of the stock. The uncertain component is a stochastic process including the stocks volatility and an element of random volatility (Sengupta, 2004).

In addition to the models above, Naïve Bayes algorithm is a classification technique which generates Bayesian Networks for a given dataset based on Bayes theorem. They are statistical classifiers and can predict class membership probabilities, such as the probability that a given sample belongs to a particular class. It assumes that the given dataset contains a particular feature in a class which is unrelated to any other feature. This assumption is called class conditional independence. In this paper, we build predictive models using all of the above three modelling techniques and compare the models' performance on stock price forecasting using the Nigeria stock Exchange data.

Prediction has long been a popular field in mathematical sciences, so there is plenty of related research in the field. Thereafter, a lot of research were performed to check the models' accuracy of prediction to forecast the stock market. Hota and Dash (2021) examined the best model of stock market by comparing the performance of Random Forest (RF), Support vector machine (SVM), Long-Short Term

Memory (LSTM) and Artificial Neural Networks (ANN). From the algorithm, Neural Network performs better in all type of data values. Adeosun and Ugbebor (2021) examined the suitability of Geometric Brownian motion, symmetric and asymmetric jump-diffusion models, on the empirical log-returns of the Nigerian all-share index. The suitability analysis results showed that the symmetric jump-diffusion model fits the market indices better. Pauli et al., (2021) compared the predictive performance of multiple linear regression, Elman, Jordan, radial basis function, and multilayer perceptron architectures based on the root of the mean square error. The results showed that for all times series, considered all architectures, except the radial basis function, the artificial networks tunning provide suitable fit, reasonable predictions, and confidence interval. Islam and Nguyen (2020) compared the performance of autoregressive integrated moving average, artificial neural network and stochastic process-geometric Brownian motion. The results showed that the autoregressive integrated moving average model and the stochastic model provide better approximation for next-day stock price prediction compared to the neural network model. Azizah et al., (2020) compared the performance of geometric Brownian motion and multilayer perceptron methods on Prediction of stock prices, using Microsoft stock prices. The result of geometric Brownian motion method better than multilayer perceptron method. Huang and Liu (2019) addressed problem of predicting direction of movement of stock price and compared four prediction models; artificial neural network, support vector machine, random forest and naive-Bayes. The results suggested that for the first approach of input data where ten technical parameters are represented as continuous values, artificial neural network outperforms other three prediction models on overall performance. Udomsak (2015) investigated the performance of naive Bayes classifier and support vector machine and compare their ability to forecast. The result showed that naive Bayes is better than the support vector machine at predicting the Stock Exchange of Thailand. Adebiyi et al., (2014) compared the forecasting performance by ARIMA and artificial neural network for stock data. They analysed daily stock prices for the Dell Incorporation and found a superiority of the neural network model over the ARIMA model. Rathnayaka et al., (2014) developed a forecasting model using the geometric Brownian motion model and compared the predictions with the results from the traditional time series model ARIMA using the Colombo Stock Exchange and found that the stochastic model prediction is more significant than the traditional model.
The literature shows different opinions on the relative performances of the three models depending on data. Hence, further comparative studies of all the three models using same stock price data can assemble a consistent methodology for stock price prediction. In this paper, we study the comparative performances of the three models in predicting monthly stock prices from the Nigeria stock exchange.

MATERIALS AND METHOD

In this paper, the past closed stock price data are taken from the official website of Nigeria Stock Exchange. Kolmogorov-Smirnov (K-S) test was conducted on these data to conclude that the data are normally distributed and feasible to forecast for future stock price.
The model used in this research is Geometric Brownian Motion, Artificial Neural Network and the Naïve Bayesian Model.
Mean Absolute Percentage Error (MAPE) is calculated in order to determine the forecast accuracy as well as performance of the models.

Artificial Neural Network.

The idea of ANN came from the structure of the animal brain, more specifically, from the human neural system. It is based on the idea of how brain works, how the neurons in the brain receive information from the input neurons, analyse it, and finally identify the object or pattern. Fundamentally, the mechanism has three layers—input layer, hidden layers, and output layer. Each layer consists of neurons or nodes. The hidden part may consist of many layers, however, for the time series analysis and forecasting, the single hidden layer feed forward network is the most widely used model structure (Zhang et al. 1998). A simple three-layer neural network has the following mathematical form;

$$Y_t = W_0 + \sum_{j=1}^{q} W_{j.g}(W_{0,j} + \sum_{i=1}^{y} W_{i,j} \,.Y_{t-1}) + \epsilon_{t,} \quad (1)$$

where, $W_{i,j}$ and W_j for i = 1, 2, . . ., p, j = 1, 2, . . ., q are known as connection weights. The parameter p and q are the number of input and output nodes respectively. The network involves an activation function which plays a very important role because it converts the input signals to be used for the neurons or nodes in the next layer, eventually the output neuron. The most widely used activation functions are the logistic and hyperbolic functions (Khashei and Bijari 2010), which are shown in Equations (2) and (3)

$$sig(x) = \frac{1}{1-e^{-x}} \quad (2)$$

$$tan^{-1}(x) = \frac{1-e^{-2x}}{1+e^{-2x}} \quad (3)$$

Most of the modelers prefer the hyperbolic tangent function as the activation functions because of its faster convergence, and it makes the optimization easier. Hence, we used this activation function in our model.

Neural Network Autoregressive (NNAR)

The NNAR model is an ANN, where the input layer is just one variable input with the $lag1, lag2$ and so on models until lag to p, so it is called ANN Autoregressive (NNAR). This model is only for feedforward networks in a single hidden-layer and is denoted by NNAR(p, k), where p denotes lag-p as input and k as notes in hidden layer.
This the NNAR method uses a single hidden layer and uses a nonlinear function as in equation (3.4) to give weight and produce output from ANN.

$$Z_j = b_j + \sum_{i=1}^{n} w_{i,j} x_i \quad (4)$$

Where Z_j is the sum function of the unit bias to j on the hidden layer, b_j is a weight in the bias unit to j , $w_{i,j}$ is the weight of the layer i bias to j, and x_i is the network input to i.
The activation function uses the binary sigmoid activation function as in equation (2)

Bayes Theorem

Let $X = \{x_1, x_2, \dots, x_n\}$ be a sample, whose components represent values made on a set of n-attributes. In Bayesian terms, X is considered "evidence". Let H be some hypothesis, such as that the data X belongs to a specific class C. For classification problems, our goal is to determine $P(H/X)$, the probability that the hypothesis H holds given the" evidence", (i.e., the observed data sample X). In other words, we are looking for the probability that sample X belongs to class C, given that we know the attribute description of X. $P(H/X)$ is the a posteriori probability of H conditioned on X.

Naïve Bayesian Classifier

The naive Bayesian classifier works as follows: Let T be a training set of samples, each with their class labels. There are k class $C_1, C_2, \dots, C_n$. Each sample is represented by an n-dimensional vector, $X = x_1, x_2, \dots, x_n$, depicting n measured values of the n attributes, $A_1, A_2, \dots, A_N$ respectively. Given a sample X, the classifier will predict that X belongs to the class having the highest a posteriori probability, conditioned on X. That is $X$is predicted to belong to the class C_i if and only if $P(C_i/X) > P(C_i/X)\ for\ i \le j \le m, j \ne i$ Thus we find the class that maximizes $P(C_i/X)$. The class C_i for which $P(C_i/X)$ is maximized is called the maximum posteriori hypothesis.

By Bayes theorem; $P(C_i/X) = P\left(\frac{(X/C_i)/P(C_i)}{P(X)}\right)$ (5)

is the same for all classes, only $P(X/C_i)$need be maximized. If the class a priori probabilities, $P(C_i)$, are not known, then it is commonly assumed that the classes are equally likely, that is, $P(C_1) = P(C_2) = \cdots P(C_k)$, and we would therefore maximize $P(X/C_i)P(C_i)$. Otherwise, we maximize $P(X/C_i)P(C_i)$. Note that the class a priori probabilities may be estimated by $P(C_i) = freg(C_i, T)$/|T|. Given data sets with many attributes, it would be computationally expensive to compute $P(X/C_i)$. In order to reduce computation in evaluating $P(X/C_i)P(C_i)$,the naive assumption of class conditional independence is made. This presumes that the values of the attributes are conditionally independent of one another, given the class label of the sample.

Mathematically this means that $P(X/C_i) \approx \prod_{k=1}^{n} P\ C_k/C_i$ (6)

The probabilities $P(x_i/C_i), P(x_2/C_i), \dots, P(x_n/C_i)$ can easily be estimated from the training set. Recall that here x_k refers to the value of attribute A_k for sample X.

(a) If A_k is categorical, then $P(x_i/C_i)$ is the number of samples of class C_i in T having the value x_k for attribute A_k divided by freq(C_i, T), the number of sample of class C_i in T.

(b) If A_k is continuous-valued, then we typically assume that the values have a Gaussian distribution with a mean μ and standard deviation defined by;

$$g(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} exp - \frac{(x-\mu)^2}{2\sigma^2} \quad (7)$$

so that;

$$P(x_k/C_i) = g(x_k, \mu_{ci}\sigma_{ci}) \quad (8)$$

We need to compute μC_i and σC_i , which are the mean and standard deviation of values of attribute for training samples of class C_i . In order to predict the class label of $X, P(X/C_i)P(C_i)$ is evaluated for each class C_i. The classifier predicts that the class label of X is C_i if and only if it is the class that maximizes $P(X/C_i)P(C_i)$.

The Laplacian correction (or Laplace estimator) is a way of dealing with zero probability values. Recall that we use the estimation $P\left({}^{X}/_{C_i}\right) \approx \prod_{k=1}^{n} P(x_k/C_i)$ based on the class independence assumption. What if there is a class, C_i, and X has an attribute value, x_k, such that none of the samples in C_i has that attribute value? In that case $P(x_k/C_i) = 0$ which results in $P(X/C_i)$ even though P(x_k/C_i) for all the other attributes in X may be large. We can assume that our training set is so large that adding one to each count that we need would only make a negligible difference in the estimated probabilities, yet would avoid the case of zero probability values.

Geometric Brownian Motion

Let Ω be the set of all possible outcomes of any random experiment and the continuous time random process X_t , defined on the filtered probability space $(\Omega, F, \{F_t\}t\epsilon T, P)$.

where, $F\ is\ the\ \sigma - algebra$ of event, $\{F_t\}t\epsilon T$ denotes the information generated by the process X_t over the time interval$[0, T]$.

P is the probability measure.

Definition: A random variable 'X' has the lognormal distribution with parameters μ and σ if $log(X)$ is normally distributed. i.e.,

$log(X) \sim N(\mu, \sigma^2)$

Definition: A real valued random process $W_t = W(t, w)$ on the time interval $[0, \infty]$ is Brownian Motion or Wiener Process if it satisfies following conditions (Karlin & Taylor, 2012 and Ross, 1996).

1. Continuity:$W_0 = 0$
2. Normality:$for\ 0 \le s < t \le T, W_t - W_s \sim N(0, t - s)$
3. Markov Property: $for\ 0 \le s' <' t' < s < t \le T, W_t - W_s$ $independent\ of\ W'_t - W'_s$

A stochastic process S_t St is used to follow a Geometric Brownian Motion if it satisfies the following stochastic differential equation.

$$dS_t = \mu S_t dt + \sigma S_t dWt \quad (9)$$

where, W_t is a Wiener process (Brownian Motion) and μ & σ are constants.

Normally, it is called the percentage drift and σ is called the percentage volatility.

So, consider a Brownian Motion trajectory that satisfies this differential equation.

The right-hand side term $\mu S_t dt$ controls the trends of this trajectory and the term $\sigma S_t dWt$ controls the random noise effect in the trajectory.

After applying the technique of separation of variable, the equation becomes:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dWt \ (10)$$

Taking integration of both side;

$$\int \frac{dS_t}{S_t} = \int (\mu dt + \sigma dWt) dt \quad (11)$$

Since $\frac{dS_t}{S_t}$ relates to derivative of ln (S_t) the $It\bar{o}$ calculus becomes:

$$ln\left(\frac{dS_t}{S_t}\right) = \left[\left(\mu - \frac{\sigma^2}{2}\right) + \sigma Wt\right] \quad (12)$$

Taking the exponential in both sides and plugging the initial condition S_0, the analytical solution of Geometric Brownian Motion is given by:

$$S_t = S_0 exp\left[\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma Wt\right] \quad (13)$$

The constants μ and σ are able to produce a solution of Geometric Brownian Motion throughout time interval.

For given drift and volatility the solution of Geometric Brownian Motion in the form:

$S_t = S_o exp[x(t)]$

Where $x(t) = \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma Wt$

Now the density of Geometric Brownian Motion is given by

$$f(t,x) = \frac{1}{\sigma x(2\pi t)^{1/2}} exp[-(logx - logx_0 - \mu t)]/2\sigma^2 t \quad (14)$$

Suppose that a set of input: $t_1, t_2, t_3 \dots\dots\dots$ and a set of corresponding output: $S_1, S_2, S_t \dots\dots\dots$ from S_t and the set of data is in the mle function $L(\theta)$. Since Geometric Brownian Motion is a Markov Chain Process

$$L(\theta) = f\theta(x_1, x_2, x_3 \dots\dots\dots) = \prod_{i=1}^{n} f\theta(x_i) \quad (15)$$

Now taking derivative of the right-hand side, we get

$$\bar{m} = \sum_{i=1}^{n} \frac{x_i}{n}$$

$$\bar{v} = \sum_{i=1}^{n} \frac{(x_i - m)^2}{n}$$

Where $\bar{m}$ and $\bar{v}$ are the mle of m and v respectively and $x_i = logS(ti) - logS(ti-1)$

Estimation of Volatility and Drift

In developing the random walk algorithm, the volatility (σ) and drift (μ) of the historical stock price has to be estimated. The formula for the volatility is given as (Adeosun *et al*., 2015)

$$\mu_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \quad (16)$$

$$\bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i$$

$$v = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2} \quad (17)$$

Where;

S_i is the stock price at the end of ith trading period, u_i is the logarithm of the monthly return on the stock over a time interval,$\bar{u}$ is the unbiased estimator of the log returns (u_i), v is the standard deviation, and σ is the volatility of the monthly stock return.

The drift (μ) is given by (Adeosun et al., 2015);

$$\bar{u} = \left(\mu - \frac{1}{2}\sigma^2\right)\tau \Rightarrow \mu = \bar{u} + \frac{1}{2}\sigma^2 \quad (18)$$

Measure of Accuracy

The Mean Absolute Percentage Error (MAPE) would be used to measure the predictive accuracy of the three forecast models to check which has a better performance on monthly stock price from the Nigeria Stock Exchange.

The mean absolute percentage error (MAPE) is one of the most popular measures of the forecast accuracy due to its advantages of scale-independency and interpretability (Hanke and Reitsch, 1995). This measure is generally only used when quantity of interest is strictly positive and it is given as (Rahul and Bidyadhara, 2020);

$$M = \frac{1}{N}\sum_{t=1}^{n}\left(\frac{A_t - F_t}{A_t}\right) \quad (19)$$

Where, A_t and F_t denotes the actual and forecast value at specified time 't' respectively, and N is the number of data points.

Table 1: A scale of judgment of forecasting

MAPE	Prediction Accuracy
< 10%	Highly accurate
11% - 20%	Good prediction
21% - 50%	Reasonable prediction
> 51%	Inaccurate prediction

Source: Abidin and Jaffar (2014)

RESULTS AND DISCUSSION

This section contains results and discussion of analysis using the three market price forecast models.

Exploratory Data Analysis

To explore and gain basic understanding of the stock price data, the summary statistics and visualization plots and normality assumption test has been presented below

Table 2: Log Stock Price Summary Statistics

Statistics	Value
Mean	9.326188708
Standard Error	0.009693697
Q3	9.472145236
Median	9.345295391
Mode	#N/A
Standard Deviation	0.075087057
Sample Variance	0.005638066
Kurtosis	2. 1.016743133
Skewness	0.010408839
Range	0.286137581
Q1	9.180240154
Minimum	9.015146995
Maximum	9.672284576

From table 1, the mean and median (9.326 and 9.345 respectively) of the log of stock price for the period of 60 months (5 years) from 2015 to 2019 shows slight variations while the minimum and maximum values are 9.015 and 9.672 respectively. The standard deviation shows a little spread of the data from the mean. A value of skewedness approximately zero shows a distribution is approximately symmetric, hence, our skewedness of 0.010 shows the log stock price is approximately normal. The kurtosis reveals a distribution with flat tails which indicates a small outlier in the distribution. The normality plot in figure 1 and test give more clarity picture.

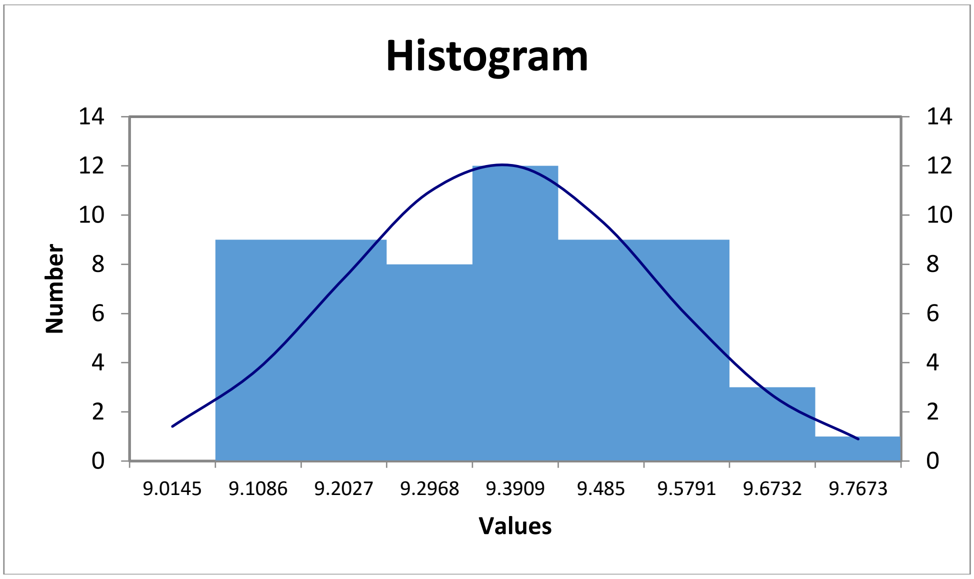

Figure 1: Histogram and Probability Plot of Log Stock Price

One-sample Kolmogorov-Smirnov test
D = 0.095, p-value = 0.617
The plot shown in figure 1 displays a normally distributed data backed-up by the K-S test which gives a p-value of 0.617 > 0.05 level of significance. This is an indication that the log stock price data is normally distributed, or rather, since the D-statistics is lesser than the p-value, it is reasonable to conclude that the log of the stock price data follows a normal distribution which satisfies the assumption of a Geometric Brownian Motion model that the log of stock prices follows a normal distribution.

Forecast Using Artificial Neural Network
The graphical representation for the neural network autoregressive model with the interpretation is given below;

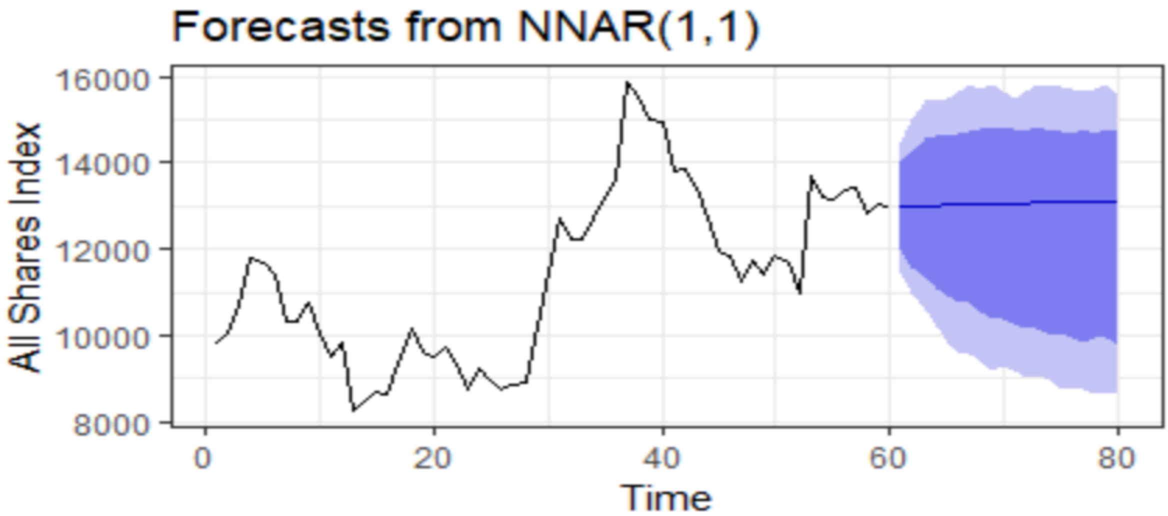

Figure 2: Artificial Neural Network Forecast Using Nigeria Stock Exchange Data

Figure 2 displays the actual stock price line graph from Nigeria Stock exchange and the artificial neural network forecast. The forecast follows a seemly constant path with a little change or growth in its path. The result from the neural network forecast depicts a minimal change in the future stock price forecasted. This is an indication of little returns on stock prices considering monthly stock price forecast from the Nigeria Stock Exchange. It also shows the future stock price has little difference in value with less than moderate level of risk making it difficult to make tangible profit.

Naïve Model Forecast of Stock Price from the Nigeria Stock Exchange data
Below is the graphical representation of the actual price and the forecast from the Nigeria Stock price using the naïve Bayesian model.

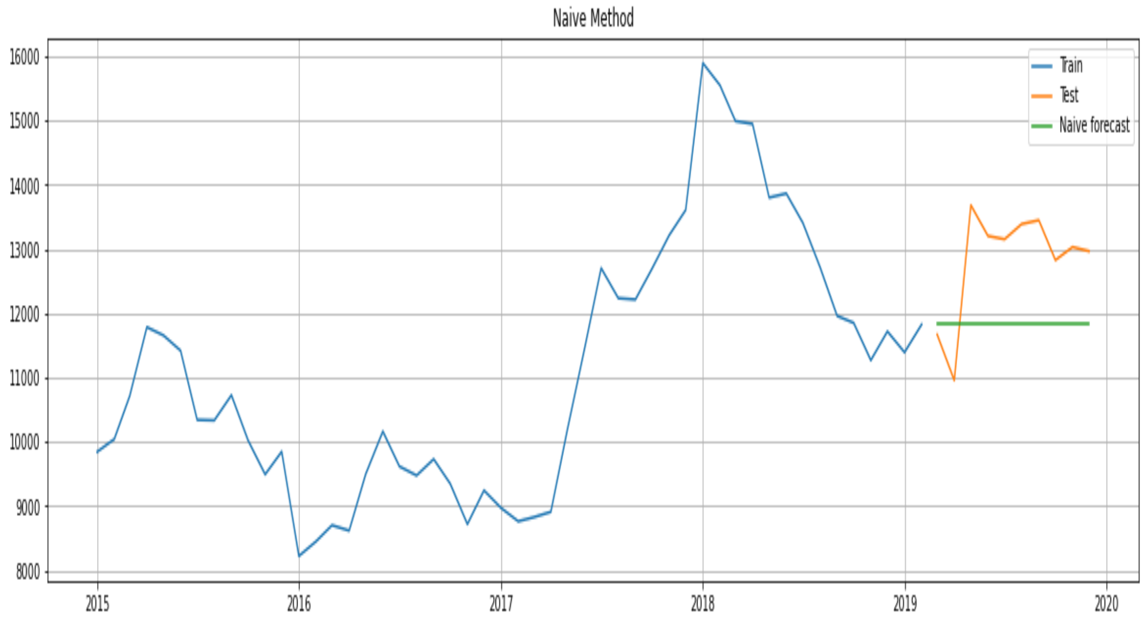

Figure 3: Naïve Model Forecast of Nigeria Stock Exchange Price

Figure 3 displays the train data (in blue), the test data (in red) and the naïve prediction (in green). The data was split into 80% training and 20% testing. The forecast (in green) gives a straight line which looks constant deviating from the pattern of the test data. Flat in the forecast is a price that is neither rising nor declining. This also implies there is little chances of making reasonable profit from the Nigeria stock exchange within the time frame of the study. The naïve Bayesian techniques gives a similar forecast when compared to forecast from the Artificial Neural Network model.

Geometric Brownian motion forecast of Nigeria Stock Exchange Price

The table 3 below, gives four (4) years (Jan. 2020 – Dec. 2023) forecast of stock price using historical stock price data from the Nigeria Stock Exchange.

Table 3: Forecasting Stock Price from 2020 to 2023

Month	Forecast
Jan’20	13,029.17
Feb’20	13,029.08
Mar’20	13,028.99
Apr’20	13,028.89
May’20	13,028.81
Jun’20	13,028.71
Jul’20	13,028.61
Aug’20	13,028.51
Sep’20	13,028.42
Oct’20	13,028.33
Nov’20	13,028.25
Dec’20	13,028.14
Jan’21	13,088.93
Feb’21	13,088.84
Mar’21	13,088.77
Apr’21	13,088.65
May’21	13,088.52
Jun’21	13,088.45
Jul’21	13,088.37
Aug’21	13,088.30
Sep’21	13,088.17
Oct’21	13,088.05
Nov’21	13,088.01
Dec’21	13,087.92
Jan’22	13,147.83
Feb’22	13,147.75
Mar’22	13,147.65
Apr’22	13,147.52

May'22	13,147.45
Jun'22	13,147.35
Jul'22	13,147.20
Aug'22	13,147.14
Sep'22	13,147.05
Oct'22	13,146.98
Nov'22	13,146.88
Dec'22	13,146.79
Jan'23	13,205.87
Feb'23	13,205.74
Mar'23	13,205.65
Apr'23	13,205.58
May'23	13,205.42
Jun'23	13,205.40
Jul'23	13,205.28
Aug'23	13,205.19
Sep'23	13,205.11
Oct'23	13,204.98
Nov'23	13,204.87
Dec'23	13,204.80

Form the result of the forecast using geometric Brownian motion displayed on table 3 above, there is little variation in the future stock prices for a period of four (4) years, with the year 2023 showing a little high value in price. The table shows that extreme price movements will rarely occur in the future. This translates to less than moderate level of risk. The riskier the stock price, the higher the chance and probability to make profit. The results show less chances of making reasonable profit from the Nigerian stock exchange, considering the period of time the study was carried out. The forecasted figures conform to the forecast using both artificial neural network and the naïve Bayesian model.

Measure of Accuracy

Below is the measure of accuracy on the models to check their performance considering monthly stock prices from the Nigeria Stock Exchange.

Table 4: Measure of Accuracy

	ME	RMSE	MAE	MPE	MAPE	ACF1
NNAR	11.4	725.0	519.	-0.30	4.60	0.0546
NAIVEB		1291.69			9.29	
GBM					12.67	

Table 4 above displays the measures of accuracy for both artificial neural network, naïve Bayesian techniques and the geometric Brownian motion. The mean absolute percentage error being the measure of accuracy preferred for the purpose of this research work shows an error of 4.60% for artificial neural network, 9.29% error for the naïve Bayesian model and 12.67% for the geometric Brownian motion.

The results of this research work are consistent with the study carried out by Hota and Dash (2021) which finds artificial neural network better than most forecast models, it is also consistent with the report by Huang and Liu (2019) which showed the artificial neural network performed better than the naïve Bayesian model but contradicts the report of Islam and Nguyen (2021) whose results showed that the geometric Brownian motion performed better than the artificial neural network in predicting next day stock prices.

CONCLUSION

This paper examined the performance of Artificial Neural Network, Naïve Bayesian techniques and the geometric Brownian motion using historical monthly stock price from the Nigeria stock exchange for over the period of five years on a monthly basis. The results of the analysis revealed that future stock price within the time frame of the research will likely experience little change or difference in monthly price values. By implication, there will be minimum probability of making profit as prices would rarely experience any high variation. The minimal difference in price will account for less than moderate level of risk on the future stock price, this in turn will affect the rate of return on investment. The three models have similar forecast results but the Artificial Neural Network techniques performed better than both the naïve Bayesian method and the geometric Brownian motion model in forecasting monthly stock prices room the Nigeria stock exchange. As a result of findings from this research, it is recommended that Artificial Neural Network techniques should be considered when forecasting monthly stock price, and also, stock price models should be used within short period of time as long-term forecast tends to give inaccurate forecast.

DECLARATION OF INTERESTS

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

REFERNCES

Abidin, S. & Jaffar, M.M. (2014). A review on geometric Brownian motion in forecasting the share prices in Bursa Malaysia. *World Applied Sciences Journal*, **17**, 87-93.

Adebiyi, A. A., Aderemi, O. A., and Charles K. A. (2014). Comparison of arima and artificial neural networks models for stock price prediction. *Journal of Applied Mathematics* 2014: 614342.

Adeosun, M.E., Edeki,S.O ., Ugbebor,O.O.,(2015) Stochastic analysis of stock market price models: A case study of the Nigerian Stock Exchange (NSE). WSEAS Transactions on Mathematics, 14: 363-363.

Adeosun, M.E., Ugbebor ,O.O. (2021) An Empirical Assessment of Symmetric and Asymmetric Jump-Diffusion Models for the Nigerian Stock Market Indices, Scientific African (2021), doi: https://doi.org/10.1016/j.sciaf.2021.e00733

Avcı, E. (2007). Forecasting daily and sessional returns of the ise-100 index with neural network models. Dogus Universitesi Dergisi 8: 128–42.

Azizah, M., Irawan, M.I., Putri, E. R. M. (2020) AIP Conference Proceedings 2242, 030016 (2020); https://doi.org/10.1063/5.0008066 Published Online: 01 June 2020 M.

Bachelier, L., (1900) Theori De La Speculation Paris: Gauthier-Villars. Reprinted in English (Bones Trans) in Cooter P.H.

Brown, R., (1827) on the particles contained in the Pollen of Plants and on the general existence of active molecules in organic and inorganic bodies", Philosophical Magazine, N. S. 4, pp. 161–173.

Dmouj, A. (2006). Stock Price Modelling: Theory and Practice. Masters's thesis, Vrije Universiteit, Amsterdam, The Netherlands.

Fama F.E. (1991). Efficient capital market II. *Journal of Finance*, 46(5) 1575 –1617

Fama, E. F. (1970). Efficient capital market: A review of theory and empirical work. *The Journal of Finance,* 25(2): 383-417. Papers and proceedings of the twenty-eighth Annual Meeting of the American finance Association New York, N. Y. December, 28-30, 1969, Published by Blackwell Publishing for the American Finance Association.

Fama, F. E. (1995). Random walks in stock market prices. *Financial Analysts Journal,* 51(1), 75- 80. http://dx.doi.org/10.2469/faj.v51.n1.1861.

Hanke, J. E., Reitsch, A. G. (1995) Business Forecasting, Englewood Cliffs, NJ: Prentice-Hall, 5th edition, ISBN 0205160050.

Hota, L., Dash, P.K. (2021) Comparative Analysis of Stock Price Prediction by ANN and RF Model. Computational Intelligence and Machine Learning Vol-2 Issue-1, April 2021, PP.1-9

Huang, C.S and Liu, Y.S. (2019) Machine Learning on Stock Price Movement Forecast: The Sample of the Taiwan Stock Exchange.

Islam, M. R. & Nguyen, N. (2020). Comparison of financial models for stock price prediction.*Journal of Risk and Financial Management* 13(181): 1-19

Khashei, Mehdi, and Mehdi Bijari. 2010. An artificial neural network (p, d, q) model for timeseries forecasting. Expert Systems with Applications 37; 479–89.

Rahul, K, Bidyadhara, B. (2020). Forecasting Short Term Return Distribution of S&P BSE Stock Index Using Geometric Brownian Motion: An Evidence from Bombay Stock Exchange. *Research Scholar, P.G. Department of Statistics, Sambalpur University*, Jyoti Vihar, Burla, Sambalpur, Odisha-768019, India.

Omar, A., & Jaffar, M.M. (2014). Forecasting share price of small size companies in bursa Malaysia using geometric Brownian motion. *Journal International Journal of Applied Mathematics & Information Sciences* 8(1): 107-112. Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, 40450 Selangor, Malaysia.

Pauli, S.T.Z., Kleina, M., Bonat, W.H. (2021) Comparing Artificial Neural Network Architecture for Brazilian Stock Market Prediction. Annals of Data Science (2020) 7(4):613–628. https://doi.org/10.1007/s40745-020-00305-w

Rathnayaka, R. M. Kapila T., Wei J., and DMK N. S. (2014) Geometric brownian motion with ito's lemma approach to evaluate market fluctuations: A case study on colombo stock exchange. Paper presented at the *2014 International Conference on Behavioral, Economic, and Socio-Cultural Computing* (BESC2014), Shanghai, China, October 30–November 2, Piscataway: IEEE, pp. 1–6.

Sengupta, C. (2004). Financial modeling using Excel and VBA. New Jersey, United States: John Wiley & Sons.

Udomsak, N. (2015). How do the naive Bayes classifier and the Support Vector Machine compare in their ability to forecast the Stock Exchange of Thailand. *arXiv preprint arXiv:1511.08987*.

Zhang, G. B. Eddy P. & Michael Y. H. (1998). Forecasting with artificial neural networks: The state of the art. *International Journal of Forecasting* 14: 35–62.