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ON EXPONENTIALLY WEIGHTED MOVING AVERAGE CONTROL CHART FOR TIME TRUNCATED LIFE TEST USING INVERSE WEIBULL DISTRIBUTION

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ABSTRACT

Statistical control charts are widely used in industry for process and measurement control to monitor production processes in order to discover any problem or issues that may arise during the production process and to help in finding solutions for these issues. This research proposes the exponentially weighted moving average (EWMA) control chart, comparing it's performance to NP and MA control chart for monitoring the number of nonconforming with the truncated lifetime of a product when the lifetimes of products follows inverse Weibull (IW) distribution. The number of failures during the life test is used as an indicator of the quality of the product. EWMA control chart was proposed for this specific situation, thus extending the applicability of control charts methodology to situations involving truncated life tests. The performance of these control charts was measured with the average run length (ARL). The simulation shows that the results obtained from the EWMA chart using IW distribution is more sensitive in detecting small shift and performs much better to monitoring shifts as when compared to other control charts considered in the study.

Keywords: Control Charts, EWMA, Inverse weibull distribution, Moving average, average rerun

INTRODUCTION

To ensure the quality standards of products, every industry must develop some mechanism by adopting suitable statistical quality control techniques and procedures, Walter Shewhart introduced the idea of a control chart during 1920's to monitor the quality of the product, which is still popular and being used with some modification to eliminate abnormal variations through separation of variations caused by special causes of variation or common causes. Montgomery(2008).The process is deemed out-of control if the points fall outside the control limits that are calculated based on a mathematical formula or if a nonrandom pattern is detected, Baklizi and Ghannam (2022). The attribute control charts are particularly important in non-manufacturing quality improvement efforts in which the targeted quality characteristics are impossible to measure on a numerical scale, Montgomery (2009). The number of defects in an item produced by a manufacturing process is common to monitor for improving the quality of the product, Alghamdi*et. al* (2017). It assigns weight to the observations in geometrically decreasing order so that the most recent observations have higher contribution, while the contribution of the oldest ones is very little. Control charts are a fundamental tool of statistical process control (SPC), control charts are now widely used, not only in industry, but also in many other areas with real applications, such as health care manufacturing processes, environmental sciences etc. Shewhart (1931), developed the first control chart considered as the main tool of SPC using statistical principles in generating. It is sometimes called the Shewhart chart, which is a chart using the data of the previous production process to scatter a plot and consider the production process. Thus, the pattern of the scatter plot cannot be seen if the production process does not change significantly. For this reason, the Shewhart chart is good at detecting larger shifts in the process. Khoo (2004), studied the Moving average (MA) control chart for detecting the fraction of non-conforming observations and showed that the Moving average (MA) chart had better efficiency than the p chart. Roberts (1959), invented a control chart that could detect changes in the production process, even if the change was only slight, called the cumulative sum (CUSUM) chart and the exponentially weighted moving average (EWMA) chart, which makes them more sensitive than Shewhart chart for detecting the smaller and moderate shifts in the process. Many authors such as focus on the designing of Exponential weighted moving average(EWMA) and moving average(MA) charts for various situations. Abbas *et. al*(2012), proposed the Exponential weighted moving average cumulative sum(EWMA–CUSUM) charts for monitoring correlated data using the Average Run Length (ARL), extra quadratic loss, and relative Average Run Length (ARL) as criteria to measure the efficiency with Shewhart, CUSUM, EWMA, Shewhart-CUSUM, and Shewhart-EWMA charts. The newly proposed control charts have efficiency in detecting better than the compared charts, Zaman *et. al*(2014), proposed the CUSUM EWMA chart to detect the change of variation in the process using the ARL, extra quadratic loss, and relative Average Run Length as criteria to measure the efficiency with Shewhart, EWMA and CUSUM charts. Johnston (1993), Carson and Yeh (2008), shows how to modify the smoothing constant for use in an Exponentially Weighted Moving Average system(EWMA), when data has to be combined either across a number of periods, or is available from only a fraction of a normal forecast review interval. Lal and Kane (2018), To the best of our knowledge, there are no studies about the on exponentially weighted moving average control chart for time truncated life test using inverse weibull distribution. The specific structure of this paper is as follows. Section1 introduction, Section 2 introduces inverse weibull distribution and the control chart method of the study, 3 results and discussion, Section 4. Result and discussion, section 5 Conclusion.

MATERIAL AND METHODOLOGY

Inverse Weibull Distribution and Control Chart

The Inverse Weibull distribution is a continuous distribution which deals with extreme events. The Inverse Weibull (IW) distribution can be readily applied to modeling processes in reliability, ecology, medicine, branching processes and biological studies. Youlong Wu et al. (2020), contributed that to monitor the failure time of systems or components, one can utilize LCL and UCL. If the time between failures plotted on the chart is below the LCL, it is an indication of increasing failure rate or system deterioration. If the plotted time is above the UCL, it shows improvement in the process. The properties and applications of IW distribution in several areas can be seen in the literature (Keller, et al (1985), Calabria and Pulcini (1989), (1990), (1994), Johnson (1984), Khan et al. (2008). A random variable X has an Inverse Weibull distribution with shape parameter α and scale parameter β having the Probability Density Function (PDF) given as;

$$f(x) = \frac{\alpha}{\beta}\left(\frac{\beta}{x}\right)^{\alpha+1} exp\left[-\left(\frac{\beta}{x}\right)^{\alpha}\right], x \geq 0, \alpha > 0, \beta > 0 \quad (1)$$

and the Cumulative Distribution Function is given as;

$$F(x) = exp\left[-\left(\frac{\beta}{x}\right)^{\alpha}\right], x \geq 0, \alpha > 0, \beta > 0 \quad (2)$$

Where α and β represents shape and scale parameter respectively. As mentioned in Aslam and Jun (2015), the shape parameter and scale parameter could be assumed to be known in practice. The shape and scale parameter can also be estimated from the data if it is unknown but in this case it is assumed to be known as α is 12.091 and β is 17.637. This assumption may follow from engineering experience we may use estimators from previous studies as x represents the life time of the products, Baklizi and Ghannam (2022). The average life time of the product is given as follows

$$\mu = \beta\Gamma\left(1 - \frac{1}{\alpha}\right) \quad (3)$$

where $\Gamma(.)$ is the gamma function.

Let μ_0 be the target mean life when the process is in control. We would like to design a control chart for monitoring the mean shift by observing the number of failed products by the specified time t_0 (truncated time). It should be noted that the subscript 0 indicates the in-control process or the specified value.

$$p = exp\left[-\left(\frac{\beta}{x}\right)^{\alpha}\right] \quad (4)$$

$t_0 = a\mu_o$ for a constant a (called a truncated time constant) and we express the unknown parameter λ in terms of m using [1], then Eq. [3] can be rewritten as

$$p_0 = exp\left[-\left(\frac{a\mu_o}{\beta}\right)^{-\alpha}\right] \quad (5)$$

$$p_0 = exp\left[-\left(\frac{a\beta\Gamma(1-\frac{1}{\alpha})}{\beta}\right)^{-\alpha}\right] \quad (6)$$

$$p_1 = exp\left[-\left(\frac{a\mu_o/c}{\beta}\right)^{-\alpha}\right] \quad (7)$$

$$p_1 = exp\left[-\left(\frac{a\beta\Gamma(1-\frac{1}{\alpha})/c}{\beta}\right)^{-\alpha}\right] \quad (8)$$

$$p_1 = exp\left[-\left(\frac{\left(a\Gamma(1-\frac{1}{\alpha})\right)}{c}\right)^{-\alpha}\right] \quad (9)$$

Maximum Estimation of Inverse Weibull Distribution

Estimation of Parameters of the Inverse Weibull with Maximum Likelihood Method from equation (3.1), the likelihood function of $X_1, X_2, \ldots, X_n$ can be constructed from the probability density function with unknown scale, β and shape, α parameters, Gebizlioglu et. al. (2011).

$$L(x,\alpha,\beta) = \prod_{i=1}^{n}\left(\frac{\alpha}{\beta}\left(\frac{\beta}{x_i}\right)^{\alpha+1} exp\left[-\left(\frac{\beta}{x_i}\right)^{\alpha}\right]\right) \quad (11)$$

The log likelihood function by taking the natural logarithm as

$$l(x,\alpha,\beta) = \sum_{i=1}^{n} log\left(\frac{\alpha}{\beta}\right) + (\alpha + 1)\sum_{i=1}^{n} log\left(\frac{\beta}{x_i}\right) - \sum_{i=1}^{n} log\left(\frac{\beta}{x_i}\right)^{\alpha} \quad (12)$$

NP Control Chart for Inverse Weibull Distribution

$$LCL = Max\left[0, np_0 - k\sqrt{np_0(1-p_0)}\right] \quad (13)$$

$$UCL = np_0 + k\sqrt{np_0(1-p_0)} \quad (14)$$

For the Out of Control

$$LCL = Max\left[0, np_1 - k\sqrt{np_1(1-p_1)}\right] \quad (15)$$

$$UCL = np_1 + k\sqrt{np_1(1-p_1)} \quad (16)$$

Moving Average

The moving average statistic of size w at time i for the number of failures (D_{oi}'s) is computed by

$$MA_i = \frac{D_i + D_{i-1} + D_{i-2} + \cdots + D_{i-w+1}}{w} \quad (17)$$

D_i is distributed as binomial distribution with mean E(D_i) = np_0

$$Var(D_i) = np_0(1-p_0) \quad (18)$$

E(MAi) = E(Di) = np_0 (19)

Var(MAi) = $\frac{1}{W^2}\Sigma_{j=1-w+1}^{1}$ Var (Di) = $\frac{1}{W^2}\Sigma_{j=1-w+1}^{1} np_0(1-p_0) = \frac{np_0(1-p_0)}{W}$ (20)

Di = Number of failures

$$LCL = Max\left[0, np_0 - k\sqrt{\frac{np_0(1-p_0)}{w}}\right] \quad (21)$$

$$UCL = np_0 + k\sqrt{\frac{np_0(1-p_0)}{w}} \quad (22)$$

For the Out of Control

$$LCL = Max\left[0, np_1 - k\sqrt{\frac{np_1(1-p_1)}{w}}\right] \quad (23)$$

$$UCL = np_1 + k\sqrt{\frac{np_1(1-p_1)}{w}} \quad (24)$$

Exponential Weighted Moving Average

The EWMA – Exponentially Weighted Moving Average chart is used in statistical process control to monitor variables (or attributes that act like variables) that make use of the entire history of a given output. This is different from other control charts that tend to treat each data point individually.

Therefore

$i = 1,\ z_1 = \lambda x_1 + (1-\lambda)z_0$

$i = 2,\ z_2 = \lambda x_2 + (1-\lambda)z_1$

$= \lambda(1-\lambda)^0 x_2 + (1-\lambda)^1\lambda x_1 + (1-\lambda)z_0$

$i = 3,\ z_3 = \lambda x_3 + (1-\lambda)z_2$

$= \lambda(1-\lambda)^0\lambda x_3 + \lambda x_2 + (1-\lambda)^2\lambda x_1 + (1-\lambda)^3 z_0$ (25)

Which can be recursively written as

$$z_i = \sum_{j=0}^{i=1}(1-\lambda)^j\ x_{i-j} + (1-\lambda)^i z_0 \quad (26)$$

$$UCL = \mu_0 + L_\delta\sqrt{\frac{\lambda}{2-\lambda}(1-\lambda(1-\lambda)^{2i})} \quad (28)$$

Centre line = μ_0

$$LCL = \mu_0 + L_\delta\sqrt{\frac{\lambda}{2-\lambda}(1-\lambda(1-\lambda)^{2i})} \quad (29)$$

Therefore, the control limits of the EWMA control chart can be given as

$$UCL/LCL = \mu_0 \pm H_2\sqrt{\delta_{\bar{X}}\left(\frac{\lambda}{2-\lambda}\right)} \quad (30)$$

Where H_2 the coefficient of control limit of EWMA control chart is, μ_0 is the mean of the process and variance is $\delta^2_{\bar{X}}$.

Average Run Length

The ARL may be defined as the average number of samples to be plotted before the process indicates an out-of-control signal C. Chananet, (2014).

$$P_{in}^0 = P\{LCD \leq D \leq UCL|p_0\}$$

$$P_{in}^0 = \sum_{d=|LCL|+1}^{UCL}\binom{n}{d}(p_0)^d(1-p_0)^{n-d} \quad (31)$$

$P_{in}^{1} = P\{LCD \leq D \leq UCL|p_1\}$

$$P_{in}^{1} = \sum_{d=|LCL|+1}^{UCL} \binom{n}{d} (p_1)^d (1-p_1)^{n-d} \quad (32)$$

$$ARL_0 = \frac{1}{1-P_{in}^{0}} \quad (33)$$

$$ARL_1 = \frac{1}{1-P_{in}^{1}} \quad (34)$$

RESULT AND DISCUSSION

The descriptive statistics of the variables, the defectives of 90 samples, Exponential Moving Average data at λ 0.3, 0.2 and 0.1, Moving Average data at weight of 3 and 5 are presented. Goodness of fit tests, estimation of the parameters with Maximum Likelihood, estimation of the LCL and UCL of the variables with in-control and out of control test will be carried out in this section.

Table 1: Descriptive Statistics of the Variables

Variable	Duration	Data Size	Mean	Std Deviation	Skewness	Kurtosis
Defeat	Jan-March 2022	90	17.778	1.7403	-0.22523	-1.3034
MA03	Jan-March 2022	90	17.789	1.1356	0.0966	-0.16197
MA05	Jan-March 2022	90	17.764	0.92776	-0.00024	0.02896
Ewma33	Jan-March 2022	90	17.748	0.70195	0.03494	-0.87867
Ewma32	Jan-March 2022	90	17.718	0.6157	-0.09536	-0.79729
Ewma31	Jan-March 2022	90	17.631	0.56551	-0.51488	-0.24959
Ewma53	Jan-March 2022	90	17.721	0.65349	-0.05727	-0.93754
Ewma52	Jan-March 2022	90	17.692	0.60504	-0.20415	-0.80249
Ewma51	Jan-March 2022	90	17.606	0.5836	-0.57322	-0.30146

Table:1 shows the variable, sample size, mean, standard deviation, skewness and kurtosis of the data.

Table 2: NP Control Chart

Size	Control Constant(a)	CL	LCL	UCL
20	0.99	3.7683	0	9.0146
30	0.99	5.6589	0	12.0872
40	0.99	7.5495	0.1251	14.9739
50	0.99	9.4491	1.1442	17.7539
60	0.99	11.3523	2.2507	20.4539
90	0.99	17.0229	5.8771	28.1687

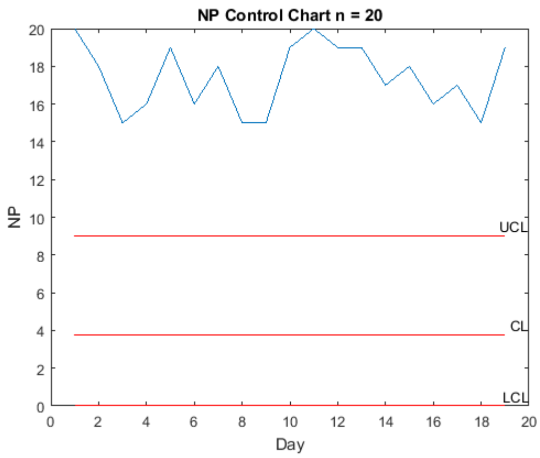

Figure 1: NP Control Chart n = 20

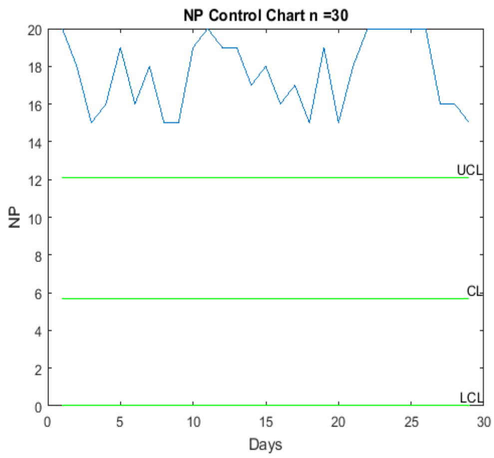

Figure 2: NP Control Chart n = 30

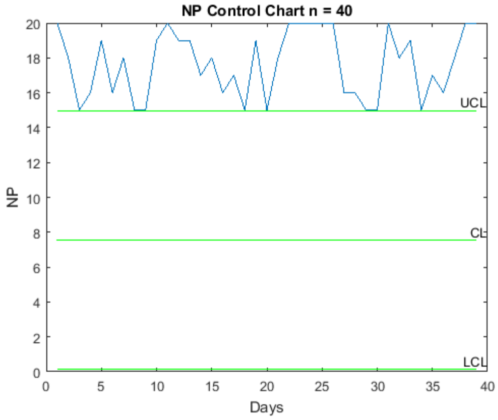

Figure 3: NP Control Chart n = 40

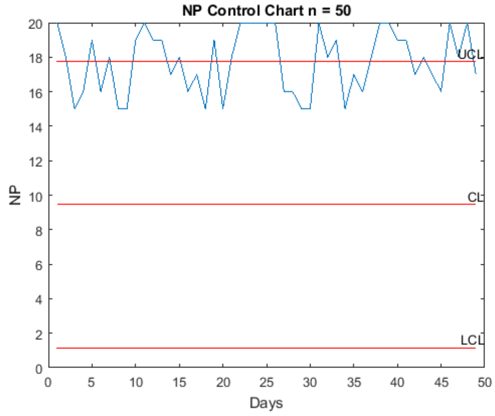

Figure 4: NP Control Chart n = 50

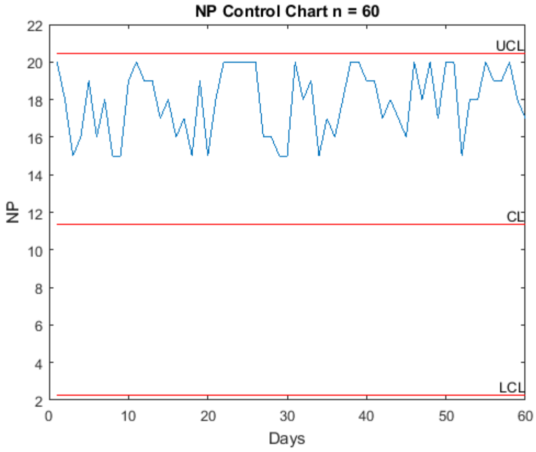

Figure 5: NP Control Chart n = 60

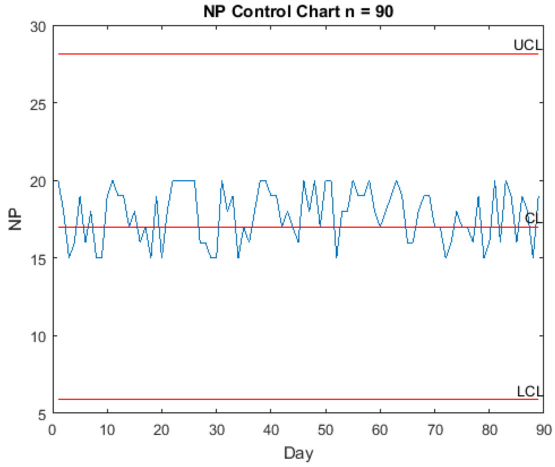

Figure 6: NP Control Chart n = 90

Table 3: Moving Average Control Chart at w = 5

Size	Control Constant(a)	CL	LCL	UCL
20	0.99	3.7808	0	9.0338
30	0.99	5.6827	0	12.1214
40	0.99	7.5749	0.1409	15.0089
50	0.99	9.4805	1.1651	17.7959
60	0.99	11.3894	2.2764	20.5024

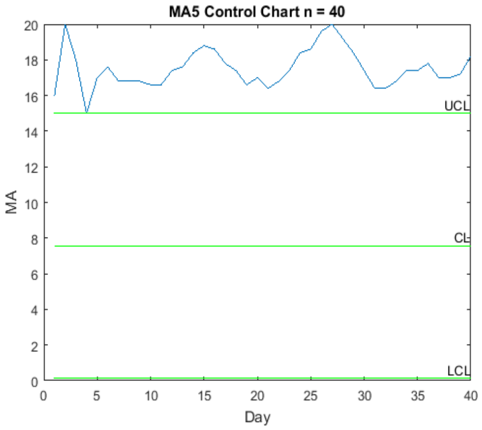

Figure 7: MA05 Control Chart n = 40

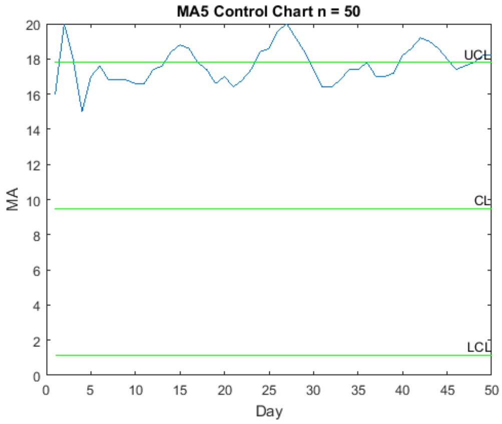

Figure 8: MA05 Control Chart n = 50

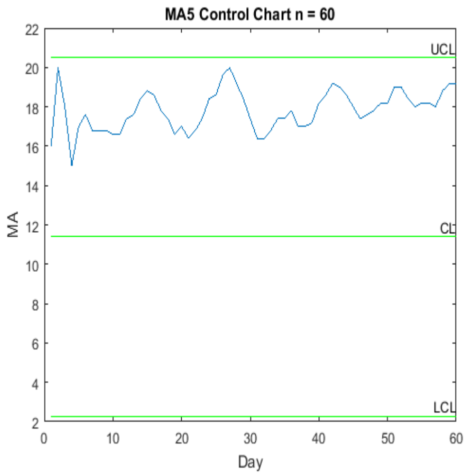

Figure 9: MA05 Control Chart n = 60

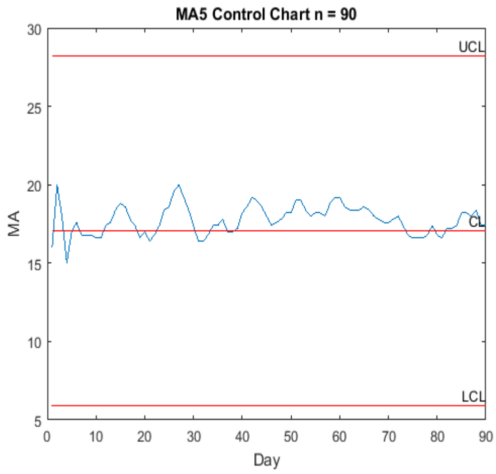

Figure 10: MA05 Control Chart n = 90

Table 4: Exponential Weighted Moving Average at λ=0.2

Size	Control Constant(a)	CL	LCL	UCL
20	0.99	3.7857	2.0338	5.5375
30	0.99	5.6863	3.5396	7.8331
40	0.99	7.5845	5.1053	10.0637
50	0.99	9.4895	6.7167	12.2623
60	0.99	11.3969	8.3584	14.4353
90	0.99	17.0986	13.3771	20.8202

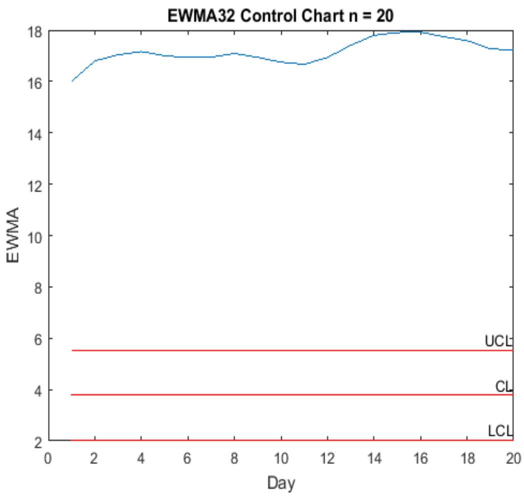

Figure 11: EWMA32 Control Chart n = 20

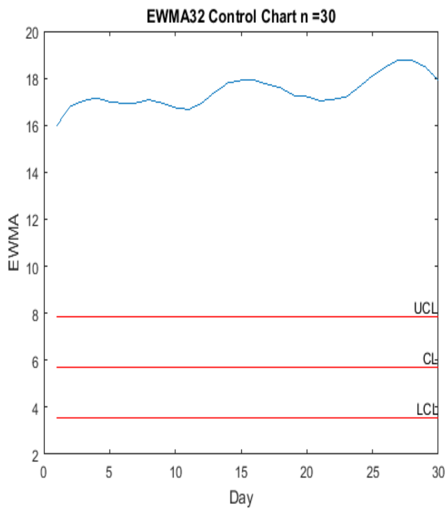

Figure 12: EWMA32 Control Chart n = 30

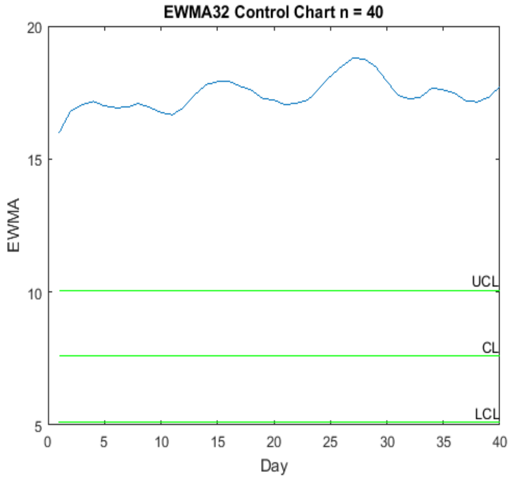

Figure 13: EWMA32 Control Chart n = 40

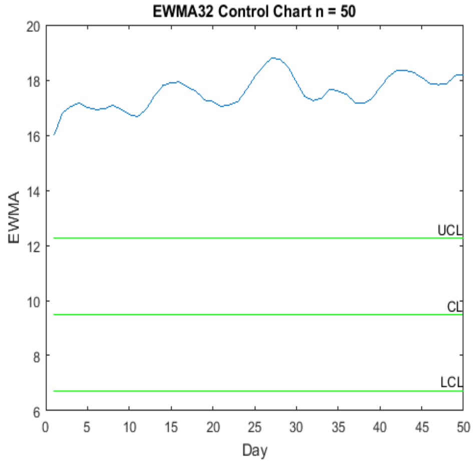

Figure 14: EWMA32 Control Chart n = 50

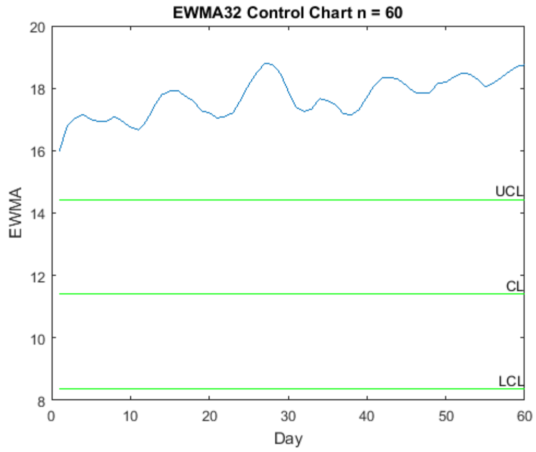

Figure 15: EWMA32 Control Chart n = 60

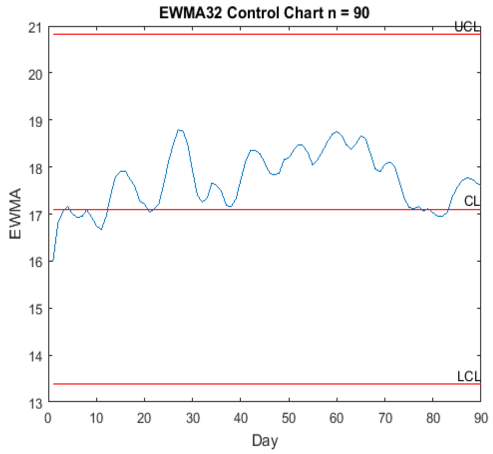

Figure 16: EWMA32 Control Chart n = 90

Average Run Length

Table 5: Average Run Length at n = 90

c	NP	MA3	MA5	EWMA33	EWMA32	EWM31	EWMA53	EWMA52	EWMA51
1	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00	370.00
0.9	1.20	37.18	46.20	156.04	40.29	36.66	115.33	40.29	1225.53
0.8	1.03	16.43	2.18	44.35	15.14	18.95	31.68	15.14	573.29
0.7	1.00	1.89	1..04	3.43	5.05	9.03	7.43	5.05	50.31
0.6	1.00	1.03	1.00	1.52	1.31	3.07	1.34	1.31	17.11
0.5	1.00	1.00	1.00	1.02	1.01	1.61	1.01	1.02	1.20
0.4	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.01
0.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Discussion

From the ARL tables above, we observe that the out-of-control ARLs are small when there is a small shift in the mean and the values increase as the mean shift increases. We noted that the simulated values of the ARL are very close to the target values (370) when the shift parameter c = 1. EWMA chart detects faster than the other charts after evaluating the performance using average run length.

CONCLUSION

A moving average (MA) and NP control charts was observed to be less effective than the proposed exponential weighted moving average (EWMA), when monitoring the number of failures under a time-truncated life test when the life of an item follows an Inverse Weibull distribution. It was observed that the EWMA control chart will be an efficient addition to the toolkit of the quality control practitioners when working on inverse Weibull distribution as it was noted to detect small process shifts more quickly than the other charts.

It is shown that the EWMA control chart outperforms the NP and MA control chart for all shift parameters after evaluating with the ARL. The result shows that the value of ARL decreases as n increases from 20 to 90 and there was also a rapid decrease as *c* decreases from 1.0 to 0.1.

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