FUDMA Journal of Sciences (FJS)
ISSN online: 2616-1370
ISSN print: 2645 - 2944
Vol. 3 No. 4, December, 2019, pp 67 –78

MODIFIED CLASS OF ESTIMATOR OF FINITE POPULATION VARIANCE

[1]Muili, J. O., *[2]Zakari, Y. and [3]Audu, A.

[1]Department of Mathematics, Kebbi State University of Sci. and Tech. Aliero, Nigeria.
[2]Department of Statistics, Ahmadu Bello University, Zaria, Nigeria.
[3]Department of Mathematics, Usmanu Danfodiyo University, Sokoto, Nigeria.

Corresponding authors email: yahzaksta@gmail.com

ABSTRACT

In this paper, we proposed a class of ratio estimators for finite population variance. The proposed estimators were obtained by transforming Javed *et al*. (2018) estimators. The mean square errors (MSEs) of the proposed estimators have been obtained up to first order of approximation using Taylor's Series Expansion and the conditions for their efficiencies over some existing variance estimators have been established. The efficiencies of proposed estimators based on optimal value of the constant, exhibit significant improvement over the estimators considered in the study. The numerical illustration was also conducted to corroborate the theoretical results. The results of the empirical study show that the proposed estimators are more efficient over existing estimators.

Keywords: Efficiency, Auxiliary variable, Mean Square Error, Unknown weight

INTRODUCTION

The use of auxiliary information in sampling has been a commonly used device for improving the precision of the estimator of finite population mean or total under study. If use intelligibly, this information provides us with the sampling strategies better than those in which no auxiliary information is used. (Rajesh *et al*., 2007) Various fields of life have been facing the problem in estimating the finite population variance. Population variance is one of the parameters which require efficient estimators because of its great significance in various fields of life like genetics, agriculture, biology and medical studies such as in matters of health: variations in body temperature, pulse beat and blood pressure which are the basic guides of diagnosis when prescribed treatment is designed to control their variation. An agriculturist requires sufficient knowledge of climatic variation to devise appropriate plan for cultivating his crop. A manufacturer needs constant knowledge of the level of variations in people's reaction to his product to be able to know whether to reduce or increase his price or improve the quality of his product. A fair understanding of variability is vitally important for better results in different walks of life.

In this paper, modified class of ratio estimators for estimating finite population variance has been proposed with objective to produce efficient estimators and their properties have been established.

Let $\Omega = (1, 2, 3, \dots, N)$ be a population of size N and Y, X be two real valued functions having values $(Y_i, X_i) \in \mathbb{R}^+ > 0$ on the i^{th} unit of $U(1 \le i \le N)$. Let S_y^2 and S_x^2 be the population mean squares of Y and X respectively and s_y^2 and s_x^2 be respective sample mean squares based on the random sample of size n drawn without replacement. The following are the other notations used throughout this paper.

N : Population size

n : Sample size

Y : Study variable

X : Auxiliary variable

$\bar{y}, \bar{x}$: Sample means of study and auxiliary variables

$\bar{Y}, \bar{X}$: Population means of study and auxiliary variables

ρ : Coefficient of correlation

C_y, C_x : Coefficient of variations of study and auxiliary variables

Q_1 : The lower quartile

Q_3 : The upper quartile

Q_r : Inter-quartile range

Q_d : Semi-quartile range

Q_a : Semi-quartile average

Q_c : Coefficient of quartile deviation

$\beta_{2(y)}$: Coefficient of kurtosis of study variable

$\beta_{2(x)}$: Coefficient of kurtosis of auxiliary variable

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i, \quad \bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i, \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \quad \gamma = \frac{1}{n}, \quad Q_a = \frac{(Q_3 + Q_1)}{2},$$

$$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2, \quad s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2, \; S_y^2 = \frac{1}{N-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2, \quad S_x^2 = \frac{1}{N-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

The sample variance estimator of the finite population variance is defined as

$$t = s_y^2 \tag{1}$$

which is an unbiased estimator of finite population variance S_y^2 and its variance is

$$Var(t) = \gamma S_y^4 \left(\beta_{2(y)} - 1\right) \tag{2}$$

Isaki (1983) proposed a ratio type variance estimator for the finite population variance S_y^2 when the finite population variance S_x^2 of auxiliary variable X is known together with its mean squared error given below:

$$S_R^2 = s_y^2 \frac{S_x^2}{s_x^2} \tag{3}$$

$$Bias\left(\hat{S}_R^2\right) = \gamma S_y^2 \left[\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right] \tag{4}$$

$$MSE\left(\hat{S}_R^2\right) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + \left(\beta_{2(x)} - 1\right) - 2\left(\lambda_{22} - 1\right)\right] \tag{5}$$

Kadilar and Cingi (2006) proposed a class of ratio type estimators for finite population variance by imposing Coefficient of variation and Coefficient of kurtosis on the work of Isaki (1983) as:

$$S_{kc_1}^2 = s_y^2 \left(\frac{S_x^2 + C_x}{s_x^2 + C_x}\right) \tag{6}$$

$$S_{kc_2}^2 = s_y^2 \left(\frac{S_x^2 + \beta_{x(2)}}{s_x^2 + \beta_{x(2)}}\right) \tag{7}$$

$$S_{kc_3}^2 = s_y^2 \left(\frac{S_x^2 \beta_{x(2)} + C_x}{s_x^2 \beta_{x(2)} + C_x}\right) \tag{8}$$

$$S_{kc_4}^2 = s_y^2 \left(\frac{S_x^2 C_x + \beta_{x(2)}}{s_x^2 C_x + \beta_{x(2)}}\right) \tag{9}$$

$$Bias\left(\hat{S}^2_{kc_i}\right)=\gamma A_i S_y^2\left[A_i\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right], \text{ where i= 1,2,3,4} \quad (10)$$

$$MSE\left(\hat{S}^2_{kc_i}\right)=\gamma S_y^4\left[\left(\beta_{2(y)}-1\right)+A_i^2\left(\beta_{2(x)}-1\right)-2A_i\left(\lambda_{22}-1\right)\right], \text{ where i= 1,2,3,4} \quad (11)$$

Here

$$A_1=\frac{S_x^2}{S_x^2+C_x}, A_2=\frac{S_x^2}{S_x^2+\beta_{2(x)}}, A_3=\frac{S_x^2\beta_{2(x)}}{S_x^2\beta_{2(x)}+C_x}, A_4=\frac{S_x^2C_x}{S_x^2C_x+\beta_{2(x)}}$$

Subramani and Kumarpandiyan (2012) proposed ratio type estimators for finite population variance using quartile and functions of quartiles as auxiliary variable given as:

$$S^2_{sk_1}=s_y^2\left(\frac{S_x^2+Q_1}{s_x^2+Q_1}\right) \quad (12)$$

$$S^2_{sk_2}=s_y^2\left(\frac{S_x^2+Q_3}{s_x^2+Q_3}\right) \quad (13)$$

$$S^2_{sk_3}=s_y^2\left(\frac{S_x^2+Q_r}{s_x^2+Q_r}\right) \quad (14)$$

$$S^2_{sk_4}=s_y^2\left(\frac{S_x^2+Q_d}{s_x^2+Q_d}\right) \quad (15)$$

$$S^2_{sk_5}=s_y^2\left(\frac{S_x^2+Q_a}{s_x^2+Q_a}\right) \quad (16)$$

$$Bias\left(\hat{S}^2_{sk_i}\right)=\gamma R_i S_y^2\left[R_i\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right], \qquad (i=1,2,3,4,5) \quad (17)$$

$$MSE\left(\hat{S}^2_{sk_i}\right)=\gamma S_y^4\left[\left(\beta_{2(y)}-1\right)+R_i^2\left(\beta_{2(x)}-1\right)-2R_i\left(\lambda_{22}-1\right)\right], \qquad (i=1,2,3,4,5) \quad (18)$$

Here

$$R_1=\frac{S_x^2}{S_x^2+Q_1},\quad R_2=\frac{S_x^2}{S_x^2+Q_3},\quad R_3=\frac{S_x^2}{S_x^2+Q_r},\quad R_4=\frac{S_x^2}{S_x^2+Q_d},\quad R_5=\frac{S_x^2}{S_x^2+Q_a}$$

Khan and Shabbir (2013) proposed a ratio-type estimator using correlation coefficient between study and auxiliary variable and the third quartile of the auxiliary variable as:

$$S_{ks}^2 = s_y^2\left(\frac{S_x^2\rho + Q_3}{s_x^2\rho + Q_3}\right) \quad (19)$$

$$Bias\left(\hat{S}_{ks}^2\right) = \gamma W S_y^2\left[W\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right] \quad (20)$$

$$MSE\left(\hat{S}_{kS}^2\right) = \gamma S_y^4\left[\left(\beta_{2(y)} - 1\right) + W^2\left(\beta_{2(x)} - 1\right) - 2W\left(\lambda_{22} - 1\right)\right] \quad (21)$$

Where $W = \frac{S_x^2\rho}{S_x^2\rho + Q_3}$

Javed *et al*. (2018) proposed a class of ratio type variance estimators utilizing different parameters of auxiliary variable as:

$$\hat{S}_{kj_1}^2 = s_y^2\left(\frac{S_x^2 Q_c^2 + Q_1}{s_x^2 Q_c^2 + Q_1}\right) \quad (22)$$

$$\hat{S}_{kj_2}^2 = s_y^2\left(\frac{S_x^2 Q_c^2 + Q_3}{s_x^2 Q_c^2 + Q_3}\right) \quad (23)$$

$$\hat{S}_{kj_3}^2 = s_y^2\left(\frac{S_x^2 Q_c^2 + Q_r}{s_x^2 Q_c^2 + Q_r}\right) \quad (24)$$

$$\hat{S}_{kj_4}^2 = s_y^2\left(\frac{S_x^2 Q_c^2 + Q_d}{s_x^2 Q_c^2 + Q_d}\right) \quad (25)$$

$$\hat{S}_{kj_5}^2 = s_y^2\left(\frac{S_x^2 Q_c^2 + Q_a}{s_x^2 Q_c^2 + Q_a}\right) \quad (26)$$

$$Bias\left(\hat{S}_{kji}^2\right) = \gamma S_y^2 K_i^*\left[K_i^*\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right] \qquad i=1,2,3,4,5 \quad (27)$$

$$MSE\left(\hat{S}_{kji}^2\right) = \gamma S_y^4\left[\left(\beta_{2(y)} - 1\right) + K_i^{*2}\left(\beta_{2(x)} - 1\right) - 2K_i^*\left(\lambda_{22} - 1\right)\right] \qquad i=1,2,3,4,5 \quad (28)$$

Where $K_1^* = \frac{S_x^2 Q_c^2}{S_x^2 Q_c^2 + Q_1}$, $K_2^* = \frac{S_x^2 Q_c^2}{S_x^2 Q_c^2 + Q_3}$, $K_3^* = \frac{S_x^2 Q_c^2}{S_x^2 Q_c^2 + Q_r}$, $K_4^* = \frac{S_x^2 Q_c^2}{S_x^2 Q_c^2 + Q_d}$, $K_5^* = \frac{S_x^2 Q_c^2}{S_x^2 Q_c^2 + Q_a}$

Many other researchers like Adewara *et al* (2012), Audul *et al* (2016), Gupta & Shabbir (2008), Olufadi & Kadilar (2014), Kazeem & Olanrewaju (2013), have suggested both ratio and product estimators for population variance.

Proposed Estimator

Motivated by the work of Javed *et al*. (2018), we proposed a class of ratio estimators of finite population variance as:

$$\hat{S}^2_{mj_1} = k_1 s_y^2 \left(\frac{S_x^2 Q_c^2 + Q_1}{s_x^2 Q_c^2 + Q_1} \right) \tag{29}$$

$$\hat{S}^2_{mj_2} = k_2 s_y^2 \left(\frac{S_x^2 Q_c^2 + Q_3}{s_x^2 Q_c^2 + Q_3} \right) \tag{30}$$

$$\hat{S}^2_{mj_3} = k_3 s_y^2 \left(\frac{S_x^2 Q_c^2 + Q_r}{s_x^2 Q_c^2 + Q_r} \right) \tag{31}$$

$$\hat{S}^2_{mj_4} = k_4 s_y^2 \left(\frac{S_x^2 Q_c^2 + Q_d}{s_x^2 Q_c^2 + Q_d} \right) \tag{32}$$

$$\hat{S}^2_{mj_5} = k_5 s_y^2 \left(\frac{S_x^2 Q_c^2 + Q_a}{s_x^2 Q_c^2 + Q_a} \right) \tag{33}$$

Where $k_i \ (i = 1,2,3,4,5)$ are unknown weights to be determined such that the MSEs of the proposed estimators $\hat{S}^2_{mj_i}$ are minimized.

In order to obtain the MSE, we defined $e_0 = \frac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 = \frac{s_x^2 - S_x^2}{S_x^2}$ such that

$$\left. \begin{array}{l} E(e_0) = E(e_1) = 0,\ E(e_0^2) = \gamma\left(\beta_{2(y)} - 1\right) \\ E(e_1^2) = \gamma\left(\beta_{2(x)} - 1\right),\ E(e_0 e_1) = \gamma\left(\lambda_{22} - 1\right) \end{array} \right\} \tag{34}$$

MSE of $\hat{S}^2_{mj_i}$, $i = 1,2,3,4,5$

In order to determine the MSE of $\hat{S}^2_{mj_i}$, $i = 1,2,3,4,5$, expressing equations (29), (30), (31), (32) and (33) in general form as:

$$\hat{S}^2_{mj_i} = k_i s_y^2 \left(\frac{S_x^2 Q_c^2 + A}{s_x^2 Q_c^2 + A} \right) \qquad i = 1,2,3,4,5 \tag{35}$$

Table 1: Values of A in proposed estimators

Estimator	A
$\hat{S}^2_{mj_1}$	Q_1
$\hat{S}^2_{mj_2}$	Q_3
$\hat{S}^2_{mj_3}$	Q_r
$\hat{S}^2_{mj_4}$	Q_d
$\hat{S}^2_{mj_5}$	Q_a

Expressing $\hat{S}^2_{mj_i}$, $i = 1,2,3,4,5$ in terms of e_0 and e_1, we have

$$\hat{S}^2_{mj_i} = k_i S_y^2 \left(1+e_0\right)\left(1+t_i e_1\right)^{-1} \quad i = 1,2,3,4,5 \tag{36}$$

Where $t_1 = \dfrac{S_x^2 Q_c^2}{S_x^2 Q_c^2 + Q_1}$, $t_2 = \dfrac{S_x^2 Q_c^2}{S_x^2 Q_c^2 + Q_3}$, $t_3 = \dfrac{S_x^2 Q_c^2}{S_x^2 Q_c^2 + Q_r}$, $t_4 = \dfrac{S_x^2 Q_c^2}{S_x^2 Q_c^2 + Q_d}$, $t_5 = \dfrac{S_x^2 Q_c^2}{S_x^2 Q_c^2 + Q_a}$

We now assume that $|e_1| < 1$ and $|t_i e_1| < 1$ so that $\left(1+e_1\right)^{-1}$ and $\left(1+t_i e_i\right)^{-1}$ are expandable. Expanding the right hand side of (36) up to second degree approximation, subtract S_y^2 from both sides, square both sides and taking expectation using the results in equation (34), we obtain the MSEs of the proposed estimators as:

$$MSE\left(\hat{S}^2_{mj_i}\right) = S_y^4 \begin{bmatrix} \left(k_i-1\right)^2 + \gamma k_i^2\left(\beta_{2(y)}-1\right) + \gamma k_i^2 t_i^2\left(\beta_{2(x)}-1\right) - 2\gamma k_i t_i\left(k_i-1\right)\left(\lambda_{22}-1\right) \\ +2\gamma k_i t_i^2\left(k_i-1\right)\left(\beta_{2(x)}-1\right) - 2\gamma k_i^2 t_i\left(\lambda_{22}-1\right) \end{bmatrix} \tag{37}$$

$$MSE\left(\hat{S}^2_{mj_i}\right) = S_y^4 \begin{bmatrix} \left(k_i-1\right)^2 + \gamma k_i^2\left(\beta_{2(y)}-1\right) + \gamma\left(3k_i^2-2k_i\right)t_i^2\left(\beta_{2(x)}-1\right) \\ -2\gamma\left(2k_i^2-k_i\right)t_i\left(\lambda_{22}-1\right) \end{bmatrix} \quad i = 1,2,3,4,5 \tag{38}$$

To obtained the expression for the value of k_i, differentiate $MSE\left(\hat{S}^2_{mj_i}\right)$ partially with respect to k_i and equate to zero

The $MSE\left(\hat{S}^2_{mj_i}\right)$, $i = 1,2,3,4,5$ expressions are minimized for the optimum values of k given by

$$k_i^{opt} = \frac{1+\gamma\left[t_i^2\left(\beta_{2(x)}-1\right) - t_i\left(\lambda_{22}-1\right)\right]}{1+\gamma\left[3t_i^2\left(\beta_{2(x)}-1\right) - 4t_i\left(\lambda_{22}-1\right) + \left(\beta_{2(y)}-1\right)\right]} = \frac{D_i}{V_i} \quad i = 1,2,3,4,5 \tag{39}$$

Replacing k_i by k_i^{opt}, $i = 1,2,3,4,5$ in equation (38), we obtain the minimum MSE as

$$MSE\left(\hat{S}^2_{mj_i}\right)_{\min} = S_y^4\left(1-\frac{D_i^2}{V_i}\right), \; i = 1,2,3,4,5 \tag{40}$$

Efficiency comparisons

In this section efficiency of the proposed estimators are compared with efficiencies of some selected estimators in the literature such as Sample variance, Isaki (1983), Kadilar and Cingi (2006), Subramani and Kumarpandiyan (2012), Khan and Shabbir (2013) and Javed *et al*. (2018).

The $\hat{S}^2_{mj_i}$ of estimators of the finite population variance is more efficient than $\hat{S}^2_R$ if,

$$MSE\left(\hat{S}^2_{mj_i}\right)_{\min} < MSE\left(\hat{S}^2_R\right)$$

$$S_y^4\left(1-\frac{D_i^2}{V_i}\right) < \gamma S_y^4\left[\left(\beta_{2(y)}-1\right)+\left(\beta_{2(x)}-1\right)-2\left(\lambda_{22}-1\right)\right] \quad (41)$$

The $\hat{S}^2_{mj_i}$ of estimators of the finite population variance is more efficient than $\hat{S}^2_{kc_i}$ if,

$$MSE\left(\hat{S}^2_{mj_i}\right)_{\min} < MSE\left(\hat{S}^2_{kc_i}\right)$$

$$S_y^4\left(1-\frac{D_i^2}{V_i}\right) < \gamma S_y^4\left[\left(\beta_{2(y)}-1\right)+A_i^2\left(\beta_{2(x)}-1\right)-2A_i\left(\lambda_{22}-1\right)\right] \quad (42)$$

The $\hat{S}^2_{MJ_i}$ of estimators of the finite population variance is more efficient than $\hat{S}^2_{sk_i}$ if,

$$MSE\left(\hat{S}^2_{MJ_i}\right)_{\min} < MSE\left(\hat{S}^2_{sk_i}\right)$$

$$S_y^4\left(1-\frac{P_i^2}{Q_i}\right) < \gamma S_y^4\left[\left(\beta_{2(y)}-1\right)+R_i^2\left(\beta_{2(x)}-1\right)-2R_i\left(\lambda_{22}-1\right)\right] \quad (43)$$

The $\hat{S}^2_{MJ_i}$ of estimators of the finite population variance is more efficient than $\hat{S}^2_{kS}$ if,

$$MSE\left(\hat{S}^2_{MJ_i}\right)_{\min} < MSE\left(\hat{S}^2_{kS}\right)$$

$$S_y^4\left(1-\frac{P_i^2}{Q_i}\right) < \gamma S_y^4\left[\left(\beta_{2(y)}-1\right)+W^2\left(\beta_{2(x)}-1\right)-2W\left(\lambda_{22}-1\right)\right] \quad (44)$$

The $\hat{S}^2_{MJ_i}$ of estimators of the finite population variance is more efficient than $\hat{S}^2_{jG}$ if,

$$MSE\left(\hat{S}^2_{MJ_i}\right)_{\min} < MSE\left(\hat{S}^2_{jG}\right)$$

$$S_y^4\left(1-\frac{P_i^2}{Q_i}\right) < \gamma S_y^4\left[\left(\beta_{2(y)}-1\right)+A_{jG}^2\left(\beta_{2(x)}-1\right)-2A_{jG}\left(\lambda_{22}-1\right)\right] \quad (45)$$

The $\hat{S}^2_{MJ_i}$ of estimators of the finite population variance is more efficient than $\hat{S}^2_{sj}$ if,

$$MSE\left(\hat{S}^2_{MJ_i}\right)_{\min} < MSE\left(\hat{S}^2_{sj}\right)$$

$$S_y^4\left(1-\frac{P_i^2}{Q_i}\right) < \gamma S_y^4\left[\left(\beta_{2(y)}-1\right)+A_{sj}^2\left(\beta_{2(x)}-1\right)-2A_{sj}\left(\lambda_{22}-1\right)\right] \quad (46)$$

When conditions (41), (42), (43), (44), (45) and (46) are satisfied, we can conclude that the proposed estimators are more efficient than some selected existing estimators.

Numerical Illustration

In order to investigate the merits of the proposed estimators, we have considered the following real populations

Table 2: [Javed *et al*. (2018)] Real Populations

Characteristics	Population 1	Population2	Population3
N	80	70	33
n	20	25	10
$\bar{Y}$	51.8267	96.7	2258.2
$\bar{X}$	11.2646	176.2671	1453.1
ρ	0.9413	0.7293	0.9690
S_y	18.3569	60.714	839.0
C_y	0.3542	0.6254	0.3715
S_x	8.4563	140.8572	277.3504
C_x	0.7507	0.8037	0.1909
$\beta_{2(y)}$	2.2667	4.7596	1.5718
$\beta_{2(x)}$	2.8664	7.0952	1.7109
λ_{22}	2.2209	4.6038	1.5117
Q_1	5.15	80.15	1221.7
Q_3	16.975	225.025	1714.2

Percentage Relative Efficiency (PRE) is a statistical tool that is used to measure and ascertain the efficiency of one estimator over another. PRE of the estimators were computed using the formula

$$PRE = \frac{MSE\left(Sample Variance\right)}{MSE\left(\hat{S}_i\right)} \; X \; 100$$

where $\hat{S}_i$ are the existing and proposed estimators in this study.

Table 3: Bias of the Reviewed and Proposed Estimators

Estimator	Bias		
	Pop-1	Pop-2	Pop-3
Sample variance	0	0	0
Isaki (1983) S_R^2	10.87589	367.3509	14022.11
Kadilar and Cingi (2006) $\hat{S}^2_{KC1}$	10.4396	367.2996	14021.95
Kadilar and Cingi (2006) $\hat{S}^2_{KC2}$	9.291509	366.8984	14020.68
Kadilar and Cingi (2006) $\hat{S}^2_{KC3}$	10.72187	367.3437	14022.01
Kadilar and Cingi (2006) $\hat{S}^2_{KC4}$	8.811388	366.7880	14014.64
Subramani and Kumarapandiyan (2012) $\hat{S}^2_{sk_1}$	8.1745	362.2715	13032.7778
Subramani and Kumarpandiyan (2012) $\hat{S}^2_{sk_2}$	3.9193	353.2657	12649.372
Subramani and Kumarpandiyan (2012) $\hat{S}^2_{sk_3}$	5.5035	358.2204	13616.573
Subramani and Kumarpandiyan (2012) $\hat{S}^2_{sk_4}$	7.8272	362.7572	13818.1863
Subramani and Kumarpandiyan (2012) $\hat{S}^2_{sk_5}$	5.7702	357.7407	12839.9972
Khan and Shabbir (2013) $\hat{S}^2_{kS}$	3.627783	348.1741	12607.21
Javed *et. al.* (2018) $\hat{S}^2_{kj_1}$	3.6289	345.3350	-2577.0819
Javed *et. al.* (2018) $\hat{S}^2_{kj_2}$	-0.6399	308.5025	-4516.475
Javed *et. al.* (2018) $\hat{S}^2_{kj_3}$	-0.4141	328.5025	3867.0818
Javed *et. al.* (2018) $\hat{S}^2_{kj_4}$	2.96	347.3958	8000.6724
Javed *et. al.* (2018) $\hat{S}^2_{kj_5}$	-0.1135	326.5505	-3694.5464
Proposed Estimator $\hat{S}^2_{MJ_1}$	-8.122453	-267.7041	-14444.35
Proposed Estimator $\hat{S}^2_{MJ_2}$	-7.76559	-262.0205	-15459.58
Proposed Estimator $\hat{S}^2_{MJ_3}$	-7.736949	-265.1357	-14295.16
Proposed Estimator $\hat{S}^2_{MJ_4}$	-7.985116	-268.0222	-15121.9
Proposed Estimator $\hat{S}^2_{MJ_5}$	-7.711719	-264.8354	-14910.4

Table 5: MSE of the Reviewed and Proposed Estimators

Estimator	Mean Square Error (MSE)		
	Pop-1	Pop-2	Pop-3
Sample variance	7191.8587	2043417.0720	28332962991
Isaki (1983) S_R^2	3924.948	1438805.6370	12848438796
Kadilar and Cingi (2006) $\hat{S}^2_{KC1}$	3849.943	1438695.9423	12848389814
Kadilar and Cingi (2006) $\hat{S}^2_{KC2}$	3658.199	1437837.9161	12847999753
Kadilar and Cingi (2006) $\hat{S}^2_{KC3}$	3898.342	1438790.1754	12848410166
Kadilar and Cingi (2006) $\hat{S}^2_{KC4}$	3580.631	1437601.7884	12846139154
Subramani and Kumarapandiyan (2012) $\hat{S}^2_{sk_1}$	3480.3515	1427962.856	12548423583
Subramani and Kumarpandiyan (2012) $\hat{S}^2_{sk_2}$	2908.7734	1408850.951	12434849780
Subramani and Kumarpandiyan (2012) $\hat{S}^2_{sk_3}$	3098.2227	1419347.720	12724277603
Subramani and Kumarpandiyan (2012) $\hat{S}^2_{sk_4}$	3424.9869	1428997.768	12785803692
Subramani and Kumarpandiyan (2012) $\hat{S}^2_{sk_5}$	3133.1398	1418329.455	12491123595
Khan and Shabbir (2013) $\hat{S}^2_{kS}$	2878.556	1398110.6273	12422455626
Javed *et. al.* (2018) $\hat{S}^2_{kj_1}$	2878.6812	1392142.665	10311288122
Proposed Estimator $\hat{S}^2_{MJ_1}$	**2737.0700**	**986808.1260**	**10167683533**
Javed *et. al.* (2018) $\hat{S}^2_{kj_2}$	2668.7908	1316648.676	11003999666
Proposed Estimator $\hat{S}^2_{MJ_2}$	**2616.8177**	**965857.3649**	**10882326305**
Javed *et. al.* (2018) $\hat{S}^2_{kj_3}$	2662.0097	1357074.728	10399624186
Proposed Estimator $\hat{S}^2_{MJ_3}$	**2607.1644**	**977340.5124**	**10062664698**
Javed *et. al.* (2018) $\hat{S}^2_{kj_4}$	2813.5423	1396473.203	11199590496
Proposed Estimator $\hat{S}^2_{MJ_4}$	**2690.7905**	**987980.6111**	**10644624064**
Javed *et. al.* (2018) $\hat{S}^2_{kj_5}$	2657.7430	1353043.931	10623377994
Proposed Estimator $\hat{S}^2_{MJ_5}$	**2598.6626**	**976233.4847**	**10495745198**

Table 5: PRE of the Estimators

Estimator	Percentage Relative Error (PRE)		
	Pop-1	Pop-2	Pop-3
Sample variance	100	100	100
Isaki (1983) S_R^2	183.2345	142.0218	220.5168
Kadilar and Cingi (2006) $\hat{S}_{KC1}^2$	186.8043	142.0326	220.5176
Kadilar and Cingi (2006) $\hat{S}_{KC2}^2$	196.5956	142.1173	220.5243
Kadilar and Cingi (2006) $\hat{S}_{KC3}^2$	184.4851	142.0233	220.5173
Kadilar and Cingi (2006) $\hat{S}_{KC4}^2$	200.8545	142.1407	220.5563
Subramani and Kumarapandiyan (2012) $\hat{S}_{sk_1}^2$	206.6417	143.1002	225.789
Subramani and Kumarpandiyan (2012) $\hat{S}_{sk_2}^2$	247.2471	145.0414	227.8513
Subramani and Kumarpandiyan (2012) $\hat{S}_{sk_3}^2$	232.1285	143.9687	222.6685
Subramani and Kumarpandiyan (2012) $\hat{S}_{sk_4}^2$	209.9821	142.9965	97.69526
Subramani and Kumarpandiyan (2012) $\hat{S}_{sk_5}^2$	229.5416	144.0721	226.8248
Khan and Shabbir (2013) $\hat{S}_{kS}^2$	249.8426	146.1556	228.0786
Javed *et. al*. (2018) $\hat{S}_{kj_1}^2$	249.8317	146.7822	274.7762
Proposed Estimator $\hat{S}_{MJ_1}^2$	**262.7576**	**207.0734**	**278.657**
Javed *et. al*. (2018) $\hat{S}_{kj_2}^2$	269.48	155.1984	257.4788
Proposed Estimator $\hat{S}_{MJ_2}^2$	**274.8322**	**211.5651**	**260.3576**
Javed *et. al*. (2018) $\hat{S}_{kj_3}^2$	270.1665	150.5751	272.4422
Proposed Estimator $\hat{S}_{MJ_3}^2$	**275.8498**	**209.0793**	**281.5652**
Javed *et. al*. (2018) $\hat{S}_{kj_4}^2$	255.6158	146.327	252.9821
Proposed Estimator $\hat{S}_{MJ_4}^2$	**267.2768**	**206.8276**	**266.1716**
Javed *et. al*. (2018) $\hat{S}_{kj_5}^2$	270.6002	151.0237	266.7039
Proposed Estimator $\hat{S}_{MJ_5}^2$	**276.7523**	**209.3164**	**269.9471**

Table 4 shows the results of Mean Square Errors (MSEs) of the proposed and some related estimators (Sample variance, Isaki (1983), Kadilar and Cingi (2006), Subramani and Kumarpandiyan (2012), Khan and Shabbir (2013) and Javed *et al*. (2018)) considered in the study for all the data sets 1, 2 and 3. The results revealed that the proposed estimators has minimum MSE. Table 5 shows the Percentage Relative Efficiency (PRE) of the proposed and some related estimators considered in the study for all the data sets 1, 2, and 3. The results revealed that the proposed estimators has the highest PRE than other estimators.

CONCLUSION

From the results of Tables 4 and 5, we infer that the proposed estimators are more efficient than the existing estimators in the sense of having least Mean Square Errors (Javed *et. al.* (2018) having 2878.6812 to Proposed Estimator having 2737.06997 in Pop-1, Javed *et. al.* (2018) having 1392142.665 to Proposed Estimator having 986808.1260 in Pop-2, Javed *et. al.* (2018) having 10311288122 to Proposed Estimator having 10167683533 in Pop-3, etc.) and highest Percentage Relative Efficiency (PRE) compare to other existing estimators (Sample variance, Isaki (1983), Kadilar and Cingi (2006), Subramani and Kumarpandiyan (2012), Khan and Shabbir (2013) and Javed *et al*. (2018)) considered in this study respectively. We therefore recommend for use in estimating finite population variance.

REFERENCES

Adewara, A.A., Singh, R. & Kumar, M. (2012). Efficiency of Some Modified Ratio and Product Estimators Using known value of Some Population Parameters, *International Journal of Applied Science and Technology.* 2(2): 76-79.

Audu, A., Adewara, A.A. and Singh, R.V.K. (2016). Class of Ratio Estimators with Known Functions of Auxiliary Variable for Estimating Finite Population Variance. *Asian Journal of Mathematics and Computer Research*. 12(1): 63-70.

Gupta, S. and Shabbir. J. (2008). Variance Estimation in Simple Random Sampling using Auxiliary Information, Hacettepe *Journal of Mathematics and Statistics* 37:57-67.

Isaki, C.T. (1983). Variance Estimation using Auxiliary Information. *Journal of the American Statistical Association* 78:117–123.

Javel K., Jamal N., Hanif M., Ali M., Shahzad U., and Luengo A.V.G. (2018). Improved Estimator of Finite Population Variance Using Coefficient of Quartile Deviation. *Asian Journal of Advanced Research and Reports*. 1(3):1-6

Kadilar, C. and Cingi, H. (2006). Ratio Estimators for Population Variance in Simple and Stratified Random Sampling. *Applied Mathematics and Computation*, 173: 1047-1058.

Kadilar, C. and Cingi, H. (2007). Improvement in Variance Estimation in Simple Random Sampling, *Communications in Statistics – Theory and Methods* 36: 2075–2081.

Kazeem, A. A., and Olanrewaju, I. S (2013). On the Efficiency of Ratio Estimator Based on Linear Combination of Median, Coefficients of Skewness and Kurtosis. *American Journal of Mathematics and Statistics*. 3(3): 130-134.

Khan, M., and Shabbir, J. (2013). A Ratio Type Estimator for the Estimation of Population Variance using Quartiles of an Auxiliary Variable. *Journal of Statistics Applications and Probability*. 2(3): 319-325.

Murthy, M.N. (1967). *Sampling Theory and Methods*. Calcutta Statistical Publishing House, India.

Olufadi, Y. and Kadilar, C. (2014). A Study on the Chain Ratio-Type Estimator of Finite Population. *Journal of Probability and Statistics.* 2014: 1-5.

Rajesh, S. Pankaj, C. Nirmala, S. and Florentin, S. (2007). A General Family of Estimators for Estimating Population Variance Using Known Value of Some Population Parameter(s). *Renaissance High Press, Ann Arbor*, USA. 65-73.

Subramani, J. and Kumarapandiyan, G. (2012) Variance Estimation using Quartiles and their Functions of an Auxiliary Variable. *International Journal of Statistics and Applications* 2 (5): 67-72.