FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645 - 2944
Vol. 3 No. 4, December, 2019, pp 501 –510

EFFECT OF FREE VIBRATION ANALYSIS ON EULER-BERNOULLI BEAM WITH DIFFERENT BOUNDARY CONDITIONS

Usman, M. A. and Muibi, B. T.

Department of Mathematical Sciences
Olabisi Onabanjo University, Ago-Iwoye, Nigeria
Corresponding author's email: usmanma@yahoo.com

ABSTRACT

This paper presents an analysis of the effect of free vibrations of a free-free beam, fixed-fixed beam and simply supported beam using the series solution. It was found that the mode shape for each of the modes has effects on the displacement or deflection of such beam so that the deflection increases as the increase of the mode. Also, a Simply-Supported beam has a lower displacement compared to the free-free beam and fixed-fixed beam which almost have the same displacement. At mode one, it is seen that a Simply Supported beam has a higher amplitude, followed by a free-free beam and then a fixed-fixed beam.

Keywords: Beam**,** Fixed-Fixed Beam, Free-Free Beam, Simply Supported Beam, Deflection.

INTRODUCTION

The vibration analysis for structures is a very important field in engineering and computational mechanics. These dynamic problems are classically described by partial differential equations associated with a set of boundary conditions. The analysis of free vibration of the beam has been a topic of interest for well over a century.

Doyle and Pavlovic (1982) solved analytically the free vibration equation of the beam on a partially elastic foundation including only bending moment effect. West and Mafi (1984) obtained the eigenvalues for free vibration of beam-column systems on an elastic foundation using a numerical approach. Catal (2002) used the separation of variables used to obtain the free vibration circular frequencies of piles partially embedded in soils. Chen and Ho (1996) examined the free and transverse vibration problems of a rotating twisted Timoshenko beam under axial loading. The method of differential transform method (DTM) was employed to solve the eigenvalue problems for free and transverse vibration problems of a rotating twisted Timoshenko beam under axial loading. In addition, the differential transform method (DTM) has been proposed to solve eigenvalue problems for free and transverse vibration problems of a rotating twisted Timoshenko beam under axial loading (Chen and Ho, 1999).

Furthermore, Ozdemir and Kaya (2006) studied the dynamic response of a tapered cantilever Bernoulli-Euler beam. They also used the differential transform method (DTM) to find the non-dimensional natural frequencies of tapered cantilever Bernoulli-Euler beam. Catal (2008) examines the free vibration equations for one end clamped and another end simply supported beam on elastic foundation. The governing equations were solved, by using the differential transform method (DTM) for various axial loads acting on the beam. The beam on the elastic foundation was investigated for these three different support conditions considering the various values of the ratio of the axial load acting on the beam to Euler Bernoulli. Also, Somia (2013) studied the dynamic response of a tapered cantilever Bernoulli-Euler beam. Differential transform method (DTM) was also used to find the non-dimensional natural frequencies of tapered cantilever Bernoulli-Euler beam.

Douka and Hadjileontiadis (2005) have investigated the dynamic behavior of a cantilever beam both theoretically and experimentally. Empirical mode decomposition and Hilbert transform were used and the instantaneous frequency was obtained. It was seen that the instantaneous frequency oscillation revealed breathing behavior. The changes in frequencies were small. Loutridis *at. el.* (2005) have developed a new method reliant on the instantaneous frequency and empirical mode decomposition using theoretical and experimental investigations that were done on a cantilever beam due to a harmonic excitation for presenting the dynamic response. It was seen that the instantaneous frequency oscillation revealed the breathing phenomenon. This time-frequency approach was better compared to Fourier analysis and is more effective for finding the dynamic response of the beam. Firouz-Abadi et al. (2007) studied the transverse free vibrations of a class of variable-cross section beams using Wentzel-Kramers-Brillouin (WKB) approximation. The governing equation of motion for the Euler-Bernoulli beam including axial force distribution was utilized to obtain a singular differential equation in terms of the natural frequency of vibration and a WKB expansion series was applied to find the solution. Ariaei, Ziaei-Rad, and Ghayour (2009) presented both analytical and calculation methods to determine the dynamic behavior of the un-damped Euler–Bernoulli breathing beams under a point moving mass using discrete element technique and the finite element method. It was observed that there were higher deflections and change in beam response was seen. The largest deflection in the beam for a particular speed takes long to build up. Huang and Li (2010) investigated the free vibration of axially functionally graded beams with variable flexural rigidity and mass density. The investigation into the dynamic response of a Bernoulli beam resting on a Winkler foundation under the action of uniform partially distributed moving load was presented by Usman (2003). The finite difference method was used to solve the coupled partial differential equation, the result revealed that the amplitude of the beam resting on the Winkler foundation increases with an increase in the value of the foundation constant. Papadimitriou *et. al.* (2005) provides a methodology for the optimal establishment of the number and location of sensors on randomly vibrating structures for the purpose of the response predictions at unmeasured locations in structural systems.

Kozien (2013) analyzed the analytical solutions of excited vibrations of the Euler-Bernoulli beam, in the general case of the external loading function. The distribution theory was applied to formulate a solution when the external functions are the concentrated force type or the concentrated moment type.

Meanwhile, Coskun *et al*. (2011); Ozturk, (2009); Snamina *et al*. (2012a, 2012b); Sriram and Craig, (1992); Trucco and Verri, 1998; Van der Avweraer *et al*, (2002) used both the variational iteration method (VIM) and homotopy perturbation method (HPM) to solve the free vibration equations of beam on elastic foundation for support conditions of one end clamped, and another end simply supported, both ends clamped and both ends simply supported considering various case studies. Civalek and Ozurk (2010) studied the dynamic behavior of the tapered column with pinned ends embedded in the Winkler-Pasternak elastic foundation.

Pesterve *et al.* (2015) developed simple tools for finding the maximum deflection of a beam for any given velocity of the traveling force. It’s shown that for given boundary conditions, there exists a unique response velocity dependence function. They suggested a technique to determine this function which is based on the assumption that the maximum beam response can be adequately approximated by means of the first mode.

Also, the maximum response function is calculated analytically for a simply supported beam and constructed numerically for a clamped-damped beam. Friction dampers are another common passive vibration control system that dissipates energy through friction forces. These forces are generated with moving parts by sliding over each other. The energy dissipated by a friction damper reduces the energy demand on the structure and damps the structural response. The friction damper system includes the friction unit and a structural system in order to integrate the friction unit with the structure. The structural system can be either steel braces bolted to corner regions of the open bay space in the frame or an infill wall with gaps around the edges to prevent stiffness interaction of the wall with the frame members. Friction dampers are used as sacrificial or non-sacrificial elements. Their utilization as sacrificial elements is a very common attitude in a civil engineering environment. In earthquake engineering applications, some of the structural members might be sacrificed in order to prevent the collapse of the entire structure. These structural members absorb and dissipate the transmitted energy through plastic deformation in specially detailed regions. Location of the friction damper and stiffness of the braces which are used in order to install dampers are the main factors that affect the design parameters of the damper.

Usman *et al.* (2018) presented an analysis of free vibrations of a cantilever beam and simply supported beam using a series solution. It was found that the deflection of the beam increases as the length of the beam increases for a cantilever beam but decreases for the case of a simply supported beam. The response amplitude of a cantilever beam is greater than that of a simply supported beam.

The deflection of the beam can be studied using different beam theories approach which includes the Euler-Bernoulli beam, Shear beam, Rayleigh beam and Timoshenko beam This study investigates the deflection of the free vibration of Timoshenko beam. The aim of this paper is to investigate the effect of various parameters on the vibration of the free Euler-Bernoulli beam with different boundary conditions. In order to achieve the set aim, the following are the objectives of this study, which are:

1 .To give a comprehensive analysis of free vibration of Euler-Bernoulli beam
2 . To present a very simple and practical technique for determining the response of Euler-Bernoulli beams with different boundary conditions (Simply Supported, Free-Free and Fixed-Fixed Condition).

3. To compare between Simply Supported, Free-Free and Fixed-Fixed Euler-Bernoulli Beam.

MATHEMATICAL FORMULATION

In this work, a uniform beam of free vibration is considered the free vibration of a beam of finite length *L*, the differential equation for the free vibration of a beam when the beam is of constant flexural rigidity *EI* is given as

$$EI\frac{\partial^4 V(x,t)}{\partial x^4} + \rho A\frac{\partial^2 V(x,t)}{\partial t^2} = 0 \quad (1)$$

where:

E = Coefficient of elasticity, *I* = is the moment of inertia of the beam cross-section, *ρ* =Density of the mass, *A* = Surface area of the beam *t* = Time coordinate *x* = Spatial coordinate *ρA* = Mass per unit length, *V* (*x*,*t*) = is the deflection of the beam.

The boundary conditions:

Three cases are considered in this work, namely beams which are Fixed-beam, free beam and simply supported beam respectively.

$$V(x,t) = \frac{\partial v(x,t)}{\partial x} = 0 \quad at \;\; x = 0 \quad or \;\; x = L \quad (2)$$

$$V(x,t) = \frac{\partial^2 v(x,t)}{\partial x^2} = 0 \quad at\; x = 0 \quad or \;\; x = L \quad (3)$$

$$\frac{\partial^2 v(x,t)}{\partial x^2} = \frac{\partial^3 v(x,t)}{\partial x^3} = 0 \quad at\; x = 0 \quad or \;\; x = L \quad (4)$$

$$\frac{\partial v(x,t)}{\partial x} = \frac{\partial^3 v(x,t)}{\partial x^3} = 0 \quad at\; x = 0 \quad or \;\; x = L \quad (5)$$

and finally the initial conditions are:

$$V(x,0) = V_0(x) \quad (6)$$

$$\frac{\partial V(x,0)}{\partial t} = v_0(x) = 0 \quad (7)$$

METHOD OF SOLUTION

In this section, the initial-boundary value problem described by equations (1),(2),(3),(4),(5),(6) and (7). To the effect, is assume that the unknown Lateral deflection $V(x,t)$ of the beam resting on a foundation can be expressed as

$$V(x,t) = F(x)\sin(\omega t + \alpha)$$

The equation can be written as;

$$F^{iv}\sin(\omega t + \alpha) = \frac{1}{\gamma^2}F\omega^2\sin(\omega t + \alpha) \quad (8)$$

$$F^{iv} = \frac{1}{\gamma^2}F\omega^2$$

Solution of the Spatial Function/Mode Shape Function

we shall consider the solution of the spatial function and then apply the boundary conditions considering two cases of the beam, the simply supported beam, and the cantilever beam. From the equation, with $K = \lambda^4$

$$F^{ive} - \lambda^4 F = 0$$

The general solution now becomes

$$F_n(x) = A_n\sin\lambda_n x + B_n\cos\lambda_n x + C_n\sinh\lambda_n x + D_n\cosh\lambda_n x$$

$$\frac{dF_n(x)}{dx} = \lambda\left(A_n\cos\lambda_n x - B_n\sin\lambda_n x + C_n\cosh\lambda_n x + D_n\sinh\lambda_n x\right)$$

$$\frac{d^2F_n(x)}{dx^2} = \lambda^2\left(-A_n\sin\lambda_n x - B_n\cos\lambda_n x + C_n\sinh\lambda_n x + D_n\cosh\lambda_n x\right)$$

$$\frac{d^3F_n(x)}{dx^3} = \lambda^3\left(-A_n\cos\lambda_n x + B_n\sin\lambda_n x + C_n\cosh\lambda_n x + D_n\sinh\lambda_n x\right) \quad (9)$$

Simply Supported Beam

For a simply supported beam, we have that the displacement and the bending moments ar zero at both ends, this translate into the following boundary conditions

$$V(0,t) = 0 = V(L,t) \quad (10)$$

$$\frac{\partial^2 V(0,t)}{\partial x^2} = 0 = \frac{\partial^2 V(L,t)}{\partial x^2}$$

Substituting the expression *V* in the boundary conditions, we have

$$V(x,t) = \sum_{n=1}^{\infty} \left(\sin \frac{n\pi}{L} x\right) \sin(\omega t + \alpha) \tag{11}$$

as the free vibration solution of a simply supported beam.

Free-Free Beam

For a free-free beam, we have that the bending moment and shear forces are zero at both ends which translate into the following boundary conditions

$$\frac{\partial^2 V(0,t)}{\partial x^2} = 0 = \frac{\partial^2 V(L,t)}{\partial x^2} \tag{12}$$

$$\frac{\partial V^3(0,t)}{\partial x^3} = 0 = \frac{\partial V^3(L,t)}{\partial x^3} \tag{13}$$

Substituting the expression *V* in the boundary conditions, we have

$$V(x,t) = \left[-\frac{\cos\lambda L + \cosh\lambda L}{\sinh\lambda L - \sin\lambda L}(\sin\lambda x - \sinh\lambda x) + (\cos\lambda x - \cosh\lambda x)\right]\sin(\omega t + \alpha) \tag{14}$$

$$1 - \cos\lambda L \cosh\lambda L = 0$$

Equation (15) is the free vibration of a Free-Free beam.

Fixed-Fixed Beam

For a fixed-fixed beam, we have that the displacement and slope are zero at both ends which translate into the following boundary condition

$$V(0,t) = 0 = V(L,t)$$

$$\frac{\partial V(0,t)}{\partial x} = 0 = \frac{\partial V(L,t)}{\partial x} \tag{16}$$

Substituting the expression *V* in the boundary conditions

$$V(x,t) = \left[-\frac{\cos\lambda L + \cosh\lambda L}{\sinh\lambda L - \sin\lambda L}(\sin\lambda x - \sinh\lambda x) + (\cos\lambda x - \cosh\lambda x)\right]\sin(\omega t + \alpha) \tag{17}$$

$$1 - \cos\lambda L \cosh\lambda L = 0$$

Equation (18) is the free vibration of a Fixed-Fixed beam.

RESULTS AND DISCUSSION

In order to validate our model in the previous section, the following beam dimension and specification are used:

The beam was made of steel

E=2.1×$10^{11}N$

Length (L)=10 m

Density of the mass (ρ) = 7800kg/m^3

The surface area of the beam cross-section A=0.01×0.01m^2

Moment of Inertia $I = 8.33 \times 10^{-17} m^4$ alpha(α)=0.005;

DISCUSSIONS

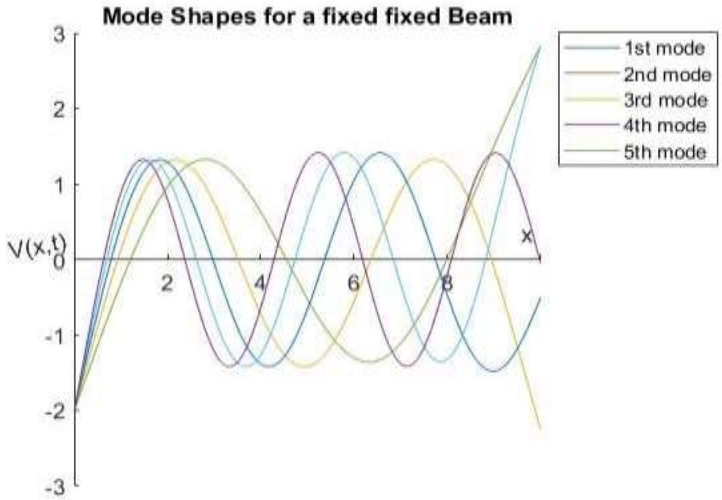

Figure 1: First five mode shapes for Fixed-Fixed Beam

Figure 1 shows the mode shape for Fixed-Fixed Beam while figure 2 display the mode shape for Free-Free Beam, it is found that the resonance frequencies are the same for both cases (Free-Free beam and Fixed-Fixed beam) except that in the case of Fixed-Fixed beam, there is no translation and rotation at $\omega = 0$ since it is not allowed by the boundary conditions.

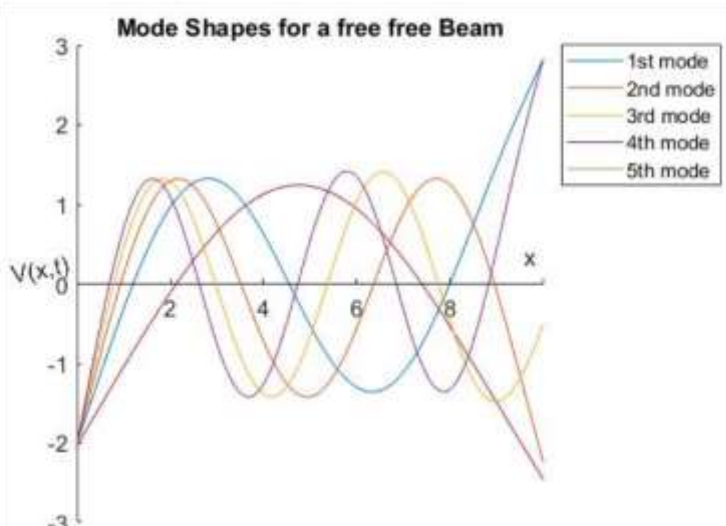

Figure 2: First five mode shapes for Free-Free Beam

Figure 2 shows the modal shapes for the simply supported beam.

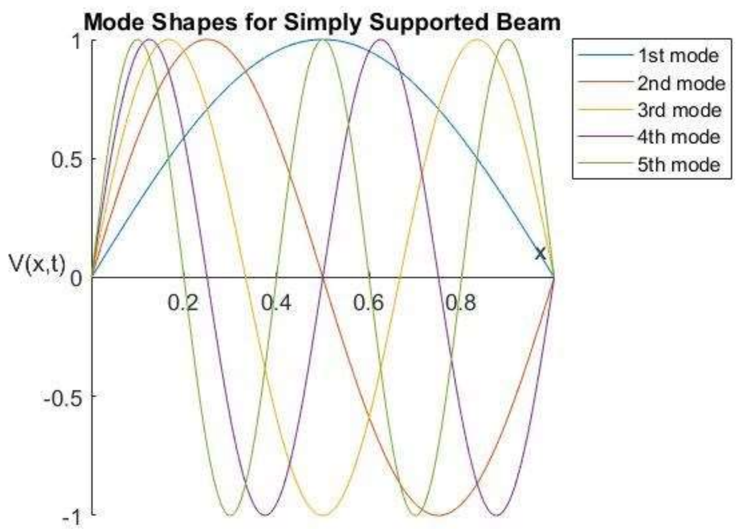

Figure 3: First five mode shapes for Simply Supported Beam

Figure 3 shows the mode shape for a Simply Supported Beam, It can be seen that as the number of mode increases the amplitude also increases which shows that the mode shape for each of the modes has effects on the displacement or deflection of such beam so that the deflection increases as the modes increases.

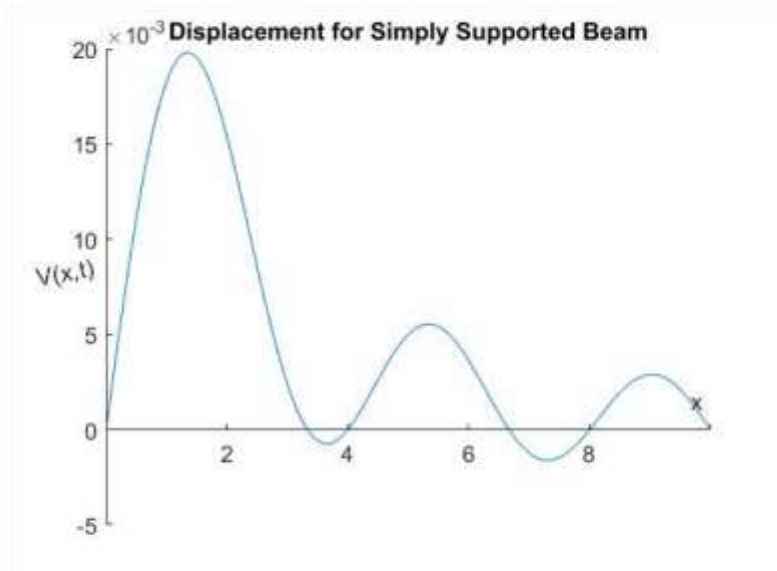

Figure 4: Displacement of a Simply Supported Beam for n=1...5

Figure 4 shows displacement for the simply supported beam.

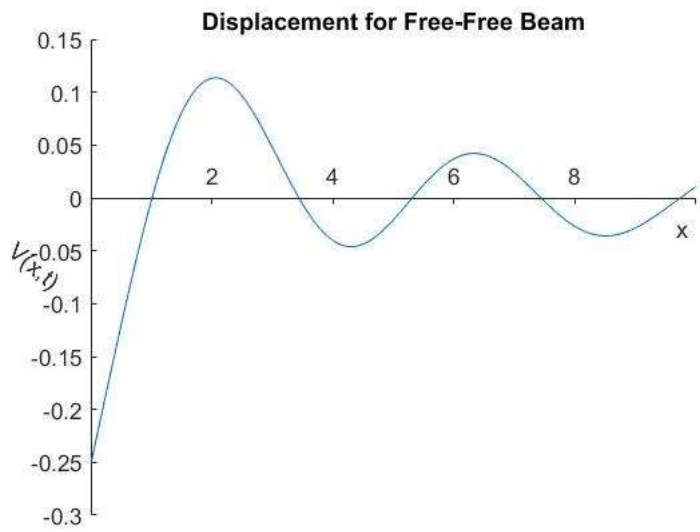

Figure 5: Displacement of a Free-Free Beam for n=1...5

Figure 5 SHOW displacement for a force-free and fixed beam for the values of n=1 to 5

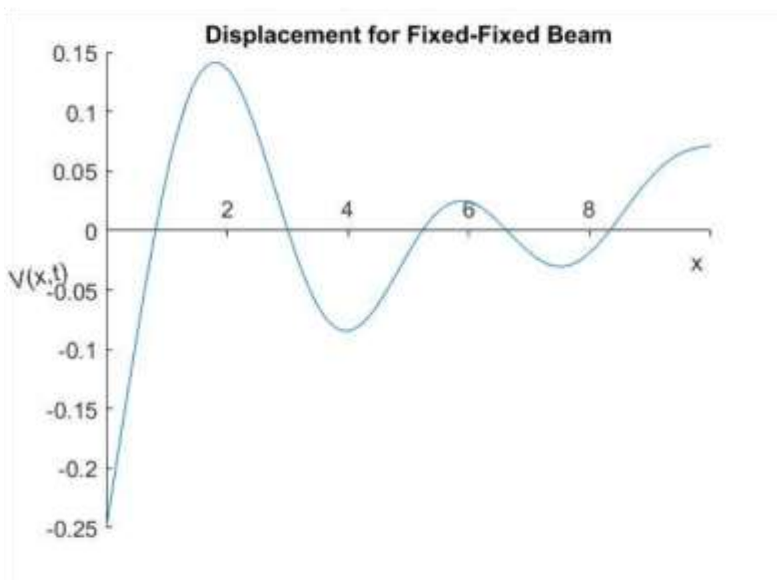

Figure 6: Displacement of a Fixed-Fixed Beam for n=1…5

Figure 4, 5 and 6 Shows the displacement of a Simply Supported Beam, Free-Free Beam, and Fixed-Fixed respectively for the summation of n=1 to 5. It is found that the amplitude decreases along the axis. Figure 7 displays the comparison between the displacement of a Simply Supported, Free-Free and Fixed-Fixed Beam for n=1 to 5. It is found that a Simply-Supported beam has a lower displacement compared to the Free-Free Beam and Fixed-Fixed Beam which almost have the same displacement.

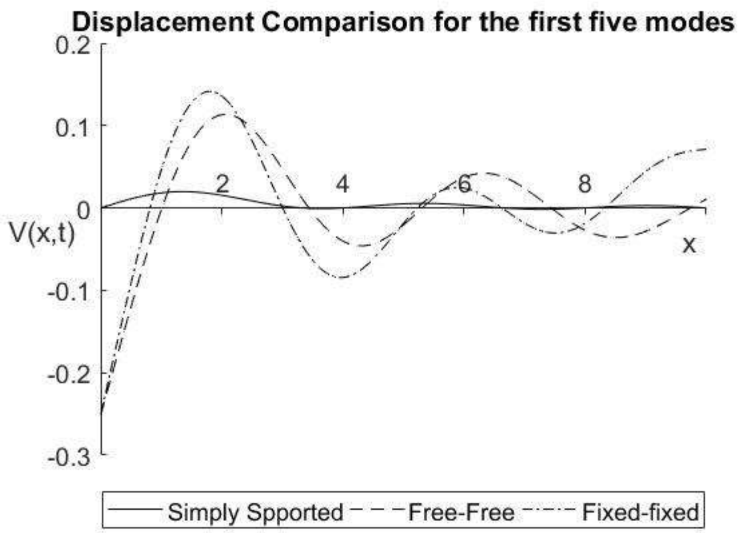

Figure 7: Displacement Comparison for n=1..5 between Simply Supported, Free-Free and Fixed-Fixed Beam.

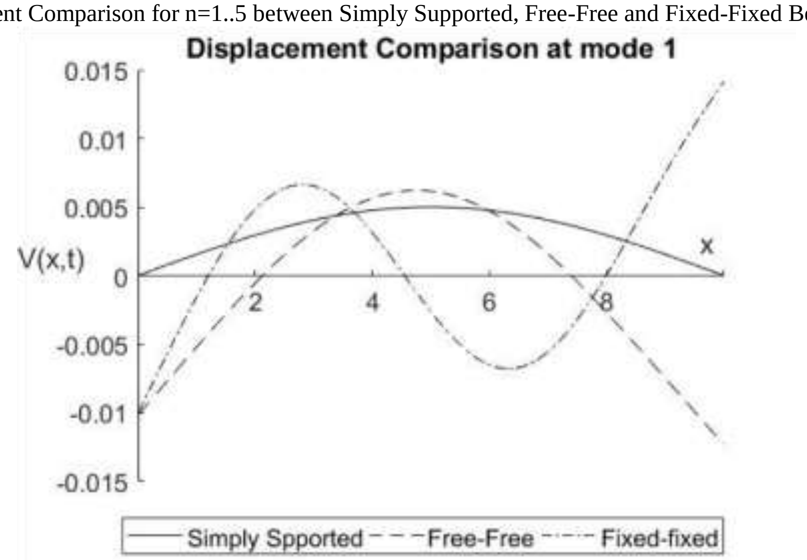

Figure 8: Displacement Comparison for n=1 between Simply Supported, Free-Free and Fixed-Fixed Beam

Figure 8 displays the comparison for n=1 between the displacement of a Simply Supported, Free-Free and Fixed-Fixed beam. At mode one, it is seen that a Simply Supported beam has a higher amplitude, followed by a Free-Free beam and then a Fixed-Fixed beam.

CONCLUSION

The effect of free vibration of the Euler-Bernoulli beam for various support conditions is considered in this paper. The governing equation of the fourth-order partial differential equation was solved using a series solution. The deflection for various values of the length of the beam was considered for each of the beams and was plotted against x using a computer program (MATLAB).

It can be concluded from the Figures (4.1)-(4.8) above that the mode shape for each of the modes has effects on the displacement or deflection of such beam so that the deflection increases as the modes increase. Also, a Simply-Supported beam has a lower displacement compared to the Free-Free Beam and Fixed-Fixed Beam which almost have the same displacement. At mode one, it is seen that a Simply Supported beam has a higher amplitude, followed by a Free-Free beam and then a Fixed-Fixed beam.

REFERENCES

Ariaei, A., Ziaei-Rad, S. and Ghayour, M. (2009). Vibration analysis of beams with open and breathing cracks subjected to moving masses. *Journal of Sound and Vibration*, 326: 709-724.

Catal H. H. (2002). Free vibration of partially supported piles with the effects of bending moment, axial and shear force, *Engineering Structures*, 24(12):1615–1622.

Catal S. (2008). The solution of free vibration equations of the beam on elastic soil by using differential transform method. *Applied Mathematical Modelling*, 32(9):1744–1757.

Chen C.K. and Ho S.H. (1996). Application of differential transformation to eigenvalue problems. *Applied Mathematics and Computation*, 79(2-3):173–188.

Chen C. K. and Ho S. H. (1999). Transverse vibration of a rotating twisted Timoshenko beams under axial loading using differential transform. *International Journal of Mechanical Sciences* 41(11):1339–1356.

Civalek O. and Ozturk B. (2010). Free vibration analysis of tapered beam-column with pinned ends embedded in the Winkler-Pasternak elastic foundation. *Geomechanics and Engineering*, 2(1):45–56.

Coskun S. B., Atay M. T. and Ozturk B.(2011). *Transverse vibration analysis of Euler-Bernoulli beams using analytical approximate techniques*. Advances in Vibration Analysis Research, chapter 1, pp. 1–22, In-Tech, Vienna, Austria.

Douka, E. and Hadjileontiadis, J.L. (2005). Time-frequency analysis of the free vibration response of a beam with a breathing crack. *NDT&E International*, 38: 3-10.

Doyle P. F. and Pavlovic M. N.(1982). The vibration of beams on partial elastic foundations. *Earthquake Engineering and Structural Dynamics*, 10(5):663–674.

Firouz-Abadi R. D., Haddadpour H., and Novinzadeh A. B. (2007). An asymptotic solution to transverse free vibrations of variable-section beams. *Journal of Sound and Vibration*, 304(3-5): 530–540.

Huang Y. and Li X.F. (2010). A new approach for free vibration of axially functionally graded beams with non-uniform cross-section. *Journal of Sound and Vibration*, 329(11):2291–2303, 2010.

Kozien M. S. (2013). Analytical solution of exciting vibrations of a beam with the application of distribution. *Acta Physica Polonica A*, 126(6):1029-1033.

Ozdemir O. and Kaya M. O. (2006). Flaps bending vibration analysis of a rotating tapered cantilever Bernoulli-Euler beam by differential transform method. *Journal of Sound and Vibration*, 289(1-2):413–420.

Ozturk B. (2009). Free vibration analysis of beam on an elastic foundation by the variational iteration method. *International Journal of Nonlinear Sciences and Numerical Simulation*, 10(10):1255–1262.

Papadimitrou C., Haralampidis Y., and Sobczyk K. (2005). Optimal experimental design in stochastic structural dynamics. *Probabilistic Engineering Mechanics*, 20:67-78.

Sioma A. (2013). The rope wear analysis with the use of 3D vision system, Control Engineering, 60(5):48-49

Snamina J.and Sapin´s ski B., Romaszko M. (2012a). Vibration parameter analysis of a sandwich cantilever beams with multiple MR fluid segments. *Engineering Modeling*, 43:247-254

Snamina J. and Sapin´ski B., Wszol Ek W., Romaszko M. (2012b). Investigation on vibrations of a cantilever beam with magnetorheological fluid by using the acoustic signal. *Acta Physica Polonica A*, 121:1-8

Sriram P. and Hanagud S., Craig J.I. (1992). Mode shape measurement using a scanning laser Doppler vibrometer. *The International Journal of Analytical and Experimental Modal Analysis*, 7(3):169-178

Trucco E. and Verri A. (1998). *Introductory Techniques for 3D Computer Vision*, Prentice-Hall

Tuma J. and Cheng F. (1983). *Theory and Problems of Dynamic Structural Analysis*, Schaum's Outline Series, McGraw-Hill, New York, NY, USA.

Usman M,A. (2003). Dynamic response of a Bernoulli beam on the Winkler foundation under the action of moving partially distributed moving load. *Nigeria Journal of Mathematics and Application*, 16:128-147

Usman M.A., Hammed F.A., Olayiwola M. O, Okusaga S. T. and Daniel D. O. (2018). Free Vibrational Analysis of Beam with Different Support Conditions. *Transaction of the Nigerian Association of Mathematical Physics*, 7:27-32.

Van der Auweraer H., Steinbichler H., Haberstok C., Freymann R. and Storer D. (2002). Integration of pulsed laser ESPI with spatial domain modal analysis. *Shock and Vibration*, 9(1/2):29-42

West H. H. and Mafi M.(1984). Eigenvalues for beam-columns on elastic supports. Journal of Structural Engineering, 110(6):1305–1320.