INVESTIGATION OF ARRHENIUS-CONTROLLED CHEMICAL REACTION THROUGH A SUPERHYDROPHOBIC MICROCHANNEL

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ABSTRACT

The consequence of fully developed free convection with Arrhenius-controlled heat flow for a viscous and an electrically-conducting fluid flowing along an isothermally heated vertical parallel plate in a slit micro-channel is presented in this article. One wall had super-hydrophobic slip (SHS) and a temperature spike, while the other did not. A semi-analytical technique (perturbation series) was utilized to analyze the primary equations. The analytical solutions were thoroughly presented, and the functions of the pertinent parameters were illustrated with the help of various plots. It is revealed from this work that the action of chemically reacting factors is noticed to substantially strengthen the fluid movement in the micro-channel for a constant pressure gradient. In the fields of engineering and medicine, it is essential to understand these fluids’ characteristics. Due to the lubrication of micro-channel boundary walls where conductivity and viscosity interact with thermos-physical behavior, the outcomes of the current research can substantially enhance the operations of micro-electromechanical systems (MEMS) and micro-devices that rely on micro-fabrication processes.

Keywords: Arrhenius-controlled fluid, free convection, super-hydrophobic slip, temperature jump, slit micro-channel

INTRODUCTION

The staggering development of technology as a result of people’s desires for smaller and more lightweight machines has drawn the curiosity of researchers, engineers, and creative minds to conceptual investigation in mini-technology, micro-technology, and eventually nanotechnology. This is what prompted researchers in computational fluid dynamics to shift their attention away from studying flows in macro-channels and toward looking at flows in mini-channels, micro-channels, and nano-channels. The importance of micro-channel inward temperature emission, micro-jet boundary-layered cooling, massive density transistors in high-performance computer systems, and other machines has given rise to a tremendous growth in micro-channel fluid and thermally transported flows in recent years. These components also play important roles in aerospace technology, micro energy pipes, fabrication activities, and transistors with high power densities. Understanding the flow characteristics is important since the majority of these compositions include internal micro-channel streams (Al-Nimr & Khadrawi 2004; Jha et al., 2014a). The impact of the flow pattern on micro-structure in a variety of physical settings has been the subject of several studies that have been reported. With the concerns in mind, the consequence of Arrhenius-driven thermal flow caused by an induced electromagnetic field in a microchannel was lately the subject of a theoretical investigation by Hamza et al. (2023). Ojemer and Hamza (2022) employed the homotopy perturbation technique to analyze Arrhenius kinetically induced heat from a source or sink fluid in a microchannel. The effects of Hall current and ion-slip on hydro-magnetic thermal flow in an upward micro-channel impacted by a magnetic field being applied were discussed by Jha and Malgwi (2019a). Jha et al. (2017) discussed the consequence of Hall effects on hydro-magnetic natural convection in a vertical microchannel as part of their investigation. Some other research conducted on this area include (Chen and Weng 2005, Jha and Aina 2015, Buonomo and Manca, 2012, Weng and Chen 2009, Jha et al. 2014a, Jha et al. 2014b), to highlight a few. The field of magneto hydrodynamic (MHD) has drawn growing interest in recent years due its usefulness in several MHD applications, such as MHD generators, cooling baths with cooling metallic plates, electric transformers, and MHD injectors. Chemical engineering aspect, which includes the utilization of hydro magnetic pumps to transfer electrically conducting fluids, is currently employed in some nuclear power plants (Jha et al. 2015). Several studies on hydro-magnetic convective flow have been executed under a variety of natural conditions. Omokhualie & Dange (2023) numerically determined the action of heat sink on time-dependent MHD Jeffery flow of a chemically reactive fluid using the finite difference method (FDM). Ojemer et al. (2023) recently deliberated on the hydro magnetic flow of an electrically conductive Casson fluid driven by heat radiation factor in an upstanding porous channel. Osman et al. (2022) used the Laplace transformation approach to describe the influence of free hydro magnetic flow across an infinitely inclined plate. Siva et al. (2021) worked on a thermal transfer investigation of electrokinetic flow in a circulating microfluidic channel and demonstrated a precise response to the MHD action. Choudhary (2012) investigated heat and mass transfer in viscoelastic hydro magnetic boundary surface flow toward an upward parallel plates. Sandeep and Sugunanamma (2013) researched the effect of an angled magnetic field on the unsteady free convection of a dusty reactive fluid inside two immeasurable flat plates packed with a porous substance. When Joseph et al. (2015) analyzed the time-dependent hydro magnetic poiseuille flow through two infinitely parallel porous plates in an inclined magnetic field, they also took heat and mass transmission into account. They found out that as the magnetic number increases, so does the rate at which it moves. The thermal Grashof number Gr and the solutal Grashof number GC increase velocity. Geethan et al. (2016) discussed the actions of heat radiation, chemical reactions, and thermal diffusion on hydro magnetic natural
convection slip flow down an inclined plate at a uniform temperature due to heat generation. Sivaiah and Reddy (2017) conducted a theoretical study of hydro magnetic free convective mass and energy transfer fluid with thermal diffusion in the coexistence of Hall current and radiation effects.

The assessment of the outcome of the novel combination utilizing MHD-free convection in a super-hydrophobic (SHO) micro-channel is receiving greater focus from engineering scientists and technologists. SHO surfaces have a tendency to minimize drag in a flow due to the immense slip generated by liquid/solid interactions, which makes it an essentially pertinent variable to determine the degree of frictional force based on the slip size. SHO surfaces are used by oil and gas industries, semiconductor manufacturing operations, and businesses that build tiny machines. In light of these factors, Jha and Gwandu (2017) performed a mathematical research of a natural hydro magnetic flow in a upstanding slit micro-channel exhibiting SHS and temperature jump. Their studies revealed that heating the super-hydrophobic material produced the highest upward velocity. Employing non-linear Boussinesq approximation techniques, Jha and Gwandu (2019) researched the free convection of an electrically conducting fluid in an upstanding microchannel with super-hydrophobic slip and temperature jump. Based on their theoretical results, increasing the temperature jump coefficient causes the temperature to go down when the SHS is heated and up when the no-slip surface (NSS) is heated. Jha and Gwandu (2020) expanded their previous article by providing a theoretical characterization of free convection airflow over permeable plates heated on both sides, one channel with NSS and the other SHS. Ramanuja et al. (2020) investigated natural convection flow in an isothermally heated channel with a temperature rise and super-hydrophobic slip on one surface while the other side have no slip. Hatte and Pitchumani (2021) carefully and explicitly described the effect of thermal flow inside a cylinder with non-wetting walls using a fractional rough surface assessment. The method investigates the fluid interaction’s dynamical stability in the gaps of air-infused SHS. Their results demonstrate that, opposed to popular perception, super-hydrophobicity, which is indicated by the highest contact angles, rarely leads to an increase in the maximum convective heat transfer performance and that, depending on fluid flow circumstances, hydrophobic coatings can give good thermal conductivity.

The influence of Arrhenius-controlled fluid on micro-channel having a SHS in the presence of internal heat generation/absorption have not been investigated in any of the afore-mentioned works, hence the motivation for this article. As a result, the focus of this paper is to present a theoretical investigation of a steady natural convection of chemically reacting fluid in a vertical slit micro-channel with SHS. The perturbation procedure is employed to analyze the dimensionless nonlinear and coupled leading equations. The findings from this study can be very helpful in areas such as in micro-devices made with micro-fabrication processes, in MEMS, in the lubrication industry, in biomedical sciences, in the fabrication and extraction industries, to mention a few.

Problem formulation

Imagine a steady, fully developed free convection flow of an electrically conducting fluid flowing steadily upward along an upstanding parallel plate micro-channel affected by Arrhenius kinetic and a heat source/sink. Because of a specific micro-engineering process, one of the surfaces is extremely tough to wet (super-hydrophobic). The other side (slip-resistant surface) was not tempered with either. As demonstrated in Figure 1, the SHS is maintained at $y_0 = 0$ whereas, the non-slip surface is maintained at $y_0 = L$. Since the super-hydrophilicity of a surface is more important than how it flows, different temperature jumps and slip factors were used on the various plates. Following Jha and Gwandu (2017), the primary equations for this particular problem are as follows, presuming that the fluid is viscous and chemically reactive, and using the typical Boussinesq buoyancy approximation with boundary requirements.

### Boundary Conditions

#### Dimensionless Forms

\[
\frac{d^2 T}{dy^2} = M^2 U = -\theta 
\]  \quad (1)

\[
\frac{d^2 \theta}{dy^2} + \lambda \frac{\theta}{1+\theta} = 0 
\]  \quad (2)

The boundary conditions in dimensionless forms are:

\[
\theta(0) = 1 + \Gamma \frac{\partial u}{\partial y} \quad \text{and} \quad u(0) = -\gamma \frac{\partial \theta}{\partial y} 
\]  \quad (3)

---

**Figure 1** The schematic of the flow configuration

---
\( \theta(1) = 0 \), \( u(1) = 0 \)

The non-dimensional quantities used in deriving Equations 1-3 are given below:

\[
\begin{align*}
&u = \frac{u}{U}, \quad y = \frac{y}{L}, \quad T = \frac{T - T_0}{\Delta T}, \quad x = \frac{x}{L}, \\
&M^2 = \frac{\sigma G^2 \lambda^2}{\nu}, \quad \epsilon = \frac{T_0}{\Delta T}, \quad \lambda = \frac{\alpha \gamma}{\nu}, \quad \beta = \frac{\gamma}{\nu}, \\
&(Y, y, \Gamma) = (Y', y', \Gamma')/h,
\end{align*}
\]

Where \( \lambda \) is the chemical reactant parameter, \( \Gamma \) is the temperature jump parameter, \( y \) is the velocity slip parameter and \( M \) is the magnetic field intensity.

**METHODOLOGY**

**Method of Solution**

The momentum and temperature equations are resolved semi-analytically by perturbation method.

we assume

\[
U = U_0 + \lambda U_1
\]

Substituting eq (4) into eqs (1), (2) and (3) and taking the coefficient of \( \lambda^0 \) and \( \lambda \), the following is the derivation of the sets of ordinary differential equations and accompanying boundary conditions for temperature and velocity:

\[
\begin{align*}
\theta_1 &= -\frac{\chi^2}{2} - V_1 \frac{\gamma^2}{6} - V_2 \frac{\gamma^2}{2} - C_1 V_2 / 12 - 2C_1 V_2 \frac{\gamma^2}{6} - C_1 V_2 \frac{\gamma^2}{2} + V_3 y + V_4 \\
\end{align*}
\]

The solution for velocity gradient is also obtained as follows:

\[
\begin{align*}
U_0 &= W_0 \cosh (M y) + W_2 \sinh (M y) + K_3 y + K_4 \\
U_1 &= W_0 \cosh (M y) + W_2 \sinh (M y) + K_3 y + K_4 + K_5 y + K_7
\end{align*}
\]

Recall that the expressions for temperature and velocity distributions are represented as follows:

\[
\begin{align*}
\theta &= \theta_0 + \lambda \theta_1 \\
U &= U_0 + \lambda U_1
\end{align*}
\]

The rates of heat transfer and skin friction at both micro-channel walls are obtained as follows:

\[
\begin{align*}
\frac{dT}{dy} \bigg|_{y=0} &= V_1 + \lambda V_3 \\
\frac{dT}{dy} \bigg|_{y=1} &= V_1 + [-1 - \frac{V_1}{2} - V_2 - C_1 V_2 / 3 + C_1 V_2 / 2 + V_3 \\
\frac{dU}{dy} \bigg|_{y=0} &= W_0 M + K_3 + \lambda \{W_6 M + K_6 \} \\
\frac{dU}{dy} \bigg|_{y=1} &= W_2 M \sinh (M) + W_2 M \cosh (M) + K_3 + \lambda \{W_4 M \cosh (M) + 4K_3 + 3K_4 + 2K_2 + K_6 \}
\end{align*}
\]

**Validation of results**

As approaches \( \lambda \) zero, the work of Jha and Gwandu (2017) is effectively recovered, indicating a strong agreement between the present study and their work. The empirical evaluation of Jha and Gwandu (2017) and the current study is shown in Table 1.

**RESULTS AND DISCUSSION**

Theoretical investigation of an Arrhenius-driven heat transfer fluid is carried out on a steady hydro magnetic flow of an incompressible fluid moving along an isothermally heated parallel plate micro-channel, with one surface exhibiting SHS and temperature jump and the other not having slip. For determining the steady state equations, a perturbation procedure (semi-analytical method) is used. Various plots were sketched to illustrate the functions of pertinent parameter namely chemically reacting parameter, \( \lambda \), velocity slip \( \gamma \), temperature \( \Gamma \) and magnetic field effect \( M \) on the fluid velocity and temperature gradient. The reference values selected for this investigation are \((\gamma = 1, \ M = 0.5, \ \lambda = 0.001)\), except otherwise stated, as it relates to real life situation.

The attributes of chemical reaction on the temperature profile is showcased in Figure 2. Clearly, as the value of \( \lambda \) improves, the temperature jumps significantly. According to Hanza (2016), raising the levels of \( \lambda \), the temperature equation’s viscous heating and chemical reactant factors are strengthened remarkably, causing a considerable temperature increase.

The function of \( \lambda \) on fluid velocity is demonstrated in Figure 3. It was evident that as \( \lambda \) mounts, an escalation in the fluid velocity was noticed. Also, it was shown that the fluid wall effect decreases as velocity slip rises at super-hydrophobic walls. This makes the gas move faster near the wall. A decline in fluid viscosity and a subsequent rise in fluid velocity are caused by the remarkable increase in temperature in reaction to the higher \( \lambda \).

The implication of MHD on the velocity variation is portrayed in Figure 4. As the magnetic number increases (when both and are each equal to 1), the trend demonstrates a deteriorating outcome on the velocity (particularly the maximum velocity), which is expected because the Lorentz force in the magnetic field affects the fluid speed.

Figure 5 describes the pattern of the velocity distribution against displacement \( y \), and the results agree with prior research by other researchers. This suggests that as particles move away from the SHS, their velocity rises briefly before diminishing as they reach the center. If neither surface is super-hydrophobic, the velocity does not climb much and
Figure 6 displays the action of a magnetic field on shear stress. MHD has a similar impact on shear stress at both plates, however the effect is greater on the plate where \( y = 0 \).

Figure 2: Variation of temperature for \( \lambda \) for constant values of \( \gamma = \Gamma = 1, M = 0.5 \)

Figure 3: Variation of velocity for \( \lambda \) for constant values of \( \gamma = \Gamma = 1, M = 0.5 \)

Figure 4: Variation of velocity for MHD for constant values of \( \gamma = \Gamma = 1, \lambda = 0.001 \)
INVESTIGATION OF ARRHENIUS

Figure 5: Variation of velocity for the displacement y for constant values of M=0.5, λ =0.001

Figure 6: Variation of sheer stress for MHD for constant values of γ = Г= 1, λ=0.001

Table 1: Illustrates the numerical computations of Jha and Gwandu’s (2017) work and the current investigation for velocity and temperature distributions for γ = Г= 1, Gre=1, M = 0.5 when λ approaches 0.

<table>
<thead>
<tr>
<th>Y</th>
<th>θ(Y) Jha and Gwandu (2017)</th>
<th>U(Y)</th>
<th>θ(Y) Current work</th>
<th>U(Y)</th>
</tr>
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<td>0.1</td>
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<tr>
<td>0.5</td>
<td>0.2500</td>
<td>0.0686</td>
<td>0.2499</td>
<td>0.0686</td>
</tr>
</tbody>
</table>

CONCLUSION
The paper investigated the implications of chemically reacting fluid on the steady free convection of an electrically conducting fluid moving upwardly within an isothermally heated parallel plate in the micro-channel due to heat generation/absorption, with one wall exhibiting super-hydrophobic slip and temperature jump and have NSS. A semi-analytical method is applied to address the steady state system of primary equations, and various illustrated plots demonstrating the functions of relevant parameters on flow patterns are shown. A sound understanding of the behavior of these kinds of fluids is particularly significant in both engineering and medicine. A summary of the key results is highlighted below:

i. It was revealed that a surge in the fluid thermal distribution is seen as the chemical reactant parameter is mounting.
ii. Uplifting the chemical reaction parameter promotes the fluid velocity.
iii. The fluid velocity diminishes as the magnetic parameter numbers increase as a result of Lorentz forces manifested by the MHD phenomenon.
iv. The frictional force at both microchannel wall is observed to demonstrate similar downward trend
v. In the future, this study will be expanded to incorporate unsteady state (time-dependent) case, heat generating/absorbing effect or by using a different physical geometry.

REFERENCES


**Nomenclature**

- $B_0$: constant magnetic flux density [kg/s$^2$.m$^2$]
- $g$: gravitational force [m/s$^2$]
- $h$: channel size [m]
- $C_p, C_v$: specific heats at constant pressure and constant volume [Jkg$^{-1}$K$^{-1}$]
- $M$: magnetic field
- $e$: activation energy
- $Nu$: dimensionless heat transfer rate
- $T$: dimensionless temperature of the fluid [K]

- $T_0$: reference temperature [K]
- $u$: dimensionless velocity of the fluid [ms$^{-1}$]
- $y$: dimensionless distance between plates
- $U_0$: reference velocity [ms$^{-1}$]

**Greek letters**

- $\gamma$: dimensionless slip length parameter
- $\Gamma$: dimensionless temperature jump parameter
- $\beta$: thermal expansion coefficient [K$^{-1}$]
- $\beta_o, \beta_v$: dimensionless variables
- $\mu$: variable fluid viscosity [kgm$^{-1}$s$^{-1}$]
- $k$: thermal conductivity [m.kg/s$^3$.K]
- $\alpha$: thermal diffusivity [m$^2$s$^{-1}$]
- $\gamma_s$: ratios of specific heats ($C_p/C_v$)
- $\sigma$: conductivity of the electric fluid [s$^{-3}$m$^2$/kg]
- $\rho$: fluid density[kgm$^{-3}$]
- $\nu$: viscosity of the fluid [m$^2$s$^{-1}$]