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SINE-LOMAX DISTRIBUTION: PROPERTIES AND APPLICATIONS TO REAL DATA SETS

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ABSTRACT

In this study, a novel distribution called the two-parameter Sine Lomax distribution was introduced. The distribution was developed by combining the Sine generalized family of distributions with the Lomax distribution. Various statistical properties of this new distribution were investigated, including the survival function, hazard function, quantile function, rth moment, entropy, moment generating function, and order statistics. The probability density function (PDF) plot indicated that the distribution is skewed to the right. Additionally, the hazard plot of the Sine Lomax distribution showed both monotonic increase and monotonic decrease. To estimate the parameters of the newly proposed distribution, the maximum likelihood approach was employed. A simulation study was conducted to evaluate the consistency of the estimators. The simulation results indicated that the estimators are consistent, as the bias and mean square error decrease with increasing sample sizes. The performance of the Sine Lomax distribution was compared to other extensions of Lomax distributions and the baseline distribution which is the Lomax distribution using various evaluation criteria, including the Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQIC). The proposed distribution demonstrated the lowest scores among the competing models, indicating its potential for accurately modeling real-world data sets. Based on the results, the proposed Sine Lomax distribution is recommended as a superior alternative to the competing models for modeling certain real-world data sets.

Keywords: Maximum likelihood estimation, entropy, Lomax distribution, hazard function, Moment

INTRODUCTION

The accuracy of parametric statistical inference and data set modeling relies heavily on the goodness of fit between the probability distribution and the given data sets, assuming all distributional assumptions are met. Numerous studies have been conducted to develop distributions with more desirable and flexible properties to effectively model real-world data sets of varying density and failure rate functions. Currently, researchers are focused on creating new hybrid distributions that generalize existing ones, aiming to achieve better data modeling capabilities. These hybrid distributions are formed by combining a baseline distribution with a family distribution. Several authors have extensively reviewed different families of distributions (Hamedani et al., 2018).

The main objective behind constructing this distribution family is to enhance the flexibility of classical distributions, enabling them to provide improved fits for survival data sets compared to other candidate distributions with the same number of parameters. This family should be capable of modeling various types of failure rates, including monotonic and non-monotonic patterns. The Lomax distribution, also known as the Pareto distribution of the second kind with two parameters (α, λ), has attracted significant attention from theoretical and statistical researchers due to its applications in reliability and lifetime testing studies. Lomax first introduced and studied this distribution in 1954, and it has since been utilized for analyzing business failures, as well as in economic, behavioral, scientific, and traffic modeling.

Researchers such as Falgore and Doguwa (2020) employed the Lomax distribution to model firm size data, while Hassan and Al-Ghamdi (2009) used it for reliability and life testing, while Zweig and Cambell (1993) applied it to analyze receiver operating characteristic (ROC) curves. Ijaz et al. (2019) suggested this distribution as a heavy-tailed alternative to the exponential, Weibull, and gamma distributions. Statistical distributions are widely utilized to describe real-life phenomena, and as a result, the field of statistical distribution theory is extensively explored, leading to the development of novel distributions. Statisticians often seek distributions that offer greater flexibility. This has led to a demand for generalized or extended distributions capable of simulating lifetime data with monotonically increasing, decreasing, constant, or more importantly, unimodal bathtub-shaped failure rates. Consequently, there is a necessity to extend traditional distributions or construct new ones, as certain distributions can only accommodate monotonically increasing or decreasing failure rates.

Therefore, in this study, we propose the Sine-Lomax distribution, which can effectively handle datasets exhibiting monotonically or non-monotonically increasing, decreasing, constant, and unimodal bathtub-shaped failure rates. The Lomax distribution, with its heavy-tailed nature, serves as the foundation for this two-parameter distribution.

MATERIALS AND METHODS

Lomax Distribution

Given a non-negative random variable X which follows the Lomax distribution with parameters α and λ, with the CDF is given as:

$$G(x,\alpha,\lambda) = 1 - \left(1+\frac{x}{\lambda}\right)^{-\alpha}, x > 0, \alpha > 0, \lambda > 0 \qquad (1)$$

and the corresponding PDF is expressed as:

$$g(x,\alpha,\lambda) == \frac{\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)} \qquad (2)$$

where, $\alpha > 0$ and $\lambda > 0$ are the shape and scale parameters respectively

The Lomax distribution has been applied to some real-world data sets by researchers in recent times. It was applied to breaking stress of carbon fibers data by Ijaz et al. (2019), in losses due to wind catastrophes recorded in 1977 by Ijaz et al.

(2020), remission times of bladder cancer patients by Chesneau and Jamal (2020).
Some extensions of the Lomax distribution that exists in the literature include: Transmuted Lomax distribution by Ashour and Eltehiwy (2013), Power Lomax distribution by Rady et al. (2016), Type II half logistic Exponentiated Lomax by Bukoye et al. (2021) and Slashed Lomax distribution by Li and Tian (2022).
The rapid progress and accessibility of processing power has spurred recent advancements in stochastic modeling. They have permitted direct applications of existing continuous distributions with some functional complexity for a variety of statistical purposes. Additionally, these have accelerated the development of new and flexible distribution families. Kumar et al. (2015) pioneered the sinusoidal transformation that leads to the sine generated (S-G or Sin-G) family. Some trigonometric families of distribution include Cos-G family by Souza (2015), Polyno-Expo-Trigonometric distributions by Jamal and Chesneau (2019), New Sine-G family by Mahmood et al. (2019), Sine Topp-Leone-G family by Al-Babtain et al. (2020), Hyperbolic Tan-X family by Ampadu (2021), Arcsine family of distribution by Rahman (2021) and Tangent Topp-Leone-G family by Nanga et al. (2022). In this paper, we focused on Sine G family proposed by Kumar et al. (2015) to develop a new probability distribution called the Sine-Lomax Distribution. Some compounding of baseline distribution with the Sine G family proposed by Kumar et al. (2015) includeSine Power Lomax by Nagarjuna et al. (2021), Sine Modified Lindley distribution by Tomy et al. (2021),Sine-Exponential distribution by Isa et al. (2022a), and Sine Burr XII by Isa et al. (2022b) among others.

Sine Family of Distribution
The CDF and PDF of the Sine G family of distributions proposed by Kumar et al. (2015) are defined by the following equations:

$$F_S(x;\xi) = \sin\left[\frac{\pi}{2}G(x;\xi)\right], \quad x \in \mathbb{R} \qquad (3)$$

$$(x;\xi) = \frac{\pi}{2}g(x;\xi)\cos\left[\frac{\pi}{2}G(x;\xi)\right], \quad x \in \mathbb{R} \qquad (4)$$

Where $G(x;\xi)$ and $g(x;\xi)$ in equation (3) and (4) above are the CDF and PDF of the base line distribution with parameter(s) vector denoted by ξ, respectively.

Development of the Proposed Sine-Lomax Distribution
The cumulative density function (CDF) of the newly proposed probability distribution (Sine Lomax) and the probability density function (PDF) are presented in equation (5) and (6):

$$F(x,\alpha,\lambda) = \sin\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\} \qquad (5)$$

and the pdf is given by

$$f(x,\alpha,\lambda) = \frac{\pi}{2}\frac{\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}\cos\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\} \qquad (6)$$

The survival function S(x), the hazard function h(x), the reverse hazard function r(x), the cumulative hazard function H(x) and the quantile function u_q are given in equation (7) to (10):

$$S(x) = 1 - sin\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\} \qquad (7)$$

$$h(x) = \frac{f(x)}{1-F(x)} = \frac{\frac{\pi\alpha}{2\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}\cos\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}}{1-sin\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}} \qquad (8)$$

$$r(x) = \frac{\pi}{2}\frac{\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}\cot\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\} \qquad (9)$$

$$H(x) = -\ln\left\{1 - sin\left(\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right)\right\} \qquad (10)$$

$$Q(u) = G^{-1}\left\{\lambda\left[\left(1-\frac{2sin^{-1}u}{\pi}\right)^{-\frac{1}{\alpha}}-1\right]\right\} \qquad (11)$$

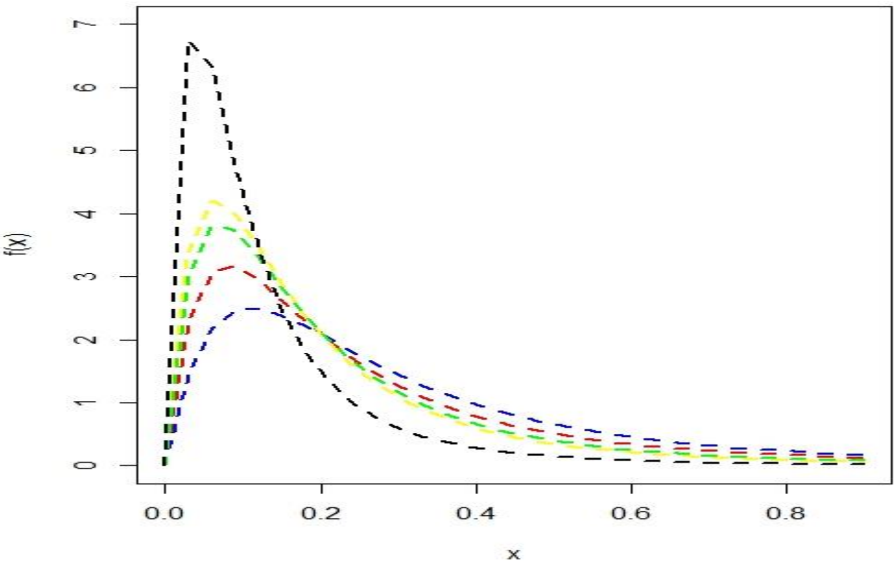

Figure 1. PDF plot of Sine – Lomax Distribution
Figure 1 shows the PDF plot of the Sine Lomax distribution and the plot reveals that the distribution is positively skewed

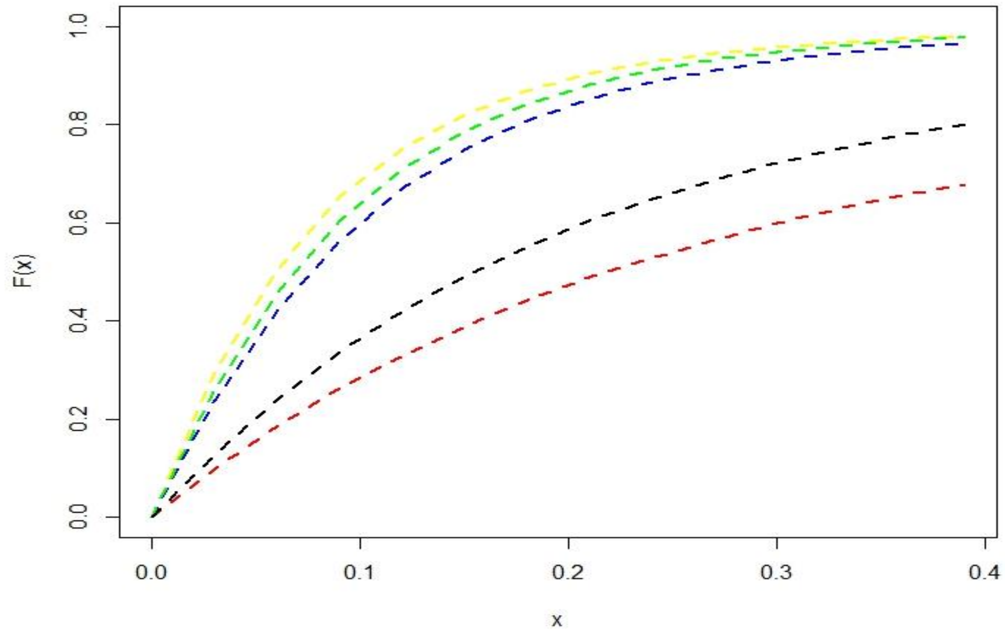

Figure 2: CDF plot of Sine Lomax Distribution

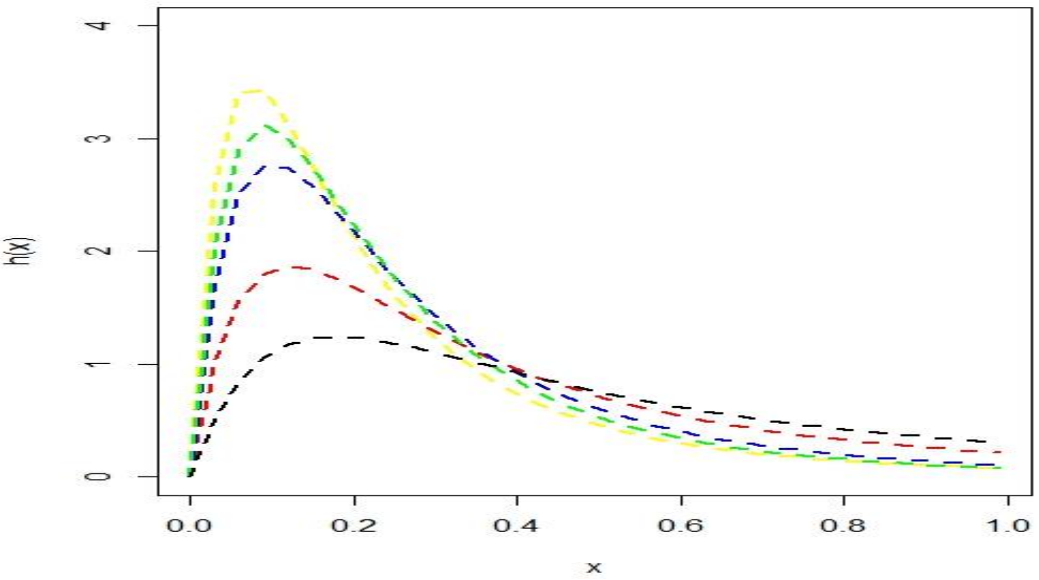

Figure 3: Hazard plot of Sine Lomax distribution

The hazard plot of the sine Lomax distribution shows that the distribution has an inverted bathtub shape

Parameter Estimation

In this section, we consider maximum likelihood estimation (MLE) to estimate the involved parameters.

Method of Maximum Likelihood Estimation

Let $x_1, x_2, \dots, x_n$ be a random sample of size n from the Sine-Lomax distribution with pdf given in equation (6), the log-likelihood function $l(\alpha,\ \lambda)$ of the Sine-Lomax distribution is given by:

$$l(\alpha,\lambda) = n\log\left(\frac{\pi}{2}\right) + n\log\alpha - n\log\lambda - (\alpha+1)\sum_{i=1}^{n}\log\left(1+\frac{x}{\lambda}\right) + \sum_{i=1}^{n}\log\cos\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\} \quad (12)$$

Differentiating with respect to α gives:

$$\frac{\partial l(\alpha,\lambda)}{\partial\alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n}\log\left(1+\frac{x}{\lambda}\right) - \sum_{i=1}^{n}\frac{\pi}{2}\left[\left(1+\frac{x}{\lambda}\right)^{-\alpha}\ln\left(1+\frac{x}{\lambda}\right)\right]\tan\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\} \quad (13)$$

Differentiating with respect to λ gives the following expression:

$$\frac{\partial l(\alpha,\lambda)}{\partial\lambda} = -\frac{n}{\lambda} - (\alpha+1)\sum_{i=1}^{n}\frac{x}{\lambda^2}\left(1+\frac{x}{\lambda}\right)^{-1} + \sum_{i=1}^{n}\frac{\pi}{2}\frac{\alpha x}{\lambda^2}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}\tan\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\} \quad (14)$$

Equation (13) and equation (14) gives the maximum likelihood estimators of the parameters α and λ.

Useful Expansion

The pdf of the proposed Sine-Lomax Distribution can be expanded as follows:

$$f(x,\alpha,\lambda) = \frac{\pi}{2}\frac{\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)} \cos\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}$$

We first expand $\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}$, we will have the following:

$$\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)} = \sum_{i=0}^{\infty}(-1)^i\binom{\alpha+1+i-1}{i}\left(\frac{x}{\lambda}\right)^i$$

$$\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)} = \sum_{i=0}^{\infty}(-1)^i\binom{\alpha+i}{i}\left(\frac{x}{\lambda}\right)^i$$

Therefore, the pdf will be given by:

$$f(x,\alpha,\lambda) = \frac{\pi}{2}\frac{\alpha}{\lambda}\sum_{i=0}^{\infty}(-1)^i\binom{\alpha+i}{i}\left(\frac{x}{\lambda}\right)^i \cos\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}$$

$$\cos\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\} = \sum_{j=0}^{\infty}\frac{(-1)^j}{(2j)!}\frac{\pi^{2j}}{2^{2j}}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]^{2j}$$

The pdf will be reduced to:

$$f(x,\alpha,\lambda) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{(-1)^{i+j}}{(2j)!}\frac{\pi^{2j+1}}{2^{2j+1}}\frac{\alpha}{\lambda}\binom{\alpha+1}{i}\left(\frac{x}{\lambda}\right)^i\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]^{2j}$$

Again,

$$\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]^{2j} = \sum_{k=0}^{\infty}(-1)^k\binom{2j}{k}\left(1+\frac{x}{\lambda}\right)^{-\alpha k}$$

The pdf is also reduced to:

$$f(x,\alpha,\lambda) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^{i+j+k}}{(2j)!}\frac{\pi^{2j+1}}{2^{2j+1}}\frac{\alpha}{\lambda}\binom{\alpha+1}{i}\left(\frac{x}{\lambda}\right)^i\binom{2j}{k}\left(1+\frac{x}{\lambda}\right)^{-\alpha k}$$

$$\left(1+\frac{x}{\lambda}\right)^{-\alpha k} = \sum_{l=0}^{\infty}(-1)^l\binom{\alpha k+l-1}{l}\left(\frac{x}{\lambda}\right)^l$$

$$f(x,\alpha,\lambda) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^{i+j+k+l}}{(2j)!}\frac{\pi^{2j+1}}{2^{2j+1}}\frac{\alpha}{\lambda^{i+l+1}}\binom{\alpha+1}{i}\binom{\alpha k+l-1}{l}\binom{2j}{k}x^{i+l}$$

Let $\psi = \frac{(-1)^{i+j+k+l}}{(2j)!}\frac{\pi^{2j+1}}{2^{2j+1}}\frac{\alpha}{\lambda^{i+l+1}}\binom{\alpha+1}{i}\binom{\alpha k+l-1}{l}\binom{2j}{k}$

Therefore, the pdf can be expressed as:

$$f(x,\alpha,\lambda) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\psi\, x^{i+l} \tag{15}$$

The expansion of the cdf is also given below:

$$F(x;\alpha,\lambda) = sin\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\} = \sum_{m=0}^{\infty}\frac{(-1)^m}{(2m+1)!}\frac{\pi^{2m+1}}{2^{2m+1}}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]^{2m+1}$$

$$\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]^{2m+1} = \sum_{q=0}^{\infty}(-1)^q\binom{2m+1}{q}\left(1+\frac{x}{\lambda}\right)^{-\alpha(2m+1)}$$

$$\left(1+\frac{x}{\lambda}\right)^{-\alpha(2m+1)} = \sum_{p=0}^{\infty}(-1)^p\binom{\alpha(2m+1)+p-1}{p}\left(\frac{x}{\lambda}\right)^p$$

$$F(x;\alpha,\lambda) = \sum_{m=0}^{\infty}\sum_{q=0}^{\infty}\sum_{p=0}^{\infty}\frac{(-1)^{2m+q+p-1}}{(2m+1)!}\frac{\pi^{2m+1}}{2^{2m+1}}\binom{2m+1}{q}\binom{\alpha(2m+1)+p-1}{p}\left(\frac{x}{\lambda}\right)^p$$

Let $\Theta = \frac{(-1)^{2m+q+p-1}}{(2m+1)!}\frac{\pi^{2m+1}}{2^{2m+1}}\frac{1}{\lambda^p}\binom{2m+1}{q}\binom{\alpha(2m+1)+p-1}{p}$

Therefore, the cdf will be reduced to:

$$F(x;\alpha,\lambda) = \sum_{m=0}^{\infty}\sum_{q=0}^{\infty}\sum_{p=0}^{\infty}\Phi x^p \tag{16}$$

Equation (15) and equation (16) gives the reduced form of the PDF and the CDF of the Sine-Lomax Distribution and they were used to derive some of the mathematical properties of the newly developed distribution.

Mathematical Properties

Some of the mathematical properties such as the rth moment, moment generating function, the entropy and order statistics are derived.

rth Moment

Moments are required and vital in any statistical study, particularly in applications. It can be used to investigate some properties of a distribution such as skewness, kurtosis and measures of dispersion (Halid and Sule, 2022). The r^{th} moment of the random variable X with PDF $f(x)$ is expressed as:

$$\mu_r' = \int_{-\infty}^{\infty} x^r f(x)dx$$

The rth moment of the Sine-Lomax Distribution is given by:

$$\mu_r' = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\psi\int_0^{\infty} x^{r+i+l}\,dx$$

Let$\Omega = \int_0^{\infty} x^{r+i+l}\,dx$

Therefore, the rth moment is given by:

$$\mu_r' = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\psi\,\Omega \tag{17}$$

Moment Generating Function

The moment generating function of a random variable X is the expected value of e^{tx} that is:

$$M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx}f(x)dx$$

We say that the moment generating function exist, if there exist a positive constant a such that $M_x(t)$ is finite for all $t \in$

$[-a, a]$. Therefore, the moment generating function for the Sine-Lomax Distribution is given by:

$$M_x(t) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\psi \int_0^{\infty} e^{tx}\, x^{i+l}\, dx$$

$$\text{Let}\Upsilon = \int_0^{\infty} e^{tx}\, x^{i+l}\, dx$$

$$M_x(t) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\psi\, \Upsilon \qquad (18)$$

Entropy

Entropy is used as a measure of information or uncertainty, which present in a random observation of its actual population. There will be the greater uncertainty in the data if the value of entropy is large. The entropy for the true continuous random variable X is defined as:

$$I_y(x) = \frac{1}{1-\theta}\log\int_{-\infty}^{\infty} f(x)^{\theta}\, dx$$

$$f(x,\alpha,\lambda)^{\theta} = \left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\psi\, x^{i+l}\right)^{\theta}$$

$$f(x,\alpha,\lambda) = \left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\psi\right)^{\theta}\Theta^{\theta}$$

Where $\Theta = \left(x^{i+l}\right)$

$$I_y(x) = \frac{1}{1-\theta}\left[\left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\psi\right)^{\theta}\log\int_0^{\infty}\Theta^{\theta}\, dx\right] \qquad (19)$$

Order Statistics (OS)

Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a continuous distribution having a PDF, $f(x)$ and CDF, F(x), Let $X_{1:n} \le X_{2:n} \le X_{n:n}$ be the corresponding order statistics (OS). The r^{th} OS is given by:

$$f_{r:n}(x) = \frac{f(x)}{B(r, n-r+1)}(F(x))^{r-1}(1-F(x))^{n-r}$$

The order statistics of the Sine-Lomax Distribution is given by:

$$f_{r:n}(x) = \frac{\left(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\psi x^{i+l}\right)}{B(r,n-r+1)}\left(\sum_{m=0}^{\infty}\sum_{q=0}^{\infty}\sum_{p=0}^{\infty}\Phi x^{p}\right)^{r-1}\left(1-\sum_{m=0}^{\infty}\sum_{q=0}^{\infty}\sum_{p=0}^{\infty}\Phi x^{p}\right)^{n-r} \qquad (20)$$

Equation (20) gives the order statistics of the newly developed Sine-Lomax Distribution

RESULTS AND DISCUSSION

Monte Carlo Simulation and Application

Simulation study

The modeling process heavily relies on assumptions that are associated with uncertainty. Monte Carlo simulation is a valuable tool for assessing the impact of risk and uncertainty in prediction and forecasting models. In this study, we employ Monte Carlo simulation to evaluate our proposed distribution in terms of its ability to model lifetime data while considering the associated risks. To evaluate the performance of the newly proposed Sine-Lomax distribution, we conduct a simulation study using the Monte Carlo Simulation method. The objective is to compute the mean, bias, and mean square error of the estimated parameters derived from the maximum likelihood estimates. The simulation generates synthetic data by utilizing the quantile function defined in equation (11) for various sample sizes, including n=20, 50, 100, 250, 500, and 1000. For each sample size, α is set to 1.0 and λ is set to 1.2. Table 1 presents the estimation results, bias, and mean square error obtained from the new distribution.

Table 1: Estimate, Bias and MSE of the new Lehmann Type II Lomax Distribution

N	Properties	$\alpha = 1.0$	$\lambda = 1.2$
20	Est.	1.3103	5.253
	Bias	0.1103	0.253
	MSE	0.3107	0.259
50	Est.	1.3036	5.261
	Bias	0.1036	0.261
	MSE	0.3046	0.267
100	Est.	1.2974	5.268
	Bias	0.0974	0.268
	MSE	0.2991	0.273
250	Est.	1.2870	5.252
	Bias	0.0870	0.252
	MSE	0.2895	0.257
500	Est.	1.2809	5.246
	Bias	0.0809	0.246
	MSE	0.2840	0.250
1000	Est.	1.2765	5.242
	Bias	0.0765	0.242
	MSE	0.2804	0.246

The results obtained from the Monte Carlo Simulations are presented in Table 1. These findings provide evidence that as the sample size increases, both the bias and mean square error (MSE) tend to approach zero. This indicates that the suggested distribution exhibits favorable characteristics and a low level of risk when applied for modeling lifetime datasets.

Application

Two data sets were used to illustrate the applicability and the practicability of the proposed model.

First Dataset

The first dataset represents to remission times (in months) of a random sample of 128 bladder cancer patients and the data was extracted from Adekunle et al. (2023). The data is given below:

"0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46, 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07, 21.73, 2.07, 3.36, 6.93, 8.65, 12.63, 22.69"

The analysis of remission time among bladder cancer patients was conducted using the newly developed Sine-Lomax Distribution model. Its performance was compared with competing models, including the Lomax distribution, exponentiated Lomax distribution (Exp. Lomax), and Beta Lomax Distribution, using evaluation criteria such as AIC, BIC, CAIC, and HGIC. The results and summary of this analysis are presented in Table 2 below.

Table 2: The Maximum Likelihood Statistic, AIC, CAIC, BIC, and HQIC of the remission times of Bladder cancer patients

Distribution	α	λ	γ	θ	MLE	AIC	CAIC	BIC	HQIC
Sine-Lomax	3.484	16.604	-	-	410.96	825.929	826.025	831.633	828.246
Lomax	9.274	77.962	-	-	413.93	831.856	831.952	837.560	834.173
Exp. Lomax	1.064	0.08	0.006	-	414.98	835.956	836.150	844.512	839.432
Beta Lomax	3.919	23.928	1.585	0.157	411.74	831.486	828.140	842.890	836.120

The analysis of remission times among bladder cancer patients is presented in Table 2. The results demonstrate that the proposed Sine-Lomax distribution exhibits superior performance compared to its competitors. It achieved the lowest maximum likelihood value, followed by the Beta-Lomax distribution and Lomax distribution, while the Exponential-Lomax distribution shows the highest maximum likelihood estimate. Additionally, when considering evaluation criteria such as AIC, CAIC, BIC, and HQIC, the proposed Sine-Lomax distribution outperforms its competitors by having the minimum values for each criterion. Therefore, the Sine-Lomax distribution is considered the preferred model for analyzing the remission times of bladder cancer patients among the competing models.

Second Dataset

The second dataset consists of survival times (in days) of guinea pigs injected with different amount of tubercle bacilli and was studied by Kharazmi and Jabbari (2017). The data is given below:

"12, 15, 22, 24, 24, 32, 32, 33, 34, 38, 38, 43, 44, 48, 52, 53, 54, 54, 55, 56, 57, 58, 58, 59, 60, 60, 60, 60, 61, 62, 63, 65, 65, 67, 68, 70, 70, 72, 73, 75, 76, 76, 81, 83, 84, 85, 87, 91, 95, 96, 98, 99, 109, 110, 121, 127, 129, 131, 143, 146, 146, 175, 175, 211, 233, 258, 258, 263, 297, 341, 341, 376"

The analysis of the second data set involved the utilization of the newly proposed Sine-Lomax Distribution. The results obtained were then compared with competing models, namely the Lomax distribution, sine-inverse Weibull distribution, and Inverse Lomax Distribution. This comparison was based on evaluation criteria including AIC, BIC, CAIC, and HQIC. The summarized analysis outcomes can be found in Table 3 below.

Table 3: The Maximum Likelihood Statistic, AIC, CAIC, BIC, and HQIC of the remission times of Bladder cancer patients

Distribution	α	λ	MLE	AIC	CAIC	BIC	HQIC
SinL	0.289	9.182	348.801	701.61	701.78	706.16	703.42
Lomax	0.200	1.912	356.821	717.64	717.82	706.16	719.45
IL	44.31	3151.9	393.251	790.50	790.67	795.06	792.32
SinIW	41.96	1.090	349.423	787.66	787.09	793.56	789.45

The analysis of the second data set using the newly proposed Sine-Lomax Distribution is presented in Table 3. The results indicate that the proposed model demonstrates superior performance compared to its competitors. This is evident from the significantly lower AIC, BIC, CAIC, and HQIC scores achieved by the Sine-Lomax distribution. Consequently, when measuring the survival times of guinea pigs injected with varying amounts of tubercle bacilli, the Sine-Lomax distribution outperforms its competitors, namely the Lomax distribution and Inverse Lomax distribution.

CONCLUSION

This study introduces an extension of the Lomax distribution known as the Sine-Lomax distribution (SL). The Sine-Lomax Distribution model was developed by incorporating the Sine-G family of distributions proposed by Kumar et al. (2015), resulting in a new trigonometric distribution. Various properties of the proposed distribution, including the survival function, hazard rate function, reverse hazard function, cumulative hazard function, moments, moment generating function, quantile function, and order statistics, were derived. The parameters of the proposed distribution were estimated using the maximum likelihood method. Additionally, a simulation study was conducted to evaluate the performance of the maximum likelihood estimators (MLEs) for the distribution parameters. Furthermore, the proposed distribution was applied to two different real-life datasets to compare its effectiveness with other well-known standard distributions such as the Lomax distribution, Exponential Lomax distribution, Sine-Inverse Weibull distribution, and Inverse Lomax distribution. The results demonstrated that the new Sine-Lomax Distribution (SLD) outperformed its competitors in terms of fitting the two datasets.

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