APPLICATION OF NON-STANDARD FINITE DIFFERENCE METHOD ON COVID-19 MATHEMATICAL MODEL WITH FEAR OF INFECTION

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ABSTRACT
This study presents a novel application of Non-Standard Finite Difference (NSFD) Method to solve a COVID-19 epidemic mathematical model with the impact of fear due to infection. The mathematical model is governed by a system of first-order non-linear ordinary differential equations and is shown to possess a unique positive solution that is bounded. The proposed numerical scheme is used to obtain an approximate solution for the COVID-19 model. Graphical results were displayed to show that the solution obtained by NSFD agrees well with those obtained by the Runge-Kutta-Fehlberg method built-in Maple 18.

Keywords: Differential Equations, COVID-19 Mathematical Model, Non-Standard Finite Difference, Approximate Solution

INTRODUCTION
The use of differential equations to model the transmission dynamics of infectious disease can be traced back to 1970 when Daniel Bernoulli justified the use of inoculation to curb the spread of smallpox (Dietz and Heesterbeek, 2002; Foppa, 2017). These models are usually nonlinear (Peter et al., 2020; Gu et al., 2023; Akinyemi et al., 2023; Kambali et al., 2023; Ochi et al., 2023) and are difficult to obtain their exact solution (Onwubuoya et al., 2018b; Riyapan et al., 2021; ur Rehman et al., 2023).

Thus, numerical methods are used to obtain approximate solutions. Some of the numerical techniques are Euler (Ashigi et al., 2021; Mohammed et al., 2021; Reza et al., 2022), Euler Predictor Corrector (Onwubuoya et al., 2018a), Non-Standard Finite Difference (Raza et al., 2022; Butt et al., 2023; ur Rehman et al., 2023).

The Non-Standard Finite Difference (NSFD) method developed by Ronald E. Mickens is a discrete representation of a continuous model (Mickens and Washington, 2012, Qui et al., 2014). Apart from predicting the behaviour of the dynamical system correctly, the NSFD method is known to preserve the dynamical properties of an epidemic model and is less difficult to implement when compared with the aforementioned numerical methods (Qui et al., 2014). Applications of NSFD method are found in financial theory (Dietz and Heesterbeek, 2002; Peter et al., 2020; Meh dizadeh et al., 2022; Meh dizadeh et al., 2023), epidemiology (ur Rehman et al., 2023; Butt et al., 2023), enzymology (Miller & O’Riordan, 2020; Zafar et al., 2023), pharmacology (Egbelowo, 2018; Egbelowo & Hoang, 2021), immunology (Costa et al., 2023; Elair et al., 2023).

The purpose of this study is to apply the NSFD scheme to solve a mathematical model presented in Ibrahim (2023). The mathematical model proposed by Ibrahim (2023), describes the spread of COVID-19 in the presence of fear of infection and is governed by the following system of nonlinear differential equations.

\[
\begin{align*}
\frac{dS}{dt} &= P - (\Omega S - K_1S + \nu_2 V) \\
\frac{dV}{dt} &= \nu_1 S - \epsilon_2 N V - K_2 V \\
\frac{dE}{dt} &= \Omega(S + eV) - K_3 E \\
\frac{dI}{dt} &= \tau_1 E - K_4 I \\
\frac{dA}{dt} &= \theta_1 E - K_5 A \\
\frac{dQ}{dt} &= \theta_2 A - K_6 \\
\frac{dF}{dt} &= \theta_3 Q + \tau_1 I - K_7 H \\
\frac{dF}{dt} &= \theta_4 Q + \tau_1 I - K_7 H \\
\frac{dH}{dt} &= \alpha_1 A + \alpha_2 I + \alpha_3 H - \mu R \\
\frac{dP}{dt} &= \phi D \\
\frac{d\theta}{dt} &= \Pi I + \Pi \theta H - \epsilon W
\end{align*}
\]

Subject to \( S(0) = 1.885103470 \times 10^8, V(0) = 2.672 \times 10^7, E(0) = 3,500, Q(0) = 400, A(0) = 1,247, I(0) = 900, H(0) = 652, R(0) = 249,911, D(0) = 3143 \) and \( W(0) = 1000 \).

\[
\Omega = \frac{2(\alpha_1 A + H \theta_2 + W \eta_3 + I) - 1}{\chi_1 D + 1}
\]

Table 1: Description of State Variables of the Model

<table>
<thead>
<tr>
<th>State Variable</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>S(t)</td>
<td>Unvaccinated Susceptible Individuals</td>
</tr>
<tr>
<td>V(t)</td>
<td>Vaccinated Susceptible Individuals</td>
</tr>
<tr>
<td>E(t)</td>
<td>Exposed Individuals</td>
</tr>
<tr>
<td>Q(t)</td>
<td>Quarantined Individuals</td>
</tr>
</tbody>
</table>

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Table 2: Description of Parameters for the Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description of Parameters</th>
<th>Hypothetical Values</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_0 )</td>
<td>Total Population of Active Humans</td>
<td>215,497,404</td>
<td>Worldometer, 2022</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Recruitment rate.</td>
<td>( \mu N_0 )</td>
<td>Estimated</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>Estimated average life span of Nigerian</td>
<td>55.75 per year</td>
<td>Worldometer, 2022</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Natural death rate.</td>
<td>( \frac{1}{\mu_0 \times 365} ) per day</td>
<td>Estimated</td>
</tr>
<tr>
<td>( \delta_{1} ), ( \delta_{2} ), ( \delta_{3} )</td>
<td>COVID-19 induced death rate for individuals in ( A, I ) and ( H ) compartments respectively.</td>
<td>0.018, 0.025, 0.01</td>
<td>Estimated</td>
</tr>
<tr>
<td>( \alpha_{1}, \alpha_{c}, \alpha_{3} )</td>
<td>The recovery rate for individuals in ( A, I ) and ( H ) compartments respectively.</td>
<td>0.0195692, 0.004165, 0.0701</td>
<td>Diagne et al., 2021</td>
</tr>
<tr>
<td>( b )</td>
<td>Rate of COVID-19 efficacy</td>
<td>0.6309</td>
<td>WHO, 2021</td>
</tr>
<tr>
<td>( v_1 )</td>
<td>Vaccination rate for susceptible individuals</td>
<td>0.4</td>
<td>Diagne et al., 2021</td>
</tr>
<tr>
<td>( v_2 )</td>
<td>Waning rate of COVID-19 vaccine</td>
<td>0.095</td>
<td>Paul and Kuddus, 2022</td>
</tr>
<tr>
<td>( \tau_1 )</td>
<td>Quarantine rate for exposed individuals</td>
<td>0.012</td>
<td>Nana-Kyere et al., 2022</td>
</tr>
<tr>
<td>( \theta_{1}, \theta_{2} )</td>
<td>Progression rate for individuals in ( E ) to ( A ) and ( A ) to ( I ) compartments respectively.</td>
<td>0.70.08</td>
<td>Srivastav et al., 2021</td>
</tr>
<tr>
<td>( \theta_{3}, \tau_{2} )</td>
<td>The hospitalization rate for individuals in ( Q ) and ( I ) compartments respectively.</td>
<td>0.06, 0.02</td>
<td>Nana-Kyere et al., 2022</td>
</tr>
<tr>
<td>( \eta_{1}, \eta_{c}, \eta_{3} )</td>
<td>Modification parameters associated with reduction of infectiousness for individuals in ( A, H ) and ( W ) as compared to ( I ) class respectively.</td>
<td>0.75, 0.5, 0.33</td>
<td>Garba et al., 2020</td>
</tr>
<tr>
<td>( \pi )</td>
<td>Shedding rate of coronavirus into the environment.</td>
<td>0.002</td>
<td>Garba et al., 2020</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Modification parameters associated with reduction of shedding for individuals in ( H ) as compared to ( I ) class respectively.</td>
<td>0.5</td>
<td>Garba et al., 2020</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>The decay rate of coronavirus in the environment.</td>
<td>0.85</td>
<td>Garba et al., 2020</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Burial rate of dead infectious individuals.</td>
<td>0.2276 (0.1)</td>
<td>Aba Oud et al., 2021</td>
</tr>
<tr>
<td>( \chi_1 )</td>
<td>Level of fear associated with COVID-19 infection.</td>
<td>0.2276 (0.1)</td>
<td>Estimated</td>
</tr>
<tr>
<td>( \beta )</td>
<td>COVID-19 transmission coefficient</td>
<td>( \frac{\beta_0}{N_0} )</td>
<td>Estimated</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \beta_0 = 0.1086 )</td>
<td>Adewole et al., 2021</td>
</tr>
</tbody>
</table>

The rest of this paper is arranged as follows: Section 2 presents the material and methods. Results and discussion are addressed in Section 3. Section 4 gives the conclusion of the study.
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MATERIALS AND METHODS
This section deals with the introduction to NSFD, the dynamical properties of Model (1) and the application of NSFD on Model (1).

Basic Concept of NSFD
First, we consider an autonomous ordinary differential equation of the form
\[ \frac{dx}{dt} = f(x(t)) \]  
(2)
Definition 1: A discretized form of (2) is called an NSFD scheme provided at least one of these conditions is satisfied.
1. The discretized representation of (2) is
\[ \frac{dx}{dt} \rightarrow \frac{\Delta x(t)}{\Delta t} \quad n = 0, 1, \cdots M - 1 \]  
(3)
such that \( t_n = t_0 + \Delta t \), \( x_n = x(t_n) \), \( h = \frac{T}{M} \) the numerator function \( G(h) = 1 + O(h) \), and the denominator function \( Z(h) = 1 + O(h^2) \).
2. The nonlinear term \( f(x) \) in (2) should be approximated using the nonlocal discretized form. For instance,
\[ x^2 \approx x_n x_{n+1} \]  
(4)
Here, \( T \) is the final time, \( h \) the time step size and \( M \) the number of iterations. Again, we consider a system of first-order nonlinear differential equations
\[ \begin{align*}
\frac{dx_1}{dt} &= -\alpha_1 x_1 x_2 - b_1 x_1 \\
\frac{dx_2}{dt} &= \alpha_1 x_1 x_2 + b_2 x_2
\end{align*} \]  
(5)
subject to \( x_1(0) = c_1 \) and \( x_2(0) = c_2 \).
Discretized (5) using the semi-implicit finite scheme while ensuring that the above condition conditions are met to have
\[ \begin{align*}
\frac{X_{1,n+1} - g_{1} X_{1,n}}{z_i} &= -a_1 X_{1,n+1} X_{2,n} - b_1 X_{1,n+1} \\
\frac{X_{2,n+1} - g_{2} X_{2,n}}{z_i} &= a_1 X_{1,n+1} X_{2,n} - b_2 X_{2,n+1}
\end{align*} \]  
(6)
Following Ahmed (2011) and Sweilam et al. (2017), to have \( G_1 = G_2 = 1 \) and an exponential denominator function \( Z_1 = e^{\beta_{1} t} \) and \( Z_2 = e^{\beta_{2} t} \), are used. Hence, (6) becomes
\[ \begin{align*}
X_{1,n+1} &= \frac{x_n(n x_{n+1} + b_2 + 1)}{z_i} \\
X_{2,n+1} &= \frac{x_n(n x_{n+1} + b_2 + 1)}{z_i}
\end{align*} \]  
(7)
Remark [Sweilam et al. (2017)]: Whenever the denominator function \( Z(h) = h \), the scheme is called NSFD-I, otherwise it is called NSFD-II.
Thus, this study utilizes the NSFD-II scheme. Next, the dynamical properties such as the existence and uniqueness, positivity and boundedness solution of Model (1) are examined.

Existence and Uniqueness Solution of the Covid-19 Model
Theorem 2.1: The system (1) has a unique solution in the region \((S, V, E, Q, A, I, H, R, D, W) \in \mathbb{R}_{+}^{10}\)
Proof: We write the right-hand side of Model (1) as
\[ \begin{align*}
f_1 &= P - \alpha S - K_1 S + \nu V \\
f_2 &= \nu S + \sigma V - \alpha E - K_2 E \\
f_3 &= \sigma V - \alpha E - R - K_3 E \\
f_4 &= \sigma E + \alpha D + \alpha_3 H - R - K_3 E \\
f_5 &= \alpha V + \alpha_2 D + \alpha_4 E - K_3 E \\
f_6 &= \alpha D + \alpha_5 D + \alpha_6 H - K_3 E \\
f_7 &= \alpha H + \alpha_7 H + \alpha_8 H - K_3 E \\
f_8 &= \alpha R + \alpha_9 R + \alpha_10 R + \alpha_11 R + \alpha_12 R - K_3 E \\
f_9 &= \alpha I + \alpha_13 I + \alpha_14 I + \alpha_15 I - K_3 E \\
f_{10} &= \alpha_16 I + \alpha_17 I + \alpha_18 I + \alpha_19 I - K_3 E
\end{align*} \]  
(8)

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The solution of (15) yields

\[ \frac{\partial f_0}{\partial S} = \frac{\partial f_0}{\partial V} = \frac{\partial f_0}{\partial E} = \frac{\partial f_0}{\partial Q} = \frac{\partial f_0}{\partial K} = 0 \leq \infty, \quad \frac{\partial f_0}{\partial A} = |\alpha_1| \leq \infty, \quad \frac{\partial f_0}{\partial I} = |\alpha_2| \leq \infty, \quad \frac{\partial f_0}{\partial R} = |\alpha_3| \leq \infty. \]

Therefore as \( S \leq \infty \), \( V \leq \infty \), \( E \leq \infty \), \( Q \leq \infty \), \( I \leq \infty \), \( R \leq \infty \), \( D \leq \infty \), \( W \leq \infty \), the state variables are non-negative.

Hence, the state variables are non-negative since their initial conditions are non-negative. Therefore, the solution set

\[ \{ (S(t), V(t), E(t), Q(t), A(t), I(t), H(t), R(t), D(t), W(t)) \} \]

of Model (1) is non-negative \( \forall t \geq 0 \), provided the initial conditions are non-negative.

**Proof:** It is readily seen that the first equation of Model (1) satisfies

\[ \frac{dS}{dt} \geq -(\alpha + K_1)S \quad (9) \]

Solve (9) using the separable variable techniques to obtain

\[ S(t) \geq S(0)e^{-\left(\int^{t_0}_0 (\alpha + K_1) \right)} \geq 0 \quad \forall t \geq 0. \]

Similarly, the second equation of system (1) gives

\[ \frac{dV}{dt} \geq -(e\Omega + K_2)\frac{V}{V(t)} = \frac{dV}{dt} \geq -(e\Omega + K_2)V \quad (10) \]

The solution of (10) gives

\[ V(t) \geq V(0)e^{-\left(\int^t_0 (e\Omega + K_2) \right)} \geq 0 \quad \forall t \geq 0. \]

Following a similar argument, the rest equations of system (1) yields

\[ E(t) \geq E(0)e^{-\varepsilon t} \geq 0 \quad \forall t \geq 0, \quad Q(t) \geq Q(0)e^{-\varepsilon t} \geq 0 \quad \forall t \geq 0, \quad A(t) \geq A(0)e^{-\mu t} \geq 0 \quad \forall t \geq 0, \quad I(t) \geq I(0)e^{-\mu t} \geq 0 \quad \forall t \geq 0, \quad H(t) \geq H(0)e^{-\varepsilon t} \geq 0 \quad \forall t \geq 0, \quad R(t) \geq R(0)e^{-\mu t} \geq 0 \quad \forall t \geq 0, \quad D(t) \geq D(0)e^{-\mu t} \geq 0 \quad \forall t \geq 0, \quad W(t) \geq W(0)e^{-\varepsilon t} \geq 0 \quad \forall t \geq 0. \]

Hence, the state variables are non-negative since their initial conditions

\[ \{ (S(0), V(0), E(0), Q(0), A(0), I(0), H(0), R(0), D(0), W(0)) \} \] are not negative. Hence, we conclude the proof.

**Bounding of Solution**

**Theorem 3:** The set \( \Theta = \{(S, V, E, Q, A, I, H, R, D, W) \in \mathbb{R}^{10}_+; N \leq \frac{P}{\mu}; D \leq \frac{d_1}{\phi \mu}; W \leq \frac{P(1+\theta)}{\mu} \} \) is positively invariant and attractive to Model (1).

**Proof:** Since \( N(t) = S(t) + V(t) + E(t) + Q(t) + I(t) + H(t) + R(t) \) then the rate of change of the total active population has been obtained by adding the first-eight equations of the system (1) to get

\[ \frac{dN}{dt} = P - \mu N - \delta_A A - \delta_I I - \delta_H H \quad (12) \]

It is readily seen that (12) becomes

\[ \frac{dN}{dt} \leq P - \mu N \quad (13) \]

Solve (13) by integrating factor to have

\[ N(t) \leq \frac{P}{\mu} + \left( N(0) - \frac{P}{\mu} \right) e^{-\mu t} \quad \forall t \geq 0. \]

(14)

Therefore as \( t \to \infty \), \( 0 \leq N(t) \leq \frac{P}{\mu} \).

It is readily seen that \( A(t) \leq \frac{P}{\mu} \), \( I(t) \leq \frac{P}{\mu} \) and \( H(t) \leq \frac{P}{\mu} \) since \( 0 \leq N(t) \leq \frac{P}{\mu} \). Then, the ninth equation of the system (1) gives

\[ \frac{dD}{dt} \leq \frac{d_1}{\phi \mu} - \phi D \quad (15) \]

Where \( d_1 = \delta_1 + \delta_2 + \delta_3 + 3\mu \)

The solution of (15) yields

\[ D(t) \leq \frac{d_1}{\phi \mu} + \left( D(0) - \frac{d_1}{\phi \mu} \right) e^{-\phi t} \quad \forall t \geq 0. \]

(16)
As \( t \to \infty \), \( 0 \leq D(t) \leq \frac{d_\theta \rho}{\phi \mu} \).

Similarly, the solution of the last equation of system (1) is obtained as

\[
W(t) \leq \frac{p \eta (1 + \theta)}{\mu \varepsilon} + \left( W(0) - \frac{p \eta (1 + \theta)}{\mu \varepsilon} \right) e^{-\mu t} \quad \forall t \geq 0. 
\]

Hence, as \( t \to \infty \), \( 0 \leq W(t) \leq \frac{p \eta (1 + \theta)}{\mu \varepsilon} \). Therefore, \( \theta \) is positively invariant since \( N \), \( D \) and \( W \) are bounded.

**Application of NSFD2**

The continuous dynamical model (1) is converted to its discrete form based on the rules and steps outlined in Section 2. Thus (1) becomes

\[
S_{n+1} - S_n = \frac{-(a_n \eta_1 + H_n \eta_2 + W_n \eta_3 + l_n) S_n + K_i S_n + V_n \eta_2 + P}{\mu \varepsilon} - K_1 S_{n+1} + \frac{V_n \eta_2 + P}{\mu \varepsilon} \quad \forall t \geq 0. 
\]

\[
V_{n+1} - V_n = \frac{-(a_n \eta_1 + H_n \eta_2 + W_n \eta_3 + l_n) V_n + K_i V_n}{\mu \varepsilon} - K_2 V_{n+1} + S_{n+1} V_1 
\]

\[
E_{n+1} - E_n = \frac{-(a_n \eta_1 + H_n \eta_2 + W_n \eta_3 + l_n) E_n + K_i E_n}{\mu \varepsilon} - K_4 E_{n+1} \quad \forall t \geq 0. 
\]

\[
Q_{n+1} - Q_n = \frac{\tau_1 E_{n+1} - K_{q} Q_{n+1}}{\tau_2} 
\]

\[
A_{n+1} - A_n = \frac{\beta_2 A_{n+1} - K_5 A_{n+1}}{\tau_2} 
\]

\[
I_{n+1} - I_n = \frac{\beta_2 A_{n+1} - K_5 A_{n+1}}{\tau_2} \quad \forall t \geq 0. 
\]

\[
H_{n+1} - H_n = \frac{\beta_2 Q_{n+1} + \tau_2 I_{n+1} - K_7 H_{n+1}}{\tau_2} \quad \forall t \geq 0. 
\]

\[
R_{n+1} - R_n = \frac{\alpha_2 A_{n+1} + \alpha_3 H_{n+1} - \mu R_{n+1}}{\tau_2} \quad \forall t \geq 0. 
\]

\[
D_{n+1} - D_n = \frac{I_{n+1} + I_{n+1} + l_2 H_{n+1} - \phi D_{n+1}}{\tau_2} \quad \forall t \geq 0. 
\]

\[
W_{n+1} - W_n = \frac{\mu (I_{n+1} + I_{n+1} + l_2 H_{n+1} - \phi D_{n+1})}{\tau_2} \quad \forall t \geq 0. 
\]

\[
\text{Where,} \quad Z_0 = e^{\mu z_{10}}, \quad Z_0 = e^{\mu z_{10}}, \quad Z_0 = e^{\mu z_{10}}, \quad \text{and} \quad Z_i = e^{\mu z_{10}} \quad \forall i = 1, \ldots, 7. 
\]

Make \( S_{n+1}, V_{n+1}, E_{n+1}, Q_{n+1}, A_{n+1}, I_{n+1}, H_{n+1}, R_{n+1}, D_{n+1}, \text{and} \ W_{n+1} \) subject formula from (18)-(27) respectively to have

\[
S_{n+1} = \left[ (V_n \eta_2 + P) (Z_n + Z_1) \right] (Z_n + Z_1) 
\]

\[
V_{n+1} = \left[ (V_n \eta_2 + P) (Z_n + Z_1) \right] (Z_n + Z_1) 
\]

\[
E_{n+1} = \left[ (V_n \eta_2 + P) (Z_n + Z_1) \right] (Z_n + Z_1) 
\]

\[
Q_{n+1} = \left[ (V_n \eta_2 + P) (Z_n + Z_1) \right] (Z_n + Z_1) 
\]

\[
A_{n+1} = \left[ (V_n \eta_2 + P) (Z_n + Z_1) \right] (Z_n + Z_1) 
\]

\[
I_{n+1} = \left[ (V_n \eta_2 + P) (Z_n + Z_1) \right] (Z_n + Z_1) 
\]

\[
H_{n+1} = \left[ (V_n \eta_2 + P) (Z_n + Z_1) \right] (Z_n + Z_1) 
\]

\[
R_{n+1} = \left[ (V_n \eta_2 + P) (Z_n + Z_1) \right] (Z_n + Z_1) 
\]

\[
D_{n+1} = \left[ (V_n \eta_2 + P) (Z_n + Z_1) \right] (Z_n + Z_1) 
\]

\[
W_{n+1} = \left[ (V_n \eta_2 + P) (Z_n + Z_1) \right] (Z_n + Z_1) 
\]

\[
\text{Where} \quad \psi_i = \beta (A_n \eta_1 + H_n \eta_2 + W_n \eta_3 + I_n) \quad \forall i = 1, \ldots, 7. 
\]

\[
\text{RESULTS AND DISCUSSION} 
\]

We simulated the COVID-19 model (29)-(38) for \( T = 150 \) days while using the initial conditions mentioned above by setting the stepsize \( h = 0.01 \) for NSFD-II. To validate the reliability of NSFD-II, the result obtained by NSFD-II was compared with the Runge-Kutta-Fehlberg (RKF45) method built-in Maple 18 software.

The results generated by NSFD-II and RKF45 methods for the population of susceptible individuals are displayed in Figure 1. Both methods show a gradual decrease in the population of susceptible humans for about 10 days and become steady for the remaining simulation period.
The population profile for the vaccinated humans obtained by NSFD-II and RKF45 methods is shown in Figure 2. The figure also shows that both methods agree that the population of vaccinated humans gradually increases first, before becoming steady.

Figures 3-4 depict the population profile for the exposed and quarantined humans respectively. The figures show that both methods describe that the population of the exposed and quarantined decreases to zero.
Figures 5-6 present the population profile for the asymptomatic and symptomatic humans respectively. Both figures show that NSFD-II and RKF45 methods convey that the population of the individuals in the A and I compartments gradually increases first before declining to zero.
A graphical comparison between the results obtained by NSFD-II and RKF45 for the population of hospitalized humans is displayed in Figure 7. It is observed from Figure 7 that first there was a drop in the number of hospitalized humans, shortly followed by an increase before decreasing to zero.
The population profile for the recovered individuals generated by NSFD-II and RKF45 is shown in Figure 8. The figure shows that the number of those who recovered from COVID-19 infection increases for about 60 days and later begins to decrease.

The population and concentration profiles for COVID-19 deceased individuals and COVID-19 viruses in the environment are depicted in Figures 9-10 respectively. Figures 9-10 show that $D(t)$ and $W(t)$ decreases to zero.
Finally, Figures 1-10 show that the result obtained by NSFD-II agrees excellently well with those of RKF45 despite that the default RKF45 built-in Maple 18 software makes use of adaptive step size to ensure that the absolute and relative errors for each iteration are not above $10^{-7}$ and $10^{-6}$. Thus, this makes RKF45 more cumbersome and difficult to implement when compared with NSFD-II.

**CONCLUSION**

An application of NSFD-II to solve a deterministic mathematical model for the transmission dynamics of COVID-19 in the presence of fear of infection was considered in this study. This model was shown to possess a unique solution that is positive and bounded. The solution obtained by NSFD-II was compared graphically with those obtained by the default Runge-Kutta Fehlberg (RKF45) built-in Maple 18 software. The comparison shows that both methods are in excellent agreement even though the RKF45 is more cumbersome and not easy to implement when compared with NSFD-II. Thus, the use NSFD-II method is reliable and efficient and should be applied to solve other nonlinear real phenomena.
REFERENCES


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