



# A PROBABILISTIC MODELLING APPROACH TO QUANTIFY AND ASSESS THE IMPACT OF CLIMATE CHANGE ON THE GROWTH OF YAM IN MAKURDI, NIGERIA

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### ABSTRACT

Probabilistic Models that quantify and assess the short and long run impact of climate change on the growth of yam in Makurdi, Nigeria are rare; this is because most existing models only consider the distribution pattern of the climatic variables without crop requirements. This work has been able to develop a probabilistic modeling approach for this purpose. Data on daily rainfall amount in the study area for the period of 34 years(1977-2010) were transformed into zeros (0) and ones (1) representing binary outcome of a random variable defined as the climatic condition for the growth of yam in respect of rainfall. The outcome "1" represent a favourable climatic condition while the outcome "0" represent an unfavourable climatic condition. The sequence of favourable and unfavourable outcomes formed a discrete time stochastic process which was modeled as aMarkov Chain. The n-step matrix of transition probabilities shows in probabilistic terms the results of the short run impact of climate change on the growth of yam while the converged or steady probability matrix shows the long run impact. For the seven – month phase, the study area is at risk of 79% negative impact of climate change on the growth of 21% every four years. It was recommended that farmers in Makurdi metropolis should invest less in yam production yearly and more every five years and that the modeling approach be generalized for quantifying and assessing the impact of climate change on any crop and other climatic variables.

Keywords: Climate, Climatic Condition, favourable, Crop, discrete time stochastic process, Markov Chain

# INTRODUCTION

Climate change modeling and prediction are essential input in agriculture as bumper harvest depends largely on the variability and quantity of these variables. The yield of crops particularly in dry land depends on the rainfall pattern, dry spells occur due to insufficient rainfall throughout the rainy season (Behera and Subudhi, 2018). The rates of change in climate have made several researchers to develop interest on how to control or adapt to these changes using statistical models. Among them is Ayinde et al. (2011) who studied the effect of climate change on agricultural product in Nigeria using the co-integration model approach. The multiple regression model was used for the assessment of food crop production in relation to climate variation, for instance (Olajire et al., 2018; Mijinyawa and Akpenpuun 2015; Aondoakaa 2012) The study by Zakari et al. (2017) aimed at establishing the relationship that exists between rainfall, temperature and yam yield, the correlation analysis showed there is a weak correlation between each of the climatic parameter and the yield of yam .Another work in this line is the one by Uger (2017), the author found that the trend of yam production in Benue state showed fluctuations over the years and the highest production was in 2015 with the production rate of 2874.80 metric tons and this was as a result of the variations in temperature and rainfall which showed that

there is positive, but weak relationship between temperature and rainfall variability on yam production in the study area. Enete (2014) studied the impacts of climate change on agricultural production in Enugu, Nigeria. Rainfall data for thirty years (1981–2010) was used for the study. The data was analysed using descriptive statistics and correlation analysis. The study revealed a general change in the seasonal rainfall regime with a long drier season. The study also showed that all the traditional crops with the exception of cassava and pepper had a significant yield decrease as rainfall continued to be more inconsistent.

Some researchers have studied effect of climate change on climatic parameters and the yield of crops using Markov chain models. Yoo et al. (2016) evaluated the effect of climate change on daily rainfall on the mean number of wet days and the mean rainfall amount. A three-state Markov chain was employed to examine the pattern and distribution of daily rainfall in Uyo metropolis of Nigeria (Raheem *et al.*, 2015).

Yusuf *et al.* (2014) analyzed annual rainfall distribution for crop production using the Markov chain model. A four state model for annual rainfall distribution in Minna, Nigeria with respect to crop production was used. The results showed that in the long run 14% of annual rainfall shall be low, 34% annual rainfall will be moderate also well spread, 47% of the annual

rainfall shall be high and 5% of the annual rainfall shall be moderate but not well spread respectively.

Kar *et al.* (2014) studied weekly rainfall pattern for Crop Planning in Kandhamal District of Odisha, India using Markov's Chain Model. The results revealed that, there are total of 48 weeks where rainfall exceeds more than 20 mm, so harvesting excess runoff water for future supplemental irrigations was encouraged. The authors also drew attention towards soil erosion measures to control for erosion. Other works in the vein include those of Nuga and Adekola (2018); Yusuf *et al.* (2016) Makokha *et al.* (2016).

Agada et al. (2018) developed statistical Indicators of climate change in Makurdi Nigeria. Data on rainfall, air temperature and relative humidity for the period of 1977-2010 was used for the analysis. The statistical methods employed in developing the indicators were the Kruskal-Wallis one-way ANOVA test, the Linear trend and the Markov chain model. Two Functional Statistical Indicators were developed which showed significant difference in the distribution of wet and dry days, warmer and colder maximum air temperature days and high and low relative humidity days over the period. The indicator function revealed that climate change is fast setting into the study area. Agada et al. (2019) extended the work of Agada et al. (2018), probabilistic indicator functions were developed to two quantify climate change impact on the chance of occurrence of the climatic conditions for the growth and storage of crops in Makurdi, Nigeria. The study established the fact that climatic condition for crop growth and storage crop has been affected by climate change and opined that if climate change is not checked, the study area will lose its growth condition for crops. This study is motivated by the work of Agada et al. (2018). The authors used statistical indicator functions to study climate change as mentioned earlier, the long run probability of wet and dry days of the Markov chain model was used as an indicator of climate change setting into Markudi, Nigeria. The study did not quantify the impact of climate change on the

Let the random variable X<sub>t</sub> represent rainfall amount at time t. Mathematically; growth of specific crops in the area, but only discussed the implication of their result to crop production based on existing literature. In addition, Agada *et al.* (2019) was able to quantify the chance of occurrence of climate change conditioned on the climatic crop requirements only without quantifying the long term behavior of the climate.

Using the model by Agada *et al.* (2018) as a guide, this research focuses on the impact of climate change on crop production. It is concerned with the use of Markov chain model to assess the favourabilty in the climatic condition (in respect to rainfall) for the growth of yam in Makurdi, Nigeria. Many statistical models have been used to analyse the impact of climate change on crop production. However, one may not be able to lay a finger on any of them that have considered the short run and long run behavior of the climatic parameters conditioned on the climatic requirements of the specific crops. This gap in literature the researchers intend to fill.

# METHODS

### Data Description and Transformation

This study employed secondary data sourced from the Nigerian Meteorological Agency Headquarters, Tactical Air command, Makurdi, Benue state. The data is a climatic data on rainfall amount (mm) for Makurdi metropolis for a period of 34 years (1977-2010).

The climatic condition for the growth of yam requires an annual rainfall amount of at least 1250 mm (Ayoade, 1983) within a growing phase of 7, 8 and 9 months. This translates into at least 5.9mm/per day, 5.1 mm/per day and 4.6mm/per day for the respective growing phases. The rainfall amount was converted into a sequence of binary events such that: if the daily rainfall amount meets the requirement for the growth of yam it is said to be favourable and coded 1 and otherwise unfavourable and coded 0.

$X_t = \left\{ \begin{array}{l} 1, \\ 0, \end{array} \right.$	if rainfall amount $\geq 5.9$ mm otherwise	(1)
$X_t = \left\{ \begin{array}{l} 1, \\ 0, \end{array} \right.$	if rainfall amount $\geq 5.1 \text{ mm}$ otherwise	(2)
$X_t = \begin{cases} 1, \\ 0, \end{cases}$	if rainfall amount $\geq$ 4.6mm otherwise	(3)

for the growth phase of 7, 8 and 9 months respectively, t = 1, 2, ..., n (days).

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## Markov Chain Model

A Markov chain is a sequence of random variables  $X_1, X_2, X_3, ...$  with the Markov property: given the present state, the future and past states are independent. Formally,

 $Pr(X_{t+1} = x_{t+1} | X_1 = x_1, X_2 = x_1, ..., X_t = x_t) = Pr(X_{t+1} = x_{t+1} | X_t = x_t)$ (4) The current status of the system can fall into any one of N +1 mutually exclusive category called states. For notational convenience, these states are labeled 0, 1, ..., N. The random variable X<sub>t</sub> represents the state of the system at time t, so it's only possible values are 0, 1, ..., N. The system is observed at particular times, labeled t = 0, 1, 2, ..., N. Thus, the stochastic process X<sub>t</sub> = {X<sub>0</sub>, X<sub>1</sub>, X<sub>2</sub>, ..., X<sub>n</sub>} provides a mathematical representation of how the status of the physical system evolves over time. This kind of process is referred to as being a discrete time stochastic process with a finite state space (Hillier and Lieberman, 2010).

#### **State and Transition Probability Matrix**

Predicting the future state involves knowing the system likelihood or the probability of the chain from one state to another (Sharma, 2010).  $f_{ij}$  represents the frequency of the process of a step through which the starting state i transfers to the state j in the sequence  $X_0, X_1, X_2, ..., X_n$ . The transition probability is specified by a matrix  $P = (P_{ij})$ . If S contains N states, then P is an N × N matrix whose entries are all nonnegative and whose rows sum to 1. The number  $P_{ij}$  is the conditional probability, given that the chain is in state i at time t, the chain jumps to the state j at time t + 1. For a Markov chain with the states  $s_1, s_2, ..., s_N$ , the matrix of transition probability is written as:

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1N} \\ p_{21} & p_{22} & \cdots & p_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ p_{N1} & p_{N2} & \cdots & p_{NN} \end{pmatrix}$$

where,  $P_{ij} = Pr(X_{t+1} = x_{t+1} | X_t = x_t)$ 

The transition probability  $P_{ij}$  is estimated as:

$$p_{ij} = \frac{\sum_{i=1}^{n} f_{ij}}{\sum_{i=1}^{n} \sum_{i=1}^{n} f_{ij}}$$
(6)

#### Long-run properties of Markov chains

This property of Markov chain states that regardless of the initial state of the system, when the number of transition steps is sufficiently large, then the transition probability from state *i* to state *j* becomes settle down to some constant value (Bhusal, 2017).

(5)

(7)

Hence, 
$$\lim_{n \to \infty} P_{ij}^{(n)} = \pi_j > 0$$

where the  $\pi_j$  uniquely satisfy the following steady-state equations

$$\pi_{j} = \sum_{i=1}^{M} \pi_{i} P_{ij}$$
 for  $j = 0, 1, 2, ..., N$ 

$$\sum_{i=1}^N \pi_j = 1$$

 $\pi_{j} = (\pi_{0}, \pi_{1}, \pi_{2}, \dots, \pi_{N})$ 

The  $\pi_j$  are called the steady-state probabilities of the Markov chain. The term steady state probability means that the probability of finding the process in a certain state, say j, after a large number of transitions tends to the value  $\pi_i$  (Hillier and Lieberman, 2010).

#### **Expected First passage times**

Let  $f_{ij}^{(n)}$  denote the probability that the first passage time from state *i* to *j* is equal to *n* and  $\mu_{ij}$  denote the expected first passage time from state i to state j, which is defined by

$$\mu_{ij} = \begin{cases} \infty, & if \sum_{n=j}^{\infty} f_{ij}^{(n)} < 1 \\ \\ \sum_{n=j}^{\infty} n f_{ij}^{(n)}, & if \sum_{n=j}^{\infty} f_{ij}^{(n)} = 1 \end{cases}$$
(8)

when

$$\sum_{n=j}^{\infty} f_{ij}^{(n)} = 1$$
(9)

 $\mu_{ii}$  uniquely satisfies the equation

$$\mu_{ij} = 1 + \sum_{k \neq j}^{\infty} p_{ik} \mu_{kj}.$$
(10)

For the case of  $\mu_{ij}$  where j = i, is the expected number of transitions until the process returns to the initial state i, and so is called the expected recurrence time for state i. After obtaining the steady-state probabilities ( $\pi_0$ ,  $\pi_1$ ,  $\pi_2$ ,...,  $\pi_N$ ), these expected recurrence times can be calculated immediately as

$$\mu_{ii} = \frac{1}{\pi_i} for i = 0, 1, ..., N$$

#### Modeling the Sequence of Favorable and Unfavourable Climatic Condition for the Growth of Yam

The 0 and 1 realization of the random variable  $X_t$  in equations 3.1-3.3 automatically define a two-state markov chain for modeling the sequence of favourable and unfavourable climatic condition for the growth of yam. This means that initially the process may be in any of the two states and thereafter transit to the other state. The probability of this state is what is known as the transition probability.

Let  $\{X_t, t \in T\}$  be a Markov chain with order set T and state space S. Particularly, for this work, since  $S = \{0,1\}$  then  $\{X_t, t \in T\}$ . The most common of this is the first order which is defined as  $P(X_{n+1} = j | X_0 = i_0, X_1 = i_0, ..., X_n = i_n)$ 

$$= P(X_{n+1} = j|, X_n = i_n)$$
(11)

for all 
$$i_0, i_1, \dots, i_n \in S$$

The total number of states, S of Xt is finite. In this study, S is equals two as earlier mentioned.

It is important to mention that a Markov chain of order 1 is completely determined by its initial state and set of transition probabilities  $P_{ij}$ . The permutations of the sequence of transitions ij for ij = 0,1 completely specify the sequence of favourable and unfavourable states of the Markov chain of order 1.

#### **RESULTS AND DISCUSSION**

Result based on the analysis of data collected is presented in this session. The result include the estimated n-step state-transition probabilities, steady state probabilities and mean first passage time of the states for the seven month phase, eight month phase and nine month phase respectively.

	P <sub>11</sub>	P <sub>10</sub>	P <sub>01</sub>	P <sub>00</sub>	
Seven month growth phase	0.1992213	0.80078	0.21521	0.78479	
Eight month growth phase	0.2066667	0.79333	0.19599	0.80401	
Nine month growth phase	0.2100939	0.789906	0.177456	0.822545	

#### **Table 1: Actual State Transition Probability**

Table 1 shows the transition probabilities for each of the growth phases 7, 8 and 9 months respectively. For the seven month growth phase, the probability of having a favourable year succeeding favourable year is 0.1992(20%), an unfavourable year succeeding favourable year 0.80078 (80%), a favourable year succeeding an unfavourable year is 0.21521 (22%) and unfavourable year succeeding unfavourable year is 0.78479 (78%). The eight month phase has a 20% chance of a favourable year succeeding a favourable year, 79% chance of an unfavourable year succeeding favourable year, 20% chance of a

favourable year succeeding unfavourable year and an 80% chance of an unfavourable year succeeding an unfavourable year, while the nine months phase has 21%,79%18% and 82% chance of transiting from a favourable year to a favourable year, a favourable year to an unfavourable year, unfavourable year to a favourable year and an unfavourable year to year yam growth transition in the short-run.

		Actual State	
	Preceding State	favourable	Unfavourable
2 <sup>nd</sup>	favourable	0.2120227	0.7879773
	unfavourable	0.2117671	0.7882329
3 <sup>rd</sup>	favourable	0.2118181	0.7881819
	unfavourable	0.2118221	0.7881779
4 <sup>th</sup>	favourable	0.2118213	0.7881787
	Unfavourable	0.2118213	0.7881787

Table 2: N-Step Transition Probability Values for the Seven Month Phase

# Table 3: N-Step Transition Probability Values for the Eight Month Phase

		Actual State	
	Preceding State	favourable	unfavourable
2 <sup>nd</sup>	Favourable	0.1981945	0.8018055
	Unfavourable	0.1980804	0.8019196
3 <sup>rd</sup>	Favourable	0.1981040	0.8018960
	Unfavourable	0.1981028	0.8018972
4 <sup>th</sup>	Favourable	0.198103	0.801897
	Unfavourable	0.198103	0.801897

### Table 4: N-Step Transition Probability Values for the Nine Month Phase

Preceding State         favourable         Unfavourable           2 <sup>nd</sup> favourable         0.1843126         0.8156874           unfavourable         0.1832474         0.8167526           3 <sup>rd</sup> favourable         0.1834712         0.8165288           unfavourable         0.1834364         0.8165636           4 <sup>th</sup> favourable         0.1834437         0.8165563           Unfavourable         0.1834426         0.8165574           5 <sup>th</sup> Favourable         0.1834428         0.8165572			Actual State	
2       Involution       0.10101010       0.00100011         unfavourable       0.1832474       0.8167526         3 <sup>rd</sup> favourable       0.1834712       0.8165288         unfavourable       0.1834364       0.8165636         4 <sup>th</sup> favourable       0.1834437       0.8165563         Unfavourable       0.1834426       0.8165574		Preceding State	favourable	Unfavourable
3rd       favourable       0.1834712       0.8165288         unfavourable       0.1834364       0.8165636         4 <sup>th</sup> favourable       0.1834437       0.8165563         Unfavourable       0.1834426       0.8165574	2 <sup>nd</sup>	favourable	0.1843126	0.8156874
4 <sup>th</sup> favourable         0.1834364         0.8165563           Unfavourable         0.1834437         0.8165563           Unfavourable         0.1834426         0.8165574		unfavourable	0.1832474	0.8167526
4 <sup>th</sup> favourable         0.1834437         0.8165563           Unfavourable         0.1834426         0.8165574	3 <sup>rd</sup>	favourable	0.1834712	0.8165288
Unfavourable 0.1834426 0.8165574		unfavourable	0.1834364	0.8165636
	4 <sup>th</sup>	favourable	0.1834437	0.8165563
5 <sup>th</sup> Favourable 0.1834428 0.8165572		Unfavourable	0.1834426	0.8165574
	5 <sup>th</sup>	Favourable	0.1834428	0.8165572
Unfavourable 0.1834428 0.8165572		Unfavourable	0.1834428	0.8165572

Table 2, 3 and 4 show the n-step transition probabilities for each of the growth phases 7, 8 and 9 months respectively. The  $2^{nd}$ , 3rd,  $4^{th}$  and  $5^{th}$  step are the transition probabilities from one state to another in two year, three years and four years respectively.

### **Table 5: Steady State Probabilities**

	$\pi_{\rm F}$	$\pi_{F'}$
Seven month growth phase	0.2118213	0.7881787
Eight month growth phase	0.198103	0.801897
Nine month growth phase	0.1834428	0.8165572

The steady state transitions probabilities of the rainfall condition for growth of yam for the 7, 8 and 9 months growth phases is captured in table 5 above. In the long run the rainfall amount in Makurdi will be 21% favourable and 79% unfavourable for the growth of yam for seven months phase, 20% favourable and

80% unfavourable for eight months phase and 18% favourable and 82% unfavourable for nine months phase respectively.

This steady state transition probability for rainfall in Makurdi reveals the following information: The probability that the amount of rainfall will be favourable for the growth of yam in near future irrespective of its initial states of being favourable or unfavourable is 0.2118(21%), there is 0.7882 (79%) chances that the rainfall amount will be unfavourable in near future irrespective of its initial states being favourable or unfavourable.

It was observed that the entire transition matrix contains a more likelihood of probabilities in the second column, this shows that irrespective of what state the chain occupies, there is a high probability that the next state in the chain will be an unfavourable state. This is an indication that in the long run, rainfall amount will not be favourable for the growth of yam. This might lead to reduction in yam production if measures are not taken.

# Table 6: Mean First Passage Time

	F to F	F to F'	F <sup>,</sup> toF	F' to F'
Seven month growth phase	4.72096	1.24878	4.64668	1.26875
Eight month growth phase	5.04788	1.26050	5.10237	1.24704
Nine month growth phase	5.45129	1.26597	5.63352	1.22247

## F is favourable and F' is unfavourable

It is desirable to know the number of transitions made by the process in going from favourable state to unfavourable state for the first time. Table 6 shows the mean first passage time for the seven, eight and nine months growth phase. It can be observed that expected time until the rainfall amount is unfavourable, given that the process started when rainfall amount is favourable is about a year while expected time until the rainfall amount is favourable, given that the process started when rainfall amount is unfavourable is favourable, given that the process started when rainfall amount is unfavourable, given that the process started when rainfall amount is unfavourable is 5 years.

The expected return time to the same state is reciprocal of the steady state probabilities. For the seven months growth phase, the expected return time to the favourable state, starting from the same favourable state is 5.047879. This result shows that the chain for the rainfall amount should visit the favourable state on an average of five years. In the similar way, the expected return time to unfavourable state, starting from the same state is 1.247043. This means the chain for rainfall amount should visits the unfavourable state on an average of one year. This implies that the study area will experience favourable climatic conditions in respect of rainfall for the growth of yam and unfavourable climatic condition every five (5) years and every one year respectively. The eight months phase is also captured in table 9. It can be seen that, on the average the time until the rainfall amount is unfavourable, given that the process started when rainfall amount is favourable is about one year while expected time until the rainfall amount is favourable, given that the process started when rainfall amount is unfavourable is 5 years. The mean return time is every five (5) years and every one year for favourable and unfavourable climatic conditions in respect of rainfall for the growth of yam respectively. The nine months phase also implies a mean return time of every five (5) years and every one year for favourable and unfavourable climatic conditions in respect of rainfall for the growth of yam too. In a nutshell the study shows the area will experience

favourable climate for the growth of yam every four to five years.

### CONCLUSION

The following conclusions were drawn from the study:

- (i) a probabilistic modeling approach for quantifying and assessing the impact of climate change on the growth of yam was developed in this study.
- (ii) the analysis of the first order Markov chain employed showed that the chain is finite and irreducible.
- (iii) Makurdi, Nigeria is at 79% risk of unfavourable yam yield and a favourable yam yield of 21%, 80% risk of unfavourable yam yield and a favourable yam yield of 20% and an 82% risk of unfavourable yam yield and a favourable yam yield of 18% for the seven, eight and nine-month growth phases respectively.
- (iv) Makurdi, Nigeria experiences unfavourable yam yield yearly and favouable yam yield in every five years.

### RECOMMENDATIONS

Climate change has been established to have adverse effects on crop production. Hence, there is need to device effective strategies that will help to control the effect of climate change to enhance food sustainability. Based on the findings of this work, it is recommend that:

- (i) farmers in Makurdi metropolis should invest less in yam production yearly and more every five years.
- (ii) the modeling approach should be generalized for quantifying and assessing the impact of climate change on any crop and other climatic variables.

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