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N-LEVEL AND N-UPPER LEVEL SOFT SETS

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ABSTRACT
In this paper, the concept of n-upper level soft set is introduced together with some of its properties. It is shown that some properties holding in n-level soft set do not hold in n-upper level soft set. It is further demonstrated that both the first and the second decomposition theorems fails in n-upper level soft set.

Keywords: Soft set, Multiset, Soft multiset, n-level Soft set, n-upper level Soft set

INTRODUCTION

The issue of handling various problems arising in environmental science, medicine, engineering, social sciences etc., which has various uncertainties has become a great concern to scientist. Therefore, the theories of soft set and multiset emerged which are useful mathematical tools in dealing with uncertainties. Researchers such as (Blizard, 1991; Molodtsov, 1999; Maji et al., 2002; Singh et al., 2007; Ali et al., 2009; Qin and Hong, 2010; Sezgin and Atagun, 2011; Alkhazaleh et al., 2011; Majumdar, 2012; Babitha and Sunil, 2013; Tokat and Osmanoglu, 2013; Isah and Tella, 2015; Singh and Isah, 2016) immensely contributed to the emergence and development of these theories.

The concept of n-level set and n-upper level set were first introduced in (Nazmul et al., 2013) and (Ibrahim et al. 2016), respectively, together with some of their properties. However, the concept of n-level Soft set was first initiated in (Isah, 2019). In this paper n-upper level soft set is introduced and some of its properties characterized.

Preliminaries

Definition 1 Soft set (Molodtsov, 1999)

Let U be an initial universe set and E a set of parameters or attributes with respect to U. Let $P(U)$ denote the power set of U and $A \subseteq E$. A pair $(F, A)$is called a *soft set* over U, where F is a mapping given by $F: A \longrightarrow P(U)$.

In other words, a soft set (F, A) over U is a parameterized family of subsets of U. For $e \in A$, $F(A)$ may be considered as the set of e-elements or e-approximate elements of the soft set (F, A). Thus (F, A) is defined as

$$(F, A) = \{F(e) \in P(U): e \in E, F(e) = \emptyset \text{if } e \notin A\}.$$

Definition 2 Multisets (Jena et al., 2001; Girish and John, 2009)

An mset M drawn from the set X is represented by a function $Count\ M$or $C_M$defined as$C_M: X \longrightarrow \mathbb{N}$.

Let M be a multiset from X with x appearing $n$times in M. It is denoted by $x \in^n M$. $M = \{k_1/x_1, k_2/x_2, \dots, k_n/x_n\}$ where M is a multiset with x_1 appearing k_1 times, x_2 appearing k_2 times and so on.

Let M and N be two msets drawn from a set X. Then

$M \subseteq N$ iff $C_M(x) \leq C_N(x)$ for all $x \in X$.

$M = N$ if $C_M(x) = C_N(x)$ for all $x \in X$.

$M \cup N = max\{C_M(x), C_N(x)\}$ for all $x \in X$.

$M \cap N = min\{C_M(x), C_N(x)\}$ for all $x \in X$.

$M - N = max\{C_M(x) - C_N(x), 0\}$ for all $x \in X$.

Let M be a multiset drawn from a set X. The support set of Mdenoted by M^*, is defined as $M^* = \{x \in X: C_M(x) > 0\}$.

The power multiset of a given mset M, denoted by $P(M)$ is the multiset of all submultisets of M, and the power set of a multiset M is the support set of $P(M)$, denoted by $P^*(M)$.

Let $\{M_i : i \in I\}$ be a nonempty family of msets drawn from a set X. Then

(i) Their Intersection, denoted by $\bigcap_{i \in I} M_i$ is defined as

$$C_{\bigcap_{i \in I} M_i}(x) = \bigwedge_{i \in I} C_{M_i}(x), \forall x \in X,$$

where $\wedge$is the minimum operation.

(ii) Their Union, denoted by $\bigcup_{i \in I} M_i$ is defined as

$$C_{\bigcup_{i \in I} M_i}(x) = \bigvee_{i \in I} C_{M_i}(x), \forall x \in X,$$

where $\vee$ is the maximum operation.

Definition 3 Soft Multiset (Tokat and Osmanoglu, 2013)

Let U be a universal multiset, E be a set of parameters and $A \subseteq E$.Then a pair (F, A) is called a soft multiset where F is amapping given by $F : A \longrightarrow P^*(U)$. For all $e \in A$, the mset $F(e)$ is represented by a count function $C_{F(e)}: U^* \longrightarrow \mathbb{N}$.

Let (F, A) and (G, B) be two soft multisets over U. Then
(a) (F, A) is a soft submultiset of (G, B) written $(F, A) \sqsubseteq (G, B)$ if
i. $A \subseteq B$
ii.$C_{F(e)}(x) \leq C_{G(e)}(x), \forall x \in U^*, \forall e \in A$.

(b)$(F, A) = (G, B) \Leftrightarrow (F, A) \sqsubseteq (G, B)$ and $(G, B) \sqsubseteq (F, A)$.
Also, if $(F, A) \sqsubset (G, B)$ and $(F, A) \neq (G, B)$ then (F, A) is called a proper soft submset of (G, B) and (F, A) is a whole soft submset of (G, B) if $C_{F(e)}(x) = C_{G(e)}(x), \forall x \in F(e)$.
(c) Union:
$(F, A) \sqcup (G, B) = (H, C)$ where $C = A \cup B$ and $C_{H(e)}(x) = max\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in C, \forall x \in U^*$.
(d) Intersection:
$(F, A) \sqcap (G, B) = (H, C)$ where $C = A \cap B$ and $C_{H(e)}(x) = min\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in C, \forall x \in U^*$.
(e) Difference:
$(F, E)\backslash(G, E) = (H, E)$ where $C_{H(e)}(x) = max\{C_{F(e)}(x) - C_{G(e)}(x), 0\}, \forall x \in U^*$.
(e) Null:
A soft multiset (F, A) is called a Null soft multiset denoted by Φ, if $\forall e \in A\ F(e) = \emptyset$.
(f) Complement:
The complement of a soft multiset (F, A), denoted by $(F, A)^c$, is defined by $(F, A)^c = (F^c, A)$ where $F^c: A \longrightarrow P^*(U)$is a mapping given by$F^c(e) = U\backslash F(e), \forall e \in A$ where $C_{F^c(e)}(x) = C_U(x) - C_{F(e)}(x),\ \forall x \in U^*$.

Definition 4 n-Level Soft Set (Isah, 2019)
Let (F, A) be a Soft multiset over a universal multiset U and a set of parameters E. Then, we define the *n-level soft set* of (F, A), denoted $(F, A)_n$ as

$$(F, A)_n = \{(e, \{x\}) \big| C_{F(e)}(x) \geq n, n \in \mathbb{N}, \forall e \in A, \forall x \in U^*\}.$$

Definition 5 (Isah, 2019) Let $(F, A)_n$be the *n-level soft set* of (F, A), then

$$F_n(e) = \{x \in U^* \big| C_{F(e)}(x) \geq n, n \in \mathbb{N}, \forall e \in A\}.$$

Theorem 1 (Isah, 2019) Let (F, A) and (G, B) be Soft multisets over U and E, suppose $m, n \in \mathbb{N}$. Then,

(i) $((F, A) \sqcup (G, B))_n = (F, A)_n \sqcup (G, B)_n$,
(ii) $((F, A) \sqcap (G, B))_n = (F, A)_n \sqcap (G, B)_n$,
(iii) IF$(G, B) \sqsubseteq (F, A)$ then $(G, B)_n \sqsubseteq (F, A)_n$,
(iv) IF $m \leq n$ then $(F, A)_n \sqsubseteq (F, A)_m$,
(v) $(G, B) = (F, A) \Longrightarrow (G, B)_n = (F, A)_n, \forall e \in A, \forall x \in U^*$

Definition 6 (Isah, 2019) Let $S(U, E)$ be the class of all Soft multisets over U and E i.e. $S(U, A) = \{F: A \longrightarrow P^*(U),\ A \subseteq E\}$. Let $Q \subseteq U^*$, then, we define a soft multiset ${}_n(F, A) \in S(U, A)$ as

$${}_n(F, A) = \{(e, nQ) | C_{nQ}(x) = n, \forall e \in A, \forall n \in \mathbb{N}.$$

Theorem 2 (First Decomposition Theorem) (Isah, 2019)
Let $(F, A)_n$ be a n-level soft set of a soft multiset (F, A), over U and E. Then,
$C_{(F,A)}(x) = C_{F(e)}(x), \forall e \in A = \sum_{n\in\mathbb{N}} \mathcal{X}(F_n(e))(x), \forall e \in A = \sum_{n\in\mathbb{N}} \mathcal{X}(F, A)_n\ (x)$ where $\mathcal{X}(F_n(e))$ is the characteristic function of $(F_n(e)), \forall e \in A$ and $\mathcal{X}(F, A)_n$ is the characteristic function of $(F, A)_n$.

Proof
Let $x \in F_r(e), \forall e \in A, r = r_1, r_2, \dots, r_m, m = cad(A)$ for $x \in U^*$. Observe that $x \notin F_{r+n}(e), n \in \mathbb{N}$. Then
$C_{(F,A)}(x) = C_{F(e)}(x) = r, \forall e \in A$. Now

$$\sum_{n\in\mathbb{N}} \mathcal{X}(F,A)_n\,(x) = \sum_{n\in\mathbb{N}} \mathcal{X}(F_n(e))(x)\,, \forall e \in A = \sum_{n=1}^{r} \mathcal{X}\big(F_n(e)\big)(x) + \sum_{n\in\mathbb{N}} \mathcal{X}(F_{r+n}(e))(x)\,, \forall e \in A$$
$$= [1+1+\cdots r \text{ times}] + [0+0+\cdots] = \text{r}, \forall e \in A.$$

Hence, $C_{(F,A)}(x) = \sum_{n\in\mathbb{N}} \mathcal{X}(F,A)_n(x)$.

Theorem 3 (Second Decomposition Theorem) (Isah, 2019)
Let $(F,A)_n$ be the n-level soft set of a soft multiset (F,A) over U and E. Then
$(F,A) = \coprod_{n\in\mathbb{N}} {}_n(F,A)_n$ where $\sqcup$ is the soft multiset union.

Proof
Let $x \in U^*$ and $C_{(F,A)}(x) = t, \forall e \in A$. This imply that $x \in (F,A)_n$, for $n = 1,2,\dots,t$ and $x \notin (F,A)_n, \forall n \geq t+1,\ \forall e \in A$.
Now,

$$C_{(\coprod_{n\in\mathbb{N}} {}_n(F,A)_n)}(x) = \coprod_{n\in\mathbb{N}} {}_n(F,A)_n\,(x)$$
$$= {}_1(F,A)_1 \sqcup {}_2(F,A)_2 \sqcup \dots \sqcup {}_t(F,A)_t \sqcup_{t+1}(F,A)_{t+1} \sqcup \dots$$
$$= \cup\{1,2,\dots,t,0,0,\dots\} = t, \forall e \in A.$$
$$= C_{(F,A)}(x), \forall e \in A, \forall x \in U^*$$
$$= (F,A).$$

Therefore, $(F,A) = \coprod_{n\in\mathbb{N}} {}_n(F,A)_n$.

In the next section, we introduce the concept of n-upper level Soft set and show that some properties of n-level Soft set do not hold in n-upper level Soft set.

n-Upper Level Soft Set

Definition 7 Let (F,A)be a Soft multiset over a universal multiset U and a set of parameters E. Then, we define the *n-upper level soft set* of (F,A), denoted $(F,A)^n$ as

$$(F,A)^n = \{(e,\{x\}) \big| C_{F(e)}(x) \leq n, n \in \mathbb{N}, \forall e \in A, \forall x \in U^*\}.$$

Example 1 Let $E = \{e_1,e_2,e_3,e_4,e_5,e_6,e_7\}, A = \{e_1,e_2,e_3\}, B = \{e_1,e_3\}, U = \{9/x, 5/y, 4/z\}$,
$(F,A) = \{(e_1,\{3/x,2/y,1/z\}),(e_2,\{2/x,4/y\}),(e_3,\{1/x,2/z\})\}$ and
$(G,B) = \{(e_1,\{3/x,1/y\}),(e_3,\{1/x,2/z\})\}$. Then,

$$(F,A)^1 = \{(e_1,\{z\}),(e_3,\{x\})\}$$
$$(F,A)^2 = \{(e_1,\{y,z\}),(e_2,\{x\}),(e_3,\{x,z\})\}$$
$$(F,A)^3 = \{(e_1,\{x,y,z\}),(e_2,\{x\}),(e_3,\{x,z\})\}$$
$$(F,A)^4 = \{(e_1,\{x,y,z\}),(e_2,\{x,y\}),(e_3,\{x,z\})\}$$
$$(F,A)^n = (F,A)^4, n \geq 5.$$

and

$$(G,B)^1 = \{(e_1,\{y\}),(e_3,\{x\})\}$$
$$(G,B)^2 = \{(e_1,\{y\}),(e_3,\{x,z\})\}$$
$$(G,B)^3 = \{(e_1,\{x,y\}),(e_3,\{x,z\})\}$$
$$(G,B)^n = (G,B)^3, n \geq 4.$$

Definition 8 Let $(F,A)^n$be the n-upper level soft set of (F,A), then

$$F^n(e) = \{x \in U^* \big| C_{F(e)}(x) \leq n, n \in \mathbb{N}, \forall e \in A\}.$$

Example 2 Let $(F,A) = \{(e_1,\{3/x,2/y,1/z\}),(e_2,\{2/x,4/y\}),(e_3,\{1/x,2/z\})\}$, then

$$(F,A)^1 = \{(e_1,\{z\}),(e_3,\{x\})\}$$

and $F^1(e_1) = \{z\},\ F^1(e_2) = \emptyset,\ F^1(e_3) = \{x\}$.

Remark 1 Theorem 1 (iii) of n-level Soft set in (Isah, 2019) above, fails for n-upper level Soft set.

Counter Example
Let $(F,A) = \{(e_1,\{3/x,2/y,1/z\}),(e_2,\{2/x,4/y\}),(e_3,\{1/x,2/z\})\}$ and

$(G,B) = \{(e_1, \{3/x, 1/y\}), (e_3, \{1/x, 2/z\})\}$. Then,

$$(F,A)^1 = \{(e_1, \{z\}), (e_3, \{x\})\}$$

and

$$(G,B)^1 = \{(e_1, \{y\}), (e_3, \{x\})\}.$$

Observe that, $(G,B) \sqsubseteq (F,A)$ but neither $(G,B)^1 \sqsubseteq (F,A)^1$ nor $(F,A)^1 \sqsubseteq (G,B)^1$.

Theorem 4 Let (F,A) and (G,B)be Soft multisets over U and E, suppose $m, n \in \mathbb{N}$. Then,

(i) $((F,A) \sqcup (G,B))^n \sqsubseteq (F,A)^n \sqcup (G,B)^n$

(ii) $(F,A)^n \sqcap (G,B)^n \sqsubseteq ((F,A) \sqcap (G,B))^n$

(iii) If $m \leq n$ then $(F,A)^m \sqsubseteq (F,A)^n$

(iv) $(G,B) = (F,A) \Longrightarrow (G,B)^n = (F,A)^n$.

Proof

(i) Let $x \in ((F,A) \sqcup (G,B))^n \Longrightarrow x \in (F,A) \sqcup (G,B),\ C_{F(e)}(x) \geq n, C_{G(e)}(x) \geq n$

$\Longrightarrow x \in (F,A), C_{F(e)}(x) \geq n$ or $x \in (G,B),\ C_{G(e)}(x) \geq n$

$\Longrightarrow x \in (F,A)^n$ or $x \in (G,B)^n$

$$\Longrightarrow x \in (F,A)^n \sqcup (G,B)^n$$

i.e., $((F,A) \sqcup (G,B))^n \sqsubseteq (F,A)^n \sqcup (G,B)^n$

Conversely, let $x \in (F,A)^n \sqcup (G,B)^n$

$\Longrightarrow x \in (F,A)^n$ or $x \in (G,B)^n$

$\Longrightarrow C_{F(e)}(x) \leq n, \forall e \in A$ or $C_{G(e)}(x) \leq n, \forall e \in B$

$\Longrightarrow x \in (F,A), C_{F(e)}(x) \leq n$ or $x \in (G,B),\ C_{G(e)}(x) \leq n$

$\Longrightarrow x \in (F,A)$ or $x \in (G,B),\ C_{F(e)}(x) \geq n, C_{G(e)}(x) \leq n$

$$\Longrightarrow x \in (F,A) \sqcup (G,B), \qquad C_{F(e)}(x) \leq n, C_{G(e)}(x) \leq n$$

$$\Longrightarrow x \in ((F,A) \sqcup (G,B))^n$$

i.e., $(F,A)^n \sqcup (G,B)^n \sqsubseteq ((F,A) \sqcup (G,B))^n$

(ii) Let $x \in (F,A)^n \sqcap (G,B)^n \Longrightarrow x \in (F,A) \sqcap (G,B),\ C_{F(e)}(x) \leq n$ and $C_{G(e)}(x) \leq n$

$\Longrightarrow x \in (F,A), C_{F(e)}(x) \leq n$ and $x \in (G,B),\ C_{G(e)}(x) \leq n$

$\Longrightarrow x \in (F,A)^n$ and $x \in (G,B)^n$

$$\Longrightarrow x \in ((F,A) \sqcap (G,B))^n$$

i.e., $(F,A)^n \sqcap (G,B)^n \sqsubseteq ((F,A) \sqcap (G,B))^n$

(iii) Let $m \leq n$ and suppose $x \in (F,A)^m$

$$\Longrightarrow C_{F(e)}(x) \leq m, \forall e \in A$$

$$\Longrightarrow C_{F(e)}(x) \leq n, \forall e \in A$$

$$\Longrightarrow x \in (F,A)^n$$

i.e., $(F,A)^m \sqsubseteq (F,A)^n$.

(iv) Let $(G,B) = (F,A)$

$\Longrightarrow A = B$ and $C_{F(e)}(x) = C_{G(e)}(x), \forall e \in A, \forall x \in U^*$

$\Longrightarrow \forall n \in \mathbb{N}$, if $C_{F(e)}(x) \leq n$ it imply $C_{G(e)}(x) \leq n, \forall e \in A, \forall x \in U^*$ and vice versa

Thus, $(G,B)^n = (F,A)^n$.

Remark 2 The converse of (iv) above is not always true.

Counter example

Let $(F,A) = \{(e_1, \{3/x, 2/y, 1/z\}), (e_3, \{1/x, 2/z\})\}$ and

$(G,B) = \{(e_1, \{3/x, 1/y, 1/z\}), (e_3, \{1/x, 2/z\})\}$. Then,

$$(F,A)^1 = \{(e_1, \{z\}), (e_3, \{x\})\}$$

$$(F,A)^2 = \{(e_1, \{y, z\}), (e_3, \{x, z\})\}$$

$$(F,A)^3 = \{(e_1, \{x, y, z\}), (e_3, \{x, z\})\}$$

$$(F,A)^n = (F,A)^3, n \geq 4.$$

and

$$(G,B)^1 = \{(e_1,\{y,z\}),(e_3,\{x\})\}$$
$$(G,B)^2 = \{(e_1,\{y,z\}),(e_3,\{x,z\})\}$$
$$(G,B)^3 = \{(e_1,\{x,y,z\}),(e_3,\{x,z\})\}$$
$$(G,B)^n = (G,B)^3, n \geq 4.$$

However, $(F,A)^2 = (G,B)^2$ and $(F,A)^3 = (G,B)^3$, but $(G,B) \neq (F,A)$.

Theorem 5 The First and Second Decomposition Theorems fails for n-upper level Soft set.

Proof

Counter example (First Decomposition Theorem)

Let $(F,A) = \{(e_1,\{3/x,2/y,1/z\}),(e_3,\{1/x,2/z\})\}$, then

$$(F,A)^1 = \{(e_1,\{z\}),(e_3,\{x\})\}$$
$$(F,A)^2 = \{(e_1,\{y,z\}),(e_3,\{x,z\})\}$$
$$(F,A)^3 = \{(e_1,\{x,y,z\}),(e_3,\{x,z\})\}$$
$$(F,A)^n = (F,A)^3, n \geq 4.$$

Now,

$$C_{(F,A)}(z) = C_{F(e)}(z), \forall e \in A$$
$$= C_{F(e_1)}(z) + C_{F(e_2)}(z) = 1 + 2 = 3.$$

However,

$$\mathcal{X}(F^1(e_1))(z) = 1, \mathcal{X}(F^2(e_1))(z) = 1, \mathcal{X}(F^3(e_1))(z) = 1, \mathcal{X}(F^n(e_1))(z) = 0, n \geq 4,$$
$$\mathcal{X}(F^1(e_3))(z) = 0, \mathcal{X}(F^2(e_3))(z) = 1, \mathcal{X}(F^3(e_3))(z) = 1, \mathcal{X}(F^n(e_3))(z) = 0, n \geq 4.$$

and thus,

$$\sum_{n\in\mathbb{N}} \mathcal{X}(F,A)^n(z) = \sum_{n\in\mathbb{N}} \mathcal{X}(F^n(e))(z), \forall e \in A = \sum_{n=1}^{3} \mathcal{X}(F^n(e_1))(z) + \sum_{n=1}^{3} \mathcal{X}(F^n(e_3))(z)$$
$$= \mathcal{X}(F^1(e_1))(z) + \mathcal{X}(F^2(e_1))(z) + \mathcal{X}(F^3(e_1))(z) + (F^1(e_3))(z) + \mathcal{X}(F^2(e_3))(z) + \mathcal{X}(F^3(e_3))(z)$$
$$= 1 + 1 + 1 + 0 + 1 + 1 = 5 \neq C_{(F,A)}(z) = 3.$$

Counter example (Second Decomposition Theorem)

Let $(F,A) = \{(e_1,\{3/x,2/y,1/z\}),(e_3,\{1/x,2/z\})\}$, and

$$(F,A)^1 = \{(e_1,\{z\}),(e_3,\{x\})\}$$
$$(F,A)^2 = \{(e_1,\{y,z\}),(e_3,\{x,z\})\}$$
$$(F,A)^3 = \{(e_1,\{x,y,z\}),(e_3,\{x,z\})\}$$
$$(F,A)^n = (F,A)^3, n \geq 4.$$

Then,

$$_1(F,A)^1 = \{(e_1,\{z\}),(e_3,\{x\})\}$$
$$_2(F,A)^2 = \{(e_1,\{2/y,2/z\}),(e_3,\{2/x,2/z\})\}$$
$$_3(F,A)^3 = \{(e_1,\{3/x,3/y,3/z\}),(e_3,\{3/x,3/z\})\}$$

and $_1(F,A)^1 \sqcup {}_2(F,A)^2 \sqcup {}_3(F,A)^3 = \{(e_1,\{3/x,3/y,3/z\}),(e_3,\{3/x,3/z\})\} \neq (F,A)$.

CONCLUSION

The concepts of n-level soft set and n-upper level soft set are aiding tool for addressing various problems of uncertainties that characterized environmental science, medicine, engineering, social sciences and so on.

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